

values (most tables, including the recent ones, give values of t for which $P(|u| \geq t)$ has selected values) have proceeded by steps of .1 for t , the two principal ones being due to W. S. GOSSETT ("Student") [*Metron*, v. 5, No. 3, 1925, p. 105-108], which in its main part gives values to 4D for $n = 1(1)20$ and $t = 0(.1)6$, and to KARL PEARSON [*Tables for statisticians and biometricians*, Part I, 3rd edition, London, 1930, p. 36]. The present tables were calculated by linear interpolation for $0 \leq t \leq 6$ and $1 \leq n \leq 20$ from Gossett's 1925 tables; for higher values of t and n , approximate methods given by Gossett were used. [Some spot checking by direct calculation of the corresponding incomplete β -functions for $t = 1(.01)1.09$ with $n = 1$ and 10 gave in 8 cases values differing from Gilbert's by 1 in the third decimal place. For a more critical value $n = 20$, $t = 0.5$ a direct evaluation of the incomplete β integral confirmed Gilbert's value of 0.623. C. C. C.]

AUTOMATIC COMPUTING MACHINERY

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TECHNICAL DEVELOPMENTS

Our contribution under this heading, appearing earlier in this issue, is "The California Digital Computer," by PAUL L. MORTON.

DISCUSSIONS

"Floating Decimal" Calculation on the IBM Card Programmed Electronic Calculator

There is a wide variety of problems occurring in run-of-the-mill computing in which it is extremely helpful to represent numbers x as $x_0 \cdot 10^p$ where x_0 has the same significant digits as x within the capacity of the machine used, $1 \leq x_0 < 10$, and p is an integer. Three types we might mention are:

- 1) Problems in which certain computed quantities have magnitudes difficult to estimate.
- 2) Problems in which one or more quantities have such a wide range of magnitude that no single fixing of the decimal point will suffice for the entire range.
- 3) Problems in which the setup time involved in estimating magnitudes is not justified by the saving in machine time made possible by fixed decimal calculation.

For application of the IBM Card Programmed Electronic Calculator to such problems, we have devised a calculator programming of the general-purpose type based on the "floating decimal" representation of numbers. A brief explanation of the way the machine is instructed will help in an understanding of what follows.

The calculating unit (IBM 604) of the Card Programmed Electronic Calculator is, for one step in the calculation, instructed to receive numbers

A and B , to perform on them an operation $f_i(A, B)$, and to leave result C where it can be used for further calculation or can be stored. The numbers A , B , and C may have up to ten digits and a sign. The choice of operations f_i is determined to suit the needs of the group using the machine. The more calculation one can put into one operation, the faster the C.P.E.C. can do the problem. Hence, for a large problem, it is sensible to design the operations f_i with this in mind. A more versatile but slower setup is one in which the same set of operations will suffice for any problem. It will rather obviously include among its operations addition, subtraction, multiplication, and division. What additional operations are included is up to the needs of the group using the machine. The choice of operations is put into effect by wiring of plugboards. Most operations are completely instructed by wiring of the plugboards for the 604, but on occasion, it is useful to alter A or B before they are delivered to the calculator or C as it comes from the calculator. This implies special wiring on the tabulator plugboard.

In our programming of the C.P.E.C. for general-purpose floating decimal calculation, we represent a number x by a ten-digit number X . In the representation $x = x_0 \cdot 10^p$, we carry eight significant digits in x_0 and allow p the range $-50 \leq p \leq 49$. The first eight digits of X are the digits of x_0 . The last two digits of X are those of $p + 50$. The sign of X is that of x_0 . Examples are:

x	X
$4.1752139 \cdot 10^4$	4175213954
$-1.9920367 \cdot 10^{-20}$	-1992036730

The set of operations which is available in our setup is as follows:

$$\begin{aligned}
 f_0(A, B) &= A \\
 f_1(A, B) &= A + B \\
 f_2(A, B) &= A - B \\
 f_3(A, B) &= A \cdot B \\
 f_4(A, B) &= A/B \\
 f_5(A, A) &= \sqrt{A} \\
 f_6(A, B) &= |A| \cdot B \\
 f_7(A, B) &= |A|/B
 \end{aligned}$$

Corresponding to $i = 3, 4, 5, 6, 7$ there is an operation f'_i such that $f'_i = -f_i$.

If no number is delivered to the calculator to be used as A or B in a calculation, then the number already in that position is used. Because numbers delivered to storage cannot be read from storage for use as A or B immediately, it is often convenient to store the result of a calculation in the A or B position in the calculator before the next instructions are given. For all operations except f_0 , we automatically store C in the A and B positions. For f_0 , C is stored in the B position, and the unused B in the A position.

Operations 6 and 7 are made possible by subsidiary wiring on the tabulator plugboard which replaces A by $-|A|$. Hence, neither of these operations can make use of holding the previous result within the calculator in A position.

Our floating decimal programming for the calculator relies heavily on the program repeat feature of the 604 used in the C.P.E.C. This feature

allows the machine, given a set of instructions, to repeat it several times in one operation and, if desired, to use one subset of these instructions on one time through a set, a different subset the next time. This use of program repeat will make addition or subtraction take two or three times the normal computing time if A and B are different by several orders of magnitude or if C has a magnitude much smaller than that of A and B . As is to be expected, the square root operation will always be slow. The other operations will always be finished in the minimum time for a single operation.

Although it is rather expensive in selectors on the tabulator plugboard, one can easily instruct the machine to make a reasonable (within a factor of 4) guess at the q th root of a number which then is improved by iteration.

The guess at the q th root of $x_0 \cdot 10^p$ is $3 \cdot 10^{\left[\frac{p}{q} + 50\right] - 50}$ where $[x]$ is the greatest integer $\leq x$.

The zero of our "floating decimal" setup cannot conveniently have all its digits zero. An explanation of our addition will demonstrate the origin of our representation. Suppose two numbers are added which differ in absolute value only in the eighth significant digit but which are of opposite sign. Then the digits of the result will lead off with zeros. In order to get the answer back to standard representation, we examine the left-hand digit. If this digit is zero, we shift the digits of the answer one position to the left and decrease the calculated p by one. This shifting is continued until the left-hand digit is non-zero. To enable the machine to stop shifting if all eight digits of the answer are zero, we subtract one in the right-hand digit of the answer each time a shift is made after the first. Thus on our machine,

$$x_0 \cdot 10^p - x_0 \cdot 10^p = -1.1111111 \cdot 10^{p-9}.$$

This shifting device gives an obvious means for converting fixed decimal numbers of eight digits or less to floating decimal numbers. Suppose the number of largest magnitude in a column of figures is $x \cdot 10^p$. We punch $x \cdot 10^p$ into the card with standard "floating decimal" representation and all the other numbers with the same last two digits and the same decimal position. Then transferring these numbers through the 604 will convert them to standard "floating decimal" representation.

One additional operation which is useful only in the printing or punching of final results has been included in our planning. If calculations made on the C.P.E.C. are to be used on other IBM machines, it may be desirable to convert results to the fixed decimal form. This procedure will often replace key punching by reproducing. The board is so arranged that if $A = x_0^A \cdot 10^{p_A}$, $B = x_0^B \cdot 10^{p_B}$, $p_B \geq p_A$, and the operations code is 2 and 7, then A will come out on Channel C with exponent p_B and the left-hand $p_B - p_A$ digits will be zeros. The digits of x_0^B do not affect this rounding of x_0^A ; thus it is not necessary to save an entire storage unit to carry p_B .

When Channels A and B are transmitted to the 604, the significant digits of Channel A are entered in General Storage 1 and 2, its exponent in Factor Storage 4; the significant digits of Channel B are entered in Factor Storage 1 and 2, and its exponent in Factor Storage 3. The sign entries of FS 1 and 2 and GS 1 and 2 are wired from the channel sign hubs, in normal fashion.

The sign wiring of FS 3 and FS 4 is unusual and important to the operation of the board. Its effect is that these two units always read true figures with negative sign. Two pilot selectors are used. The first is picked up whenever a spread read-in card is at the third brushes. The second is picked up immediately by the impulse from "Transfer and S.P. \times Control Plus" hub for counter 2A, which is wired to total on all cycles. FS 3 sign hub is wired to common of one position of the first selector. The transferred hub of that selector is wired to the 10 impulse emitter, and the normal hub is wired to common of one position of the second selector. Transferred of the second selector is wired to the 10 impulse emitter, normal is wired to the sign of Channel B. FS 4 is wired similarly in the other position of each of the two pilot selectors, with the final connection to sign of Channel A.

The significant digits of Channel C are in GS 3 and 4, and the exponent in the two lowest order positions of the 604 counter. Channel C sign is to be taken from GS 3 and 4.

The absolute value device is this: an x -punch is used to instruct the machine; it picks up a pilot selector, which transfers the sign of GS 1 and 2 from Channel A sign hub and connects it to the sign of FS 4. Then GS 1 and 2 contains the negative of the absolute value of A.

The coding that has been used for this board is:

No Instruction	Addition	(f_1)
2	Subtraction	(f_2)
3	Multiplication	(f_3)
4	Division	(f_4)
4 and 5	Square Root	(f_5)
7	Transfer	(f_6).

The effect of instruction 2 is just to substitute $-B$ for B . Therefore we also have:

2 and 3	Negative Multiplication	(f'_3)
2 and 4	Negative Division	(f'_4)
2, 4, and 5	Negative Square Root	(f'_5).

If we use the x -punch for absolute value in addition to operation codes, we have:

x , 2, and 3	(f_6)
x , 2, and 4	(f_7)
x and 3	(f'_6)
x and 4	(f'_7).

Calculate selectors 2, 3, 4, 5, and 7 may be picked up in the ordinary way. Selector 1 is to be picked up whenever either operation code 3 or 4 is present. Selector 6 is to be coupled to selector 5. Selector 8 is to be coupled to selector 4.

On the programming chart, some positions are marked with an asterisk. These are positions where the instruction must be selected according to which operation is being performed. The selection is indicated in the "remarks" column. In that column the symbol 012, for instance, refers to program 01, exit 2.

The significance of the symbols at the program positions is determined by the heading of the column in which they appear. For instance, GS 1 and 2 in the Exit 1 column means General Storage 1 and 2 read-out. The same symbol in the Exit 2 column means General Storage 1 and 2 read-in. The abbreviations R & R and RO refer to the counter. We have used the symbol # to help distinguish between abbreviations for General Storage and those for Group Suppression. Thus we have GS 3 and 4 (General Storage 3 and 4) and GS #4 (Group Suppression number 4). We have written miscellaneous orders (Balance Test for Step Suppression, group suppression controls, etc.) in Exit 3 where possible.

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APPENDICES

Suppression Types for General-Purpose Floating Decimal Programming for C.P.E.C. 604

Types of suppression:	
I	Group suppress # 1
II	Group suppress # 2
III	Group suppress # 3
IV	Group suppress # 4
V	Suppress on negative balance.
VI	Suppress on positive balance.
VII	Suppress on non-zero balance.
X	Suppress if selector 5 is normal.
XI	Suppress if selector 5 is up.
XII	Suppress on negative balance; suppress if selector 5 is normal.
XIII	Suppress on negative balance; suppress if selector 1 is up.
XIV	Suppress on negative balance if selector 1 is normal; group suppress # 1 if selector 1 is up.
XV	Group suppress # 1 if selector 4 is up.
XVI	Suppress on negative balance if selectors 1 and 5 are normal; suppress on positive balance if selector 5 is up; group suppress # 1 if selector 1 is up and 5 is down.
XVII	Suppress if selector 4 is up.
XVIII	Group suppress # 1; suppress if selector 5 is up.
XIX	Group suppress # 1; suppress if selector 4 is up.
XX	Group suppress # 4 if selector 5 is normal.
XXI	Group suppress # 1; suppress if selector 4 is normal.
XXII	Suppress on zero balance if selectors 3 and 7 are normal; suppress if selector 3 is up and 7 is normal; group suppress # 1 if selector 7 is up.
XXIII	Suppress on zero balance; suppress if selector 1 is up.
XXIV	Suppress on negative balance; suppress if selector 7 is up.
XXV	Suppress if selector 5 is up; group suppress # 2 if both selector 2 and selector 7 are up.

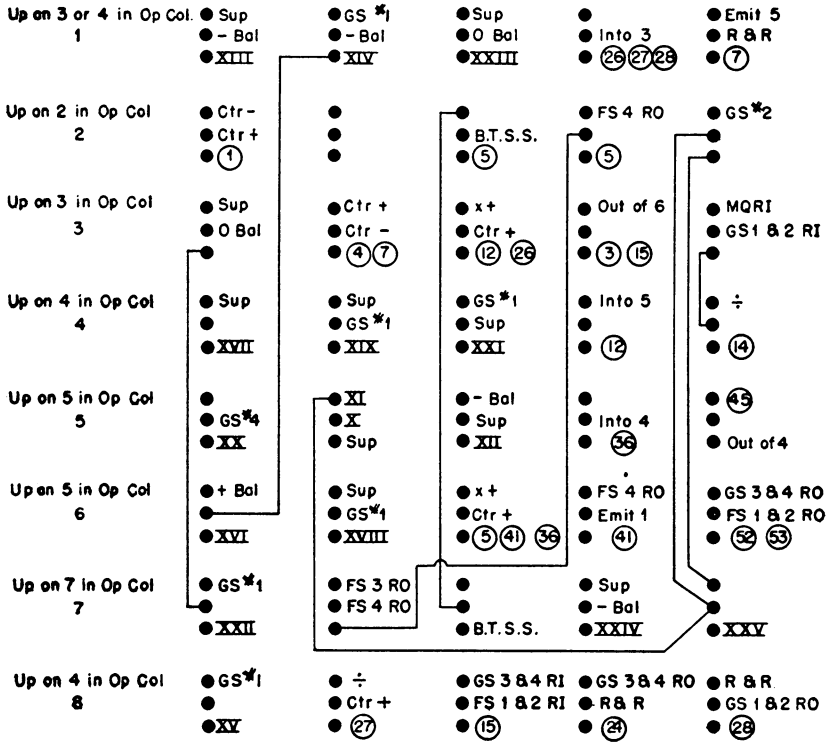
PROGRAMMING CHART

	Read Out Exit 1	Read In Exit 2	Shift Exit 3	Type Sup.	Remarks
1	FS 1 & 2	CTR +*	GS #4 PU	II	012 CTR - if sel 2 is up
2	R 0	MQ		XVII	
3	R & R	FS 1 & 2	*	II	033 out of 6 if sel 3 is up
4	FS 3	CTR -*		XVIII	042 CTR + if sel 3 is up

PROGRAMMING CHART—Continued

	Read Out Exit 1	Read In Exit 2	Shift Exit 3	Type Sup.		Remarks
5	FS 4*	CTR +*	*	none	051 FS 3	if sel 7 is up and sel 2 is down
6			Zero test	XXIII	052 x +	if sel 5 is up
7	R & R*	CTR -*	into 2	XI	053 B.T.S.S.	if sel 7 is down or sel 2 is down
8	FS 4	CTR +		XIII	071 emit 5	if sel 1 is up
9	FS 3	FS 4		XIII	072 CTR +	if sel 3 is up
10	R & R	FS 3		XIV		
11	FS 3	CTR +	B.T.S.S.	X	122 x +	if sel 3 is up
12	GS 1 & 2	CTR +*	*	XVI	123 into 5	if sel 4 is up
13	GS 1 & 2	CTR +	into 4	XII	142 MQ	if sel 3 is up
14	FS 1 & 2	GS 1 & 2*		XIV	142 ÷	if sel 4 is up
15	R & R	FS 1 & 2*	*	XIV	152 GS 3 & 4	if sel 4 is up
16	FS 1 & 2	CTR +		XXIII	153 out of 6	if sel 3 is up
17	Half adj.			XXIII		
18	R & R	FS 1 & 2	out of 2	XXIII		
19	FS 3	CTR +		XXIII		
20	Emit 1	CTR -		XXIII		
21	R 0	FS 3		XXIII		
22	FS 4	CTR -		XXIII		
23			Zero test	XXIII		
24	R & R*	CTR +	into 6	XV	241 GS 3 & 4	if sel 4 is up
25	MQ	GS 3 & 4	into 2	XXI	262 x +	if sel 3 is up
26	GS 1 & 2	CTR +*	into 3*	XIX	263 into 1	if sel 1 is up
27	FS 1 & 2	CTR +*	into 3*	I	272 ÷	if sel 4 is up
28	GS 1 & 2*	CTR -	into 3*	XXII	273 into 1	if sel 1 is up
29	Emit 1	CTR -	into 3	XX		
30	GS 3 & 4	CTR +	into 6	XXI	281 R & R	if sel 4 is up
31	MQ	CTR +	into 2	XXI	283 into 1	if sel 1 is up
32	Half adj.	GS #3 PU	into 2	I		
33	R & R	GS 3 & 4	out of 3	X		
34	Emit 5	MQ	into 3	none		
35	FS 1 & 2	x +	GS #3 DO	X		
36	GS 3 & 4	CTR +*	into 4*	III	362 x +	if sel 5 is up
37	R 0	GS 3 & 4	out of 3	XI	363 into 1	if sel 5 is up
38	Half adj.		into 3	none		
39	R & R	FS 1 & 2	out of 4	none		
40	Emit 5	FS 3	into 2	X		
41	Emit 1*	CTR +*	GS #4 DO	III	411 FS 4	if sel 5 is up
42	Emit 1	CTR -	GS #2 DO	XIX	412 x +	if sel 5 is up
43	FS 3	CTR +		XI		
44	FS 3	x -		X		
45	R & R	FS 3	*	none	453 out of 4	if sel 5 is up
46	FS 1 & 2	CTR -	GS #1 PU	VII		
47	GS 3 & 4	CTR +	GS #1 DO	X		
48	R 0	GS 3 & 4	out of 3	X		
49	R & R		B.T.S.S.	none		
50	MQ	CTR -	into 5	XXV		
51	MQ	CTR -	into 5	XXV		
52	FS 1 & 2*	CTR +		VI	521 GS 3 & 4	if sel 5 is up
53	FS 1 & 2*	CTR -		V	531 GS 3 & 4	if sel 5 is up
54						
55	R & R	GS #3 DO	B.T.S.S.	none		
56		GS #2 PU	Prog. Rpt.	VI		
57	FS 1 & 2	GS 1 & 2		XXIV		
58	FS 3	FS 4		XXIV		
59	FS 3	CTR +		V		
60	FS 1 & 2	GS 3 & 4		V		

DIAGRAM OF THE CALCULATOR SELECTORS



*Notes on Numerical Analysis—4
Relaxation and Step-by-Step Methods*

Classical methods for the numerical solution of ordinary differential equations are all of the step-by-step type, directly applicable only to equations with conditions specified at one end of the range of integration. The comparatively recent development of relaxation methods provides a new technique for the solution of differential equations for which conditions are given at two distinct points in the range of integration. We shall describe a combination of the two methods which has proved effective in certain cases, for instance, when the behavior of the solution is exponential in one part of the range and oscillatory in another.

To fix ideas consider a second-order equation, from which the first derivative has been removed, of the form

$$(1) \quad y'' + fy = g,$$

where f and g are functions only of the independent variable x . We can establish a finite-difference equation to replace (1), connecting values of y at three adjacent pivotal points, separated by an interval h , of the form

$$(2) \quad y_{r+1} + y_{r-1} - (2 - h^2 f_r) y_r - h^2 g_r + \Delta(y_r) = 0.$$

Here, Δ , the difference-correction, is given by

$$(3) \quad \Delta = -\delta^4/12 + \delta^6/90 + \dots$$

(We could use a more powerful three term formula, involving a much smaller Δ , but these equations are adequate for our present illustrative purpose.)

An equation of type (2) is to be satisfied at every pivotal point in the range of integration. These equations, moreover, are quite independent of the initial or boundary conditions of the problem, but the latter will determine the method by which such equations are solved.

If the conditions are of the boundary-value type, we shall commonly have specified values y_0 and y_n at the two ends of the range of integration. With initial neglect of Δ , we can therefore solve equations (2) as a set of linear simultaneous equations.

If the conditions are of the initial-value type, we shall be given, or can find fairly easily, two adjacent values y_0 and y_1 at the start of the range. Again, neglecting Δ , we can in this case solve equations (2) by recurrence, obtaining in succession values of y_2, y_3, \dots , etc.

In each case the approximate solution so obtained can be differenced, an approximation to Δ calculated at every point, and with these values inserted as additional terms in equations (2), the respective processes can be repeated to obtain better approximate solutions y . This cycle is carried on until the equations (2) and (3) are accurately satisfied.

Now the linear equations are most easily solved when the coefficients are dominant in the principal diagonal. The use of iteration or relaxation methods is then particularly convenient, and the results are relatively free from error. This dominant diagonal is present whenever the solution is of the exponential type, when f is negative. If the solutions are oscillatory, with f positive, the linear equations are relatively ill-conditioned and their solution much more difficult and less precise.

The recurrence method has precisely the opposite features. In exponential regions there is an uncontrollable growth of error; in oscillatory regions the error is comparatively negligible.

We would therefore like to use relaxation in exponential regions, recurrence in oscillatory regions. In some problems of practical interest the solution has different forms in different regions, and sometimes the given conditions allow us to use the appropriate methods.

Consider, for example, an equation of the form

$$(4) \quad y'' + A(x)y = 0, \quad y(0) = 0,$$

in which $A(x)$ is large and negative near the origin, later increasing, passing through zero, and having an asymptotic value of say unity. The asymptotic solution, for large x , then has the form

$$y \sim a \sin x + b \cos x,$$

and the problem is to determine the ratio a/b .

The solution begins by increasing rapidly, then decreases and begins to oscillate, as shown in figure 1.

The finite-difference expression for (4) is given by

$$(5) \quad y_{r+1} + y_{r-1} - (2 - h^2 A_r)y_r + \Delta(y_r) = 0.$$

Now the coefficient of y , is initially greater than 2 in absolute value, and begins to be less than 2 when $A(x)$ passes through its zero. Let the point at which this occurs be called x_c . The solution is exponential for $x < x_c$, oscillatory for $x > x_c$.

Equation (4) is homogeneous in y , so that y can be given any arbitrary value at one particular point other than the origin. Fix the value $y = y_c$ at $x = x_c$. For $x < x_c$ we then have a boundary-value problem easily soluble

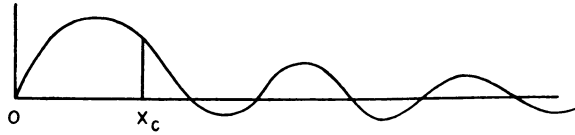


Fig. 1.

by relaxation methods. Let us by this means obtain the first approximation, with Δ neglected, in this range. This solution provides at least two starting values, at x_c and x_{c-h} , from which we can carry forward the solution by recurrence into the oscillatory region.

Since the same equation (5) has been used throughout, the approximation so obtained is "smooth" throughout the entire range. We can therefore difference and calculate Δ everywhere. A better solution can now be obtained in the exponential region by relaxation. The improved value at x_{c-h} , together with the given value at x_c and the difference-corrections, enable us again to carry this better approximation into the oscillatory region, as far as necessary. The process can be repeated until equations (5) are everywhere satisfied.

There is here a nice computational point. The exponential region is completely self-contained, and the final solution, with Δ included, could be obtained before ever proceeding into the oscillatory region. If this were done, however, the accurate exponential solution would not fit well with the first approximate oscillatory solution: there would be some sort of discontinuity at x_c , the differences would not be smooth, and the calculation of Δ uncertain. For this reason the application of Δ is deliberately deferred until it can be effected smoothly over the whole range.

The same technique will help us to throw light on a matter hitherto obscure in relaxation. From a first approximation to the solution of a boundary-value problem the difference correction is not immediately calculable near the ends of the range, since central differences do not exist there.

The first approximation, however, enables us to extend this approximate solution, by recurrence methods, outside the chosen range in both directions (provided, of course, that the function exists outside this range). We can then compute Δ not only inside, but also at points outside the range. The internal values enable us to improve our internal solution, and these new internal values, together with the external Δ 's, allow us to obtain a new solution externally, to the same degree of approximation as the new internal values. The process is repeated until there is no further correction. The number of values determined outside the range depends, of course, on the order of significant differences in Δ .

If the solution is everywhere of exponential type, the external recurrence may not be very accurate, the error at a point growing with its distance from the particular end point. Since these values are required, however, only for the computation of Δ , which has small coefficients multiplying the differences, some error can be tolerated.

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BIBLIOGRAPHY Z-XV

1. ANON., "Digital computer," *Radio Electronics*, v. 21, June 1950, p. 9.

A brief description of General Electric's OMIBAC (Ordinal Memory Inspecting Binary Automatic Calculator), a binary machine having a magnetic cylinder memory particularly useful for repetitive problems.

2. E. C. BERKELEY, "Simple Simon," *Scientific American*, v. 183, Nov. 1950, p. 40-43, illustrs.

The author describes a simple digital relay computer called Simon which was constructed to illustrate the principles of automatic digital computers [*MTAC*, v. 4, p. 239].

M. M. ANDREW

NBSMDL

3. EDMUND C. BERKELEY, "The relations between symbolic logic and large-scale calculating machines," *Science*, v. 112, p. 395-399.

This is an elementary discussion of some of the logical problems arising in constructing and programming for a modern automatic computer. A representative machine, capable of certain simple logical and arithmetical operations is described, and the "program" for calculating square roots on it is discussed in some detail.

MICHAEL MONTALBANO

NBSMDL

4. J. L. HILL, *Design features of a magnetic drum information storage system*, Engineering Research Associates, Inc., St. Paul 4, Minnesota, 1950, 8 p. (Transcript of a paper delivered at the Association for Computing Machinery Conference, March 28-29, 1950, Rutgers University), figs.

An expository article (for a supplementary description of this information storage system see *MTAC*, v. 4, 1950, p. 31-39).

5. CUTHBERT C. HURD, *The Selective Sequence Electronic Calculator of the International Business Machines Corporation*, Carbide and Carbon Chemicals Corporation, Engineering Development Division, Theoretical Analysis Department, Report No. K-134, Feb. 4, 1948, 6 pages, diag.

An expository paper.

6. SHERMAN C. LOWELL, *Digital Computer Research at the T. R. E., Malvern*, Office of Naval Research, London, Technical Report ONRL—35-50, Mar. 29, 1950, 3 p.

The main design features of the Telecommunications Research Establishment computer are described. It is a general-purpose, high-speed, parallel, and binary machine.

7. SHERMAN C. LOWELL, *The Manchester University Digital Computer*, Office of Naval Research, London, Technical Report ONRL—34-50, Mar. 29, 1950, 4 p.

This is an expository article which describes a high-speed, serial, and binary machine and its applications and also a larger machine of similar design being built by Ferranti Brothers. The Ferranti computer will be superior from the engineering and maintenance standpoint.

8. OFFICE OF NAVAL RESEARCH, *Digital Computer Newsletter*, v. 2, no. 4, December 1950, 5 p.

The present status of the following digital computer projects is treated briefly in this number.

1. NRL Computer (NAREC)
2. Moore School Automatic Computer (MSAC)
3. Naval Proving Ground Calculators
4. SEAC (National Bureau of Standards Eastern Automatic Computer)
5. Aberdeen Proving Ground Computers:
 - Coupling of IBM Relay Calculators
 - The ENIAC
6. SWAC (National Bureau of Standards Western Automatic Computer)
7. The Institute for Advanced Study Computer
8. Project Whirlwind
9. The ACE Pilot Model, Teddington, England
10. Data Handling and Conversion Equipment:
 - Digital Reader
 - Digital Data Recording System
 - Analog Digital Converter.

NEWS

American Statistical Association.—On Thursday December 28, 1950, at a meeting of the Association in Chicago, a round table discussion was held which had as its topic, "What can high-speed electronic computing equipment do for and to statistics?" The moderator was WILLIAM G. MADOW, University of Illinois. Mr. S. N. ALEXANDER, NBS, discussed the subject from the viewpoint of an electronic engineer while Mr. B. SCHREINER, A. C. Nielsen Company, talked about the problem as seen by an expert user. A discussion followed by H. C. GRIEVES, Bureau of the Census, and JOHN J. FINELLI, Metropolitan Life Insurance Co. Organizations joining in the meeting were the Psychometric Society, the Institute of Mathematical Statistics, and the Association for Computing Machinery.

Centre National de la Recherche Scientifique.—An international colloquium on “Modern calculating machines and the human mind” was held at the CNRS on January 8 through 13th at Paris. The program for the meeting was as follows:

- First Session, *Recent Technical Progress in Large-Scale Calculating Machines*
- LOUIS DE BROGLIE, Institut Blaise Pascal, Secretary of Academy of Sciences, *Chairman*
- “The Mark II, III, IV Machines” H. H. AIKEN, Director of the Computation Laboratory, Harvard University
- “A magnetic automatic calculating machine” A. D. BOOTH, Birkbeck College, University of London
- “Development of electronic digital computers at the National Bureau of Standards” E. W. CANNON, National Bureau of Standards, Washington, D. C.
- “Research activities of the National Physical Laboratory” E. M. COLEBROOK, National Physical Laboratory, Teddington, England
- “The Institut Blaise Pascal Machine” L. COUFFIGNAL, Director of the Computer Laboratory, Institut Blaise Pascal
- “Mathematical machines in Sweden” STIG EKELOFF, Chalmers Tekniska Högskola, Goteborg
- “Research in progress at the University of Brussels” P. GERMAIN, Université Libre, Brussels
- “Short-cut multiplication in a parallel-decimal automatic calculating machine” E. J. PETHERICK, S. H. HOLLINGDALE, Royal Aircraft Establishment, Farnborough, Hants, England
- Second Session, *Applications of Mathematical and Scientific Problems to Large-Scale Calculating Machines*
- M. CAQUOT, Institut Blaise Pascal, *Chairman*
- “Numerical integration of the wave equation” F. H. VAN DEN DUNGEN, University of Brussels
- “Explanation of numerical methods of integration of systems of linear partial differential equations and the results obtained” M. PICONE, University of Rome
- “Different problems a machine can solve” A. M. UTTLEY, Telecommunications Research Establishment, Worcester, England
- “Les erreurs de chute dans les calculs systematiques” A. VAN WIJNGAARDEN, Mathematische Centrum, Amsterdam
- “Operational experience with the EDSAC” M. V. WILKES, Director, University Mathematical Laboratory, Cambridge
- Third Session, *Large-Scale Calculating Machinery and the Logic and Physiology of the Nervous System*
- L. LAPICQUE, Institut Blaise Pascal, *Chairman*
- “A presentation of the homeostat” W. R. ASHBY, Department of Research, Barnhouse, England
- “Some new analogies between the structure of computers and the structure of the brain” L. COUFFIGNAL, Institut Blaise Pascal, Director, Laboratoire de Calcul Mécanique
- “Large-scale calculating machines and the physiology of the nervous system” H. GASTAUT, Faculté de Médecine de Marseille
- “Mechanical realization of models of the brain structure; presentation of artificial animals” W. GREY WALTER, Burden Neurological Institute
- “Work in Sweden” G. KJELLBÖRG, Tekniska Högskolan, Stockholm

- Discussion
- Discussion
- "Developments in automatic computing machinery in the Spanish school"
- "Présentation des appareils de Leonardo Torrès-Quevedo:
1. le joueur d'échecs automatique
 2. le télékine, premier appareil construit pour le radio-guidage des bateaux
 3. les fusées logarithmiques de la machine à résoudre les équations logarithmiques"
- R. LORENTE DE NO, Laboratories of the Rockefeller Institute
- W. McCULLOCH
- P. PUIG-ADAM, Academy of Sciences, Madrid
- G. TORRÈS-QUEVEDO, Ingénieur des Ponts et Chaussées de Madrid

Panel on Electron Tubes.—The Conference on Electron Tubes for Computers was held in Atlantic City on December 11th and 12th, 1950, under the joint sponsorship of the AIEE and the IRE in collaboration with the Panel. The program was as follows:

- Computer Experience with Electron Tubes*
- "Review of AIEE conference on electron tubes for instrumentation and industrial use"
- "Experience with receiving-type vacuum tubes on the Whirlwind Computer Project"
- "Electron tube experience at IBM"
- "Performance of electron tubes in the ENIAC"
- Electron Tube Problems*
- "Design and operation of tubes for long life"
- "The JETEC approach to the tube reliability problem"
- "Cathode inter-face impedance and its effects in aged vacuum tubes"
- "Cathode impedance and tube failure"
- "6SN7WGE and 6AN5; mutual conductance dispersion and the effect of low duty cycle operation on long life performance"
- "Open discussion in improved electron tubes for computers"
- Special Purpose Computer Tubes*
- "A stable binary electrostatic storage system"
- "Recent experiences with the selective electrostatic memory tube"
- "The MIT storage tube"
- "The development of the Rogers Additron"
- "A proposal for a binary adder tube utilizing beam deflection principles"
- "Special cold cathode discharge tube for counting and switching applications"
- MINA REES, ONR, *Chairman*
- W. R. CLARK, Leeds and Northrup Co.
- E. S. RICH, MIT
- J. A. GOETZ and A. W. BROOKE, IBM
- WRIGHT E. ERION and HOMER W. SPENCE, Aberdeen Proving Ground, Maryland
- S. N. ALEXANDER, NBS, *Chairman*
- J. O. McNALLY, BTL
- J. R. STEEN, Sylvania Electric Products, Inc.
- H. B. FROST, MIT
- L. S. NERGAARD, RCA
- I. LEVY, Raytheon Manufacturing Co.
- A. L. SAMUEL, IBM, *Chairman*
- A. M. CLOGSTON, BTL
- J. RAJCHMAN, RCA
- P. YOUTZ, MIT
- T. VAN DYK, Rogers Majestic, LTD.
- D. H. GRIDLEY, NRL
- M. W. WALLACE and J. HENEY, Federal Telecommunication Laboratories

- "A decimal counting tube" T. R. KOHLER, Philips Laboratories
- "New improvements and applications in Remtron counter tube" FRANK J. COOKE, Remington Rand Corp.
- Tube Manufacture and Crystal Diode Experience* J. G. BRAINERD, University of Pennsylvania, *Chairman*
- "Design and manufacture of electron tubes for electronic computer service" R. E. HIGGS and H. E. STUMMAN, RCA
- "Problems in the manufacture of special tubes for computer usage" R. L. McCORMACK, Raytheon Manufacturing Co.
- "Development of the 7AK7" R. W. SLINKMAN, Sylvania Electric Products, Inc.
- "Some problems involved in the manufacture of germanium diodes" N. DeWOLFF, General Electric Co.
- "Experience with germanium diodes in the SEAC program" H. WRIGHT, NBS
- "Crystal diode life experience in the Whirlwind Computer circuits" H. B. FROST, MIT
- Williams Type Storage* J. H. BIGELOW, Institute for Advanced Study, *Chairman*
- "Tube experience in the SWAC" H. D. HUSKEY, NBS
- "The SEAC memory using Williams storage" W. W. DAVIS, NBS
- "The selection of cathode-ray tubes for Williams storage" J. H. POMERENE, Institute for Advanced Study
- "Methods of testing cathode-ray tubes for service in Williams storage systems" D. FRIEDMAN, NBS
- "Theory of storage in cathode-ray tubes" J. KATES, University of Toronto
- "Progress report on the electron mechanism on the Williams storage technique" A. W. HOLT, NBS
- "Space charge effects in Williams storage tube" L. BRILLOUIN, IBM

OTHER AIDS TO COMPUTATION

A New Differentiating Machine

The first step in the design of a differentiating machine is the selection of a mechanical analogue for the derivative of a plotted function. The incorporation of this mechanical differential ratio into a machine that will draw the derived curve of a plotted function will follow with the use of the proper linkages, gears, etc., that will transfer the relative value of this mechanical $\frac{dy}{dx}$ to a writing pen for tracing the derived curve.

The most common type of differentiating machine that has been built uses a tangent line analogue based on the geometric concept of the derivative. The differentiating machine built in 1904 by J. E. MURRAY¹ employs this tangent line analogue in the form of two dots on a celluloid plate. So long as the dots remain on the curve to be differentiated, the chord connecting them is approximately parallel to the tangent to the curve at the mid point between the dots, then by a system of connecting linkages, a writing pen is caused to draw the derived curve.