2. 12 separate square wave voltage sources (0 to 12 volts, or one 72 volt source in 2 volt steps)
3. 5 arbitrary functions of independent variable
4. 5 arbitrary current sources (controllable from any voltage source)

D. *Two Metering Desks*

1. 3 dynamometer-type meters and 2 vacuum tube meters for metering sinusoidal current, voltage, power, etc.
2. 3 cathode-ray oscillographs for transient solution (current, voltage, or charge)
3. 2 three-wire metering circuits (and selector system) for metering all passive circuit elements and source forcing functions
4. 2 manual metering selector circuits for 16 of 25 main busses
5. Circuits for connecting 36 synchronous switches to any of 26 main busses

E. *Amplifier Cabinets* (portable)

1. 5 negative gain (0 to 50) d-c amplifiers (15 ohm impedance), each having an RC time-delay circuit for servo-problems
2. 15 positive gain (0 to 100) d-c amplifiers with input and one ohm output impedance
3. 10 d-c isolators and preamplifiers

F. *Multiplier Cabinets* (portable)

1. 10 multipliers with isolated inputs*

G. *Arbitrary Functions of Dependent Variable* (portable)

1. 3 voltage limiters
2. 2 current limiters
3. 1 arbitrary function of dependent variable—Type I
   10 arbitrary functions of dependent variable—Type II

* Additional elements are contemplated. The Table lists only elements of the computer in its present operating form.

RECENT MATHEMATICAL TABLES

658[A–D, Q, R].—ITALY, ISTITUTO IDROGRAFICO DELLA R. MARINA, *Tavole Logaritmiche a Cinque Cifre Decimali.* Sixth ed. Genoa, 1941, xxii, 512. 19.7 × 27.8 cm. Full cloth, and gilt titles.

Contents: **T. I** (p. 1–57).—Mantissae of logarithms and cologarithms of numbers from 1 to 10 080; **T. II** (p. 60–179)—Logarithms of the absolute values of the trigonometric functions $0(1'')2°$; **T. III** (p. 182–361)—Logarithms of the absolute values of the trigonometric functions $0(15'')90°$; **T. IV** (p. 364–382)—Addition and subtraction logarithms; **T. V** (p. 384–428)—Values of the natural trigonometric functions $0(1')90°$ or $0(4^s)6^h$; T. **VI–XIII** (430–464)—Conversion of common logarithms to natural, and vice versa; n^2, n^3, $n^{\frac{1}{2}}$, $n^{\frac{1}{3}}$, $\ln n$, $1000\ n^{-1}$, πn, $\frac{1}{4}\pi n^2$, $n = 0(1)1020$; table of arcs, altitude of sector, chord for $0(1°)180°$, also arcs corresponding to $0(1')60'$, $0(1'')60''$; conversion of centesimal grades, minutes and seconds into sexagesimals; conversion of arcs into time and vice versa; conversion of sexagesimal minutes and seconds into decimals of degrees, and vice versa; geodetic elements relative to the international ellipsoid; values of international normal gravity, g, for ϕ (geogr. latitude) $= 0(15')90°$. Constants and units of measure (p. 467–480). Formulae (p. 483–512)—algebra, plane analytic geometry, differential and integral calculus, plane and spherical trigonometry, series, interpolation, astronomy, geodesy, least squares.

EDITORIAL NOTE—The first edition of this work was published in 1913; the second, 1916; third, 1922; fourth, 1930; fifth, 1937. There was a "nuova edizione, ridotta," 128 p., in 1939, and a copy may be seen in Library of Congress.

659[A–E, J–L].—JOHN BORTHWICK DALE, *Five-figure Tables of Mathematical Functions comprising Tables of Logarithms*, Powers of Numbers, *Trigonometric*, Elliptic, and other *Transcendental Functions*. Second edition, London, Edward Arnold & Co., 1949, viii, 121 p. 13 × 21.6 cm. 6 shillings. New York, Longmans, Green & Co., $1.50.

This popular work, first published in 1903 (xv, 92 p.) was reprinted in 1905, 1908, 1909, 1911, 1913, 1917, 1919, 1922, 1927, 1932, 1937, 1942, 1943, 1945. It was prepared with the idea of meeting the requirements of workers in Physical Science and Applied Mathematics. For the new edition the type has been reset, two new tables have been added (IA (p. 14–18), logarithms of numbers from 1000 to 2999, and XVIIA (p. 88–90) logarithms of hyperbolic functions), and other tables have been rearranged in place and in forms more suitable for ready use. It would seem as if many errors had been made in resetting the tables; here are five random illustrations: $\ln 4 = 1.38629$ (instead of 1.38269); $3.98^{\frac{1}{3}} = 1.5848$ (instead of 1.5484); $4.55^{\frac{1}{3}} = 1.6571$ (instead of 1.6751); $5.49^{\frac{1}{3}} = 1.7641$ (instead of 1.7461); p. 112, last col. $x = .87$, .83424 (instead of .83244).

The Tables are still numbered I–XXXIX: I–VII (p. 10–53), logs of numbers, antilogs, squares, square roots, cubes, cube roots, reciprocals; VIII–XIV (p. 54–81), the natural trigonometric functions and their logs, circular or radian measure; XV–XVIII (p. 82–91), natural logs—to base e, nat. log $10^{\pm n}$, exponential and hyperbolic functions, log e^x; XIX (p. 92), $gd^{-1}\theta$; page 93 is blank; XX–XXII (p. 94–102), $F(\theta, \phi)$, $E(\theta, \phi)$, complete elliptic functions (the rounding error of $F_1(88°)$ in the final digit, 1 instead of 2, *MTAC*, v. 3, p. 262, still persists); XXIII–XXV (p. 103–105), gamma function, logs of factorials, zonal surface harmonics, $P_n(x)$, $n = 1(1)7$, $x = 0(.01)1$; XXVI–XXVII (p. 106–107), $J_0(x)$, $J_1(x)$, $x = 0(.1)15$, $J_n(x)$, $n = 0(1)12$, $x = 0(1)20$ (the error in $J_0(5.9)$ in the first ed. has been corrected); XXVIII (p. 108–109), $I_n(x)$, $n = 0(1)11$, $x = 0(.2)5$, Lodge, 1889; XXIX (p. 110), $J_n = 0$, first 9 roots $n = 0(1)5$ (here are 27 end-figure errors of 1 to 25 units); first 6 roots of $J_0(x)$ and $J_1(x)$, $j_{0,s}/\pi$ and $j_{1,s}/\pi$ (Stokes 1851 tables); XXX (p. 110), $J_n(x)$, $n = \nu + \frac{1}{2}$, $\pm 2n = 1(2)11$; XXXI (p. 111), Erf(x); XXXII (p. 112–113), Ei$(\pm x)$, Ci(x), Si(x); XXXIII–XXXIV (p. 114–116), binomial coefficients for interpolation by differences, and proportional parts; XXXV–XXXVII (p. 117), reciprocals of factorials, Bernoulli numbers (B_1–B_{20}); conversion of time and angular measure; XXXVIII (p. 118), decimal equivalents, fractions of Julian year of $365\frac{1}{4}$ days; XXXIX (p. 119–121), numbers used in calculations, Euler's numbers, Euler's constant; 10D values, for $n = 1(1)20$, of $S_n = 1^{-n} + 2^{-n} + 3^{-n} \cdots$, $s_n = 1^{-n} - 2^{-n} + 3^{-n} - \cdots$, $T_n = 1^{-n} + 3^{-n} + 5^{-n} + \cdots$; English and metric equivalents.

Dale does not give the source of any of his tables.

In conclusion it may be noted that in the first ed. [1913, "sixth impression"], p. 27, arg. 993, *for* 3.9766, *read* 9.9766; p. 91, l. 20, *for* 3.0205, *read* 3.0255.

R. C. A.

660[B–E, M, N].—ANDRÉ SAINTE-LAGÜE, *Mathématiques*, p. A30–A110, in *Techniques de l'Ingénieur, directeur général de la rédaction* C. MONTEIL. I: *Généralités*. Paris, Techniques de l'Ingénieur, 1947, 24.3 × 29.4 cm.

There are various tables of powers, quarter squares, interest, logs, antilogs, natural and hyperbolic trigonometric functions, etc., p. A30–A60, 43 p.; table of indefinite integrals, p. A80–5 to A80–14. Nomography, p. A110–12 to A110–18.

661[C, D, K].—ANDRÉ DANJON, *Tables des Fonctions Trigonométriques. Valeurs naturelles à 6 décimales de centième en centième du degré nonagésimal*. Paris, Librairie Hachette, 1948, ii, 126 p. 18.4 × 27.2 cm. Stiff cover. 850 francs.

This is the third publication, within a year, in which are tables of natural trigonometric functions in the nonagesimal system with the arguments taken at interval 0°.01. COMRIE'S tables (RMT **631**), and NBSCL (RMT **662**) are the others. The further publication of tables using minutes and seconds, instead of decimals of a degree, is likely to become increasingly rare. Thus we are now harking back to tables such as nearly 350 years ago HENRY BRIGGS thought were the most useful he could prepare.

Here are 6D tables (p. 6–95), with Δ, of sin, csc, tan, cot, sec, and cos. In connection with sines and cosines linear interpolation can be used throughout, and in the case of tangents and secants from 0 to 75°.8; the method for dealing with later angles is indicated. If one is content with 6S, linear interpolation is legitimate up to 87°. On p. 5 are interpolation tables for trigonometric functions, including *S* and *T*. There is also a loose card for 11(1)200(100)900 in a proportional parts table.

The present tables were prepared from a 10D manuscript. The author states that the proofs were carefully compared with the tables in *Trigonometria Britannica*, 1633, of Briggs and for each 0°.05 with ANDOYER, *Nouvelles Tables Fondamentales* (1915–1918).

On p. 96–99 are tables for the conversion of (a) sexagesimal fractions of a degree into decimal fractions; (b) decimal fractions of a degree into sexagesimal fractions; (c) hours into degrees and decimal fractions of a degree; (d) degrees into hours. A 6D table of logs 1000(1)-9999 occupies p. 100–117, and various mathematical formulae and constants, p. 119–125. On p. 126 are reprinted two statistical tables, Normal distribution of Laplace-Gauss, and Distribution of χ^2 from R. A. FISHER & F. YATES, *Statistical Tables*, third ed., 1948.

The work presents a neat appearance. The one-column "Directions for the use of the tables" is in four languages: French, Spanish, English and German.

R. C. A.

662[D].—NBSCL, *Table of Sines and Cosines to Fifteen Decimal Places at Hundredths of a Degree.* (*NBS Applied Math. Series*, no. 5.) Washington, Govt. Printing Office, 1949. viii, 95 p. For sale by the Superintendent of Documents, Washington, D. C. 40 cents. 19.8 × 25.9 cm.

We have here an original valuable table of sines and cosines to 15D, δ^2, throughout the quadrant, at interval 0°.01. A preliminary manuscript to 18D was checked by differencing and then rounded to 15D and typewritten. The final manuscript was again checked by differencing.

The introduction by H. E. SALZER includes a discussion of direct and inverse interpolation, with examples, and the usual interpolation tables, of $E(p)$ and $F(p)$, are given at the end of the volume, p. 94–95.

The appearance of the volume would have been decidedly improved by the omission of the tautological headings, "Table of Sines and Cosines," on 90 pages.

It is more than 300 years ago since tables such as these were first computed and published. About 1601 HENRY BRIGGS (1561–1631) finished the calculation of a remarkable series of tables at interval 0°.01, throughout the quadrant, including those of sines and cosines to 15D, Δ. These tables were posthumously published by HENRY GELLIBRAND (1633). We have already indicated (*MTAC*, v. 1, p. 129–130) that Briggs was led to the decimal choice of such parameter by perusal of a publication of Vieta. In the first part of this 1633 volume (p. 43–44) there is also a table of sines for each 0°.625 in the quadrant to 19D, Δ.

Many years earlier than Briggs RHETICUS (died 1576) computed an extraordinary table of sines for each 10″ of the quadrant, 15D, Δ^3 or Δ^2; compare N**109**. Hence we may say that this table includes the values of sines and cosines to each 0°.125 of the quadrant. This table was first published by PITISCUS[2] in 1613 along with a Rheticus table of sines at interval 1″, from 0 to 1° and from 89° to 90°, 15D, Δ^2, and hence giving values at interval 0°.0625 in this range.

In the work under review there is no reference to these Rheticus tables or to the ANDOYER[3] tables (1915) of sines and cosines covering the same range, 15D, Δ, and hence including values to each 0°.125 of the quadrant.

There are three further tables of sines and cosines to at least 15D, as follows: by HERRMANN[4] (1848), [1°(1°)89°; 30D], and reprinted, p. 92–93 in the work under review; by J. T.

PETERS[5] (1911), [0(10′)45°; 21D], and therefore including a table at interval 0°.5; and SPENCELEY & SPENCELEY[6] (1947), [1°(1°)90°; 25D].

On checking the first 50 values each of sines and cosines of the NBSCL table and that of Briggs, it was found that among the 50 sine values of the latter 42 differed from the former only in the fifteenth place, and by 1 to 4 units (20 unit errors, 5 2-unit errors, 5 3-unit errors, and 12 4-unit errors). Among the values of the 50 cosines there were only 11 unit errors in the fifteenth decimal place.

R. C. A.

[1] H. BRIGGS & H. GELLIBRAND, *Trigonometria Britannica*, Gouda, 1633.

[2] G. J. RHETICUS, ed. by B. PITISCUS, *Thesaurus Mathematicus*, Frankfort, 1613.

[3] H. ANDOYER, *Nouvelles Tables Trigonométriques Fondamentales*, v. 1, Paris, 1915.

[4] HERRMANN, "Bestimmung der trigonometrischen Functionen aus den Winkeln und der Winkel aus den Functionen, bis zu einer beliebigen Grenze der Genauigkeit," Akad. d. Wissen., Vienna, *Math. naturw. Kl., Sitzb.*, v. 1, "second unchanged ed.," 1848, p. 477–478; a similar table of tangents and cotangents is given on p. 479–480. In this v. the pages are numbered consecutively from the beginning to the end. These page numbers agree with those given in the *Royal Soc. Cat. Sci. Papers*. Yet the references given by Mr. SALZER, p. v, are entirely different, namely: pt. IV, p. 176–177, and 178–179. It would therefore seem as if the five parts of the volume were originally issued with separate paging but when assembled in a volume were paged consecutively.

[5] J. T. PETERS, "Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus zur genauen Berechnung von zwanzigstelligen Werten sämtlicher trigonometrischen Funktionen eines beliebigen Arguments sowie ihrer Logarithmen," Akad., d. Wissen., Berlin, *Phys.-math. Cl., Abh.*, 1911, p. 12–18.

[6] G. W. & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables*. Washington, 1947.

663[D, H, M].—HARRY D. HUSKEY, "On the precision of a certain procedure of numerical integration"; Appendix: D. R. HARTREE, "Note on systematic rounding-off errors in numerical integration," NBS, *Jn. of Research*, v. 42, 1949, p. 57–62. 19.7 × 26 cm.

Summary: An example of numerical integration (illustrated by three trigonometric tables) is given and shows very systematic effects in the less significant digits. This lack of randomness gives rounding-off errors that exceed the predicted standard deviation by a factor of three.

The example considered shows that systematic rounding-off errors can occur in numerical integration, irrespective of the number of digits kept in the contributions to the integral. In the Appendix this phenomenon is examined, and criteria are set up to detect the cases in which it may arise to a serious extent.

664[D, I].—JØRGEN RYBNER & K. STEENBERG SØRENSEN, *Tabel til Brug ved Addition af Komplekse Tal. Table for use in the Addition of Complex Numbers*. Copenhagen, Jul. Gjellerups Forlag, 1948, xiv, 95 p. 23.6×31.5 cm. 20 Danish crowns. American agent: Scandinavian Book Service, P.O. Box 99, Audobon Sta., New York 32. $5.50.

The addition of two complex numbers can be carried out by means of a table by dividing by the number having the larger numerical value (in KENNELLY's notation $\underline{/\phi} = e^{i\phi} = \cos\phi + i\sin\phi$):

$$z_3 = z_1 + z_2 = r_1\underline{/\phi_1} + r_2\underline{/\phi_2} = r_1\underline{/\phi_1}[1 + (r_2/r_1)\underline{/\phi_2 - \phi_1}],$$
$$r_2 \le r_1 = r_1\underline{/\phi_1}(1 + r\underline{/\phi}) = r_1\underline{/\phi_1}\cdot R\underline{/\alpha} = r_1 R\underline{/\phi_1 + \alpha}.$$

Subtraction is reduced to addition by changing the negative sign of z_2 into an alteration of its phase angle by ±180°:

$$z_3 = r_1\underline{/\phi_1} - r_2\underline{/\phi_2}, \qquad = r_1\underline{/\phi_1} + r_2\underline{/\phi_2 \pm 180^\circ}.$$

The table represents the function

$$R\underline{/\alpha} = 1 + r\underline{/\phi}$$

giving R(5D) and α(3D) as functions of r and ϕ, for $r = 0(.01)1$, $\phi = 0(1°)180°$.

$R^2 = 1 + r^2 + 2r \cos \phi$, $\sin \alpha = r \sin \phi/R$, $\tan(\frac{1}{2}\phi - \alpha) = [(1 - r)/(1 + r)]\tan \frac{1}{2}\phi$.

The computations were started by calculating R and α directly from these formulae at intervals .05 for r, and 5° for ϕ. These calculations were carried out by means of the seven-place trigonometrical tables of PETERS,[1] R being determined to 7D and α to 5D, that is, in both cases two more places than given in the final table. The other values of the table were found by interpolation.

Pages 2–93 are devoted to tabulation of R and α for $r = 0(.01)1$, $\phi = 0(1°)180°$. Differences d_r, d_ϕ are given throughout. At the bottoms of the R and α pages are the interpolation formulae:

$$R(r_0 + xh, \quad \phi_0 + yk) = R_{00} + xd_r + yd_\phi + xyd_{r\phi}{}^2 + B''(x)d_{rr}{}^2 + B''(y)d_{\phi\phi}{}^2$$
$$\alpha(r_0 + xh, \quad \phi_0 + yk) = \alpha_{00} + xd_r + yd_\phi + xyd_{r\phi}{}^2 + B''(x)d_{rr}{}^2 + B''(y)d_{\phi\phi}{}^2$$

and on each page are the ranges of values of $d_{r\phi}{}^2$, $d_{rr}{}^2$, $d_{\phi\phi}{}^2$. On p. 95 is a table of interpolation coefficients $B''(u) = -\frac{1}{4}u(1 - u)$, $u = x$ or y.

The introductory pages include a number of worked out examples illustrating the table's use; for example: the values of R and α are found when $r = .5082$, $\phi = 136°.59$.

It is obvious that great care was taken to attain accuracy in preparation of the volume.

In an earlier work by Professor Rybner,[2] which we reviewed, *MTAC*, v. 3, p. 174–175, there was a nomogram for $R\underline{/\alpha} = 1 + r\underline{/\phi}$ but the present table was necessary because it was not found possible to construct the corresponding nomograms, with an accuracy sufficient for practical calculations.

A small table of $R\underline{/\alpha} = 1 + r\underline{/\phi}$ was given by F. EMDE[3] but the angles are measured in fractions of a right angle, 0(.05)2, which is inconvenient, since this unit is not commonly used.

R. C. A.

[1] J. T. PETERS, *Siebenstellige Werte der trigonometrischen Funktionen von Tausendstel zu Tausendstel des Grades*. Berlin, 1938.

[2] J. RYBNER, *Nomograms of Complex Hyperbolic Functions*. Copenhagen, 1947.

[3] F. EMDE, *Tables of Elementary Functions. Tafeln elementarer Funktionen*. Leipzig, 1940. American reprint 1945, p. 32–33. See *MTAC*, v. 1, p. 384–385. The table is also given in JAHNKE & EMDE, *Tables of Functions*, e.g. 1945, p. 18–19 of the second part.

665[E].—HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 20, *Tables of Inverse Hyperbolic Functions*, By the staff of the Laboratory, H. H. Aiken, director. Cambridge, Mass., Harvard University Press, 1949, xx, 290 p. 19.5 × 26.7 cm. $10.00. Offset print.

These tables give 9D values with first and second forward differences of the inverse hyperbolic sine, cosine, and tangent. More precisely, **T. I** (p. 3–47), from which **T. II** and **T. III** were derived, gives $\tanh^{-1} x$ for $0 \leq x < 1$. For $x < .9$ the values of the argument are

$$x = 0(.001).5(.0005).75(.0002).9.$$

Only 39 percent of the table is devoted to 90 percent of the unit interval. The singularity of $\tanh^{-1} x$ at $x = 1$ forced the tabulation for $.9 < x < 1$ to proceed by finer and finer intervals in order to preserve the adequacy of second order interpolation. The final entry is for $x = .99999$.

T. II (p. 51–66) is straightforward and gives $\sinh^{-1} x$ for

$$x = 0(.002)3(.005)3.495.$$

T. III (p. 69–103) gives $\cosh^{-1} x$ for $1 < x < 3.5$. Here trouble is encountered at $x = 1$ so that one-half of the table has to be devoted to 6 percent of the entire range of x. Beyond

$x = 1.15$ the argument values are

$$x = 1.15(.0005)1.4(.001)1.8(.002)3.498.$$

Finally **T. IV** (p. 107–290) gives $\sinh^{-1} x$ and $\cosh^{-1} x$ for $3.5 \leq x \leq 22980$ at various intervals ranging from .005 to 20 keeping the second differences under about 2000 units in the 9th decimal place.

The accuracy of the whole volume is to within .9 units in the 9th decimal. The computation time was only 12 days, of 24 hours each. This represents an increase in speed of the Automatic Sequence Controlled Calculator due to an improved printing mechanism.

The fact that the table was produced by automatic methods accounts for a number of its features which would have puzzled table makers of a decade ago. For example **T. IV** has the luxurious feature of tabulating two functions so nearly alike that their values differ by only a few hundred units in the last place of decimals and their second differences are practically indistinguishable. This goes on for more than 150 pages and would be an unthinkable waste of time and paper, not to mention an added source of typographical error, had the table been hand-made. In fact, by past standards an extensive table of these three functions has no doubt been considered unworthy of much effort in view of the fact that their values can be obtained from a good table of logarithms.[1] At any rate the bibliography of 17 titles[2] (p. xvi–xx) included mostly trivial tables. The well known table of Hayashi[3] is perhaps the only one approaching the present one in extent. The former table, however, is poorly arranged for these functions and is unreliable. The present volume, therefore, represents a significant advance in the tabulation of hyperbolic functions.

D. H. L.

[1] $\text{Sinh}^{-1} x = \ln[x + (x^2 + 1)^{\frac{1}{2}}] = \text{csch}^{-1}(1/x), \quad -\infty < x < \infty$
$\cosh^{-1} x = \pm \ln[x + (x^2 - 1)^{\frac{1}{2}}] = \text{sech}^{-1}(1/x), \quad 1 < x < \infty$
$\tanh^{-1} x = \frac{1}{2} \ln[(1 + x)/(1 - x)] = \coth^{-1}(1/x), \quad -1 < x < +1.$

[2] After listing the tables of inverse hyperbolic functions in M. BOLL, *Tables Numériques Universelles*. Paris, 1947, p. 475, there is the remark: "This single page of tables is replete with errors." See *MTAC*, v. 2, p. 336–338; v. 3, p. 466–467.

[3] K. HAYASHI, *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen*, Berlin, 1926, p. 9–201.

666[F].—D. H. LEHMER, "On the converse of Fermat's theorem," *Amer. Math. Mo.*, v. 43, 1936, p. 347–354; v. 56, 1949, p. 300–309.

Fermat's theorem states that if a is any integer, and p a prime, then $a^p - a$ is divisible by p. For $a = 2$, the strict converse, attributed to the ancient Chinese, would assert that if n divides $2^n - 2$, then n is a prime. SARRUS in 1819 disproved this by the counter-example $n = 341 = 11 \times 31$; $2^{341} - 2$ is divisible by 341. A true converse of Fermat's theorem, due to LUCAS, states that if n divides $a^x - 1$ for $x = n - 1$, but not for x a proper divisor of $n - 1$, then n is prime. In both theoretical and practical work it is frequently required to settle whether a fairly large number is prime or not. No general usable test is available (WILSON's theorem is unusable). "One may ask for a criterion which is almost characteristic of primes and then tabulate those relatively few composite numbers which also satisfy this condition." The false Chinese converse is such a criterion. But to find the composite exceptions, like 341, factorizations of large numbers are frequently demanded. The author's long experience in this enabled him in the first paper to publish a list of composite exceptions[1] between 10^7 and 10^8 whose least factor exceeds 313. The smallest prime factor of each is listed. In the second paper the list is extended[2] from 10^8 to $2 \cdot 10^8$. The Army Ordnance's ENIAC all-electronic digital computer made the work possible in a remarkably short time. Incidentally it is proved that there is an unlimited number of n's dividing $2^n - 2$ that are the products of three primes.

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[1] This list contained 526 entries believed to contain all exceptional numbers; no number having more than two prime factors, each in excess of 313. In the second paper it was dis-

covered after recomputation that in this list there were 2 entries to delete and 7 to insert (*MTAC* v. 2, p. 279) so that there are exactly 531 exceptional numbers in question less than 10^8.—EDITOR.

[2] In the second list are 327 entries—320 two prime-factor numbers and 7 three prime-factor numbers. Thus there are in all 858 exceptional numbers $< 2 \cdot 10^8$.—EDITOR.

667[F].—D. H. LEHMER, "On the partition of numbers into squares," *Amer. Math. Mo.*, v. 55, 1948, p. 476–481.

$P_k(n)$ denotes the number of partitions of n into k integral squares $\geqq 0$. Partitions differing only by permutations of the squares are not counted as distinct. The author is concerned with the solutions of $P_k(n) = 1$. It has long been known that $P_k(n) > 0$ for $k \geqq 4$. The author solves completely $P_4(n) = 1$, and finds all numbers that can be partitioned into 4 squares in precisely 2 ways. For $P_3(n) = 1$ he finds the solutions $4^a m$, $a \geqq 0$, for $m \leqq 427$. Any further values of m exceed 10,000.

E. T. BELL

668[F].—NILS PIPPING, "Diagonalkettenbrüche," Aabo, Finland, Akademi, *Acta, Mathem. et Phys.*, v. 16, no. 5, 1949, 23 p.

The conclusion of this paper (p. 16–23) presents a table of the expansion of $D^{\frac{1}{2}}$ in a diagonal continued fraction for non-square integers D from 501 to 1000. This is an extension of a previous table (*MTAC*, v. 3, p. 96). In 97 of the 491 expansions the diagonal continued fraction is regular.

D. H. L.

669[F].—D. YARDEN & A. KATZ, "Luaḥ sedarot nesiga linariyot binariyot misseder 3" [Table of binary linear recurring sequences of order 3]. *Riveon Lematematika*, v. 2, p. 54–55, 1948.

Four recurring sequences defined by

$$\begin{array}{llll} U_n = U_{n-2} + U_{n-3} & U_0 = 0, & U_1 = 0, & U_2 = 1 \\ V_n = V_{n-2} + V_{n-3} & V_0 = 3, & V_1 = 0, & V_2 = 2 \\ \bar{U}_n = -\bar{U}_{n-2} + \bar{U}_{n-3} & \bar{U}_0 = 0, & \bar{U}_1 = 0, & \bar{U}_2 = 1 \\ \bar{V}_n = -\bar{V}_{n-2} + \bar{V}_{n-3} & \bar{V}_0 = 3, & \bar{V}_1 = 0, & \bar{V}_2 = -2 \end{array}$$

are tabulated for $\pm n = 0(1)64$. The factorizations of the numbers are also given with the exception of $V_{62} = 2 \cdot 9803919989$. Prime values of these four functions are underlined.

D. H. L.

670[I].—H. E. SALZER, *Table of Coefficients for Inverse Interpolation with Central Differences.* NBS, *Mathematical Table*, no. MT 27. Second ed., June 1, 1949. Washington, D. C., ii, 15 p. 14.9 × 23.2 cm.

This table, previously reviewed in *MTAC*, v. 1, p. 315–316, is here reprinted with permission of the editors of *Jn. Math. Phys.* No changes were made in the text or tabular material.

671[K].—HAROLD JEFFREYS, *Theory of Probability.* Second ed., Oxford, Clarendon Press, 1948, viii, 412 p. 15.4 × 23.5 cm. 30 shillings; New York branch office price \$9.00. Appendix, p. 396–404, "Tables of *K*."

The author uses K as a generic term for the ratio of probabilities $P(q\,|\,\theta H)/P(q'\,|\,\theta H)$, where q is the initial hypothesis, q' the alternative, H the previous information, and θ the observational evidence. The actual seven tables given are double-entry tables, entered with values of n, or ν, and of K, to read values of χ^2, t^2 or z, depending upon the table. The particular function K is separately defined in each table. Each resulting table gives, of course, selected values of a specified mathematical function of two arguments, and does not depend for its calculation upon any particular theory of probability. The values of K are 1, $10^{-\frac{1}{2}}$, 10^{-1}, $10^{-\frac{3}{2}}$, 10^{-2}, in each table. In T. I–IV, n or $\nu(= n - 1)$ ranges from 5 to 100,000 (save in T. II, where n starts at 7). The entries proceed by one unit up to 20, by ten units to 100, thence the numbers go by successive factors of 2 and 5, up to 100,000. The functional values to be read are to one place of decimals, and hence to two or three significant figures. In T. IIIA $\nu = 1(1)9$, T. IVA, $\nu = 1(1)8$, and in T. V, $\nu = 1(1)10(2)20$, 50, while z is read to 2D. In T. IIIA, IVA, and V, the values of K for $t = 0$, or $z = 0$, are also listed for the given values of ν. The argument, K, is related to n or ν and the function to be read, as follows:

T. I: $K = (2n/\pi)^{\frac{1}{2}} \exp(-\frac{1}{2}\chi^2)$; reading χ^2.
T. II: $K = \frac{1}{2}\pi n^{\frac{1}{2}}\chi \exp(-\frac{1}{2}\chi^2)$; reading χ^2.
T. III: $K = (\pi\nu/z)^{\frac{1}{2}}[1 + (t^2/\nu)]^{\frac{1}{2}(1-\nu)}$; reading t^2.

T. IIIA: $K^{-1} = (2/\pi)\int_0^\infty {}_1F_1\left\{\frac{1}{2}(1 - n),\ \frac{1}{2},\ -\frac{nv^2\bar{x}^2}{2(\bar{x}^2 + s'^2)}\right\} \exp\left\{-\frac{ns'^2v^2}{2(\bar{x}^2 + s'^2)}\right\}\frac{dv}{1 + v^2}$

and ${}_1F_1(\alpha, \gamma, x)$, is the confluent hypergeometric function, $1 + \frac{\alpha x}{\gamma} + \frac{\alpha(\alpha + 1)x^2}{2!\gamma(\gamma + 1)} + \dots$,
$t = \nu^{\frac{1}{2}}\bar{x}/s'$; reading t^2.
T. IV: $K = \frac{1}{2}\nu^{\frac{1}{2}}\pi t[1 + (t^2/\nu)]^{-\frac{1}{2}\nu}$; reading t^2.

T. IVA: $K^{-1} = 2\pi^{-1}\int_0^\infty {}_1F_1\left\{1 - \frac{1}{2}n,\ 1,\ -\frac{n(a^2 + b^2)v^2}{2(2s'^2 + a^2 + b^2)}\right\} \exp\left\{-\frac{ns'^2v^2}{2s'^2 + a^2 + b^2}\right\}$

$\times \frac{dv}{1 + v^2}$, where ${}_1F_1(\alpha, \gamma, x)$ is as given above, and $a^2 + b^2$ replaces $\bar{x}^2$; reading t^2.

T. V: $K^{-1} = 2^{\frac{1}{2}}\pi^{-1}\int_0^\infty [(u^2 + 1)/(u^4 + 1)]u^n \exp\{\frac{1}{2}nb^2(1 - u^2)\}du$ where $b = e^z$; reading z.

In general these tables serve to indicate with what degree of accuracy it is worth while to carry out the reduction of a given set of observations. Though differing in principle from the use of probability integrals, they lead to substantially the same numerical estimates.

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672[K].—N. V. SMIRNOV, "Table for estimating the goodness of fit of empirical distributions," *Annals Math. Statistics*, v. 19, 1948, p. 279–281. 17.4 × 25.2 cm. Reprinted from N. V. SMIRNOV, "On the estimation of the discrepancy between empirical curves of distribution for two independent samples," Moscow, Univ., *Bull. Math.*, *série internationale*, v. 2, fasc. 2, 1939, p. 15–16. 14.7 × 21.8 cm.

This is a table of $L(z) = 1 - 2\sum_{\nu=1}^{\infty}(-1)^{\nu-1}e^{-2\nu^2z^2} = \sum_{-\infty}^{\infty}(-1)^{\nu}e^{-2\nu^2z^2} = (2\pi)^{\frac{1}{2}}z^{-1}$

$\times \sum_{\nu=1}^{\infty} e^{-(2\nu-1)^2\pi^2/8z^2}$ mostly 6D, for $z = .28(.01)2.5(.05)3$. The values vary from .000001 for

for $z = .28$, to .999 99997 for $z = 3$.

It may be noted that in the *Annals*, p. 178 and 279 (W. FELLER) the expression for $L(z)$ in the first of the three forms given above is incorrect, since it has $e^{-\nu^2z^2}$. This error is copied in *Math. Reviews*, v. 9, p. 599. The second and third forms, given by Smirnov, are correct.

R. C. A.

673[K, L].—J. I. MARCUM, *Tables of Hermite Polynomials and the Derivatives of the Error Function.* U. S. Air Force. Project RAND no. P-90. Santa Monica, Cal., 29 December 1948, 241 leaves, text on one side only. Hektographed. 20.5 × 28 cm. Not available for distribution at present.

Introductory text: The error function may be defined by $\phi(x) = E'(x) = (2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}x^2}$. The n^{th} derivative of this function is then

$$\phi^n(x) = D^nE'(x) = (2\pi)^{-\frac{1}{2}}d^ne^{-\frac{1}{2}x^2}/dx^n.$$

The following tables give values of the derivatives of the error function so defined for

$$n = 1(1)10, \qquad x = 0(.01)12; 6\text{S}.$$

In the process of computing these tables, the Hermite polynomials, defined by

$$h_n(x) = (-1)^n e^{\frac{1}{2}x^2} d^n e^{-\frac{1}{2}x^2}/dx^n,$$

were generated as a by-product. The tables also include the values of these polynomials.

The specific purpose for computing these tables was for use in GRAM-CHARLIER series approximations to certain distribution functions for a sine wave plus random noise [see J. I. MARCUM, *A Statistical Theory of Target Detection by Pulsed Radar—mathematical appendix.* Project RAND, Report no. R-113, July 1, 1948 (restricted)]. For use in the Gram-Charlier series, n should go at least to 9 and x to at least 10 in interval .01. No combination of available tables could fulfill these requirements. Hence these tables were computed on the IBM machinery at Project RAND.

A table of exact values of $h_n(x)$ for $n = 2(1)6$, $x = 0(.01)4$ was given by N. R. JØRGENSEN *Undersøgelser over Frekvensflader og Korrelation.* Diss. Copenhagen, 1916, p. 196–205. There is also a table of $\phi^n(x)$ in T. C. FRY, *Probability and its Engineering Uses.* New York, 1928, p. 456–457, $n = 1(1)6$, $x = [0(.1)4; 5\text{D}]$; and in New York, W.P.A., ms., $n = 1(1)14$, $x = [0(.1)8.4; 20\text{D}]$.

EDITORIAL NOTE: In the Marcum tables 5D uniformity in display for both $h_n(x)$ and $\phi^n(x)$ is attained by supplying extra columns for each function indicating the number of places the decimal point is to be moved to the right or to the left. Thus for $x = 12$ and $n = 10$ the first 6 figures in a 11 figure value for $h_n(x)$ are given, while for $\phi^n(x)$ we have a value to 27D.

674[L].—FREDERICK W. GROVER, *Inductance Calculations. Working Formulas and Tables,* New York, Van Nostrand, 1946, xiv, 286 p. $5.75. 15 × 23 cm.

Among 55 tables in this volume Dr. FLETCHER has already referred to T. 13, 36, 37, 40, 49–51 in his "Guide to tables of elliptic functions," *MTAC,* v. 3, p. 235, 238, 239, 253.

Auxiliary tables 3–4, p. 238–247 are of zonal harmonic functions and differential coefficients of zonal harmonics. **T. 3**: $P_n(x)$, $n = 2(1)8$, and $x^{-1}P_n(x)$, $n = 3(2)7$; **T. 4**: $P_n'(x)$, $n = 3(2)7$, Δ, Δ^2, or Δ^3; each table giving exact values for $x = 0(.01)1$. Such tables are also to be found in A. H. H. TALLQVIST, *Grunderna af Teorin för sferiska Funktioner.* Helsingfors, 1905.

R. C. A.

675[L].—D. R. HARTREE & S. JOHNSTON, "On a function associated with the logarithmic derivative of the gamma function," *Quart. Jn. Mech. Appl. Math.,* v. 1, 1948, p. 29–34. 15.3 × 23.4 cm.

Here is a table of $\text{Re}[\psi(k) - \ln k + \frac{1}{2}k^{-1}] = \text{Re}\psi(10 + i|k|) - \log|k| - |\gamma| \times \left[\frac{1}{1^2|\gamma| + 1} + \frac{2}{2^2|\gamma| + 1} + \cdots + \frac{9}{9^2|\gamma| + 1}\right]$, where $\psi(z) = \Gamma'(z)/\Gamma(z)$, for $\gamma = k^{-2} = [-1(.01) + 1; 8\text{D}]$, Δ^2. The values were evaluated to 10D. The maximum error in a tabulated value should not be more than .6 in the eighth decimal. Linear interpolation is adequate to give 6D accuracy.

Extracts from text

676[L].—VERA HUCKEL, *Tables of Hypergeometric Functions for Use in Compressible-flow Theory*, U. S. National Advisory Committee for Aeronautics, *Technical Note*, no. 1716, Oct. 1948, 13 p. 19.9 × 26.3 cm.

In the hodograph method of treating plane potential compressible flows the differential equation, originally obtained by CHAPLYGIN in his study on gas jets, plays a significant role. This paper tabulates various hypergeometic functions which arise as particular solutions of Chaplygin's differential equation. The tables should prove useful in the tabulation of other auxiliary functions which may arise in various compressible-flow problems. The adiabatic index for air has been taken as 1.4.

Chaplygin's differential equation is

$$(1) \qquad \tau(1-\tau)d^2Y_k/d\tau^2 + [(k+1) - (k+1-\beta)\tau]dY_k/d\tau + \tfrac{1}{2}\beta k(k+1)Y_k = 0$$

where $\beta = 2.5$, $M^2 = 5\tau(1-\tau)^{-1}$, M being the Mach number, and $Y_k(\tau) \to 1$, as $\tau \to 0$.

The tables of numerical values have been prepared for a selected number of the complete set of solutions of (1). These solutions extend the results of Chaplygin into the supersonic range and to negative values of the index k.

Tables 1–2 are for the functions Y_k, $k = -15(.5)+15$, $\tau = [0(.01).5; 5D]$, with corresponding Mach numbers.

Tables 3–4 are for the functions $dY_k/d\tau$, $k = -15(.5)+15$, $\tau = [0(.01).5; 4D]$, with corresponding Mach numbers.

REFERENCES

I. E. GARRICK & CARL KAPLAN, *On the Flow of a Compressible Fluid by the Hodograph Method II—Fundamental Set of Particular Flow Solutions of the Chaplygin Differential Equation*. NACA, *Report*, no. 790, 1944, 21 p.

S. A. CHAPLYGIN, *Gas Jets*, NACA, *Technical Memorandum*, no. 1063, 1944, 112 p. +3 leaves. Translated by S. REISS from the Russian in Moscow Univ., *Uchenyia Zapiski*, Otd. Fiz.-Mat., v. 21, 1904, p. 1–121.

Extracts from text

EDITORIAL NOTE: Miss HUCKEL has acknowledged to us that her statement regarding what is tabulated in Tables 3–4 is incorrect, that what she really has tabulated is $-(2/\beta k)dY_k/d\tau$, as suggested by Dr. J. C. P. MILLER in *Math. Reviews*, v. 10, May 1949, p. 329.

In the Report of GARRICK & KAPLAN there are a number of Tables including a 5D table of Y_k for k from -5 to $+5$ and with M as parameter varying from .1 to 2, and with corresponding values of τ.

677[L].—A. N. LOWAN & WILLIAM HORENSTEIN, *On the Function $H(m, a, x) = \exp(-ix)F(m+1-ia, 2m+2; ix)$*. NBS, *Mathematical Table*, no. MT 19. Second ed., June 15, 1949. Washington, D. C., ii, 20 p. 14.9 × 23.2 cm.

This table, previously reviewed in *MTAC*, v. 1, p. 156, is here reprinted, with permission of the editors of *Jn. Math. Phys.* No changes were made in the text or tabular material.

678[L].—WILHELM MAGNUS & FRITZ OBERHETTINGER, *Formulas and Theorems for the Special Functions of Mathematical Physics. Translated from the German by* JOHN WERMER. *Published and distributed in the Public Interest by authority of the Attorney General under License No. A–1341.* New York, Chelsea Publ. Co., 231 West 29th St., 1949, viii, 172 p. $3.50; 15% discount to teachers and members of the Amer. Math. Soc. on direct purchase from the publishers. Offset print.

The first German edition of this valuable work (1943), from which this English translation was made, was reviewed in *MTAC*, v. 3, p. 103–105. Since we there gave the detailed table of contents it will not be necessary to repeat them here. Most of the errata which we then listed have been corrected in this translation, but those noted below still remain. The translation and original correspond page for page throughout.

But we reviewed also the second enlarged and greatly improved German edition (1948), in *MTAC*, v. 3, p. 368–369, which may be procured comparatively easily. Hence many people who might have purchased the English edition may now prefer the better German work.

The errata referred to above are the following:

P. 24, l. 13, *for* u^3, *read* w^3.
P. 46, l. 13, *before* $=0$, *insert* y.
P. 73, exponents μ, of lines 3 and 5, are badly placed.
P. 83, l. 11, *for* $\alpha\gamma$ are real and $\alpha + 1 > \gamma > 1$, *read* α and γ are real, and $\gamma > 0, \alpha > \gamma - 1$; l. 14, *delete*.
P. 88, l. -1, *for* 0 ± 1, *read* $0, \pm 1$.
P. 108, l. 3, *for* x[3 times], *read* t.
P. 119, heading, *for* §2, *read* §1; col. 2, l. 5, *for* $y|$, read $|y|$.
P. 131, l. -1, for the image function of the original function $t^\nu J_\mu(at)$ in the Laplace transform table, the second edition has

$$p^{-\mu-\nu-1}\frac{\Gamma(\mu+\nu+1)}{2^\mu\Gamma(\mu+1)}a^\mu {}_2F_1[\tfrac{1}{2}(\mu+\nu+1),\quad \tfrac{1}{2}(\mu+\nu+2);\quad \mu+1;\quad -a^2/p^2],$$

$$\textit{not } (-1)^m\frac{\Gamma(\mu+\nu+1)}{(p^2+a^2)^{\frac{1}{2}(\nu+1)}}P_\nu{}^{-\mu}(p/\sqrt{p^2+\alpha^2})\mathrm{Re}(\nu+m) > -1.$$

R. C. A.

679[L].—NBSCL, *Tables of the Confluent Hypergeometric Function* $F(\frac{1}{2}n, \frac{1}{2}; x)$ *and Related Functions*. (NBS, *Applied Mathematics Series*, no. 3.) Washington, D. C., 1949, xxii, 73 p. 19.8 × 26.1 cm. For sale by the Superintendent of Documents, Washington, D. C., 35 cents. See *MTAC*, v. 3, p. 414, 483.

The confluent hypergeometric function $F(\alpha, \gamma; x)$ is defined by

$$F(\alpha, \gamma; x) = \sum_{j=0}^{\infty} \Gamma(\gamma)\Gamma(\alpha+j)x^j/[\Gamma(\alpha)\Gamma(\gamma+j)j!]. \tag{1}$$

It is easily verified that it is the solution of the differential equation

$$xy'' + (\gamma - x)y' - \alpha y = 0 \tag{2}$$

satisfying the initial conditions $y(0) = 1$ and $y'(0) = \alpha/\gamma$. KUMMER's formula

$$F(\alpha, \gamma; x) = e^x F(\gamma - \alpha, \gamma; -x) \tag{3}$$

is a direct consequence of this property and may be verified by substitution into (2). In this review we use the abbreviation $F_n = F(\frac{1}{2}n, \frac{1}{2}; x)$. Using the series expansion (1) for the right side in (3) it is readily seen that when n is a positive odd integer, then F_n equals e^x multiplied by a polynomial of degree $\frac{1}{2}(n-1)$. In particular

$$F_1 = e^x, \qquad F_3 = e^x(1+2x), \qquad F_5 = e^x(1+4x+\tfrac{4}{3}x^2). \tag{4}$$

The ratios $g_n = F_n/F_{n-2}$ satisfy the recurrence relation

$$g_{n+2} = (2x + 2n - 1)/n - (n-1)/ng_n. \tag{5}$$

Formulae (4) and (5) permit to calculate successively all $F_n(x)$ for odd n, and this method was used in the preparation of the Tables. Various other relations were used to check the accuracy of the tables, and it is believed that the error is less than one unit of the last decimal place.

In the present tables the subscript n of F_n is always an odd integer. The original objective was the tabulation of $\ln F_n$, but it turned out that the function $(2nx)^{-\frac{1}{2}} \ln F_n$ is more convenient for interpolation and has the added advantage of being of the order of magnitude of unity, over a considerable range for n and x. The main part of the tables (p. 14–73) is devoted to the tabulation of this function for $n = 3(2)201$ and $x = .1(.01).6(.1)2(.2)7(1)$-$45(5)100$. For $n = 3(2)21$, the range .6 to 2 is covered in steps of .05. Rows correspond to values of x, columns to n. On p. 2–6 the function F_n itself is tabulated for $x = 0(.01).1$ and $n = 3(2)201$, and on p. 8–11 we find $F_n/\cosh\{(2n-1)x\}^{\frac{1}{2}}$ for the same range of x and $n = 43(2)201$. The latter table is given for interpolation purposes since in the range covered F_n varies very rapidly. All tables are to 6D, except the table of F_n on p. 3–6 which is to 7S.

A pleasing feature of the volume is a comparatively large collection of formulae and asymptotic expansions for F_n and certain related functions. (Most of these formulae are not directly connected with the tables.) An interpolation chart shows how many terms of the bivariate Lagrange interpolation formula are required for various n and x in order to obtain an accuracy to 6D.

Confluent hypergeometric functions appear in many applications. The present tables were designed especially for statisticians. Their primary purpose is in connection with the sequential t-test designed by A. WALD. It is also of great use in the analysis-of-variance and the ordinary t-test.

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680[L, M, S].—C. J. BOUWKAMP, "On the effective length of a linear transmitting antenna," *Philips Research Reports*, Eindhoven, Holland, v. 4, June 1949, p. 179–188. 15.4 × 23.4 cm.

$F_1(H) = \int_0^H J_1(x) \cos x dx/x$, $F_2(H) = \int_0^H J_2(x) \sin x dx/x$, $C(4H) = \int_0^{4H} (1 - \cos x) dx/x$, $H_{e,s} = 2F_1(H) \sin H$, $H_{e,a} = 2F_2(H) \cos H$, $R = 30[C(4H) - \sin^2 2H]$, $R_s \approx 80|H_{e,s}|^2$, $R_a \approx 64|H_{e,a}|^2$.

In Table I are 6D values of $F_1(H)$, $F_2(H)$, $C(4H)$ for $H = 0(.1)3$.

In Table II are given 4D values of $H_{e,s}$, $H_{e,a}$, R_s, R_a, $R_s + R_a$, R, for $H = .1(.1)3$.

Extracts from text

681[L, M].—HAROLD LEVINE & JULIAN SCHWINGER, "On the theory of diffraction by an aperture in an infinite plane screen. I," *Phys. Rev.*, s. 2, v. 74, 1948, p. 958–974. 19.8 × 26.6 cm.

On p. 972–974, $\int_0^1 v^{-n-m}(1 - v^2)^{\frac{1}{2}} J_{n+\frac{1}{2}}(\alpha v) J_{m+\frac{1}{2}}(\alpha v) dv$ is evaluated for $m = 1, 2$, and $n = 1, 2$.

682[L, S].—J. MEIXNER, *Lamé's Wave Functions of the Ellipsoid of Revolution*. Translated by MARY L. MAHLER from "Die Laméschen Wellenfunktionen des Drehellipsoids," Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters (ZWB), Berlin-Adlershof, *Forschungsbericht* no. 1952, June, 1944, for the National Advisory Committee for Aeronautics. *Technical Memorandum*, no. 1224, Washington, D. C., April 1949, iv, 102 p. 19.8 × 26.3 cm.

The basic differential equation treated is

$$\frac{d}{d\eta}\left[(1-\eta^2)\frac{df}{d\eta}\right] + \left(\frac{-\mu^2}{1-\eta^2} - k^2c^2\eta^2 + \lambda\right) f = 0. \tag{1}$$

Corresponding to given parameters μ and $\gamma = kc$, there exists a countably infinite set of eigenvalues $\lambda_\nu{}^\mu(\gamma)$ corresponding to which (1) has a solution, called the eigenfunction, which can be represented by

$$X_\nu{}^{\mu(1)}(\xi;\gamma) = \sum_{r=-\infty}^{\infty} i^r a_{\nu,r}^\mu(\gamma) P_{\nu+r}^\mu(\xi), \tag{2}$$

where $P^{\mu}_{\nu+r}(\xi)$ is the associated LEGENDRE function of the first kind and r is always even. It should be noted that if in (1) we set $f = (1 - \eta^2)^{\frac{1}{2}\mu}w$, we obtain

$$(3) \qquad (1 - \eta^2)w'' - 2(\mu + 1)\eta w' + [-\mu(\mu + 1) - k^2c^2\eta^2 + \lambda]w = 0.$$

If μ is an integer m, (3) yields the *prolate* spheroidal wave functions (*oblate* if $\gamma = iC$, C real). Fairly extensive tabulations of both the eigenvalues and coefficients proportional to $a^m_{n,r}$ for integral m, are available.[3] In fact, the author refers to this text, and his mathematical development closely follows it—at least so far as the determination of the eigenvalues, and the derivation of the relations existing between the various solutions of (2). The author notes that if μ in (1) is equal to $-\frac{1}{2}$, we obtain MATHIEU'S equation (this was already noted.[3]) Although the mathematical treatment applies to general values of μ, the tabular material is restricted to integral values of μ, which are designated by m, as in 3. Before describing the tables, it will be necessary to define a second solution of (1):

$$X_n^{m(2)}(\xi;\gamma) = \sum_{r=-\infty}^{r'} i^r a^m_{n,r}(\gamma)P^m_{n+r}(\xi) + \sum_{r=r'+2}^{\infty} i^r a^m_{n,r}(\gamma)Q^m_{n+r}(\gamma).$$

In the above $Q^m_{n+r}(\xi)$ is the associated Legendre function of the second kind, and r' is some finite integer.

It may be shown that

$$X_n^{-m(1)}(\xi;\gamma) = \sum_{r=-\infty}^{\infty} i^r b^m_{n,r}(\gamma)P^{-m}_{n+r}(\xi),$$

where

$$b^m_{n,r}(\gamma) = [(n+m+r)!(n-m)!/(n+r-m)!(n+m)!]\,a^m_{n,r}(\gamma) = a^{-m}_{n,r}(\gamma).$$

The tables comprise the following:

Table 1. Coefficients of $\gamma^2, \gamma^4, \gamma^6, \gamma^8$, and γ^{10} in the power-series expansion of $\lambda_n{}^m(\gamma)$, $m = 0(1)9$, $n = m(1)9$; 10D.

Tables 2, 3, 4, and 5. Coefficients of γ^2, γ^4 and γ^6 in the power-series expansion of $a^m_{n,r}/a^m_{n,0}$ of (2), $m = 0(1)9$, $n = m(1)9$, $\pm r = [2, 4, 6;$ 10D]. (The coefficient of γ^{2k} is zero if $k < |r|$).

Table 6. The coefficients of γ^2, γ^4 and γ^6 in the expansion of $\alpha^m_{n,r}/a^m_{n,0}$; $m = 0(1)2$, $n = m(1)(5 - m)$; $r = [-6(2) - 2$; 10D].

Tables 7, 8, 9, 10. The coefficients of γ^2, γ^4, and γ^6 in the power-series expansion of $b^m_{n,r}/b^m_{n,0}$; $m = 1(1)9$, $n = m(1)9$, $\pm r = [2(2)6;$ 10D]. (For $m = 0$, $b_{n,r} = a_{n,r}$).

The author points out that these truncated power series are useful only for relatively small values of γ, when n is of low order, but that more accuracy is obtainable for high values of n.

In addition to the above tabular material, the author inserted some of BOUWKAMP'S tables of eigenvalues[1] for $m = 0$, and values of the corresponding eigenfunction or its derivative at $\xi = 0$ and 1. There are seven tables of this character, comprising seven pages. Corrections which Bouwkamp made after publication were apparently not known to Meixner; as a result the last two places of these tables are totally unreliable, and more serious errors also remain uncorrected. It should be noted that nearly all values of 1 which Meixner used[5] were corrected and amplified[2] (see review[4]). Since 2 is relatively accessible, Meixner's reproduction of Bouwkamp's erroneous values should be discarded; it is unfortunate that the translator did not replace these seven pages with the more useful tables.[2]

Meixner's new results are contained in Tables 1–10; they duplicate some results of 1 only for $m = 0$. Since Meixner gives no *tabular* values for the eigenvalues or coefficients, his results do not duplicate the tables in 3 but rather add to them. Some asymptotic formulae for the eigenvalues and functions are also given, and there is a graph of $\lambda_n(\gamma)$ for $m = 0$, $n = 0(1)9$. The text contains a valuable compilation of theoretical results, with some discussion of their application to various problems.

The NACA performed a genuinely useful service in making this work available in English.

GERTRUDE BLANCH

NBSINA

[1] C. J. BOUWKAMP, *Theoretische en numerieke Behandeling van de Buiging door een ronde Opening*. Diss. Groningen, 1941

[2] C. J. BOUWKAMP, "On spheroidal wave functions of order zero," *Jn. Math. Physics*, v. 26, 1947, p. 79–92.

[3] J. A. STRATTON, P. M. MORSE, L. J. CHU, & R. A. HUTNER. *Elliptic Cylinder and Spheroidal Wave Functions*. New York, 1941. See *MTAC*, v. 1, p. 157–160.

[4] *MTAC*, v. 3, 1948, p. 99–101; review of items 1 and 2.

[5] The values corresponding to $\gamma^2 = 15, 20, 25, 50$, and 100 are the only ones not reproduced in 2.

683[M].—ERIK HALLÉN, *Iterated Sine and Cosine Integrals*, Roy. Inst. Techn., Stockholm, *Trans.*, no. 12, 1947, 6 p. 17.6 × 24.7 cm.

In antenna theory there are some integral functions which might also be of interest elsewhere. They derive from sine and cosine integrals in the same manner as these derive from sine and cosine. The following are tabulated:

$$L_{11}(x) = C_{11}(x) + iS_{11}(x) = \int_0^x t^{-1}L(t)dt,$$

$$L_{21}(x) = C_{21}(x) + iS_{21}(x) = \int_0^x t^{-1}e^{it}[L(2t) - L(t)]dt,$$

$x = [0(.2)14; 5D]$, where $L(x) = C(x) + iS(x) = \int_0^x t^{-1}(1 - e^{-it})dt$, $C(x) = \gamma + \ln x - Ci(x)$ and $S(x) = Si(x)$. There is also a table of $L_{11}(x)$, for $x = [14.2(.2)28; 5D]$. The tables were computed by mechanical integration. The fifth decimal is uncertain within two units.

Extracts from text

EDITORIAL NOTE: For a less extended 6D table of L_{11}, by C. J. BOUWKAMP, see *MTAC*, v. 3, p. 303–304.

684[M].—J. LE CAINE, *A Table of Integrals involving the functions* $E_n(x) = \int_1^\infty e^{-xu}u^{-n}du$. National Res. Council, Div. of Atomic Energy, *Report* no. MT 131. 1948, i, 45 leaves. 20.3 × 27.4 cm.

The definite and indefinite integrals involve products of two or more factors $E_n(kx)$, $E_n(ax + b)$, $E_n(k|x - y|)$, e^{kx}, $e^{-k|x-y|}x^m$. In the single integrations x is the integration variable, and x and y in the double integrations. For numerical tables of such integrals see *MTAC*, v. 2, p. 260, 272 and v. 3, p. 303–304, and RMT **683**.

R. C. A.

685[M, Q].—H. C. VAN DE HULST, "Scattering in a planetary atmosphere," *Astrophys. Jn.*, v. 107, 1948, p. 220–246. 16.9 × 24 cm.

In this paper the problem of multiple scattering in a planetary atmosphere, both with and without a diffusely reflecting bottom surface, is discussed. We assume that the atmospheric scattering is isotropic with an albedo a, and that the ground surface reflects the radiation according to Lambert's law, with an albedo b. The mathematical functions occurring in the calculations are of three types. **T. 1**: for $E_n(x) = \int_1^\infty t^{-n}e^{-xt}dt$, $E_1^{(2)}(x) = \int_x^\infty t^{-1}E_1 dt$ the functions tabulated are $E_1(x)$, $E_1^{(2)}(x)$, $E_1^{(0)}(x) = e^{-x}$, $E_2(x)$, and also $\ln x + \gamma$ (γ being Euler's constant) are each tabulated to 6D; and $2E_3(x)$ to 5D. **T. 2–3**: For $F_n(b, x) = \int_0^x e^{bt}E_n(t)dt$, there are 4D tables for $F_n(-s, x)$, $s = 10, 5, 2, 1$; and of $e^{-sx}F_n(s, x)$, $s = 0, 1, 2, 5, 10$, in each case $n = 1, 2$. **T. 4**: For $G_{mn}(x) = \int_0^x E_m(t)E_n(t)dt$, $G'_{mn}(x) = \int_0^x E_m(t)E_n(x - t)dt$, there are 5D values for $m, n = 1, 2$; also there are 4D values for

$g(x) = 2 - 2E_2(x) - G_{12}(x) - G_{12}'(x)$ and $c(x) = [x - \frac{1}{2} + E_3(x)]/x$. In each of the tables $x = .01, .02, .05, .1(.1).4(.2)1(.5)3, \infty$.

Extracts from text

EDITORIAL NOTE: For considerably more extensive tables of $E_n(x)$ see *MTAC*, v. 2, p. 272.

686[N].—FINANCIAL PUBLISHING COMPANY, **A.** *Eighth Coupon Rate Monthly Bond Values showing accurate monthly values on bonds and other redeemable securities paying interest semi-annually. For eighth coupon rates only from* $1\frac{1}{8}\%$ *to* $3\frac{7}{8}\%$. Boston, Financial Publishing Co.; London, Routledge & Kegan Paul Ltd., 1949 [the printing and binding were not completed until Feb. 1949], x, 537 p. 25 × 36.8 cm. Bound in red imitation leather, thumb index. $125.00. In the back of the volume is a loose, slightly smaller (24.5 × 36 cm.) paper covered reprint, 48 p., dated 1948, from the second edition (p. 1109–1154, 0% coupon) of *Monthly Bond Values*, 1941, and entitled: *Tables of Present Worth* of 100, *from* 1 *month to* 110 *years*.
B. *Executive Bond Values Tables. Desk Edition, Showing net returns on Bonds and other redeemable Securities paying Interest semi-annually.* Boston, Financial Publishing Co.; London, George Routledge & Sons, Ltd., 1947, 1984 p. 11 × 18.6 cm. Bound in leather, full gilt, thumb index. $25.00.

A. In the 8 years since the second edition of *Monthly Bond Values* was published (see *MTAC*, v. 1, p. 114–115), continuing low interest rates and the development of sale at competitive bidding of corporate bonds have produced a number of issues with $\frac{1}{8}\%$ coupons which were not covered by complete tables in the second edition. Hence the need for this new volume which is essentially supplemental to the second edition.

The 12 coupon rates here are $1\frac{1}{8}\%(\frac{1}{4}\%)3\frac{7}{8}\%$. The yields are from 0 to 5.70%, at interval .05%, as before. Maturities, as in the earlier volume, progress by monthly intervals up to 40 years 3 months, from $40\frac{1}{2}$ years to 50 years semi-annually, 50 years to 65 years annually, and 70 years to 110 years by 5 years. All tabulation is to 6D, adjusted from at least 8D computation.

As formerly, on every page are 12 columns (one for each month) and 115 lines corresponding to 0 (.05%) 5.70%, the various yields (in black face type on each side of the page), making 1380 entries all arranged in groups of five, admirably spaced for ready use. Also in black face type are given under the appropriate columns the amounts of accrued interest on $100. for 1, 2, 3, 4, 5 months. Pages 536–537 contain a 6D table of accrued interest on $100. for any number of days 1(1)180, in 6 months, for each of the coupon rates. An 8D "Table of factors" is provided (p. 532–534) for computing values for 1 to 30 days, between the even months, for each of the yields.

In the foreword to the volume are explained various methods of using and applying the tables which are largely a reprint of the second edition. Part of what was said in review of the earlier volume is applicable to the present book, and the comments made at that time apply also to this volume.

The manner of preparing the tables was almost wholly dictated by trade practice and by what would be most intelligible to bond dealers. The method of computation involved the addition of the constant difference between values at successive coupon rates. This constant difference was computed to a minimum of two decimal places beyond the six decimal places contained in the table, and in most cases to more than two extra decimals. The mathematician notes with interest that trade practice calls for the addition to every invoice of simple interest at the coupon rate. Hence it became necessary to compute the theoretically correct "flat" value and from this to subtract simple accrued interest. Thus when an invoice is made for bonds sold at a yield basis the sum which changes hands will be the theoretically correct amount, but it will be itemized in two distinct elements neither of which may be considered mathematically exact. This feature is most readily observed in the case of any bond selling to yield its coupon rate. The value is shown as 100 on coupon dates only, and for intermediate dates at something less than 100; but if the accrued interest at the bottom of the column be added a correct result will be obtained.

Another curious feature is that in spite of solicitude for correct semi-annual compound interest in the body of the book, values for less than six months' maturity are shown at true discount, in accord with trade custom of dealers and banks. Again, the editors believed that labelling a present-worth table as the value of a 0% bond evokes a clearer concept to the average bond dealer, than a proper labelling would.

Without a volume of this character, dealing in municipal securities would be a somewhat difficult process. For example, a dealer offers a $3\frac{1}{2}$% coupon municipal having 20 years and 4 months to run on a 1.95% yield basis. What dollar price must the purchaser pay? Turning to the $3\frac{1}{2}$% coupon table, find the page "20 years, column 4 months, and on the margin the yield 1.95%," and the price is given as 125.913848 or $1259.14 per $1000 bond. Also at the bottom of the column is given the accrued interest in monthly intervals, $.583333 per $100 or $5.83 per $1000 bond. The methods of adjusting for fractional parts of a month, and for many other special cases of practice, are explained clearly in the introductory pages. These pages contain also quite explicit instructions involving the use of adding or accounting machines where these are available.

These 1941 and 1949 tables are the most extensive of their kind, and are of value particularly to dealers and those handling state and municipal securities where price is usually quoted on a yield basis, also for insurance companies where exact computations of yield are essential. The tables are obviously of importance also to the actuary or accountant in making up amortization schedules.

B. Since the 1941 and 1949 volumes are too large and bulky for use outside an office, two small volumes have earlier found considerable favor. One of these, *Comprehensive Bond Values*, 1936, had the advantage of containing tables for odd eighth coupons, now being increasingly used. The other, *Pocket Edition of Monthly Bond Values*, 1939, had the advantage of monthly maturity intervals but with no eighth coupons. The volume under review contains a combination of the material in these volumes.

It covers coupon rates $\frac{1}{2}$%($\frac{1}{8}$%)4%($\frac{1}{4}$%)5%. The maturities for $\frac{1}{4}$% coupons are monthly to 20 years, quarterly to 30 years, semi-annually to 50 years, annually to 56 years, bi-annually 56 years to 60 years, and at 5 year intervals from 60 years to 100 years. On the $\frac{1}{8}$% coupons maturities are shown monthly to 10 years, semi-annually to 42 years, annually to 56 years, bi-annually to 60 years, and at 5 year intervals to 100 years. Coupon rates for $\frac{1}{2}$% and $\frac{3}{4}$% stop at 28 years. For $\frac{5}{8}$%, $\frac{7}{8}$%, $1\frac{1}{8}$% and $1\frac{3}{8}$% they stop at 30 years. The tables are in condensed forms and show values to 2D only.

In the foreword to the book are descriptions of methods of use of the tables and at the end are tables of factors to be used on callable bonds and tables of current yields on bonds or stocks, given the price and coupon or dividend rate. There are also included tables for accrued interest on a 365 day basis from $\frac{1}{2}$% ($\frac{1}{8}$%) 4% ($\frac{1}{4}$%) 6% for use with government securities. The volume is very valuable for the convenience of salesmen and others in determining quick approximate figures.

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687[R].—ITALY, ISTITUTO GEOGRAFICO MILITARE, Florence, A. [GIOVANNI BOAGA], *Sulla Rappresentazione Conforme di Gauss*, 1941, ii, 65 p. B. [GIOVANNI BOAGA], *Sulla Rappresentazione Conforme di Gauss, Allegati*, (i) *Parte I, Tavole Numerici*, 1942, vii, 25 p.; (ii) *Parte II, Tavole Ausiliarie . . .*, 1942, xiv, 49 + *Errata Corrige* 4 p.; (iii) *Parte III, Applicazioni*, 1942, 38 p. 15.5 × 21.6 cm.

This work treats the conformal representation of the earth considered as an ellipsoid of revolution onto the Euclidean plane. The tabular material is for that part of the representation which lies between latitudes 36° and 48°, the range in which Italy lies. The main treatise gives a theoretical development of the representation with special attention to the preparation of formulae well adapted to calculation.

Let λ be the longitude in seconds of a given point P on the ellipsoid, measured from a convenient meridian, and let B be the length of arc on the meridian from the equator to a reference point. Next let x, y be the rectangular coordinates of the point P' in the plane corresponding to the point P. Then one has approximately

$$\log (x - B) = 2 \log \lambda + T_0 + C_2\lambda^2$$

$$\log y = \log \lambda + T_1 + C_3\lambda^2$$

where the quantities T_0, T_1, C_2, C_3 depend on the latitude of the point P. This dependence is tabulated in Part I. More precisely the quantities B, T_0 and T_1 and their differences per second together with C_2, C_3, and their logarithms are given for every minute of latitude between 36° and 48°. This table is based on the HAYFORD ellipsoid. B is given to a millimeter, the T's to 7D and the C's to 4S or 5S.

Part II is based on the BESSEL ellipsoid and gives slightly different values for the above quantities. Both Parts I, II give three other coefficients used to compute the "convergence of the meridian," the "modulus of linear deformation" and "angular reduction." These are also tabulated for each minute of latitude. The use of the tables is illustrated in Part III.

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688[U].—GEORGE G. HOEHNE, *Practical Celestial Air Navigation Tables. Volume II, Latitudes 20° to 39° North inclusive.* Miami, The Navigation Publishing Co., 1943, 256 p. 15 × 24.3 cm. $3.65.

This volume was reviewed earlier in *MTAC* v. 2, p. 129–131; the copyright difficulties which formerly prevented its sale to the public have been taken care of and it is now generally available at the price indicated above. In this connection, it may be pointed out that many of the altitudes in the volume were taken directly from the British ANT (or our H. O. 218 which was a direct copy of it); during the war, both of these sets of tables were restricted. With the closing of the war, these restrictions were lifted and the way was open for arrangements to meet British Crown copyright requirements.

It is hoped that another volume, similar to this but covering latitudes 40°–59° North, will soon be available.

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689[U].—O. A. DE AZEREDO RODRIGUES, *Gráficos Usados na Navegação. D. N. 7-1.* Rio de Janeiro, Diretoria de Hidrografia e Navegação, 1948, 48 p. 16.5 × 24.5 cm. 35 cruzeiros.

This volume contains twenty nomograms useful in the ordinary computations of navigation. The author is a Capitão-de-Corveta in the Brazilian Navy; he already has one other navigation table to his credit, *Tábuas para Retas de Altura* (RMT **516**, *MTAC*, v. 3, p. 114–115). The nomograms are well designed for practical use and are clearly printed on good heavy white paper. The volume is bound in light green cardboard and presents a neat appearance.

Following a brief preface and a list of abbreviations, there is an index. The remainder of the volume is given over to the nomograms and to explanations of the use of a few of the more complicated ones. For each one except the last, one or more examples are given with the correct answers.

The first nomogram provides the difference of latitude, dL, and departure, P, for a rhumb line, given the distance, $D = 0(1')60'(2')150'(5')220'$, and the course angle, $C = (0\text{–}90°)$. The corresponding equations are $dL = D \cos C$, $P = D \sin C$.

The second converts departure, $P = 0(1')100'$ into difference of longitude, $dLo = 0 - 220'$, given the mean latitude, $L_m = 0 - 90°$. $dLo = P \sec L_m$.

The third provides the distance, $D = 0 - 100$ nautical miles to an object of known elevation, $E = 0 - 2400$ meters, given the vertical angle $\alpha = 0(5')6°$ and the height of eye of the observer, 1–45 meters. A small table converts height of eye into dip of the horizon, $\beta = 2'(1')12'$. The basic equation is $D = kE \cot (\alpha - \beta)$ where k is the fraction of a nautical mile equal to one meter. The fourth yields the distance, $D = 100 - 3500$ meters, given the elevation $E = 0(1)52$ meters of an object and the vertical angle $\alpha = 0°30'(5')3°30'$ subtended by it. $D = E \cot \alpha$.

The fifth gives the distance, $D = 3 - 1000$ meters travelled in $S = 1 - 60$ seconds at a velocity, $v = 4(0.5)36$ knots. $D = kvs$ where $k =$ number of meters per second in one knot.

The sixth yields the distance $D = 12$ to 880 nautical miles travelled in $H = 2^h(10^m)$-$6^h(20^m)24^h$ at a speed $V = 6(0.5)36$ knots. $D = VH$.

Number 7 provides the correction $\gamma = 0(0°.1)13°.5$ to be applied to a radio bearing before plotting it on a Mercator map, having given the difference in longitude $dLo = 0(0°.5)31°$ and the mean latitude $L_m = 0(0°.5)65°$. The corresponding equation is $\gamma = \frac{1}{2}dLo \sin L_m$.

The eighth nomogram is somewhat larger than the remaining charts; it requires two facing pages, which are in reality a single sheet folded in the middle. Even though it has been placed in the middle of the book and special care has been taken in binding it, the crease will probably cause some trouble until the volume is limp from use. This nomogram allows one to find the azimuth angle $A = 16°$ to $164°$ of a celestial body when the local hour angle $t = 5°$ to $170°$, the declination $d = 75°$ same name $-25°$ opposite name to latitude and the latitude $L = 0$ to $70°$ are given. The equation solved by the nomogram is

$$\cot A \sec L = \tan d \csc t - \tan L \cot t.$$

The PAGEL coefficient, $\cot A \sec L$, may thus be evaluated with this nomogram.

Number 9 gives the azimuth angle $A = 0 - 180°$ as a function of local hour angle, $t = 0$ to $180°$, declination, $d = 0(1°)90°$, and altitude $h = 0(1°)90°$. The equation here is $\csc A = \csc t \sec d \cos a$.

Nomograms 10 and 11 provide respectively the altitude, $h = 0(1°)90°$ and local hour angle $t = 0$ to $90°$ of a celestial body when it crosses the prime vertical. The arguments are in both cases latitude and declination; in the altitude nomogram, $L = 1°$ to $90°$, $d = 1°$ to $90°$ and in the local hour angle chart, $L = 0(1°)90°$ and $d = 0(1°)60°$. The equations are $\csc h = \csc d \sin L$ and $\sec t = \cot d \tan L$.

Nomograms 12 and 13 give respectively the altitude, $h = 0(1°)90°$ and the local hour angle $t = 10°(1°)90°$ of a body at maximum azimuth angle. The arguments in the former nomogram are $L = 1°(1°)90°$ and $d = 1°(1°)90°$; in the latter, they are $L = 0(1°)60°$ and $d = 0(1°)90°$. The equations are $\csc h = \csc L \sin d$ and $\sec t = \tan d \cot L$.

Nomograms 14 and 15 yield the azimuth angle $A = 0 - 180°$ and local hour angle $t = 5°$ to $175°$ of a body which is rising (or setting). The arguments are respectively $L = 0$ to $89°$, $d = 1°$ to $90°$ and $L = 0(1°)90°$, $d = 0(1°)60°$. The equations are $\sec A = \csc d \cos L$ and $\sec t = \cot d \cot L$.

Nomogram 16 provides the correction $c = 0(1')80'$ to be applied to a circum-meridian altitude, given the local hour angle $t = 0(15')17°30'$ and the declination $d = 60°$ same name to $30°$ opposite name h, and latitude L with the same range as d.

Nomogram 17 gives the change in altitude Δh per minute of arc change in local hour angle Δt. With $L = 0$ to $90°$ and azimuth angle 0 to $90°$, $\Delta h/\Delta t = 0(0'.05)1'$. The equation used is $\Delta h/\Delta t = \sin A \cos L$.

Nomogram 18 yields the change in altitude $\Delta h = 0(1')30'$ corresponding to a change in latitude $\Delta L = 0(1')30'$ and an azimuth angle of 0 to $180°$. $\Delta h = \Delta L \cos A$.

Nomogram 19 is a simple nomogram for multiplication and division. Chart no. 20 is intended for plotting lines of position and obtaining fixes. On it the scales of distance and latitude vary with the mean latitude, while the scale of longitude is constant. This is in contrast to the usual charts of this type in which the scale of longitude varies and the other two scales are equal and constant; there is much to recommend this approach.

It should be pointed out that where the smallest division of a scale in the nomogram is uniform, the interval of the corresponding argument is indicated; otherwise the range of values covered by the scale is noted. It will be clear that the accuracy of a given nomogram may often be increased by interpolating between the marks provided.

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690[U].—D. H. SADLER, "Tables for astronomical polar navigation," Institute of Navigation, *Jn.*, v. 2, 1949, p. 9–24. 15.5 × 24.9 cm.

Summary: The special features of polar navigation are examined with a view to the design of tables for astronomical navigation in polar regions. The use of long intercepts and curved position lines is thoroughly investigated. It is shown that considerable economy of presentation can be achieved; as an example, a one-page table covering all bodies and both polar caps is given. Making the fullest use of polar astronomy, samples are given of permanent tables for the sun and stars independent of the *Air Almanac.*

MATHEMATICAL TABLES—ERRATA

References to Errata have been made in RMT **659** (Dale), **662** (Briggs), **665** (Boll), **666** (Lehmer), **672** (Feller), **676** (Huckel), **678** (Magnus & Oberhettinger), **682** (Bouwkamp, Meixner), **683** (Hallén); N **106** (Watson), **107** (FMR, Gauss), **109** (DeMorgan, Rheticus).

160.—D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, 1941.

P. 14, table, col. 3, the second and third entries from the bottom should be interchanged.

P. 15, l. 9, *for* Creak 1, *read* Creak 2.

P. 47, l. −9, *for* 195 [twice], *read* 295.

P. 74, l. 18, *for* $29 \leq p$, *read* $17 \leq p$.

P. 76, l. 7, *for* VORONOˇ *read* VORONOĬ.

P. 95, l. 17, *for Haupt Exponents, read Haupt-Exponents.*

P. 102, Glaisher 16, l. 2, *for* 127, *read* 125.

P. 109, l. 10–11, *for milia accuratis, read millia accuratius.*

P. 114, in reference Ostrogradsky 1 *for* v. 1, 1838, *read* v. 3, 1836.

P. 129, the error for $n = 1019681$, occurs in Burckhardt 1, *not* 2. The error for $n = 2012603$ occurs in Burckhardt 2, *not* 3. *Delete* l. 4 (446021) and 5 (446023) since these errata were listed by Burckhardt himself, v. 1, p. IV.

P. 134, l. −6, *for* 61·330413, *read* 1399·14407.

P. 134, Cunningham 28, p. 243, col. 11, l. 18, *read* 9683, *instead of* 4683; p. 244, in tables giving elements y, of primes $(y^2 + 1)/13$, omit the entry 671, and *for* 3930 in col. 4, l. 6, *read* 2930.

P. 139, Euler 2_3, 193 factor of $81^2 + 1$ *not* of $82^2 + 1$; $1068^2 + 1 = 5^6 \cdot 73$, *not* $5^5 \cdot 73$; 773 factor of $1090^2 + 1$, *not* of $1080^2 + 1$.

P. 156, Kraitchik 4[f_1] *for* $p = 116537$, *read* 116337.

P. 174–6, the following items in the index are slightly out of alphabetical order: Forms, quartic; Ince; Logarithms; Pierce; Sum of powers.

A few careful readers have pointed out the carelessness of the first definition of the function $\psi(n)$ on p. 7. This function has only half the value attributed to it by this definition whenever n is divisible by 8.

D. H. L.

161.—D. N. LEHMER, "A complete census of 4 × 4 magic squares," Amer. Math. Soc., *Bull.*, v. 39, 1933, p. 764–767.

This paper contains a list, p. 767, of the number of normalized squares with various first columns and last rows.