bound offset print of the first edition, authorized by license of the Soviet military powers. The first 70 p. include a description of various desk calculating machines, of problems with which they may deal, and also lists of references to literature appearing before 1942.

R. C. A.

NOTES

106. George Neville Watson—Table Maker.—This distinguished British Mathematician and Table Maker was born in Westward Ho, Devonshire, 31 January 1886. He was a student at Trinity College, Cambridge, senior wrangler, 1907; class I (div. ii), mathematical tripos, part II, 1908; Smith's Prizeman, 1909; fellow of Trinity, 1910–1916.

He was an assistant professor of pure mathematics at University College, London, 1915–18; professor of mathematics at the University of Birmingham since 1918, of pure mathematics since 1937; gold medallist of the Danish Royal Academy of Sciences, 1912; a fellow of the Royal Society of London, 1919; honorary secretary, 1919–33, and president, 1933–35, of the London Mathematical Society; president of the Mathematical Association, 1932–34; Sylvester medallist of the Royal Society, 1946; and De-Morgan medallist of the London Mathematical Society, 1947. He is also a Sc.D. of Univ. of Cambridge, D.Sc. Univ. of London, M.Sc. Univ. of Birmingham, Hon. LL.D. Univ. of Edinburgh, Hon. Sc.D. Univ. of Dublin.

The following is a list of Professor Watson's tables, and of some papers with numerical results of some importance:

1. (with G. A. Schott) "Asymptotic formulae occurring in electron theory," *Quart. Jn. Math.*, v. 47, 1916, p. 311–333. Various numerical results are given.

2. "The sum of a series of cosecants," *Phil. Mag.*, s. 6, v. 31, 1916, p. 111–118. $S_n = \sum_{m=1}^{n-1} \csc(m\pi/n)$ is tabulated for n = [2(1)30(5)100, 360, 1000; 5D].

3. [Tables connected with gamma functions], BAAS, *Report*, 1916, p. 123–126. Four 10D tables: (a) $10 + \ln \Gamma(1 + x)$, $x = .005(.005)1$; (b) $10 + \int_0^x \log \Gamma(1 + t)dt$, $x = .01(.01)1$; (c) $\psi(x) = d \ln \Gamma(x)/dx$, $x = 1(1)101$; $\psi(x)$ for $x = 1.5(1)100.5$.

4. "The zeros of Bessel functions," R. Soc. London, *Proc.*, v. 94A, 1918, p. 190–206. Tables of $J_{\pm\frac{1}{3}}(x)$, $U_{\frac{1}{3}}(x)$, $-V_{\frac{1}{3}}(x)$, $-V_{\frac{1}{3}}(x)/U_{\frac{1}{3}}(x)$, for x = [0(.05)2(.2)8; 4D].

5. "Bessel functions of equal order and argument," *Phil. Mag.*, s. 6, v. 35, 1918, p. 364–370. Table of $n\int_0^1 J_n(nx)dx$, n = [1(2)23; 7D].

6. *A Treatise on the Theory of Bessel Functions*, Cambridge, England, 1922; second ed., New York, 1944; reprinted Feb. 1945, Apr. 1948; tables, p. 665–752. For tabular errors in this v. see *MTAC*, v. 2, p. 49–51.

T. I: $J_0(x)$, $Y_0(x)$, $J_1(x)$ and $Y_1(x)$, for x = [0(.02)16; 7D]. The values of $J_0(x)$, $J_1(x)$ up to 15.5 were taken from Meissel's 12D table (1889) while the values of $Y_0(x)$ and $Y_1(x)$ were computed partly by interpolation in Aldis' table of $G_0(x)$ and $G_1(x)$ (1900). T. I gives also, for the same range of argument, 7D values of $|H_n^{(1)}(x)|$, $\mathbf{H}_n(x)$, $n = 0, 1$, and of arg $H_n^{(1)}(x)$, $n = 0, 1$ to the nearest $0''.01$.

T. II consists of tables of $e^{-x}I_0(x)$, $e^{-x}I_1(x)$, $e^xK_0(x)$, and $e^xK_1(x)$, e^x, for x = [0(.02)16; 7D]. The 8S or 9S table of e^x was constructed with the help of Newman's 12–18D table of e^{-x} (1883).

T. III consists of $J_{\frac{1}{3}}(x)$, $Y_{\frac{1}{3}}(x)$, $|H_{\frac{1}{3}}^{(1)}(x)|$, and arg $H_{\frac{1}{3}}^{(1)}(x)$, of the same scope as T. I; a table of $e^xK_{\frac{1}{3}}(x)$ is also included.

T. IV gives 7D values of $J_n(x)$, for $n=2(1)5$, $x=.1(.1)5$; 6D values of $J_n(x)$, $n=0(1)20$, for $x = 1(1)12$; 7S at least, or 7D values of $Y_n(x)$ for $n = 0(1)10$, and $x = 0(.1)5$, and 7D for $n = 0(1)13$, $x = 6(1)12$; 7D values of $e^{-x}I_n(x)$ for $n = 2(1)5$, $x = .1(.1)5$; 7S at least, or 7D values of $K_n(x)$ for $n = 0(1)10$, $x = 0(.1)5$. The 6D values of $J_n(x)$ are taken from LOMMEL's table (1885) with some corrections, and some values of $K_n(x)$ were taken from ISHERWOOD's table (1904).

T. V gives Lommel's 6D table (1886) of $J_n(x)$, $n = -6\frac{1}{2}(1)+6\frac{1}{2}$, $x = 1(1)50$, and $n = 7\frac{1}{2}(1)18\frac{1}{2}$, $x = 1(1)20$, and FRESNEL's integrals, $x = .5(.5)50$, and $x = .02(.02)1$ with some modifications and corrections, and with the first 16 maxima and minima of the integrals. There are about 30 errors here.

T. VI gives the 7D values of $J_n(n)$, $-Y_n(n)$, $J_n'(n)$, $Y_n'(n)$ and $n^{\frac{1}{3}}J_n(n)$, $-n^{\frac{1}{3}}Y_n(n)$, $n^{\frac{1}{3}}J_n'(n)$, $n^{\frac{1}{3}}Y_n'(n)$ for $n = 1(1)50$.

T. VII gives the first 40 zeros of $J_n(x)$ and $Y_n(x)$ to 7D, for $n = 0(1)5, \frac{1}{3}$; and also of $J_{-\frac{1}{3}}(x) + J_{\frac{1}{3}}(x)$, and of $J_{-\frac{1}{3}}(x) - J_{\frac{1}{3}}(x)$.

T. VIII gives the 7D values of $\frac{1}{2}\int_0^x J_0(t)dt$, $\frac{1}{2}\int_0^x Y_0(t)dt$ for $x = .02(.02)1$, with the first 16 maxima and minima of the integrals, to 7D.

7. "The sum of series of cosecants," *Phil. Mag.*, s. 6, v. 45, 1923, p. 577–581. Compare no. 2. $S_n^{(3)} = \sum_{m=1}^{n-1} \csc^3 (m\pi/n)$, table for $n = [2(1)30(5)100, 360, 1000; 5D]$.

8. "Theorems stated by Ramanujan (V): approximations connected with e^x," London Math. Soc., *Proc.*, s. 2, v. 29, 1929, p. 293–308. Solutions of $ue^{1-u} = Ue^{1-U} = e^{-t}$ with values of $\phi(t) = \frac{135}{8}\left\{\frac{U}{(U-1)^3} - \frac{u}{(1-u)^3}\right\}$, tables of u, U, $\phi(t)$, $-t^{-1}\ln\phi(t)$, for $t = [0(.02)\text{-}1(.1)11; 7D]$.

9. "The constants of Landau and Lebesgue," *Quart. Jn. Math.*, Oxford s., v. 1, 1930, p. 310–318. Contains certain numerical results.

10. "The use of series of Bessel functions in problems connected with cylindrical wind-tunnels," R. Soc. London, *Proc.*, v. 130A, 1930, p. 29–37. A few numerical results.

11. "Du Bois-Reymond's constants," *Quart. Jn. Math.*, Oxford s., v. 4, 1933, p. 140–146. Various computed results are given.

12. "Two tables of partitions," London Math. Soc., *Proc.*, s. 2, v. 42, 1937, p. 550–556. Tables of the number of partitions of n into odd parts and into distinct odd parts for $n = 0(1)400$.

13. "Tabellazione di una particolare funzione definita da un integrale improprio," R. Accad. naz. Lincei, *Cl. d. sci. fis., matem. e nat., Rendiconti*, s. 6, v. 27, 1938, p. 525–528. Table of $\phi(\lambda) = \int_0^\infty[x + \frac{1}{2} - (x^2 + x)^{\frac{1}{2}}]\sin \lambda x dx$, $\lambda = [0(.01).4(.1)32; 7D]$.

14. "Three triple integrals," *Quart. Jn. Math.*, Oxford s., v. 10, 1939, p. 266–276. Certain numerical results.

15. "A table of Ramanujan's function $\tau(n)$," London Math. Soc., *Proc.*, s. 2, v. 51, 1949, p. 1–13. See *MTAC*, v. 3, p. 468. Gives a table of $\tau(n)$ for $n = 1(1)1000$; also $\tau(n)n^{-11/2}$ to 5D.

We are indeed grateful to Professor BROADBENT, the editor of the *Mathematical Gazette*, for consenting to allow us to reproduce the portrait of Professor Watson published as a frontispiece to the *Gazette*, v. 18, 1934.

THE EDITORS

107. THE LEMNISCATE CONSTANT.—By this constant $\bar{\omega}$ is usually meant twice the number

$$\tfrac{1}{2}\bar{\omega} = A = \tfrac{1}{4}(2\pi)^{-\frac{1}{2}}\{\Gamma(\tfrac{1}{4})\}^2 = \tfrac{1}{4}B(\tfrac{1}{4}, \tfrac{1}{2}) = F(\tfrac{1}{4}, -\tfrac{1}{2}; \tfrac{5}{4}; 1)$$
$$= \left\{\binom{-\frac{1}{4}}{\frac{1}{4}}\right\}^{-1} = \tfrac{1}{2}\pi\vartheta_4^2(0|i) = \Pi(\tfrac{1}{4})\Pi(-\tfrac{1}{2})/\Pi(-\tfrac{1}{4}) = \int_0^1 (1 - x^4)^{-\frac{1}{2}}\, dx.$$

It derives its name from the fact that it plays the rôle of π in the rectification of the lemniscate. In fact the length of the lemniscate of diameter D is

$\bar{\omega}D$. This constant occurs in a good many connections in the theory of elliptic functions and in number theory. The values

$$\begin{aligned}
&(1)\ A = 1.311031,\\
&(2)\ A = 1.31102\ 87771\ 46059\ 87,\\
&(3)\ A = 1.31102\ 87771\ 46059\ 90680\ 3207,
\end{aligned}$$

were given by EULER, STIRLING[1] and GAUSS[2] respectively. The value (3) is given in the FMR *Index*. WRENCH [*MTAC*, v. 3, p. 202] has pointed out that this value is incorrect beyond the 17th decimal.

In order to verify certain identities involving this constant with a number of numerical functions the writer has made two independent redeterminations of A using the two lacunary series

$$A = \tfrac{1}{2}\pi\{\sum_{n=-\infty}^{\infty}(-1)^n e^{-\pi n^2}\}^2 = 2^{\frac{1}{2}}\pi e^{-\frac{1}{3}\pi}\{\sum_{n=-\infty}^{\infty}(-1)^n e^{-2\pi(3n^2+n)}\}^2$$

the first of which was used by Gauss. These calculations agree to 52D and give

$$\begin{aligned}
A = 1.31102\ &87771\ 46059\ 90523\ 24197\\
94945\ &55970\ 68413\ 77475\ 71581.
\end{aligned}$$

Gauss gave also the constant

$$B = \tfrac{1}{2}\pi/\bar{\omega} = \tfrac{1}{4}\pi/A = (2\pi)^{-\frac{1}{2}}\{\Gamma(\tfrac{3}{4})\}^2 = \int_0^1 x^2(1 - x^4)^{-\frac{1}{2}}dx$$

correctly to 20D. More accurately we have

$$\begin{aligned}
B = .59907\ &01173\ 67796\ 10371\ 99612\\
46140\ &16193\ 91136\ 06331\ 60783.
\end{aligned}$$

As a byproduct of these calculations we obtain easily the following values of $\Gamma(\frac{1}{4})$ and $\Gamma(\frac{3}{4})$:

$$\begin{aligned}
\Gamma(\tfrac{1}{4}) &= 3.62560\ 99082\ 21908\ 31193\ 06851\\
&55867\ 67200\ 29951\ 67682\ 88007\\
\Gamma(\tfrac{3}{4}) &= 1.22541\ 67024\ 65177\ 64512\ 90983\\
&03362\ 89052\ 68512\ 39248\ 10807.
\end{aligned}$$

D. H. L.

[1] J. STIRLING, *Methodus Differentialis*, London, 1730, p. 58; English transl. by F. HOLLIDAY, London, 1749, p. 51.

[2] K. F. GAUSS, *Werke*, v. 3, Göttingen, 1866, p. 413.

108. NEWMAN AND GEORGE ELIOT.—We have made various references to the table-maker, FRANCIS WILLIAM NEWMAN (1805–1897), brother of the Cardinal; see for example *MTAC*, v. 1, p. 454–459, v. 3, p. 201, 257, 451. At the first of these references we noted (p. 455) that he was professor of Latin at University College, London, 1846–1869. In the *Times Literary Supplement*, London, v. 48, April 30, 1949, p. 281, there is an interesting letter by KATHLEEN TILLOTSON[1] noting, what had escaped her biographers, that GEORGE ELIOT (1819–1880) was one of the earliest students of the Ladies' College at Bedford Square, London, which opened in October, 1849 and is now Bedford College for Women. During 1851 at least, writes Mrs. Tillotson, she took courses there in History, German, French, Elocution, and Mathematics. Her mathematics, and possibly her history, was with Newman, who taught mathematics at the College January–June 1851. In a letter of March 27, 1874 George Eliot remembered her interest in Francis Newman's books "in far-off days . . . and . . . the awe I had of him as a lecturer in Mathematics at the 'Ladies' College.'" Mrs. Tillotson continues: "He had lectured in Ancient History since the opening of the college, and it is natural to suppose that it was partly his name . . . that drew MARY ANN EVANS to the Ladies' College."

In the *Times Literary Supplement*, June 3, 1949, p. 365, Prof. GORDON S. HAIGHT of Yale University quotes from a letter "which has just come to light" written by George Eliot Jan. 28, 1851. A passage quoted is as follows: "I am attending Professor Newman's course of lectures on Geometry at the Ladies' College every Monday and Thursday. You will say that I can't afford this, which is 'dreadfully true'—but the fact is that I happened to say I should like to do so and good-natured Mr. CHAPMAN[2] went straightway and bought me a ticket which he begged me to accept. I refused to accept it—and have paid for it—wherefore I must stint myself in some direction—clearly in white gloves and probably in clean collars."

R. C. A.

[1] Mrs. TILLOTSON is a coeditor of the *Works of Michael Drayton*, and wife of GEOFFREY TILLOTSON, professor of English Literature in the University of London.

[2] JOHN CHAPMAN, the publisher, at whose home George Eliot resided Jan. 8 to March 24, 1851.

109. RHETICUS, WITH SPECIAL REFERENCE TO HIS OPUS PALATINUM.—Before the twentieth century Rheticus was the greatest calculator of mathematical tables who ever lived. He was born just a century before Napier published his work on logarithms in 1614, and his tables were posthumously published in 1596 and 1613.

Since the ordinary inquirer would in more than one direction find considerable difficulty in deriving a fairly complete outline of the life and work of Rheticus, such an outline is here presented, together with a somewhat extended list of sources (everything has been personally inspected) from which fuller information may be derived. In this outline some errors in these sources have been corrected, a few new facts delineated, and our previous sketch of PITISCUS (*MTAC*, v. 3, p. 390–397) amplified.

Rheticus was one of many mathematicians named after the places of their birth. GEORG JOACHIM was born 16 Feb. 1514 at Feldkirchen in Vorarlberg, in the westernmost part of Austria-Hungary, close to the Swiss border and formerly in the Roman province of Rhaetia. Georg's mother seems to have been connected with an Italian noble family of means, named PORRI,[25] and thus a trip to Italy with his parents as a child was what might be expected, and suggested a family in comfortable circumstances. Joachim's zest for advanced education seems to have been stimulated by studies at Zürich, Switzerland. Hence in 1532 the fame of the comparatively new University of Wittenberg, where the great reformer PHILIP MELANCHTHON (1497–1560) was professor of Greek, and MARTIN LUTHER (1483–1546) was professor of Theology, drew the eighteen-year old youth to register as a student. In April 1536 he received the equivalent of a doctor of philosophy degree, the official records stating that he was "mathematicus excellens."

It was exactly at this time that he first added the surname Rheticus. (It may be noted that in an outstanding German reference work[16] there are two biographies of Rheticus, by different authors, one under "Joachim" and the other under "Rheticus.")

In May 1536 Rheticus went to Nuremberg to study with JOHANN SCHÖNER (1477–1547) professor of mathematics at the gymnasium there, and a former pupil of REGIOMONTANUS and WERNER.[22] Later he went to Tübingen where he received the call back to his University.[14] The professor of mathematics at Wittenberg having died in 1536 Melanchthon, then being Dean of the faculty, appointed for that autumn two new professors: one ERASMUS REINHOLD (1511–1553) for the higher mathematics (including astronomy), and the other, Dr. Rheticus, then 22 years of age, for elementary mathematics. After teaching there with some repute for about three years Rheticus received leave of absence in May 1539 and went to Frauenburg, Prussia, as an assistant to COPERNICUS (1473–1543), whose doctrines he soon advocated with much zeal and personal risk.

Copernicus was very reluctant to publish his epoch-making astronomical work, *De revolutionibus orbium caelestium*, but publicity achieved for its contents by Rheticus led him to change his mind and to permit publication. This publicity was a survey of the principal features of the new astronomy, cast in the shape of a letter to his former teacher SCHÖNER at Nuremberg. This was published anonymously at Dantzig in 1540, as a *Narratio prima*, and reprinted at Basel in 1541 with the name of Rheticus as author. Of this *Narratio* there were nine later editions, the eleventh being that of ROSEN in English.[28] Rheticus left Frauenburg at the end of September 1541, resumed his teaching in Wittenberg, and served as dean of the arts faculty there in the early months of 1542.[28]

Chapters 12, 13, 14 of the first of the six books of the *De revolutionibus* of Copernicus were devoted to the trigonometrical material, useful for the later discussion of astronomical problems. In the 12th chapter was a 5D table, with differences, of the sines of all angles in the quadrant at interval 10′. In 1542 Rheticus had chapters 12 and 13 printed at Wittenberg with the table replaced by his own 7D table of sines and cosines for every minute in the quadrant.

Rheticus left Wittenberg in 1542 and went to Nuremberg where the great work of Copernicus was being printed; the early part was set up under his direction. Rheticus placed the first completed copy of the work in the hands of Copernicus a few hours before he died in May 1543. The second edition of *De revolutionibus*, edited by Rheticus appeared in 1566, with the appended third edition of his *Narratio*.

In securing the publication of *De revolutionibus* Rheticus bears to Copernicus a relation similar to that of HALLEY to NEWTON, in the following century, for assuring the publication of the *Principia*.

The following statement of Müller[23] has been frequently quoted: "Without Rheticus we would know no Copernicus, without Copernicus no Kepler, without Kepler no Newton!"

At Nuremberg in 1542 Rheticus also published *Orationes de astronomia, geographia et physica*. He taught at the University of Leipzig from 1542 to 1551. In this latter year was published his *Canon doctrinae triangulorum*.[10,19]

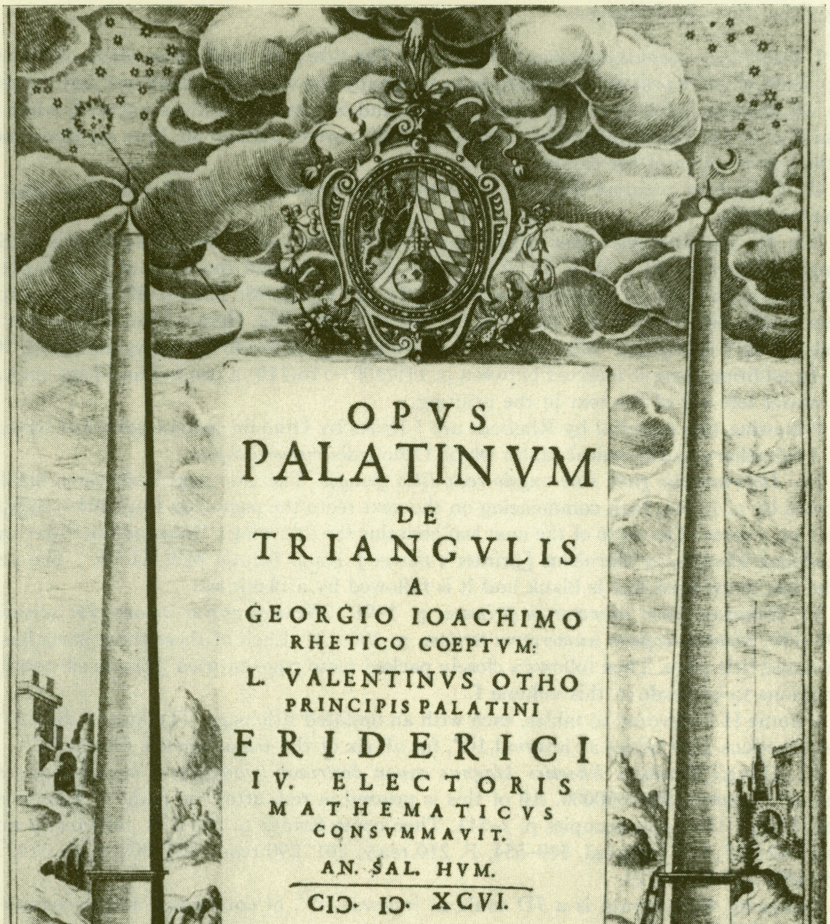
Leipzig, 24 p., to which we have earlier referred,[30] p. 394. In this quarto-format publication of 12 leaves, not listed in any of the bibliographies, p. 4–17 are occupied with a 7D Canon for all six trigonometric functions, at interval 10′, and this is followed by a dialogue between Philomathes, a supposed friend of Rheticus, and Hospes, his pupil. This was the first table (a) in which all trigonometric functions (or even sin, tan, sec) were brought together, or (b) in which the semiquadrantal arrangement was used. Both this canon and the undated reprint at Basle (apparently in 1580), are in the British Museum. With Rheticus our names for the trigonometric functions were not used; sines and cosines were perpendiculars and bases to a hypotenuse 10 000 000; the secants and tangents were hypotenuses and perpendiculars to a base 10 000 000; and the cosecants and cotangents were hypotenuses and bases to a perpendicular 10 000 000. Rheticus was the first to define his functions by means of the right-angled triangle without any reference to the circle.

In 1551 Rheticus went to Prague; and in 1554 he lectured at the University of Vienna,[24] but in 1557 he settled in Cracow, where he spent most of the rest of his life. He died during a visit to Cassau, in Hungary, on 4 Dec. 1576 in the sixty third year of his age.[31] The great scarcity of the works of Rheticus (the second edition of *De revolutionibus* is much rarer than the first) is due to the fact that in 1550 all of his works were put on the *Index Expurgatorius*.

During the last 25 years of his life Rheticus devoted himself principally to the calculation of monumental trigonometric tables. Contributions of financial assistance from MAXIMILIAN II of Austria (who died in 1576) and several Hungarian nobles enabled Rheticus to procure a number of computers for 12 years (**3** and WOLF[15]).

The Opus Palatinum. During this period of intense activity Rheticus had long known of a young mathematician, a native of Magdeburg, LUCIUS VALENTIN OTHO (1550?–1605?) well known[18] to PRÄTORIUS at the University of Wittenberg. Towards 1575 Rheticus gladly accepted the services as assistant which Otho offered. And thus it turned out that when Rheticus died in December 1576 Otho became heir to all of the scientific mss. of Rheticus. Rheticus has told us that when Otho came to him he was the same age (25) as when he (Rheticus) became an assistant to Copernicus ("Profecto in eadem aetate ad me venis, qua ego ad Copernicum veni"). In this way we learn that Otho was born about 1550. He became a professor of mathematics (following Prätorius) at the University of Wittenberg, and was continually seeking to procure the means for publishing the great tables which were practically complete. This was finally arranged with FREDERICK IV,[31] Elector Palatine, to whom the work is dedicated. Thus 20 years after the death of Rheticus, in 1596, his *Opvs Palatinvm de triangvlis a Georgio Ioachimo Rhetico coeptvm: L. Valentinvs Otho, Principis Palatini Friderici IV. Electoris Mathematicvs consvmmavit.* was published at Neustadt, now in Bavaria, in about 1446 folio pages, 22.4 × 36.7 cm. It seems to have been originally bound in vellum in two volumes. Five of the seven parts were in the first volume.

After the title-page (which we here reproduce) and 9 preliminary leaves of dedication and preface by Otho are 7 main sections each (after the


OPVS
PALATINVM
DE
TRIANGVLIS
A
GEORGIO IOACHIMO
RHETICO COEPTVM:
L. VALENTINVS OTHO
PRINCIPIS PALATINI
FRIDERICI
IV. ELECTORIS
MATHEMATICVS
CONSVMMAVIT.
AN. SAL. HVM.
CIↃ· IↃ· XCVI.

PLIN. LIB. XXXVI. CAP. IX
RERVM NATVRÆ INTERPRETA-
TIONEM ÆGYPTIORVM OPERA PHI-
LOSOPHIÆ CONTINENT.

CVM PREVILEGIO
CAES. MAIES.

Title-Page of RHETICUS, *Opus Palatinum*, 1596.

second) separately paged, and for each of the first 5 sections there is at least one separately dated title-page. The first three of these sections, by Rheticus, are as follows:

I. *De Fabrica canonis doctrinae triangvlorvm.* The title page, with verso blank, is followed by p. 3–86. P. 45 is printed and folded so as to occupy the verso of a blank leaf and the recto of p. 46, making an extra leaf. Pages 34, 35, 37 and 38 are incorrectly numbered, while the verso of p. 85 also bears the number 85 instead of the correct number 86. Here are three books on the construction of the canon.

II. *Triqvetrvm rectarvm linearvm in planitie cvm angvlo recto*, p. 86–104. One book on plane triangles.

III. *De triangvlis globi cvm angvlo recto.* 146 p. Four books on right-angled spherical triangles. The verso of p. 21 is 23, the recto of p. 23 is 24, and the verso of p. 24 is blank. P. 57 is numbered 60. P. 72–88 are numbered 81, 83, 82, 85–98. Then p. 89–106 are respectively numbered: 100–103, 95, 105, 106, 98, 108–115, 101, 102. Here a blank leaf is inserted which we shall call p. 107–108. Then corresponding to p. 109–146 are pages numbered 103–140. In addition there is inserted between p. 115(109)–116(110) a printed half-page containing matter left out of the text in the printing.

Following this material by Rheticus are 5 books by Otho on oblique spherical triangles and 3 subsidiary astronomical tables which Otho calls *meteoroscopia.*

IV. *De triangvlis globi sine angvlo recto libri qvinqve.* The title page with verso blank is followed by p. 1–264, then commencing on the next recto the pages run from 264 (repeated) to 341 on a verso. The recto of the next leaf contains the following: "Neostadii in Palatinatv. Excudebat Matthaeus Harnisius [printer's device]. Anno Salutis MD XCVI." The verso of the last mentioned leaf is blank and it is followed by a blank leaf.

V. *Meteoroscopivm numerorvm primvm.* p. 1–91. *Meteoroscopivm numerorvm secvndvm.* p. 92–105. *Meteoroscopivm numerorvm tertivm.* p. 106–121. Each of these three parts has its own dated title page. Then follows a closely packed recto page entitled "Errata sic corrige," corrections to be made in this volume I.

Volume II is devoted to tables, each with an undated title page. The first of these is the great Rheticus 10D tables at interval 10″, for all six of the trigonometric functions.

VI. *Georgii Ioachimi Rhaetici Magnus canon doctrinae triangvlorvm ad decades secvndorvm et ad partes* j0000000000. All of this is printed in red, after the name of Rheticus in black, 554 p. The table occupies p. 2–541. Then come Errata p. 542–554, incorrectly numbered 142–147, 147 repeated, 549–554. P. 210 reads 201, 290 reads 390, 309 reads 314, 398 reads 498, 512 reads 125.

Following these errata is a 7D table at interval 10″, of cotangents and cosecants for the first half of the quadrant:

VII. *Tertia series magni canonis doctrinae triangvlorvm in qvo triqvetri cvm angvlo recto in planitie minvs latvs inclvdentivm angvlvm rectum ponitur partium* 10000000. After the first two words in black, the rest of the title is red. 181 p. P. 83 is incorrectly numbered 27.

Concerning this section VII DeMorgan remarks[11] that its insertion displayed "merely the editor's want of judgment; it is clearly nothing but a previous attempt made before the larger plan was resolved on, and is much less accurate than the great table to ten places."

Delambre,[8] p. 2, states that in III–IV Rheticus and his disciple are the most prolix and obscure authors he had ever met; and that their 500 folio pages could be reduced to 10.

Pitiscus Revision of the Opus Palatinum Canon. Shortly after the *Opus Palatinum* was published it was found that in VI the tangents and secants near the end of the quadrant were very inaccurate. Possibly through representations of Pitiscus, then chaplain and former teacher of Frederick

IV, the Elector Palatine put Pitiscus in charge of the revision of this part of the canon. From the account which Rheticus gave of the construction of the canon in I, especially book 3, it is clear that the computation of the tangents and secants were based on a table of sines and cosines, at interval 10″, and extent >10D. Hence Pitiscus, perhaps about 1602 (when he was 41 years old), sought the ms. of Otho "then an old man" (say La Lande, Delambre,[8] p. 17, DeMorgan,[11] Glaisher[13]—based on the statement of Pitiscus: "ob memoriae senilis debilitatem"—who could not remember where the ms. was). He thought that perhaps he had left it at Wittenberg, and accordingly Pitiscus sent a messenger there to search for it; but after considerable expense had been incurred he returned without it. In 1602 Otho was about 52 years old, just 11 years older than Pitiscus. While the condition of his health may have affected his memory it is certainly improper[31] to refer to Otho at this time as an "old man."

After the death of Otho, at Heidelberg, about 1605, when the mss. of Rheticus which had been in his possession passed into the hands of the orientalist and astronomer at Heidelberg, JACOB CHRISTMANN, he found that they included not only the sine canon which had been considered lost, but also the original manuscript of the *De revolutionibus* of Copernicus, now preserved in Prague.

As soon as Pitiscus learned of this canon discovery he examined the mss. page for page, although they were in very bad condition, and to his great satisfaction he found

(1) the ten-second canon of sines, 15D, Δ^3;
(2) sines for every second of the first and last degrees of the quadrant, 15D, Δ^2;
(3) the commencement of a canon for every ten seconds of tangents and secants, 15D, Δ^2;
(4) a complete minute-canon of sines, tangents, and secants, 15D.

From this list of tables, taken in conjunction with VI and VII, we are enabled to view as a whole the colossal sixteenth century computations which Rheticus achieved, and which must later have been of extraordinary value to the astronomer.

With the canon (1) in hand Pitiscus recomputed to 11D all of the tangents and secants of VI in the defective region from 83° to the end of the quadrant. Then 86 pages were reprinted: p. 1, the new title-page, and 2–86, the revised table. For the new title-page to that already given in VI was added, **1**.

Recens emendatus à Bartholomaeo Pitisco Silesio. Addita est brevis commonefactio de fabrica & vsu huius Canonis. Quae est summa doctrinae, & quasi nucleus, totius operis Palatini. Canon hic, unà cum brevi commonefactione de eius fabrica & vsu etiam separatim ab opere Palatino venditur. In Bibliopoleio Harnischiano.

This is a sort of fly-title, undated; but the accompanying 19-page publication has the following title page, **2**: *Bartholomaei Pitisci Grünbergensis Silesii Brevis et Perspicua Commonefactio de Fabrica et vsv magni canonis doctrinae Triangulorum Georgii Ioachimi Rhetici.* [Printer's decoration]. *Neostadii, Typis Nicolai Schrammii.* MDCVII.

The 86 pages of **1** and the 19 pages of **2** were issued, as stated above, as a separate publication, but also with copies of the complete work with the original pages 1–86 in VI eliminated and replaced by **1**, and with **2** following the revised VI, or VII.[32] It is the latter arrangement which is found in the only three existing copies of **1–2**, which I have been able to trace, and to which I shall presently refer.

Otho's Death. Since on page 5 of **2** there seems to be definite reference to the Rheticus sine canon being in the hands of Christmann, this suggests that Otho had died before 1607 and hence I suggested that he may have died about 1605 and that Pitiscus may have questioned him concerning the table about 1602. While I arrived at this conclusion as to a possible date of Otho's death independently I was later interested to find that TROPFKE had reached the same result in the second ed. of his *Geschichte der Elementar-Mathematik*, v. 4, 1923. The only definite older reference to the death of Otho is in the "Ad lectorem" of the work we shall now consider, namely:

3. The Rheticus-Pitiscus *Thesaurus Mathematicus*. After his discovery of the new Rheticus tables Pitiscus started to prepare a second work which was finally published in 1613 and contained the following four parts.

A. (Rheticus) Canon of sines for every 10″, 15D, Δ^3, p. 2–271;

B. (Rheticus) Sines for 0(1″)1°, 89°(1″)90°, 15D, Δ^2, p. 2–61;

C. (Pitiscus) The fundamental series from which the rest were calculated to 22D, p. 1–10;

D. (Pitiscus) The sines to 22D, Δ^4 or Δ^5, for every tenth, thirtieth and fiftieth second in the first 35 minutes, p. 11–15.

Each of the parts *C*, *D* has its own correctly dated title-page 1613; but the title-page for *B* as well as that for the whole volume is given incorrectly as MDXIII. The volume title-page, in black and red, begins *Thesaurus Mathematicus sive Canon Sinuum ad radium* 1.00000.00000.00000. . . . Frankfurt. The volume is a large one 24×35 cm. It appeared shortly before the death of Pitiscus 2 July 1613 in his fifty second year. From what has been presented above the great contribution which Pitiscus made in uncovering, and presenting work of Rheticus in meticulously correct form, was indeed a notable contribution to science.

More information concerning this volume is given in *MTAC*,[30] p. 395–396.

One had to wait for nearly three and a half centuries before the tables of Rheticus were finally superseded by those of ANDOYER (1915–1918).

Known copies of the 1607 Rheticus-Pitiscus work.

The three known copies of this work are in Edinburgh, at the Crawford Library of the Royal Observatory, which we shall call the ECO copy; in Washington, at the Naval Observatory, the WNO copy; and in the private library of Mr. WILLIAM D. MORGAN of St. Paul, Minnesota—the StPM copy.

For the preparation of this article I have been deeply indebted to Mr. Morgan for allowing me to have his StPM copy for leisurely study. In at least one respect it is the finest copy in a single library. It consists of three volumes, purchased at different times from three different firms: (i) almost certainly the volume of I–V, bound in contemporary white vellum with leather tiers as originally issued in 1596, 36.7 cm. tall, and thus indicating that at least some copies of the original were bound in two v. and divided in that way; (ii) containing V–VII, as published in 1596, 1.7 cm. less tall than (i); (iii) containing **1** along with the remaining 454 pages of the trigonometric canon, **2**, and **3**. This third v. is 3 cm. less tall than (i), and at least 1.3 cm. less tall than **3** when it was originally published. In StPM the many fundamental changes in VI of Rheticus which Pitiscus made in **1** may be readily observed. In this respect and in the superb condition of (i) StPM is outstanding.

As noted above, in both ECO and WNO there is replacement of the first 86 pages of VI by **1**. The great kindness of Mrs. GRACE O. SAVAGE, the librarian at the Naval Observatory, in responding to my numerous queries concerning WNO, has been much appreciated. This copy, as tall as (i) and with wider pages, is also bound in vellum but with different ordering of material. The Library of Congress Rheticus lacking the Pitiscus revision, 36.5 cm. tall, is practically as tall as (i). Brown University has film copies of the Library of Congress Rheticus, and of Mr. Morgan's 1607 Pitiscus items.

We have earlier (*MTAC*, v. 3, p. 396) indicated the reason for the scholar's great interest in the ECO copy.

R. C. A.

[1] ADRIAAN VAN ROOMEN (or ADRIANUS ROMANUS), *Ideae mathematicae pars prima, sive methodus polygonorum*. Louvain and Antwerp, 1593. In preliminary material Roomen has (i) "lectori philomathi" in which he has words of praise for 15 of the principal mathematicians of his time (including Rheticus and VALENTIN OTHO), and (ii) a letter written by Rheticus in 1568 to PETRUS RAMUS (1515–1572), telling not only of his great tabular calculations but also of such works as the following which he planned to publish: (a) Observation of Phenomena; (b) German astronomy; (c) Natural Philosophy; (d) Foundations of Chemistry. See H. BOSMANS, "Romain (Adrien)," *Biogr. Nationale . . . de Belgique*. Brussels, v. 19, 1907, col. 855; also A. MÜLLER,[23] p. 40.

[2] J. J. C. F. DE LALANDE, "Lettre sur tables de sinus extrêmement rares," *Jn. des Sçavans*, Paris, Sept., 1771, p. 579–584. The Rheticus-Pitiscus *Thesavrvs Mathematicvs*, 1613.

[3] A. G. KÄSTNER, *Geschichte der Mathematik*. Göttingen, v. 1, 1796, p. 561f, 590f; v. 2, 1797, p. 368.

[4] C. HUTTON, *Mathematical Tables: containing Common, Hyperbolic, and Logistic Logarithms. . . . To which is prefixed a large and original history of the discoveries and writings relating to those subjects. . . .* London, 1785, p. 9–11; also in his *Tracts on Mathematical and Philosophical Subjects*. V. 1, London, 1812, p. 290–293.

[5] JEAN BERNOULLI, "Analyse de l'Opus palatinum de Rheticus & du Thesaurus mathematicus de Pitiscus," Akad. d. Wissen., Berlin, *Nouveaux Mémoires*, 1786, p. 10–33.

[6] J. B. DELAMBRE, "Rapport sur les grandes tables trigonométriques décimales du cadastre," Acad. d. Sci., Paris, *Mémoires, Sci. Math. et Phys.*, v. 5, 1804, p. 56–66.

[7] G. C. F. RICHE DE PRONY, "Éclaircissemens sur un point de l'histoire des tables trigonométriques," Acad. d. Sci., *Mémoires, Sci. Math. et Phys.*, v. 5, 1804, p. 67–93.

[8] J. B. DELAMBRE, *Histoire de l'Astronomie Moderne*, v. 2, Paris, 1820, p. 1–127: "Grandes tables trigonométriques en nombres naturels—Rheticus."

[9] A. DEMORGAN, "Rheticus," *Penny Cycl.*, v. 19, 1841, also *The English Cycl., Biography*, v. 5, London, 1857.

[10] A. DEMORGAN, "On the almost total disappearance of the earliest trigonometrical canon," RAS, *Mo. Not.*, v. 6, 1845, p. 221–228; reprinted with an addition in *Phil. Mag.*, s. 3, v. 26, 1845, p. 517–526.

[11] A. DEMORGAN, (i) "Table" in *Supplement to the Penny Cycl.*, v. 2, London, 1846, p. 596–597, 599–600. (ii) "Table," *English Cycl., Arts and Sci. Sect.*, v. 7, 1861, cols. 984, 988–989.

[12] J. C. POGGENDORFF, *Biogr.-Literar. Handwörterbuch*, v. 2, Leipzig, 1863.

[13] J. W. L. GLAISHER, *Report of the Committee on Mathematical Tables*, London, 1873, p. 43–45.

[14] F. HIPLER, "Die chorographie des Joachim Rheticus. Aus dem Autographon des Verfassers mit einer Einleitung herausgegeben," *Z. Math. Phys., Hist.-literar. Abth.*, v. 21, 1876, p. 125–150.

[15] R. WOLF, *Geschichte der Astronomie.* Munich, 1877, p. 204f, 236f, 242f, 296, 343f.

[16] *Allgemeine deutsche Biographie*, Leipzig, v. 14, 1881, p. 93–94: "Joachim, G." by K. C. BRUHNS; v. 28, 1889, p. 388–390; "Rheticus, G. J." by S. GÜNTHER.

[17] LEOPOLD PROWE, *Nicolaus Coppernicus.* Berlin, 2 v., 1883–1884; v. 1, Das Leben, v. 2, Urkunden. "For the biography of Copernicus and the social history of his times the book is extremely valuable, but Prowe's judgment in scientific matters was unreliable" (E. ROSEN).

[18] M. CURTZE, 1. "Zur Biographie des Rheticus," *Altpreussische Monatsschrift*, v. 31, 1894, p. 491–496. This article contains an extract from a Munich library ms., Codex latinus Monacensis no. 24101, written by JOHANNES PRÄTORIUS (1537–1616), inventor of the plane table used in surveying, who was professor of mathematics at the University of Wittenberg 1571–1576, and professor at the University of Altdorf from 1576 until his death. In this ms. we are told that in Aug. 1573 Otho, then mathematician for the landgrave of Hesse, came to his patron Prätorius with two notable approximations for π:

(a) $3.1415926537 > \pi > 3.14159265365$, published by VIETA 20 years later, in 1593;
(b) 355/113, which is correct to 6D.

The latter he had derived from the approximations 377/120, found by PTOLEMY, and 22/7 given by ARCHIMEDES, by subtractions of numerators and denominators. This is the earliest sure date for discovery of this approximation. This statement is made with full knowledge of the attribution of this result to: (a) the fifth Century Chinese astronomer TSU CH'UNG-CHIH by Y. MIKAMI, *The Development of Mathematics in China and Japan* (*Abh. z. Gesch. d. math. Wissen.*, Heft 30). Leipzig, 1913, p. 50; and (b) ADRIAEN ANTHONISZ (1527–1607) about 1583 by his son ADRIAEN METIUS in his *Arithmeticae libri duo et geometriae VI*, 1626, p. 51.

2. "Die abgekürzte Multiplication," *Z. Math. Phys., hist.-literar. Abt.*, v. 40, 1895, p. 7–13. Abridged multiplication and mss. of RHETICUS are here discussed. The names of OTHO, PITISCUS and WERNER, among others, also come up.

[19] K. HUNRATH, "Des Rheticus Canon doctrinae triangulorum und Vieta's canon mathematicus," *Abh. z. Gesch. d. Math.*, v. 9, 1899, p. 211–240.

[20] M. CANTOR, *Vorlesungen über Geschichte der Mathematik.* V. 2, second ed., Leipzig, 1900, p. 472–475, 600–603, etc.

[21] A. VON BRAUNMÜHL, *Vorlesungen über Geschichte der Trigonometrie.* Leipzig, v. 1, 1901, p. 140–149, 158–160, 212–226. (Das *Opus Palatinum* and Pitiscus), etc.; v. 2, 1903, 7 index references.

[22] JOHANN WERNER (1468–1528) of Nuremberg, astronomer and meteorologist, *De triangulis sphaericis libri quatuor.* Edited by A. A. BJÖRNBO; *De Meteoroscopiis libri sex, cum prooemio G. J. Rhetici.* Cracow 1557. Edited by J. WÜRSCHMIDT. *Abh. z. Gesch. d. Math. Wissen.*, v. 24, 1907, 1913.

[23] ADOLF MÜLLER, "Der Astronom und Mathematiker Georg Joachim Rheticus," *Vierteljahrsschrift für Geschichte und Landeskunde Vorarlbergs*, n. s., v. 2, 1918, p. 5–46. Valuable collection and study of material about the life of Rheticus.

[24] "Vorarlberger an in- und ausländischen Hochschulen vom Ausgange des XIII. bis zur Mitte des XVII. Jahrhunderts," *Forschungen zur Geschichte Vorarlbergs und Liechtensteins*, v. 1, 1920, p. 58, 128–130.

[25] FRANZ HAEFELE, "Zur Frage der Herkunft des Astronomen Georg Joachim de Porris," *Schriften des Vereines für Geschichte des Bodensees und seiner Umgebung*, Heft 55, Friedrichshafen a.B., 1927, p. 122–137. Some new material.

[26] MARTIN BILGERI, *Das Vorarlberger Schrifttum, und der Anteil des Landes am deutschen Geistesleben.* Vienna and Leipzig, 1936. "Georg Joachim, genannt Rheticus, der grosse Astronom und Mathematiker," p. 64–70. Nothing original.

[27] JOHANNES KEPLER, *Gesammelte Werke*, herausgegeben von MAX CASPAR, v. 1, Munich, 1938; Rheticus, *Narratio prima*, tenth edition, p. 88–131.

[28] EDWARD ROSEN, *Three Copernican Treatises: The Commentariolus of Copernicus, The Letter against Werner, The Narratio prima of Rheticus, translated with Introduction and Notes.* New York, Columbia Univ. Press, 1939. Admirable work. Introduction: "Georg Joachim Rheticus," p. 4–6; "The *Narratio Prima*," p. 9–11. Text of "*Narratio Prima*," p. 107–196, eleventh ed.

[29] ERNST ZINNER, *Entstehung und Ausbreitung der Coppernicanischen Lehre. Zum 200-jährigen Jubiläum der Friedrich-Alexander-Universität zu Erlangen.* Phys.-medizin. Soz. z. Erlangen, *Sitz.*, v. 74, 1943. Very many references to Rheticus in the index.

[30] R. C. ARCHIBALD, "Bartholomäus Pitiscus (1561–1613)," *MTAC*, v. 3, p. 390–397, 498–499, 1949.

[31] It is rare indeed that DeMorgan anywhere ever deviates in the slightest degree from exactness of statement; yet in connection with Rheticus three cases of this kind are to be noted. In no. 10 p. 228 it is stated that Rheticus died in the "sixty-first year of his age" instead of sixty-third. In nos. 9 (1857) and 11 it is stated that the Opus Palatinum was published "at the expense of the Emperor Maximilian," who had died 20 years earlier. About

1600 when Otho was about 50 years old DeMorgan refers to him in no. 11 as "then an old man."

[32] As DEMORGAN pointed out, the 86 pages of **1** are easily distinguishable by the inferiority of paper and type; the same is true of **2**; also the corrected copies may be distinguished from the uncorrected ones in a moment as follows: look at the bottom of page 7, at the running titles of the columns. The uncorrected copy will have as it ought to have Basis Differentia Hypothenusa. But the *corrected* copy will have, as it ought not to have, Hypothenusa Differentia Basis.

QUERY

32. FRENCH AND RUSSIAN TRANSLATIONS OF A VEGA-BREMIKER TABLE. In RMT **635**, reference was made to the Carl Bremiker edition of Vega's *Logarithmisch-Trigonometrisches Handbuch*, first published in 1856. It was also noted that in 1857 an English translation of this edition by W. L. F. FISCHER, and an Italian translation by LUIGI CREMONA, were published. Who were the authors of the French translation of 1857 and of the Russian translation of 1858 also mentioned? The first two translations, as well as the 1857 (second Bremiker) German edition are in the Library of Brown University. In what library may the French or Russian translations be found?

R. C. A.

QUERIES—REPLIES

41. GIRARD AND SNELL TABLES (Q**30**, v. 3, p. 451).—ALBERT GIRARD, *Tables des Sinus, Tangentes, & Secantes, selon le Raid de 100 000 Parties. Avec un traicté succinct de la Trigonometrie tant des triangles plans, que spheriques*. The Hague, 1626, 240 pages (unnumbered). 6.8 × 11.8 cm.

An examination of the copy of this book in the Library of Congress revealed that the main table contains natural sines, tangents, and secants in units of 10^{-5} for every sexagesimal minute of the first quadrant, so arranged that the functions of complementary angles appear on facing pages. Each page contains functions for a range of half a degree.

Following this principal table is a section devoted to the statement and illustration of rules for the solution of the four standard cases of oblique plane triangles. Although Girard frequently resorts to the device of dissecting oblique triangles into right triangles, he does state and use the Law of Sines. In addition, he gives in the form of rules both the Law of Tangents and the Law of Cosines,—the latter in a form involving the versed sine.

A brief section dealing with some general theorems relating to plane polygons is followed by a treatment of both right and oblique spherical triangles. The discussion of the solution of oblique spherical triangles is limited to three cases: (i) all angles given; (ii) all sides given; and (iii) two sides and their included angle given.

The author then gives to five significant figures the length of a side of each of the five regular polyhedra when inscribed in a sphere of diameter 100 000. His results for the regular tetrahedron and regular octahedron contain rounding errors.

A more extensive table is included showing the lengths to the nearest integer of the sides of regular n-gons $[n = 3(1)24]$ inscribed in a circle of diameter 200 000. Careful examination showed this table to be entirely free from error.