AUTOMATIC COMPUTING MACHINERY

Edited by the Staff of the Machine Development Laboratory of the National Bureau of Standards. Correspondence regarding the Section should be directed to Dr. E. W. Cannon, 418 South Building, National Bureau of Standards, Washington 25, D. C.

Technical Developments

Magnetic Drum Storage for Digital Information Processing Systems

Introduction.—Automatic digital computers belong to a class of devices which may be described by the term "information processing systems." Some further examples of information processing systems (hereinafter abbreviated IPS) are statistical analysis machines, airline reservation tallying systems, airport traffic control systems, and inventory record systems. A requisite component of every such device is a storage section which serves as a repository for a number of separate items of information. These items, which we shall call "words," may be numerical quantities, alphabetical material, machine instruction codes, or combinations of these.

In many applications it is required to be able to refer at random to any word in storage. Each position in storage which may be occupied by a word is therefore designated by a number called an "address." This type of information store is analogous to a function table, since to each value of the argument, or address, there corresponds a single function value, or stored quantity. In such systems it is necessary to be able to read the word stored at a given position an indefinite number of times without deterioration of that word or of its neighbors. It is also generally necessary to be able to replace or alter the word at a given storage position without disturbing the contents of neighboring positions. Alterable information storage may be physically realized in a number of different ways. Well-known examples of these are electromechanical relays and stepping switches, acoustic delay lines, electrostatic storage tubes, and various magnetic recording devices.

The choice of storage media to suit a given application is governed by several considerations. One of these is the degree of physical stability required of the stored data. In certain applications it is necessary to retain stored information for extended periods, perhaps for weeks at a time. Under such circumstances it is a distinct advantage if the retention of data in storage does not depend on or require the continued operation of electric circuits. For if the stored information is not "volatile," it is possible to shut down the equipment for overnight periods, or for the purpose of maintenance, without loss of data. The *combined* properties of alterability and non-volatility are exhibited by very few of the known physical means of storage. Among these few are stepping switches, latching relays and signals recorded on magnetizable media.

A second consideration governing the choice of storage media is the required capacity, a quantity usually expressed as the number of binary digits or bits of information to be stored. Where large storage is required, the bulky relays and stepping switches present a serious space problem. A two-position relay is capable of storing only a single binary digit of information, while an n-position stepping switch stores only $\log_2 n$ binary digits of information.

With magnetically recorded signals, on the other hand, a large quantity of information may be stored in relatively compact form, as will be shown.

A third consideration is the permissible "access time" or maximum waiting time which may be tolerated in searching for a given storage position and reading or altering the word stored there. Many applications require quick access to any position in the storage. One practical way to satisfy the need for quick random access is to scan the entire mass of stored data continuously at a rapid repetition rate. In the storage system to be described information coded in terms of the binary digits 1 and 0 is recorded on a magnetizable medium on the surface of a continuously rotating cylindrical drum. The small magnetized areas corresponding to individual digits are arranged in parallel peripheral tracks about the drum. Near each track is a single stationary magnetic head for reading and writing the digits in that track. Once in every revolution every magnetized area on the drum is thereby accessible for the effectively instantaneous operations of reading or writing. The maximum access time in this instance is equal to the rotation period of the drum.

The information is recorded in binary coded form so that it is necessary to distinguish between only two magnetic states for each elemental area. There is no need for over-all linearity of the recording and reproducing processes. Specifically, these two magnetic states correspond to positive and negative magnetization of the medium in a direction parallel to its motion. The binary coding requirement imposes no limitation on what may be stored, since information of any kind is readily expressed in a "1 − 0" or "on-off" code. Thus, decimal digits may be recorded as 4-digit binary code groups, and alphabetical characters may be recorded as 5- or 6-digit binary groups. This technique is commonly used in telegraphic systems and in electrical computing devices.

It is the purpose of the present paper to describe a practical method for the alterable, nonvolatile storage of information. For applications requiring these properties, the magnetic drum storage system provides what is felt to be a reasonable balance of the factors: access time, storage capacity, and bulk and cost of equipment.

Utilization of the Storage Surface.—Each word appears on the drum surface in parallel rather than in serial fashion. That is, each binary digit of a 30-digit word, for example, is represented by a single elemental magnetized area, or "cell," in each of 30 separate tracks, rather than by 30 cells in one track. As the drum rotates, the 30 digital cells representing one word pass simultaneously under the magnetic heads in their respective tracks. If each track should contain 4000 digital cells around its circumference, then a group of 30 tracks would store 4000 30-digit words. Several such groups of tracks may be needed to provide the required storage capacity.

The number of elemental areas per track and the number of groups of tracks on a drum are determined by the storage requirements of the application. Suppose that it is desired to store W words of b binary digits each, with maximum access time of T milliseconds. Let R represent the standard scanning rate, i.e., the number of digital cells passing a given magnetic head in a millisecond. Since there is a single read-write station per track, the drum rotation period may be made equal to T. Each head then scans RT digital cells in a revolution. In other words, each track stores RT binary digits. A group of b tracks stores RT words. To provide storage for W words, W/RT such groups of tracks are needed.

For economy of equipment and space it is desirable that the number of binary digits of stored information under the control of each magnetic head be made as large as practicable. Since each track stores RT digits, this calls for a large value of the scanning rate, R. The scanning rate is equal to the product of the drum surface velocity and the number of digital cells per unit length of track. The values at which these quantities have been standardized are 1600 inches per second and 80 digital cells per inch, respectively, corresponding to a value of 128 digital cells per millisecond for R. These are conservative design constants which have been found entirely adequate for reliable discernment of the value of every stored binary digit.

There are eight tracks per axial inch along the drum. Since 80 binary digits are stored in each peripheral inch of track, the storage capacity of the drum surface is 640 digits per square inch. In other words, each binary digit is allocated a rectangular zone having effective dimensions of 0.125 inch parallel to the drum axis and 0.0125 inch peripherally, or perpendicular to the drum axis. This rectangular zone constitutes a digital cell. Whether the stored digit is a 1 or a 0 is established by the magnetic orientation or polarity of a slightly smaller region within this zone. The magnetic polarity of the surrounding area corresponds to the convention chosen to represent 0. A plot of the magnetic intensity along the center of a peripheral track will disclose regions oriented positively and negatively in a direction parallel to the track. If the positive polarity represents 1 and the negative polarity 0, a series of 0's would be characterized by uniform magnetization along the track. A series of 1's, on the other hand, would show up as a series of spots of positive polarity separated by small regions of negative polarity.

Magnetic Heads and Drum Surface Coating.—The magnetic head is a specially designed form of electromagnet with an elongated ring-shaped core. The core has a fine gap on the side adjacent to the drum surface. To write, a winding on the core is energized with a brief pulse of current. The minute area of drum surface under the gap at that instant is magnetized in a direction determined by the polarity of the current and the sense of the winding. The same head serves for reading. As successive digital cells pass under the gap, characteristic signal voltages are induced in a pickup winding on the core. These signals have amplitudes on the order of tenths of a volt. The reading operation does not disturb the stored data in any way.

Although the tracks are spaced eight to the inch along the drum, each magnetic head in its mounting assembly occupies a circular area approximately one inch in diameter, projected on the drum surface. For this reason all of the heads are not placed in a single line parallel to the drum axis but are staggered in position.

The magnetizable medium on the drum surface is a smooth, sprayed-on coating of magnetic iron oxide, the same material which is used on magnetic sound recording tape. This surface is protected by a thin over-coating of a hard lacquer.

Each digital cell passes under its magnetic head many times per second at a 90 mile per hour relative speed. These conditions preclude the use of the contact technique commonly employed in recording on magnetic tape. A clearance of 0.002 inch is therefore maintained between the magnetic head and the drum surface. This noncontact clearance is the principal factor limiting the number of reliably resolvable digital cells per inch of track.

Functional Description of System.—The functional block diagram of a magnetic drum storage system is shown in Figure 1. The dotted boundary surrounds those units which would be considered part of the storage section of an IPS. The channels by which the storage section communicates with other sections of the IPS are shown along the lower edge of the boundary.

The external functions of the storage section are simple. If a word is to be written into storage, the information which must be transmitted to the storage section consists of: (1) the address of the desired storage position; (2) the word to be written; and (3) a control signal specifying that the operation is to "write." If the word occupying a given storage position is to be read, the required information consists of: (1) the address of the desired position; and (2) a control signal specifying that the word at that position is to be "read" to one of several possible destinations (buses to two destinations are shown).

The units with which the storage section communicates are determined by the nature of the IPS. In a computer, for example, the address and the control signals originate in the central program control of the computer. The word to be written may come from the arithmetic section. The destinations for words read out of storage may be the program control section, the arithmetic section, or a printing device.

The storage section communicates internally and externally on a parallel channel basis, in that the several binary digits of a word are transmitted at one time over as many electrical channels. Heavy lines in Figure 1 represent multi-channel buses for the transmission of words or addresses. Light lines represent single or multiple channels for control information. In each external channel, the presence of a pulse indicates a 1 and the absence of a pulse, a 0. The channels within the storage section carry more specialized forms of signals, such as d-c potentials, for example.

The word to be written and the address of the desired storage position are held, until completion of the operation, in the Insertion Register and the Address Register, respectively. These registers consist of toggle-circuits, or static flip-flops. A toggle-circuit is an electron tube circuit having two symmetrical stable states so that it is capable of holding a single binary digit of information.

Upon completion of the specified writing or reading operation, a control signal announcing completion is sent out by the storage section. At the same time, the Address and Insertion Registers are cleared, so that the storage section is then receptive to further assignment.

In the interest of clarity, the operation of the system will be described in terms of an example having a specific set of storage characteristics: (1) word size, b: 30 binary digits; (2) capacity in words, W: 8192 or 2^{13}; and (3) maximum access time, T: 16 milliseconds. The number of words which can be stored in each group of 30 tracks is equal to RT, or 2048 (128 times 16). Four track groups must therefore be provided, plus several additional tracks for location and timing purposes. These are indicated in Figure 1.

The 8192 storage positions are designated by 8192 addresses. While the addresses may be *any* set of 8192 distinct binary coded designations, that set which consists of all the 13-digit binary numbers is the least redundant and most economical of equipment.

The 13-digit address is composed of two parts, a 2-digit "group index" and an 11-digit "angular index." The group index specifies one of the four, or 2^2, groups of tracks. The angular index specifies one of 2048, or 2^{11}, angular positions of the drum.

In addition to the 120 storage tracks, there are 11 angular index tracks and one timing track. These tracks contain permanently recorded information. The angular index tracks contain the 2048 11-digit angular indices. The timing track serves as a source of timing pulses, for precisely marking the instant at which the drum passes through each of its 2048 discrete angular positions. One of these timing pulses, selected on the basis of the desired angular index, denotes the instant at which the desired storage position is available for reading or writing.

Time-selection is performed by an 11-fold coincidence detector which continuously compares the desired 11-digit angular index in the Address Register with the outputs of the circuits which read the angular index tracks. As long as the scanned angular indices do not match the desired angular index, timing pulses cannot get through the coincidence detector. When the drum passes through the angular position at which a match occurs, a single time pulse is delivered to the storage control circuits for triggering of the appropriate writing or reading operation.

The function of the Writing Circuits is to replace the word at the specified storage position with the word standing in the Insertion Register. There is a Writing Circuit associated with each of the 120 storage track magnetic heads. A Writing Circuit contains two miniature thyratrons, each of which can discharge a simple network through a winding on the magnetic head. The 30 pairs of thyratrons in the selected group are simultaneously triggered by a pulse from the storage control circuits, but only one thyratron in each pair fires, one to write a 1, the other to write a 0. One of the thyratrons is prevented from firing by application of a negative bias to its shield grid. The choice of 1 or 0 is determined by the value of the corresponding digit in the Insertion Register.

It should be noted that there is no need for "erasure," as such, since the operation of writing a word into a storage position substitutes the new word for the previous contents of that position. The word stored at a given position is simply the one that was written there last.

The use of thyratrons instead of "hard" vacuum tubes in the Writing Circuits effects a significant saving in the number of tubes. The time which must elapse between successive writing operations is admittedly longer for thyratrons. However, the duty-cycle required of the writing operation is generally so low that this limitation is of little consequence.

The reading operation consists in transmitting to the specified destination the word stored at the position specified by the address in the Address Register. The units which participate in reading are indicated in Figure 1 as Reading Gates, Reading Circuits, and Output Gates.

Track group selection is accomplished in the Reading Gates. These are preamplifiers which are either blocked or operative, as determined by control voltages. Each of the 120 Reading Gates receives signals from its associated magnetic head, but only the selected group of gates transmits signals to the Reading Circuits.

The Reading Circuits are 30 in number and consist of amplifiers and wave-form shaping circuits. These operate continuously on the signals originating in the selected group of tracks.

The amplified signals from the Reading Circuits are impressed on two sets of Output Gates, one for each destination. At the instant denoted by the selected time pulse, the appropriate set of Output Gates is pulsed. This operation, by sampling the signal stream from the Reading Circuits at the correct time, transmits the desired word to the specified destination.

The Storage Control Circuits consist of electronic switching and gating circuits for translating the group index code, the selected time pulse, and the external control signals into the appropriate group, time, and destination signals. The Storage Control Circuits also include automatic lockout delays which prevent a storage reference operation from following a previous one too closely to permit complete circuit recovery. These delays are of the order of 50 microseconds, except in the special case of a writing operation which follows a previous writing operation. In this case, the second writing operation must not take place until about 2 milliseconds after the first. If a storage reference operation is initiated too soon after a previous one and the desired angular index comes up before the lockout delays have cleared, the effect is simply to delay execution of the operation for one drum revolution.

TABLE I

Characteristics of 36 Magnetic Drum Storage Systems

CAPACITY (Wb), BINARY DIGITS	MAX ACCESS TIME (T) MILLISECONDS	DRUM DIMENSIONS, INCHES		MAGNETIC HEADS	WORD SIZE, b = 15 BINARY DIGITS				WORD SIZE, b = 30 BINARY DIGITS				WORD SIZE, b = 60 BINARY DIGITS			
		DIAMETER	LENGTH		WORDS (W)	TRACK GROUPS (W/128T)	ELECTRON TUBES	TUBES PER 1000 DIGITS	WORDS	TRACK GROUPS	ELECTRON TUBES	TUBES PER 1000 DIGITS	WORDS	TRACK GROUPS	ELECTRON TUBES	TUBES PER 1000 DIGITS
61,440	8	4.3	10	71	4,096	4	500	8.1	2,048	2	630	10	1,024	1	910	15
122,880	8	4.3	18	131	8,192	8	690	5.6	4,096	4	820	6.7	2,048	2	1090	8.9
245,760	8	4.3	33	251	16,384	16	1080	4.4	8,192	8	1200	4.9	4,096	4	1460	5.9
122,880	16	8.5	10	72	8,192	4	510	4.2	4,096	2	640	5.2	2,048	1	920	7.5
•245,760	•16	•8.5	•18	•132	16,384	8	700	2.8	•8,192	•4	•830	•3.4	4,096	2	1110	4.5
491,520	16	8.5	33	252	32,768	16	1090	2.2	16,384	8	1210	2.5	8,192	4	1480	3.0
245,760	32	17	10	73	16,384	4	520	2.1	8,192	2	650	2.6	4,096	1	930	3.6
491,520	32	17	18	133	32,768	8	710	1.4	16,384	4	840	1.7	8,192	2	1120	2.3
983,040	32	17	33	253	65,536	16	1100	1.1	32,768	8	1230	1.3	16,384	4	1490	1.5
491,520	64	34	10	74	32,768	4	530	1.1	16,364	2	660	1.3	8,192	1	940	1.9
983,040	64	34	18	134	65,536	8	720	0.73	32,768	4	850	0.87	16,384	2	1130	1.2
1,966,080	64	34	33	254	131,072	16	1110	0.56	65,536	8	1240	0.63	32,768	4	1500	0.76

The timing pulses in the storage section need not be synchronized with the clock or timing pulses in other portions of the IPS, since all digital and control information transmitted to the storage section is received and temporarily held in toggle-circuits. Information transmitted from the storage section is received on a toggle-circuit or relay register at the destination, which provides similar buffer storage. The property of asynchronism obviates the need for precise control of the angular velocity of the drum.

Characteristics of Typical Systems.—The principal characteristics of 36 typical drum storage systems are listed in Table I. These examples are all similar to the one shown in the block diagram of Figure 1, with variations in access time, storage capacity, and word size. All designs are based on a common set of physical parameters: 128 digital cells scanned by each head per millisecond; 80 digital cells per inch of track; and 8 tracks per axial inch of drum.

Each of the 12 horizontal lines of the table corresponds to a drum of given diameter and length. Four values of diameter and three values of length are represented. The four diameters correspond to drum rotation periods or maximum access times of 8, 16, 32, and 64 milliseconds. The three lengths are for drums having 60, 120, and 240 storage tracks (in addition to angular index and timing tracks). Each line of the table contains characteristics of three systems corresponding to word sizes of 15, 30, and 60 binary digits. Characteristics of the particular example described in connection with Figure 1 are identified in the table by asterisks.

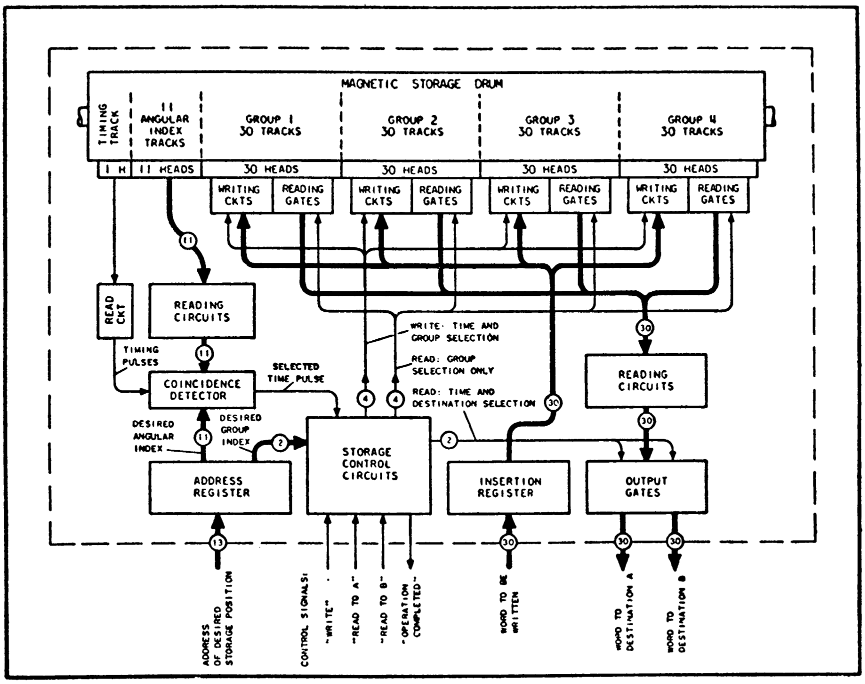

FIG. 1. Functional Block Diagram of Magnetic Drum Storage System.

The table contains information as to the number of magnetic heads and the number of electron tubes required for each example. It will be noted that the number of tubes is essentially constant for a given word size and drum length. Under these conditions the access time and storage capacity are both directly proportional to drum diameter.

Circuit cost per unit storage capacity may be expressed in terms of the number of tubes per thousand binary digits. This quantity is seen to be a decreasing function of access time and storage capacity, each considered independently, and a moderately increasing function of word size.

An idea of the space occupied by a given storage system may be gained from the tabulated data. The size of each drum is given in the table. The size of the cabinets needed to contain the electronic circuits may be estimated by allowing about one cubic foot for every 30 tubes.

Loading of Drum.—The contents of storage undergo numerous changes during the course of operation of an IPS. The entering of initial contents and the introduction of new data at occasional intervals is a function of the input section of the IPS. The choice of the input medium is governed largely by the application. Input data may be on magnetic tape or wire, punched cards, punched paper tape, or perhaps even introduced manually from a keyboard. The present storage system is capable of accepting successive items at rates up to about 500 *words* per second.

An input system using punched paper tape as the medium has been developed for use with a storage system similar to the example of Figure 1. The tape is scanned by a photoelectric reading device at a nominal speed of 75 feet per minute, corresponding to a storage insertion rate of 1800 30-digit words per minute. Even if it should be desired to load the entire drum, it would take only about five minutes to fill the 8192 storage positions. The magnetic drum rotates continuously at its normal speed during the loading operation. The tape feed need not be synchronized with drum rotation. Simple means are provided for loading sequences of data into any desired storage positions, in any order.

Some Possible Variations.—The described function table type of magnetic drum storage system embodies only the simplest and most straightforward features. Departures from these properties may be desirable to suit the needs of certain applications.

For example, it is possible to shorten the access time in a system of given capacity by assigning two or more magnetic heads to each track in place of one. Another way to shorten access time, but at the expense of storage capacity, is to repeat the stored information in several equal sectors about the drum. This method is useful only if reading is a more frequent operation than writing.

It is possible to add considerable flexibility to the manner in which stored items are located for reading out. If suitable coincidence detectors are provided, items written into storage in the standard way may subsequently be located on the basis of certain sets of digits *within* the stored words.

Although communication within the storage section is on a parallel-channel basis, the described system may readily be made part of an IPS operating serially, i.e., one in which the several digits of a word are transmitted sequentially over a single channel. This requires that the Address Register and the Insertion Register be endowed with the property of shifting. The incoming word then arrives digit by digit at one end of the receiving register. The register shifts its contents by one place upon arrival of each digit, until the complete word is assembled. Shifting registers must also be provided for transmitting words out of the storage section in serial fashion.

Status.—The developmental status of the magnetic drum storage technique at the time of this writing (May 1949) may be summarized as follows. A complete pilot model of the described function table type of system is undergoing final tests. Although this model is scaled down in capacity and word size, the standardized values of the physical parameters are used, and every basic system function is included. Tests of every operating function under every expected condition have been performed with a repeated reliability which confirms the adequacy of the selected design standards. Reliability of the basic circuits and of the magnetic and mechanical components has been further established in an extensive laboratory program of component research and in the development of other types of magnetic drum storage systems during the past two years.

ARNOLD A. COHEN

Engineering Research Associates, Inc.
St. Paul 4, Minnesota

DISCUSSIONS

Notes on Modern Numerical Analysis—I

EDITORIAL NOTE: There is a general feeling that, once the problems of construction and maintenance of automatic digital computing machines are solved, the remaining problems will be relatively simple. This may be the case if attention is confined to standard classical problems; however, if an attempt is made to use these machines fully, one is likely to encounter formidable mathematical difficulties. It is expected that these difficulties will be discussed in the current mathematical journals; but there are also smaller, more technical problems which may cause trouble. It is believed that a discussion of these smaller problems will prove beneficial in avoiding a great many difficulties which are expected to arise when the machines are in actual operation; and we should like to urge interested persons to submit technical notes of this nature for future publication in the Automatic Computing Machinery Section of *MTAC*. These notes could be by-products of or preliminaries to more constructive investigations. It would be a great advantage, for expository purposes, if the authors, even at the expense of a choice of an extravagant example, could exhibit the troubles under discussion on a manual scale.

Solution of Differential Equations by Recurrence Relations

1.1. In general the most satisfactory method for the numerical solution of ordinary differential equations is one of the "extrapolation" methods.[1] These methods have proven very efficient in the hands of a practiced computer. There is little doubt that some of the experience he uses could be codified and adapted for use on automatic digital computing machines. Nevertheless, the use of some direct recursive process is very attractive and worth investigation.

Let us consider the solution by such methods of the equation

$$y'' = -y, \tag{1}$$

with the boundary conditions $y(0) = 0$, $y'(0) = 1$, by use of the well-known formula [2]

$$h^2 y'' = (\delta^2 - \tfrac{1}{12}\delta^4 + \tfrac{1}{90}\delta^6 - \cdots)y. \tag{2}$$

1.2. First let $h = 1$, using only the first term on the right-hand side of (2). If the condition $y'(0) = 1$ is replaced by $y(1) = 1$ the following recurrence relation is obtained

$$y(n+2) = y(n+1) - y(n) \tag{3}$$

with the boundary conditions $y(0) = 0$, $y(1) = 1$. For $n = 0, 1, 2, 3, \cdots$,

$$y(n) = 0, 1, 1, 0, -1, -1, 0, 1, 1, 0, \cdots. \tag{4}$$

Two points are now worth noting. One way of introducing the circular functions analytically is to define $\sin x$ as the solution of (1); in this treatment π is defined as the least positive root of $\sin x = 0$. Observe that 3 has now been obtained as an approximation to π. Secondly, it is seen that the solution (4) is periodic.

1.3. By taking a smaller value of h, a possible improvement can be expected. If we take $h = 0.1$, the recurrence relation is now

$$y(n + 2) = 1.99y(n + 1) - y(n) \tag{5}$$

with $y(0) = 0$ and $y(1) = \sin 0.1 = 0.09983$. The solution obtained when five decimal places are used is given in column (5) of Table I and may be compared with the corresponding values of $\sin x$, to ten places, given in column (1). It will be seen that for $0 \leq x \leq 16$ the error is always negative and steadily increases in absolute value, being $-35 \cdot 10^{-5}$, when $x = 1.6$.

This solution is apparently not periodic, and we may inquire as to the existence of values of h (other than $h = 1$) for which the solution is periodic, i.e., for which the sequence of its values is periodic. It can be shown that the only values of h are $h = 2 \sin \pi/n$ for $n = 2, 3, \cdots$. When $h = 2 \sin \pi/n$, the period is n and the corresponding approximation to π is $n \sin \pi/n$ which is roughly $\pi[1 - (6n^2)^{-1}]$.

1.4. Let us next consider the possibility of improving the solution by using two terms on the right-hand side of (2). As previous experience has taught us the benefit of taking two further differences into account, it would seem plausible to expect a marked improvement. In fact, however, if we take $h = 0.1$ and work to ten places, using the natural boundary conditions, the solution of the corresponding difference equation $(\delta^2 - \frac{1}{12}\delta^4)y = -0.01y$, i.e.,

$$y(n + 4) = 16y(n + 3) - 29.88y(n + 2) + 16y(n + 1) - y(n) \tag{6}$$

rapidly diverges to $+\infty$, as is seen in column (6). The same behavior occurs if 9, 8, 7, or 6 places are used, but if 5 places are used, it will be found [see column (7)] that the solution rapidly tends to $-\infty$.

1.5. If use is made of the equation obtained by neglecting all but the first two terms on the right-hand side of (2) and substituting in (1), we find

$$(\delta^2 - \tfrac{1}{12}\delta^4)y = -h^2y. \tag{8}$$

If the term δ^4y on its left is replaced by its approximate value

$$-h^2\delta^2y \sim \delta^2(\delta^2y), \tag{9}$$

a three-term relation is obtained

$$(12 + h^2)\delta^2y = -12h^2y. \tag{10}$$

Using $h = 0.1$, the following equation replaces (6)

$$y(n + 2) = 1.99000\,83333y(n + 1) - y(n). \tag{11}$$

TABLE I

x	(1)	(5)	(6)	(7)	(11)
0	0.00000 00000	0.00000	0.00000 00000	0.00000	0.00000 00000
0.1	0.09983 34166	0.09983	0.09983 34166	0.09983	0.09983 34166
0.2	0.19866 93308	0.19866	0.19866 93308	0.19867	0.19866 93310
0.3	0.29552 02067	0.29550	0.29552 02067	0.29552	0.29552 02077
0.4	0.38941 83423	0.38939	0.38941 83685	0.38934	0.38941 83450
0.5	0.47942 55386	0.47939	0.47942 59960	0.47819	0.47942 55440
0.6	0.56464 24734	0.56460	0.56464 90616	0.54721	0.56464 24828
0.7	0.64421 76872	0.64416	0.64430 99144	0.40096	0.64421 77021
0.8	0.71735 60909	0.71728	0.71864 22373	−2.67357	0.71735 61128
0.9	0.78332 69096	0.78323	0.80125 45441		0.78332 69403
1.0	0.84147 09848	0.84135	1.09135 22239		0.84147 10261
1.1	0.89120 73601	0.89106	4.37411 56871		0.89120 74139
1.2	0.93203 90860	0.93186			0.93203 91543
1.3	0.96355 81854	0.96334			0.96355 82701
1.4	0.98544 97300	0.98519			0.98544 98328
1.5	0.99749 49866	0.99719			0.99749 51092
1.6	0.99957 36030	0.99922			0.99957 37469

The solution of this equation, using the natural boundary conditions, is shown in column (11) of the table. The error is positive and steadily increases to the value 1439×10^{-10} at $x = 1.6$. The device used here is well known and is the basis of the NUMEROV-MILNE method for the solution of second order differential equations.

1.6. The reason for the surprising results mentioned in **1.4** is clear; they are essentially due to the large coefficient 16 of $y(n+3)$ in (6). Assuming that $y(0)$, $y(1)$, $y(2)$, and $y(3)$ are correct, apart from rounding off errors, we may expect an error in $y(4)$ due either to the rounding off of the given values or to the truncation error caused by neglect of the terms involving the sixth and higher differences in (2) or to both these causes. These errors can easily be estimated. The first error cannot exceed $(16 + 29.88 + 16 + 1)\cdot\frac{1}{2}\cdot 10^{-r}$ which is about $3\cdot 10^{-r+1}$ when one is working to r decimal places. The second error may be estimated as

$$-\ 12\cdot\tfrac{1}{90}\delta^6 y\ -\ 1.\dot{3}\times 10^{-7}y$$

which is about $2.\dot{6}\times 10^{-8}$, when $x = 0.2$. When 10 decimal places are used, the truncation error is the dominant one. Examination of the rounding off errors caused by carrying out $\sin x$ (for $x = 0.1$, 0.2, and 0.3) to 9, 8, 7, or 6 places shows that the resultant error in $y(4)$ is positive, but if 5 places are used, it is negative and greater in magnitude than the truncation error.

It is this initial error which determines the ultimate behavior of $y(n)$. The error in $y(5)$ is roughly 16 times that occurring in $y(4)$, as the errors in $y(3)$, $y(2)$, $y(1)$ are negligible compared with that in $y(4)$; the error in $y(6)$ is roughly 16 times that occurring in $y(5)$, for similar reasons; and so on. This exponential increase in the error serves as the explanation of the observed divergences. It is important to note that the trouble has been caused not by an accumulation of rounding off errors but rather by a single error (caused either by rounding off or truncation) and an unfortunate choice of formula.

Let us examine this more precisely. The solution of the difference equation (6) is of the form

$$y(n) = A_1\alpha_1^n + A_2\alpha_2^n + A_3\alpha_3^n + A_4\alpha_4^n$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of the equation

$$x^4 - 16x^3 + 29.88x^2 - 16x + 1 = 0.$$

This equation has two real roots α_1, α_2 and two complex roots α_3, α_4. Since it is a reciprocal equation with real coefficients, the following relations must hold

$$\alpha_1\alpha_2 = 1, \quad \alpha_3\alpha_4 = 1, \quad \alpha_3 = \bar{\alpha}_4.$$

The solution is therefore of the form

$$y(n) = A\alpha^n + B\alpha^{-n} + C\cos n\theta + D\sin n\theta,$$

where $\alpha = \alpha_1$, and $\theta = \arg \alpha_3$. The following approximate values are found

$$\alpha = 13.94, \qquad \theta = 0.1000.$$

It is clear that, if $A \neq 0$, then the term $A\alpha^n$ will dominate the others in the solution.

It will be found that the ratio of successive errors in column (6) approaches 13.94 very rapidly.

1.7. The solution of the difference equation (10) is

$$y(n) = A\cos n\theta + B\sin n\theta$$

where, for general h, θ is defined by

$$\cos\theta = 1 - [6h^2/(12 + h^2)].$$

We are interested in the pure sine solution. A small error will introduce a component $A\cos n\theta$ which will remain small. Significant results are to be expected in this case although there will, in general, be an error caused by the accumulation of the rounding off error or by the truncation of the formula (2).

Some idea of the magnitude of the first kind of error can be obtained by consideration of the difference equation arising from (10) with $h = 0$,

$$y(n+2) = 2y(n+1) - y(n).$$

There is no rounding off here, since the coefficients are integral. For this equation, we have

$$y(n) = (-n+1)y(0) + ny(1) + (n-1)y(2) + (n-2)y(3) + \cdots + 2y(n-1).$$

If we denote by $e(n)$ the error committed in rounding off the right-hand side of the last equation, then the total error in $y(n)$ due to rounding off is

$$(-n+1)e(0) + ne(1) + (n-1)e(2) + (n-2)e(3) + \cdots + 2e(n-1) + e(n).$$

Assuming that the $e(r)$'s, where $r = 0, 1, 2, 3, \cdots n$, are independent and have a rectangular distribution, then, for large values of n, the distribution of the total error in $y(n)$ is approximately normal with zero mean and variance

$$\sigma^2 = \tfrac{1}{12}\{(n-1)^2 + n^2 + (n-1)^2 + (n-2)^2 + (n-3)^2 + \cdots + 2^2 + 1^2\} \simeq n^3/36.$$

The probable total error is thus, in units of the last decimal,

$$0.6745\sigma \simeq 0.6745n^{3/2}/6, \text{ i.e., about } 0.1n^{3/2}.$$

The maximum error of this kind cannot exceed

$$\tfrac{1}{2}\{(n-1) + n + (n-1) + (n-2) + (n-3) + \cdots + 2 + 1\} \simeq \tfrac{1}{4}n^2$$

units of the last decimal.

Some idea of the magnitude of the truncation error is obtained by noticing that, on the assumption that no rounding off errors are committed, the solution obtained is

$$y(n) = \sin n\theta = \sin nh[1 + (h^4/480) + O(h^6)]$$

instead of $y(n) = \sin nh$.

The main source of error in column (11) is due to truncation while that in column (5) is due to rounding off.

1.8. The solution of a differential equation of the form

$$y'' = -k^2y,$$

where k is a constant, can be discussed in exactly the same way as we have dealt with the case where $k = 1$. Similar considerations will apply to

$$y'' + I(x)y = 0,$$

in the oscillatory regions, i.e., where $I(x)$ is positive. In the exponential regions, where $I(x)$ is negative, there will be solutions which diverge and solutions which converge. Contamination of a divergent solution with a small component of a convergent will, in general, cause no serious trouble, but the reverse effect must be avoided, e.g., by working backwards so that the recurrence relations are used in the form

$$y(n) = a_1y(n+1) + a_2y(n+2) + a_3y(n+3) + \cdots + a_ry(n+r).$$

The behavior of the solutions in the transition case near (simple) zeros of $I(x)$ will, in general, be on the pattern of those of

$$y'' = xy$$

which are the Airy Integrals, Ai(x) and Bi(x).[3]

1.9. Recently L. Fox and E. T. Goodwin [4] suggested the following type of method for the practical solution of ordinary differential equations with one-point boundary conditions: Use a recurrence relation of the form of (5) or (11) to obtain an approximation to the solution. Difference this solution and use the differences in the untruncated formula to correct the solution at each stage of the recursion. Difference again and correct again. Repeat until satisfactory solutions are obtained. The process appears likely to be of considerable use, but when using it, as was done in this instance, care must be taken in the choice of the recurrence relation used.

1.10. Similar phenomena for partial differential equations have been investigated by L. COLLATZ,[5] who indicates some conditions under which recurrence relations can be useful in the solution of such problems.

JOHN TODD

NBSCL

[1] See, e.g., L. J. COMRIE, *Chamber's Six-figure Mathematical Tables*, London, v. 2, 1949, p. 545–549 and *Interpolation and Allied Tables*, H. M. Stationery Office, London, 1947, p. 942–943.

[2] See, e.g., W. E. MILNE, *Numerical Calculus*, Princeton Univ. Press, 1949, p. 192.

[3] See, e.g., J. C. P. Miller, *The Airy Integral*, Brit. Assn. Math. Tables, Part vol. B, Cambridge Univ. Press, 1946.

[4] L. Fox & E. T. GOODWIN, "Some new methods for the numerical integration of ordinary differential equations," Cambridge Phil. Soc., *Proc.*, v. 45, 1949, p. 373–388.

[5] L. COLLATZ, Über das Differenzenverfahren bei Anfangswertproblemen partieller Differentialgleichungen," *ZAMM*, v. 16, 1936, p. 239–247.

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1. ANON., "Commercial electronic computer," *Mechanical Engineering*, v. 71, Jan. 1949, p. 29. 20.3 × 29.8 cm.

2. ANON., "Electronic computers," *Mechanical Engineering*, v. 71, Sept. 1949, p. 746. 28.6 × 20.9 cm.

3. ANON., "NBS Interim Computer," *Technical News Bulletin*, National Bureau of Standards, v. 33, Feb. 1949, p. 16, 17, illustr. 20 × 26 cm.

4. ANDREW D. BOOTH, "A magnetic digital storage system," *Electronic Engineering*, v. 21, July 1949, p. 234–238, 19 figs. 26 × 19 cm.

5. E. M. DEELEY and D. M. MACKAY, "Multiplication and division by electronic-analogue methods," *Nature*, v. 163, Apr. 23, 1949, p. 650. 18 × 27.3 cm.

6. HERMAN H. GOLDSTINE and JOHN VON NEUMANN, *Planning and Coding of Problems for an Electronic Computing Instrument*, Institute for Advanced Study, Princeton, N. J., part II, v. 2, 1948, 68 pages, 7 figs. 21.6 × 27.9 cm.

The second volume on the planning and coding of problems for an electronic computer illustrates the preparation of several problems for the proposed Institute for Advanced Study electronic computing machine. The many steps, from the initial statement of the problem and its mathematical foundation through the final coding of the problem in machine language, are given in complete detail.

The first volume of this report [see *MTAC*, v. 3, p. 54] presented the fundamental information and explanations necessary to the understanding of the publication now under review, i.e., the instruction code of the IAS computer, the preparation of flow diagrams, and several examples of the coding of some basic arithmetic operations. The problems encountered in the second volume are numerical integration, interpolation schemes, sorting, and collating.

Automatically-sequenced computing machines are most efficiently utilized when the problem to be computed can be reduced to an iterative process. The report gives rigorous treatment to the reduction of problems to such form, which lends itself to easy translation into machine language. There are two grave difficulties which must be considered when working with computers having a fixed decimal point, which maintain the same range for resultant values as for the operands. They are (1) round-off errors, and (2) the possibility of overflow, i.e., of exceeding the magnitude of the numbers accepted by the machine. The first of these is competently considered by the authors—with an especially efficient treatment of the method derived for applying Lagrange's interpolation formula. The second difficulty is presumed to have been avoided by the application of scale factors to the problem data prior to their insertion into the computer. Although this can be accomplished easily for the problems considered here, there are many others which are too complex for such treatment and require a more complicated routine, in order to accommodate a floating decimal point.

The mathematical discussions presented as a preliminary, though most important, phase of preparing problems for an automatic computer are generally applicable to all of the high-speed machines presently contemplated or under construction. For this reason the report constitutes a valuable instruction manual for all persons interested in programming problems for a high-speed computer.

The subsequent steps of preparing flow diagrams and coded routines are especially designed to meet the specifications of the IAS machine. The authors, in their attempt to meet the requirements of an instruction manual (i.e., starting with the simple specific case and then progressing to the more difficult general case) have introduced certain inefficiencies into the method of programming and coding employed. In practice, the MDL has found it to be more efficient, both in terms of memory space and time of execution, to reverse the procedure and introduce the parameters for the specific problem into the general routine, rather than to program the modifications necessary to make the simpler routine more generally applicable.

Also, the attempt to standardize certain procedures and notations, while helpful to the novice, precludes the introduction of certain economies in coding, e.g., the notation $x_0 = 2^{-19}x + 2^{-39}x$, used for the storage of all parameters.

The increased efficiency made possible by deviating from the authors' procedure, as indicated in the previous two points, is well illustrated in Dr. SAMUEL LUBKIN'S review of volume 3 of the report [*MTAC*, v. 3, p. 541–542]. He points out that the number of words required for the problem contained in that volume can be reduced from $58 + 4I$ to $36 + 2I$.

The first two problems discussed in this volume treat the integration of a tabulated one-variable function. One method makes use of SIMPSON'S formula for the limited case of $\int_0^1 f(z)dz$; the other method involves a more general formula and integrates

$$\int_{(\lambda-\frac{1}{2})/N}^{(k-\frac{1}{2})/N} f(z)dz,$$

where k and λ may be any positive integers. It appears likely that, in a great many cases, scaling could be introduced in the function values to permit summing of terms having the same coefficients without exceeding capacity, and the common factor could be applied only once for each case, with resulting gain in time. Since this problem, as stated, does not assume such scaling, it is necessary to multiply each term of the summation individually. The authors accomplish this by taking each term in sequence as derived in the formula. By grouping those with like coefficients, however, many fewer modifications and comparisons need be programmed. Another possibility is to "spell out" the few alternatives, instead of modifying the common sequence for each successive term.

The next group of problems deals with the application of Lagrange's interpolation formula. They are:

(1) Given variables $x_1, \cdots, x_M (x_1 < x_2 < \cdots < x_M)$ and the function values $p_1, \cdots, p_M$, to interpolate this function for the value x of the variable.

(2) Given the tabulated function $p_1, \cdots, p_N$, to interpolate for the value of x, using the M points nearest x, when the values of $x_1, \cdots, x_N$ are equidistant.

(3) Same as (2) above, except that the independent variables are unrestricted.

(In problems (1) and (3) it is assumed that the variable function values are stored in two separate sequences.)

(4) Same as (3), except that the x's and p's are intermeshed in one sequence of $2N$ cells.

Although the mathematical formulation as derived by the authors is, in the opinion of this reviewer, exceptionally well-fitted for machine computation, the actual coding reflects the same general inefficiencies stated earlier. The problems (2) through (4) are obtained by adding the required modifications to the limited first case, a circumlocutory approach which is needlessly time- and space-consuming.

The last two problems are:

(1) Meshing. (Given two monotonic sequences of n and m complexes: to merge them into one monotonic sequence.)

(2) Sorting. (Given a random sequence of complexes: to arrange them in monotonic order.)

A complex $X = [x, u_1, \cdots, u_p]$ consists of $(p + 1)$ words. A monotonic sequence of complexes is arranged in the order of the principal number x.

Sorting is accomplished by considering the initial data as N sequences of one complex each and obtaining $N/2$ monotonic sequences of two complexes each. This process is continued until all of the information has been arranged in one monotonic sequence. It is evident that the first problem can be treated as a special case of the second. However the general routine again is wasteful of machine time and space.

FLORENCE KOONS

NBSMDL

7. HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 14, *Description of a Relay Calculator*, Cambridge, Mass., Harvard University Press, 1949, xvi, 366 pages, 36 plates, $8.00. 20 × 26.5 cm.

When a group of scientists has completed a project, it is faced with the job of reporting its results. Too often the effect of the report on both writer and reader is that of turkey hash the week after Thanksgiving. The Harvard Computation Laboratory, however, has made a sincere effort to write useful and well-planned descriptions of its computers. Following its *Manual of Operation* for the Mark I Computer, the Laboratory has published a description of the Mark II Relay Computer. The description is, of course, a technical report, unadorned with facile generalities. It does, however, list the names of the men and women who have contributed to the technical work. In the opinion of the reviewer, this acknowledgment is a desirable break from the attitude of many organizations that insist on Siberian isolation for scientific employees.

The reader who is primarily interested in keeping up with the expanding universe of automatic computers will want to study Chapter I entitled *The Organization of the Calculator*, with its 21 photographic plates showing the components of the computer and the general view of the machine. Those who intend to use the calculator will also want to learn about the *Operation of the Calculator* (Chapter X), *Problem Preparation* (Chapter XI), and the handling of the elementary functions (x^{-1}, $1/\sqrt{x}$, $\log_{10} x$, 10^x, $\cos x$, and $\arctan x$) which is discussed in Chapter VII.

Most of the remaining chapters are written for the engineer or mathematician who will operate this machine or who is actively working on the design of automatic computers. These chapters treat basic circuits, registers (memory), addition units, multiplication units, sequencing and control, interpolators, and input and output devices.

It may be interesting to compare the Harvard Relay Calculator, Mark II, with the five relay computers of various sizes already built by the Bell Telephone Laboratories. Of the latter, the so-called "Stibitz Computers" located at Langley Field and Aberdeen Proving Ground, are very nearly the same size as the Mark II, each of the three machines having about 12,000 relays. Each machine is divided into two calculators that can be used separately or can be combined to work on one problem. Each uses a "floating decimal" system, writing every number as a set of significant digits multiplied by a power of 10 so that the largest number of significant digits may be retained at each step.

Some of the outstanding differences are as follows:

(1) The BTL machines are self-cycled, i.e., the completion of each minor step signals the next step to start, whereas the Mark II is time-cycled, and each minor step is allotted a fixed time interval.

(2) The BTL machines represent decimal digits in a "bi-quinary" notation, with 7 relays. In this notation one relay of a pair and one of a set of five are always closed when a digit is stored. If more or less than this complement operate, a check circuit stops the machine. Mark II uses a straight binary designation with 4 relays for each decimal digit.

(3) The BTL computers carry 7 decimal places, and Mark II carries 10.

(4) The BTL computers use practically no specially-made equipment, being built almost entirely of standard telephone and teletype units, whereas the designers of Mark II have felt free to design and build special devices.

In general, it may be said that the speeds of operation and the capabilities of the BTL and Mark II computers are similar, but the engineering is distinctive. It will be interesting to see whether the operating experience will be essentially different and, in particular, whether the expenditure of 3 extra relays per digit to obtain self-checking in the BTL computers is worthwhile.

G. R. STIBITZ

Consultant in Applied Mathematics
Burlington, Vermont

8. H. R. HEGBAR, "Electronic analog computer," *Electronics*, v. 22, Mar. 1949, p. 168, 170, 172, 174, illustr. and diag. 20.3 × 29.8 cm.

9. FRANK A. METZ, JR., and WALTHER M. A. ANDERSEN, "Improved ultrasonic delay lines," *Electronics*, v. 22, July 1949, p. 96–100, bibl. 20.3 ×29.8 cm.

Forged magnesium-alloy delay lines developed as memory devices have bandwidths as great as 4 mc. at a carrier frequency of 10 mc. The attenuation is the least so far available in practical lines. Special clamping of S-cut ADP crystal transducers is described.

10. C. H. PAGE, "Digital computer switching circuits," *Electronics*, v. 21, Sept. 1948, p. 110–118, 4 figs. 20.3 × 29.8 cm.

11. LOUIS N. RIDENOUR, "Mechanical brains," *Fortune*, v. 39, May 1949, p. 109–118, illustrs. 33 × 25.8 cm.

12. T. K. SHARPLESS, "High-speed *n*-scale counter," *Electronics*, v. 21, Mar. 1948, p. 122–125, illustrs., diags., bibl. 20.3 × 29.8 cm.

13. NORBERT WIENER, "A new concept of communication engineering," *Electronics*, v. 22, Jan. 1949, p. 74–76. 20.3 × 29.8 cm.

NEWS

Eckert-Mauchly Computer Corporation.—The BINAC (Binary Automatic Computer) made its debut in the latter part of August at the Eckert-Mauchly Corporation, in Philadelphia, Pa. This all-electronic machine was built for Northrop Aircraft, Inc., of Hawthorne, California, and embodies a considerable number of improvements over the first such machine, the Army's 30-ton ENIAC, for whose construction JOHN W. MAUCHLY and J. PRESPER ECKERT, JR., were largely responsible.

Although the BINAC actually consists of twin machines, working in unison and checking each other's performance, it is small enough to fit into an ordinary-sized room, as each machine stands five feet high and occupies less than ten square feet of floor space. The most important of its many novel features is its mercury delay line "memory," whose invention contributed enormously to the reduction in the number of electronic tubes originally found necessary in the construction of the ENIAC, which is burdened with 18,000 such tubes. Each BINAC twin possesses only 700 tubes, although it is equipped with 512 memory registers (or "cells"), each capable of storing either a signed number consisting of thirty binary digits or a pair of "instructions."

The machine is constructed to follow a set of sixteen coded instructions for the execution of elementary mathematical operations. A staff of programmers and coders arrange these instructions in the proper sequence for carrying out a required computation. The resulting routine, together with the original data, is then transferred onto magnetic tape by means of a specially constructed typewriter which transforms the original material into binary-coded decimal form while yielding simultaneously a typed paper copy of the transcribed material, in the original form. By means of a manually operated switch, the information on the magnetic tape is inserted into the memory of the machine.

The very high speed, at which instructions and data are delivered from the memory to the other units within the machine and at which arithmetical operations are executed, is due to the enormous rapidity of the BINAC's "heart-beat"—its oscillator emits four million pulses every second.

During the demonstration, the BINAC gave repeat performances for two types of problems. First, the solution of a Poisson equation was computed by means of a modified Liebmann method, over a square field with 16^2 mesh points. Each transit over the 196 interior points consumed about four seconds of the machine's time. The largest difference between two successive results obtained for each point was recorded on the oscilloscope, and the computation stopped when the preassigned tolerance exceeded this difference. The entire solution, correct to about 8 decimal places, took less than ten minutes per problem.

The second part of the demonstration was devoted to obtaining the reciprocal, the square root, the reciprocal square root, and the cube root of four-decimal-digit numbers suggested by the audience. In addition to converting the original numbers into their binary equivalents to enable the Arithmetic Unit to compute the required functions, the machine also sorted the numbers, by collation, in ascending order of magnitude. The audience was invited to compare the answers with the entries in BARLOW's Tables.

Harvard University Computation Laboratory.—On September 13–16, 1949, a second Symposium on Large-Scale Digital Calculating Machinery was held at the Computation Laboratory of Harvard University under the joint sponsorship of the Navy Department's Bureau of Ordnance and Harvard University. In conjunction with this Symposium there was also a meeting of the Association for Computing Machinery. The eight sessions of the Symposium were supplemented by a continuous demonstration of the Mark III Calculator. The program was as follows:

Tuesday Morning, September 13, 1949, *Opening Addresses*, HOWARD H. AIKEN, Director of the Computation Laboratory, presiding, EDWARD REYNOLDS, Administrative Vice-President of Harvard University, and Rear Admiral F. I. ENTWISTLE, USN, Director of Research, Bureau of Ordnance.

Tuesday Afternoon, *Recent Developments in Computing Machinery*, MINA REES, Office of Naval Research, presiding:

"The Mark III calculator," BENJAMIN L. MOORE, Harvard University

"The Bell Computer, Model VI," ERNEST G. ANDREWS, Bell Telephone Laboratories

"Tests on a dynamic regenerative electrostatic memory," J. PRESPER ECKERT, JR., Eckert-Mauchly Computer Corporation

"The digital computation program at Massachusetts Institute of Technology," JAY W. FORRESTER, Massachusetts Institute of Technology

"The Raytheon electronic digital computer," RICHARD M. BLOCH, Raytheon Manufacturing Company

"A General Electric engineering digital computer," BURTON R. LESTER, General Electric Company.

Wednesday Morning, September 14, 1949, *Recent Developments in Computing Machinery*, E. LEON CHAFFEE, Harvard University, presiding:

"Design features of the NBS Interim and Zephyr Computers," S. N. ALEXANDER and H. D. HUSKEY, National Bureau of Standards

"Static magnetic delay lines," WAY DONG WOO, Harvard University

"Coordinate tubes for use with electrostatic storage tubes," R. S. JULIAN and A. L. SAMUEL, University of Illinois

"Basic aspects of special computational problems," HOWARD T. ENGSTROM, Engineering Research Associates

"Electrochemical computing elements," JOHN R. BOWMAN, Mellon Institute

"EDVAC transformation rules," GEORGE W. PATTERSON, University of Pennsylvania.

Wednesday Afternoon, *Recent Developments in Computing Machinery*, RAYMOND C. ARCHIBALD, Brown University, presiding:

"Notes on the solution of linear systems involving inequalities," GEORGE W. BROWN, Rand Corporation

"Mathematical methods in large-scale computing units," D. H. LEHMER, University of California

"Empirical study of effects of rounding errors," C. CLINTON BRAMBLE, U. S. Naval Proving Ground, Dahlgren, Va.

"Numerical methods associated with Laplace's equation," W. E. MILNE, Institute for Numerical Analysis, UCLA and Oregon State College

"An iteration method for the solution of the characteristic value problem of linear differential and integral operators," CORNELIUS LANCZOS, Institute for Numerical Analysis, UCLA

"The Monte Carlo Method," S. M. ULAM, Los Alamos Scientific Laboratory.

Thursday Morning, September 15, 1949, *Computational Problems in Physics*, KARL K. DARROW, Bell Telephone Laboratories, presiding:

"The place of automatic computing machinery in theoretical physics," WENDELL FURRY, Harvard University

"Double refraction of flow and the dimensions of large asymmetrical molecules," HAROLD A. SCHERAGA and JOHN T. EDSALL, Cornell University and Harvard Medical School

"L-shell internal conversion," MORRIS E. ROSE, Clinton Laboratories, Oak Ridge

"The use of calculating machines in the theory of primary cosmic radiation," MANUEL S. VALLARTA, University of Mexico

"Computational problems in nuclear physics," HERMAN FESHBACH, Massachusetts Institute of Technology.

Thursday Afternoon, *Aeronautics and Applied Mechanics*, HARALD M. WESTERGAARD, Harvard University, presiding:

"Computing machines in aeronautical research," R. D. O'NEAL, University of Michigan

"Problem of aircraft dynamics," EVERETT T. WELMERS, Bell Aircraft Corporation

"Statistical methods for certain non-linear dynamical systems," GEORGE R. STIBITZ, Burlington, Vermont

"Combustion aerodynamics," HOWARD W. EMMONS, Harvard University

"Application of computing machinery in research of the oil industry," MORRIS MUSKAT, Gulf Research and Development Company

"The 603–405 Computer," WILLIAM W. WOODBURY, Northrop Aircraft, Inc.

Friday Morning, September 16, 1949, *The Economic and Social Sciences*, EDWIN B. WILSON, Office of Naval Research, presiding:

"Application of computing machinery to the solution of problems of the social sciences," C. FREDERICK MOSTELLER, Harvard University

"Dynamic analysis of economic equilibrium," WASSILY W. LEONTIEF, Harvard University

"Some computational problems in psychology," LEDYARD TUCKER, Educational Testing Service, Princeton

"Computational aspects of certain econometric problems," HERMANN CHERNOFF, University of Chicago

"Physiology and computing devices," WILLIAM J. CROZIER, Harvard University

"The science of prosperity," FREDERICK V. WAUGH, Council of Economic Advisers.

Friday Afternoon, *Discussion and Conclusions*, WILLARD E. BLEICK, U. S. Naval Academy Post Graduate School, presiding:

"Computer built by the Centre Blaise Pascal," LOUIS COUFFIGNAL, Institut Blaise Pascal (read by LEON BRILLOUIN)

"The future of computing machinery," L. N. RIDENOUR, University of Illinois.

This meeting, attended by well over 500 persons, is noteworthy in that its program dealt with not only those physical sciences which are already recognized as closely akin to the large-scale computer development, but also with the increasing application of these machines to the social and economic sciences. The hosts are to be congratulated upon the cleverly-conceived meeting in which topics covering widely diverse fields were organized into a well-integrated program. Each lecture complemented the related lectures without duplication of subject matter. It was interesting to note that many of the demonstrators of the Mark III were students from foreign lands now working at the Computation Laboratory.

On Tuesday evening, September 13, 1949, a banquet was held with EDWARD A. WEEKS, JR., Editor of *The Atlantic Monthly*, as toastmaster, who introduced the speech of WILLIAM S. ELLIOTT, Elliott Brothers Research Laboratories (London) Limited, entitled "Present position of computing machine development in England."

The Cambridge University Mathematical Laboratory.—A four-day conference on automatic calculating machines held June 22 through June 25, 1949, inclusive, at the University Mathematical Laboratory in Cambridge, England, served to bring together some 150 scientists interested in the design and application of high-speed automatic calculating machines. Attention was naturally concentrated on developments in England and on the continent, although recent American developments were summarized in a paper expressly prepared for the conference by H. D. HUSKEY, National Bureau of Standards, and presented for him by J. M. BENNETT. References to American work were also made by D. R. HARTREE in a survey of the present position of work in the field and in a paper on relay machines by A. D. BOOTH, which was presented by Miss K. H. V. BRITTEN. There was a demonstration of the new Cambridge electronic calculating machine, the EDSAC, during which tables of squares and prime numbers were printed.

The full program of the conference was as follows:

Wednesday, June 22:	
Address of Welcome	M. V. WILKES, Director of the University Mathematical Laboratory
Survey of the present position of work on automatic digital computers	D. R. Hartree, Cavendish Laboratory
The EDSAC	M. V. Wilkes
Demonstration of the EDSAC	W. RENWICK, University Mathematical Laboratory
Thursday, June 23:	
The Automatic Relay Calculator	A. D. Booth, Birkbeck College. Paper presented by Miss K. H. V. Britten, British Rubber Producers' Research Association
Discussion on relay machines	
Cathode-ray tube storage	F. C. WILLIAMS, University of Manchester
Discussion on programming and coding	J. H. WILKINSON, N. P. L., presiding
French computing machine projects	L. COUFFIGNAL, Laboratoire de Calcul Mécanique, Institut Blaise Pascal, Paris
Friday, June 24:	
Checking process for large routines	A. TURING, University of Manchester
Some routines involving large integers	M. H. A. NEWMAN, University of Manchester
Discussion on permanent and semi-permanent storage facilities	E. N. MUTCH, University Mathematical Laboratory, presiding
Discussion on checking procedure and circuits	A. M. UTTLEY, T. R. E., and D. J. WHEELER, University Mathematical Laboratory, presiding
Saturday, June 25:	
Description of a machine built at Manchester	T. KILBURN, University of Manchester
Computing machines: plans, projects, and general ideas	General Discussion

In addition there were contributions from A. VAN WIJNGAARDEN, Mathematisch Centrum, Amsterdam, and G. KJELLBERG, Tekniska Högskolan, Stockholm.

Two electronic calculating machines are now in operation in England. One of these, the EDSAC, is a serial binary machine using ultrasonic tanks for storage; it has a storage capacity of 512 words of 34 digits each plus a sign digit. Five-hole teleprinter tape is used for input and a modified teleprinter for output.

The other machine, located at Manchester, has grown out of the development of a "baby" machine built to test the practicability of the cathode-ray tube storage system developed by F. C. Williams and T. Kilburn. It now has about 1400 tubes. There is no printer, but the results are read from a cathode-ray tube connected to the store. In spite of these limitations, some genuine mathematical work has been done on Mersenne numbers. Addition takes 1.8 milliseconds and multiplication up to 36 milliseconds depending on the number of digits in the multiplier. An auxiliary store using a magnetic drum whose rotation speed is locked to the clock-pulses has been developed, and transfer of blocks of numbers or orders from it to the high-speed store is possible by manipulating a series of push buttons. This machine is in a constant state of change. Its main purpose is to provide experience for those working on computers.

The Automatic Relay Calculator which has been built under the direction of A. D. Booth is a relay machine working in the binary system. The store is a magnetic drum with capacity for 256 numbers. The machine has about 800 relays all of which are of the Siemens high-speed type and have an operating time of one or two milliseconds. The word length is 20 digits, plus a sign digit. Addition takes 20 milliseconds and multiplication takes 0.4 seconds. Teleprinter tape is used for input and a teleprinter for output. This machine is complete but is not yet in working order.

Of the other British computer projects, the most advanced is the construction of a pilot model for the Automatic Computing Engine at the National Physical Laboratory. This will be a binary machine with a word length of 32 digits and will use punched cards for input and output. The store will consist of a group of ultrasonic tanks with a total capacity of 256 words. The addition time will be 32 microseconds and the multiplication time 2 milliseconds.

Work is in progress at the Telecommunications Research Establishment of the Ministry of Supply on a parallel machine using cathode-ray tubes for the high-speed store and a magnetic drum for the auxiliary store. A special feature of this machine will be the use of three-state trigger circuits, two of the states being used to represent 0 and 1 and the third being neutral. Each trigger circuit will be put in the neutral state before each transfer operation, and thus definite action will be necessary to set it to represent either a 0 or a 1. In this way the machine can be made self-checking to a large extent.

A decimal relay machine using rotary switches for the registers is being constructed at the Royal Aircraft Establishment. Each word will consist of 8 decimal digits using a floating decimal point. The addition time will be about one second and the multiplication time 1½ to 2 seconds. The machine is intended primarily to facilitate the analysis of experimental results.

A new project of Dr. Booth's, to which the name APEXC (All-Purpose Electronic X-Ray Computer) has been given, was mentioned briefly. It will use a combination of relays and electronic tubes and will have a magnetic drum store with an electromechanical auxiliary store.

L. Couffignal gave some information about a parallel binary electronic machine which he plans to build at the Institut Blaise Pascal in Paris. Some preliminary design work has already been done, and the machine will probably have a word length of 50 digits with a floating binary point.

Dr. A. van Wijngaarden described a relay machine, rather similar to the Automatic Relay Calculator, being built at the Mathematisch Centrum at Amsterdam, Holland. This will be a parallel binary machine with a word length of 30 digits. The addition time will be 15 milliseconds and the multiplication time 0.4 seconds. It will have a magnetic drum store and teleprinter input and output equipment. Another relay machine is being built in Holland at the Technische Hogeskolan in Delft. This is primarily intended for the tracing of optical rays and will have a smaller storage capacity and will be less flexible in operation than the other machines here mentioned. The addition time will be 50 milliseconds and the multiplication time 1.5 seconds. The word length will be 31 digits. In Sweden a machine known as BARK (Binar Automatish Rela-Kalkylator) is being constructed at the Tekniska Högskolan, Stockholm. This is a parallel binary machine with a word length of 32 digits and a floating binary point. It uses about 5500 relays and is programmed by means of a plug board on which a program of up to 840 orders may be plugged. Provision is made for the use of conditional orders and subroutines.

OTHER AIDS TO COMPUTATION

Addition and Subtraction on a Logarithmic Slide Rule

It does not seem to be generally known that the principle of addition logarithms can be applied to the use of an ordinary slide rule for adding. The process is, of course, not worthwhile if additions occur in isolation, but much time can be saved if additions occur in combination with multiplications or divisions, and if slide rule accuracy is sufficient.

Using the C and D scales, the sum $(a + b)$ can be found thus:

Set the index of C to the value of a on D (preferably choose $a > b$). Set the cursor to the value of b on D. Read the value of b/a on C, under the cursor. Form mentally $(1 + b/a)$ and set the cursor to this value on C. Read $(a + b)$ on D, under the cursor.

When additions are combined with another process, one of the terms can usually be arranged to appear on the D scale ready for addition, or the sum appearing on the D scale can be used there for the next process. For example $(ab + c)$ can be formed thus:

Set the cursor to a on D and move the slide so that b on CR lies below the cursor line; the index of C is now opposite ab on D. Move the cursor to the value of c on D, read c/ab on C under the cursor, add 1 mentally and set the cursor to $(1 + c/ab)$ on C. Read $(ab + c)$ on D, under the cursor.

Analogous methods apply to other combinations of operations involving addition, and subtractions can also be handled in a similar manner.

G. A. Montgomerie

7 Wood Green Road
Quinton
Birmingham 32, England

Bibliography Z–X

14. *Polnoe Sobranie Sochinenii P. L. Chebysheva* [Complete Collection of Works by P. L. Chebyshev]. Volume 4: *Teoria Mechanismov* [Theory of Mechanisms]. Moscow and Leningrad, Academy of Sciences, 1948, 254 p. + portrait frontispiece, 16.5 × 25.5 cm.

This volume which is the fourth in a series of collected works of Chebyshev, contains fourteen articles on theory of mechanisms prepared by the author during the period of 1861–1888, a brief discussion of these articles, and a brief description of model mechanisms built by the author. The author's articles included in this volume were previously published in v. 1 and 2 of the first edition, 1899–1907, of his collected works [*MTAC*, v. 1, p. 440–441]. Five of the articles were published previously in various French publications. The author's article "Theory of Mechanisms Called Parallelograms," because of its mathematical nature was included in v. 2 which is devoted to the work in mathematical analysis.