187.—NBSMTP *Table of Natural Logarithms.* V. 1, New York, 1941.

P. 184, $x = 18254$ *for* 9.18213 *read* 9.81213

LINDLEY M. WILSON

NBS

188.—WILHELM MAGNUS & FRITZ OBERHETTINGER, *Anwendung der elliptischen Funktionen in Physik und Technik.* Berlin, 1949 [*MTAC*, v. 4, p. 23].

P. 113, col. 4, last line. *For* 1,26928 *read* 1,29628

T. J. HIGGINS
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189.—A. REIZ, "On the numerical solution of certain types of integral equations," *Arkiv Mat., Astr. Fys.*, v. 29A, no. 29, 1943, 21 p. [*MTAC*, v. 3, p. 26.]

On p. 6, Reiz tabulates $\pm x_i$, p_i and α_i, where x_i are zeros of the Hermite polynomials, p_i are $\pi^{-\frac{1}{2}}\cdot$weight factors, and α_i are weight factors$\cdot\exp(x_i^2)$, for $n = 2(1)9$. The following errors of more than a unit in the last (7th) decimal place were found:

	for	*read*
$n = 3$,	1.323931*6*	1.323931*2*
$n = 4$,	1.24022*44*	1.24022*58*
$n = 5$,	1.18148*77*	1.18148*86*
$n = 6$,	1.33584*89*	1.33584*91*
	0.935580*8*	0.935580*6*
	1.136*8912*	1.136*9083*
$n = 7$,	0.89718*39*	0.89718*46*
	1.1013*979*	1.1013*307*
$n = 8$,	1.9816*821*	1.9816*568*
	0.866*8381*	0.866*7526*
	1.071*8011*	1.071*9301*
$n = 9$,	0.841*7403*	0.841*7527*
	1.04*49691*	1.04*70036*

H. E. SALZER
R. ZUCKER
R. E. CAPUANO

NBSCL

UNPUBLISHED MATHEMATICAL TABLES

[EDITORIAL NOTE. The Mathematical Tables Committee of the Royal Society wishes to announce the establishment of a depository of unpublished mathematical tables in the library of the Royal Society. Lists will be published periodically of the tables which have been accepted. The tables will be available for examination in the library and it is proposed to arrange for photo-copies to be supplied as a reasonable charge to those who desire them.

Communications should be sent to the Assistant Secretary, The Royal Society, Burlington House, Piccadilly, London W.1].

116[C, F].—NBSCL—*Radix table for the computation of logarithms to many places using logarithms of small primes.* Preliminary manuscript awaiting final check in possession of NBSCL and deposited in UMT FILE, 45 leaves, hectographed.

This table, which was calculated at the suggestion of J. B. ROSSER, gives the integers a, b, c, d, e, f, and g and the corresponding quantity

$$N = 2^a 3^b 5^c 7^d 11^e 13^f 17^g$$

for about 2000 values of N ranging from 9.996 to about $1 + 2\cdot 10^{-16}$. Exact values of N are given down to $N = 1.155$, thereafter N is given in the form $1 + v$, where v is given to 5S.

The purpose of this table is to facilitate the computation of logarithms to very many places, making use of the fact that the logarithms of 2, 3, 5, 7, 11 and 17 have been given by H. S. UHLER[1] to 330 decimals. The logarithm of 13 was calculated to 328 decimals, using Uhler's values. The logarithm of 37 is obtained as a by-product.

The method of construction and the method of using the table are described in the introduction. The entire computation and most of the preliminary checking was done by ELIZABETH GODEFROY, under the supervision of H. E. SALZER.

[1] H. S. UHLER, "Recalculation and extension of the modulus and of the logarithms of 2, 3, 5, 7, and 17." Nat. Acad. Sci. Washington, *Proc.*, v. 26, 1940, p. 205–212; "Natural logarithms of small prime numbers," *Ibid.*, v. 29, 1943, p. 319–325.

117[F].—BALLISTIC RESEARCH LABORATORY, *Table of Fermat's quotients*, 236 leaves, tabulated from punched cards. On deposit in UMT FILE.

Let $p = p_n$ be the n-th prime and let $\epsilon = \epsilon(p_n)$ be the least positive integer x such that p divides either $2^x + 1$ or $2^x - 1$ (in other words $\epsilon(p)$ is the exponent of 4 modulo p). Finally let $r(p)$ and $R(p)$ denote the least non-negative remainders on division of $2^\epsilon - 1$ by p and p^2 respectively. Then the main table gives

$$n,\ p = p_n,\ p^2,\ \epsilon = \epsilon(p),\ f = (p - 1)/\epsilon,\ r(p) \text{ and } R(p)$$

for $n = 2(1)\,2860$, that is for $p < 26000$. This table was computed in 1949 on the ENIAC by GEORGE REITWIESNER and HOMÉ MCALLISTER during check periods at the suggestion of D. H. L. and is intended to give information about Fermat's quotient[1]

$$q_2 = q_2(p) = (2^{p-1} - 1)/p.$$

This integer is related to R and f by the congruence

$$q_2 \equiv \tfrac{1}{2} fR(R + 2)/p \pmod{p}.$$

The interesting case of q_2 being divisible by p is thus equivalent to R being 0 or $p - 2$. This occurs only twice in the whole table, at $p = 1093$ and $p = 3511$. These exceptional primes were discovered by MEISSNER (1913) and BEEGER (1922); Beeger's calculations[2] are for $p < 16000$. The main table has been rearranged in three other ways, by sorting the cards on which the results were punched, as follows

1) In two parts according as $2^e \equiv 1$ or $-1 \pmod p$
2) According to values of f
3) According to values of f in each of the cases $2^e \equiv 1$ or $-1 \pmod p$.

[1] See L. E. DICKSON, *History of the theory of numbers*. V. 1, Washington, 1928, Chapter 4.
[2] N. G. W. H. BEEGER, "On a new case of the congruence $2^{p-1} \equiv 1 \pmod{p^2}$," *Messenger Math.*, v. 51, 1922, p. 149–150.
"On the congruence $2^{p-1} \equiv 1 \pmod{p^2}$ and Fermat's last theorem," *Nieuw Archif v. Wiskunde*, s. 2, v. 20, 1939, p. 51–54.

118[F].—A. GLODEN, *Tables des solutions des congruences* $X^{2^n} + 1 \equiv 0 \pmod p$ $n = 4, 5, 6$; $p < 10^4$. Typewritten manuscript, 8 leaves. Deposited in UMT FILE.

All solutions $<p/2$ of the congruences

$$x^{16} \equiv -1, \qquad x^{32} \equiv -1, \qquad x^{64} \equiv -1 \pmod p$$

are given for those primes $p < 10^4$ for which such congruences are possible.

119[F].—A. GLODEN, *Tables de décomposition des nombres premiers* $8k + 1$ *dans l'intervalle* 350000–600000 *en* $a^2 + b^2$ *et* $2c^2 + d^2$ *et tables des solutions des congruences* $l^2 + 1 \equiv 0$ *et* $2m^2 + 1 \equiv 0$ *pour ces mêmes nombres premiers*. Handwritten manuscript of 97 leaves, deposited in UMT FILE. The original manuscript deposited in the Bibliothèque National de Luxembourg.

This is an extension of the author's UMT **113** [*MTAC*, v. 5, p. 28]. The author plans a further extension to 800000.

120[F].—R. M. ROBINSON, *Stencils for the solution of systems of linear congruences modulo* 2. Box of 1024 IBM punched cards on deposit in the UMT FILE. Copies of these stencils may be obtained at cost from the Computation Laboratory, University of California, Berkeley 4, California.

Any system of 9 or fewer linear congruences may be solved modulo 2 by simply superposing and sighting 9 or fewer punched cards selected from the set. This set of stencils is used in the application of a sieve method for the solution in integers of systems of linear equations devised by H. LEWY.

121[K].—D. L. GILBERT, *Levels of significance—a direct table of the two-tail probabilities*. 38 leaves, mimeograph manuscript deposited in the UMT FILE.

This is a direct tabulation of the areas in the tails of the Student-Fisher distribution, that is, of

$$P = P(|u| \geq t) = 1 - n^{-\frac{1}{2}}(B(\tfrac{1}{2}n, \tfrac{1}{2}))^{-1} \int_{-t}^{t} (1 + u^2/n)^{-(n+1)/2} du.$$

Values of $100P$ are given to 1D for $n = 1(1)35$ and $t = 0(.01)5.99$. There is a supplementary table for $n = 1, 2, 3$ which gives the ranges of values of t to 2D for which $P(u \geqq t) = 0(.1)5.5$. Previously published tables of direct values (most tables, including the recent ones, give values of t for which $P(|u| \geqq t)$ has selected values) have proceeded by steps of .1 for t, the two principal ones being due to W. S. GOSSETT ("Student") [*Metron*, v. 5, No. 3, 1925, p. 105–108], which in its main part gives values to 4D for $n = 1(1)20$ and $t = 0(.1)6$, and to KARL PEARSON [*Tables for statisticians and biometricians*, Part I, 3rd edition, London, 1930, p. 36]. The present tables were calculated by linear interpolation for $0 \leqq t \leqq 6$ and $1 \leqq n \leqq 20$ from Gossett's 1925 tables; for higher values of t and n, approximate methods given by Gossett were used. [Some spot checking by direct calculation of the corresponding incomplete β-functions for $t = 1(.01)1.09$ with $n = 1$ and 10 gave in 8 cases values differing from Gilbert's by 1 in the third decimal place. For a more critical value $n = 20$, $t = 0.5$ a direct evaluation of the incomplete β integral confirmed Gilbert's value of 0.623. C. C. C.]

AUTOMATIC COMPUTING MACHINERY

Edited by the Staff of the Machine Development Laboratory of the National Bureau of Standards. Correspondence regarding the Section should be directed to Dr. E. W. CANNON, 225 Far West Building, National Bureau of Standards, Washington 25, D. C.

TECHNICAL DEVELOPMENTS

Our contribution under this heading, appearing earlier in this issue, is "The California Digital Computer," by PAUL L. MORTON.

DISCUSSIONS

"Floating Decimal" Calculation on the IBM Card Programmed Electronic Calculator

There is a wide variety of problems occurring in run-of-the-mill computing in which it is extremely helpful to represent numbers x as $x_0 \cdot 10^p$ where x_0 has the same significant digits as x within the capacity of the machine used, $1 \leqslant x_0 < 10$, and p is an integer. Three types we might mention are:

1) Problems in which certain computed quantities have magnitudes difficult to estimate.
2) Problems in which one or more quantities have such a wide range of magnitude that no single fixing of the decimal point will suffice for the entire range.
3) Problems in which the setup time involved in estimating magnitudes is not justified by the saving in machine time made possible by fixed decimal calculation.

For application of the IBM Card Programmed Electronic Calculator to such problems, we have devised a calculator programming of the general-purpose type based on the "floating decimal" representation of numbers. A brief explanation of the way the machine is instructed will help in an understanding of what follows.

The calculating unit (IBM 604) of the Card Programmed Electronic Calculator is, for one step in the calculation, instructed to receive numbers