reliability of the execution of every one of the computer's commands. The second part required the computer to sort a number of items obtained as a result of the operations in the first part, as well as to solve a rather complicated partial differential equation. In the last part, the tapes of the four Uniservos were run through a difficult routine of a series of movements, delivering and recording information. Two such tests, requiring twenty minutes for a flawless performance, were designated as a unit. Out of the successive nineteen units which the UNIVAC performed during the test period, sixteen were carried out perfectly; in the three units where stops occurred due to the action of the error-detection circuits, the operator was able to rectify them by the mere manipulation of the console buttons.

2. A Uniprinter test requiring a sentence and a numerical table to be printed out in proper form 200 times. This test lasted eight hours, during which five errors occurred, whereas the conditions of the test allowed eight.
3. A Card-to-Tape-Converter test, which was passed satisfactorily. Although required to process a deck of 10,000 very badly punched cards, the Converter made only one mistake while storing this information repeatedly on ten tape reels.
4. A general test, with emphasis on Uniservo performance. Various aspects of tape movements were under scrutiny such as any tendency of the tapes to move out of position during prolonged inactivity, the efficiency of flaw-detection in the magnetic coating of the tapes, the resistance of this coating to a continuous rerunning of the tape under the reading heads for 700 times, the effect that various juxtapositions of tape commands might have on their performance, and several other points of possible weakness. This test lasted ten hours during which six errors were detected by the machine.

The machine will remain on the premises of the Eckert-Mauchly Computer Corporation for about a year, performing computations for the Bureau of the Census and operating 24 hours a day.

OTHER AIDS TO COMPUTATION

Bibliography Z–XVI

10. B. P. Bogert, "A network to represent the inner ear," *Bell Lab. Record*, v. 28, 1950, p. 481–485.

An electrical network consisting of 175 sections is used to simulate the cochlea which is believed to be the frequency sensing portions of the ear. The results of measurements on this simulator are compared for a number of frequencies with the results computed from the theory developed in a paper by Peterson & Bogert.[1] The simplified model upon which a hydrodynamical theory is based consists of two parallel ducts separated by a movable membrane of varying width. Measurements on the electrical network are compared with the numerical solution of the equations of wave propagation in this model, in regard to the amplitude of the response of the membrane at different points along its length to different frequencies. The response patterns vary considerably with the frequency but good agreement was obtained between the network, the theoretical model and certain experimental results for the actual cochlea.

F. J. M.

[1] L. C. Peterson & B. P. Bogert, "A dynamical theory of the cochlea," Acoust, Soc. Amer., *Jn.*, v. 22, 1950, p. 369–381.

11. J. W. CLARK & R. E. NEUBER, "A dynamic electron trajectory tracer," I.R.E., *Proc.*, v. 38, 1950, p. 521–524.

In the gravitational analog of a vacuum tube, a surface reproduces the electric potential in the tube and electrons are simulated by metal balls. By vibrating this surface, the authors reproduce the effect of the application of radio frequency voltages to various electrodes. Relatively low mechanical frequencies may be used. The results are recorded by a stroboscopic photographic process. A number of such photographs are given as illustrations.

F. J. M.

12. H. L. CURTIS, "Determination of curvature by an osculometer," NBS, *Jn. Research*, v. 44, 1950, p. 131–134.

The radius of curvature of a curve is determined graphically by means of a series of pairs of arcs of known curvature drawn on transparent paper. The user tries to arrange a pair of arcs symmetrically on both sides of the given curve. "The curvature at a point can usually be determined to 10^{-3} reciprocal centimeters, provided the curvature is approximately constant for a distance of 2 cm. . . ." A discussion is given for the graphical determination of accelerations by this method.

F. J. M.

13. T. S. GRAY & H. B. FREY, "Acorn diode has logarithmic range of 10^9," *Rev. Sci. Inst.*, v. 22, 1951, p. 117–118.

The diode type 9004 with a d.c. filament voltage supply of 4 volts has a logarithmic current characteristic between 10^{-4} and 10^{-13} amperes for the plate voltage range of 0 to -1.75 volts.

F. J. M.

14. H. IAMS, "A method of simulating propagation problems," I.R.E., *Proc.*, v. 38, 1950, p. 543–545.

Propagation problems can be effectively simulated by propagating centimeter wavelength radio waves in large sheets of low loss dielectric materials. The conditions of the problem are simulated by imbedding objects in the sheet or utilizing the effect of varying the thickness of the sheet to simulate a change in refractive index. The propagation is recorded by means of a "phase front plotter" which records the lines of equal phase. A coupling set up to simulate waves propagated from a large distance is described.

F. J. M.

15. G. A. KORN, "Stabilization of simultaneous linear equation solvers," I.R.E., *Proc.*, v. 37, 1949, p. 1000–1002.

The author gives an experimental test involving only one equation on an amplifier such that stability in this test will permit the amplifier to be used in a linear equation solver of the Goldberg-Brown type for any number of unknowns. A discussion of this paper, by L. A. ZADEH and the author, will be found in I.R.E., *Proc.*, v. 38, 1950, p. 514.

F. J. M.

16. G. Liebmann, "Solution of partial differential equations with a resistance network analogue," *Brit. Jn. Appl. Phys.*, v. 1, 1950, p. 92–103.

The author describes a resistance network analogue which can be used for the solution of Laplace's equation. The author points out that such a network may be considered as a computing machine for carrying out the finite difference method. The expected accuracy of the resistance network solutions is influenced by the finite mesh size and by resistors which deviate from the theoretical value. The error caused by finite mesh size can be greatly reduced by an extrapolating method due to Richardson. This method is briefly discussed in the paper. An examination of the errors due to deviations in resistance values follows but in order to evaluate this error the author must assume random distribution of deviations from the theoretical value. With this assumption he finds that the errors are exceedingly small. However, since the distribution of resistance deviations cannot usually be predicted definitely, the method of the author does not permit the determination of the maximum error incurred with a network.

In describing the design of a network he presents a valuable method of overcoming the difficulty of transition from coarser to finer lumping (his Figure 11). The actual description of a network with which the author works shows that he has assembled more than 2600 resistors. The description is all too brief and gives the impression that the author has designed a network with fixed resistance values instead of using as frequently done resistors which could be set to desired values. Using fixed resistors would seriously limit the applicability of his apparatus. He uses alternating current as power supply and an oscilloscope for balance indicating. It would appear that direct current would be experimentally easier. The bibliography overlooks an important British publication which obviates the necessity in the case of a study of cylinders to replace a solid cylinder by a hollow one.[1] Moreover, he disregards the considerable literature on such networks used in this country. The Heat and Mass Flow Analyzer Laboratory at Columbia University has issued over the last 10 years some 50 papers all dealing with work on a resistance capacitance network which of course is a more inclusive system than the one dealt with in the paper under review. A reasonably complete bibliography of this laboratory up to 1947 is contained in reference.[2]

V. Paschkis

Columbia University
New York, N. Y.

[1] R. Jackson, R. J. Sargant, J. B. Wagstaff, N. R. Eyres, D. R. Hartree & J. Ingham, "Variable heat flow in steel," Iron & Steel Inst., *Jn.*, v. 150, 1944, No. 2, p. 211–268.

[2] V. Paschkis, "The heat and mass flow analyzer—tool for heat research," *Metal Progress*, v. 52, 1947, p. 813–818.

17. W. S. McEwan & S. Skolnik, "An analog computer for flame gas composition," *Rev. Sci. Inst.*, v. 22, 1951, p. 125–132.

A problem in chemical equilibrium reduces to the problem of solving a system of equations, some of which are linear in the unknowns, others are ratios which may involve square roots of the unknowns. The device described realizes the unknowns as resistances and the equations are set up as Wheatstone bridge circuits. While ganged potentiometers are used in certain instances, a switching system permits one to realize various different equations with the same resistance as unknowns in most cases. The device is manipulated until all the bridge circuits are in equilibrium. The manipulation is guided by certain physical information.

F. J. M.

18. A. MANY, "An improved electrical network for determining the eigenvalues and eigenvectors of a real symmetric matrix," *Rev. Sci. Inst.*, v. 21, 1950, p. 972–974.

An electrical network for the determination of the characteristic roots and vectors of a symmetric matrix of order ten, consisting of ten LC-circuits coupled to each other by equal condensers, is described. (The characteristic roots correspond to the resonant frequencies of the LC-circuits.) A similar but smaller and less accurate machine was described by MANY & MEIBOOM.[1] In the present paper the effects of unequal self-inductance of the coils of the tank circuits is compensated for. The stray capacitance of the wiring of the network is also compensated for by trimming the coupling condensers. Two important factors affecting the accuracy of the eigenvalues are: (1) Accuracy with which the matrix is realized, and (2) Losses in the coils. The above discussion compensates for (1). Using some results of the earlier paper, it is pointed out that the accuracy for matrices with reasonably well separated roots is mainly determined by (1) while for matrices with close eigenvalues, (2) predominates. In the first case, the accuracy obtained was about 0.01% of the difference of the greatest and smallest roots.

K. S. MILLER

New York, N. Y.

[1] A. MANY & S. MEIBOOM, "Electrical network for determining the eigenvalues and eigenvectors of a real symmetric matrix," *Rev. Sci. Inst.*, v. 18, 1947, p. 831–836.

19. R. W. MARSHALL, "An integraph for semicurvilinear coordinate paper," *Bell Lab. Record*, v. 28, 1950, p. 50–52, 65.

When a function $y(t)$ is obtained from the output of a customary recording instrument, it is plotted on curvilinear paper, i.e., the output is measured along an arc of a circle of fixed radius. The device described in this article has an input arrangement which changes this displacement to a linear displacement. This linear displacement is fed into a CORADI integraph which plots a curve corresponding to the integral of the original function.

F. J. M.

20. G. E. REYNOLDS, *A New Method for Square Root Evaluation, Particularly to Eight Significant Figures with Monromatic Calculator.* Publication E4077 Unclassified. 22 p. Air Force Cambridge Research Laboratories, Cambridge, Mass., 1951. 21.4 × 27.6 cm.

The booklet's introduction states that it stems from ideas presented by R. C. Spencer in unpublished memoranda of 1948, and also that with some changes the method is adaptable to the Fridén and Marchant calculators, as outlined in an appendix.

By use of a compact optimum-interval table of certain squares of selected numbers as well as of 4-times the numbers (in an adjacent column), a root with error not exceeding 5 in 9th place is obtained by comparatively simple calculator work. This results from combining two Newton-type iterations to a square root; i.e., the mean of quotient and divisor after dividing a number whose root is desired N by an approximate root A obtains a root substantially correct to twice the number of digits that are correct in the first approximation of the root. This is the first time so far as the reviewer knows that two successive Newton iterations have been combined for calculator usage as a continuous process. Two divisions are required, as is to be expected.

The optimum-interval table used has 138 lines of which the following are typical:

APPROX.	DIV.
11.8336	13.76
12.2500	14.00
12.6736	14.24

The left-hand number is the square of a first approximation of $N^{\frac{1}{2}}$, or A^2, and the right-hand number is $4A$, which are applied as follows:

$$(N + A^2)/4A + 0.25N/((N + A^2)/4A) = N^{\frac{1}{2}} + \epsilon$$

The selection of A^2 and $4A$ values for the table is so made that ϵ does not exceed 5 in 9th place of $N^{\frac{1}{2}}$.

The approximation amount A^2 may be either the tabular value above or below N when the latter is pointed off in the manner of the tabular values.

The method of obtaining the tabular intervals so as to minimize the size of the table is given, as are curves from which the intervals may be determined for preparation of tables for obtaining 7-, 8-, or 9-figure roots, though only the table for 8-figure roots is given. A feature of this part of the explanation is that a complicated expression involving the interval between tabular values and the approximate root for various table ranges reduces to an exceedingly simple form. The final formula used for calculator work, as above, is notable because the quotient of the first division becomes the divisor of the second. In the case of the Monromatic Calculator, a single entry of N suffices for both terms, it being held in the storage register for multiplication by 0.25 to obtain the numerator of the second term.

The appendix states that the method is applicable to late-model Fridéns without change of formula, but a change with an additional step is recommended for the Marchant. The reviewer submits that a change is not required in this instance because even though the first quotient remains in the register so the second quotient may be accumulated, the keyboard entry of N for starting the second term may still be multiplied by 0.25 to obtain $0.25N$ in the product register, provided N is then cleared from the keyboard and a negative entry of 0.25 is made in the multiplier keyboard to restore the original quotient. This is readily done on any Marchant having automatic multiplication.

The proposed machine method requires that certain exceptions have to be made to the general rule that the first significant figure of all entries is to be made at the extreme left of keyboard; no decimal markers apparently are used. The reviewer suggests that unless extreme-left entries can be made without exception, it is better practice and requires less attention by the operator if there is a suitable decimal setup, and all entries are made with reference to it. As the table shows its amounts pointed off in the range 1–100, any N may similarly be pointed off without loss of generality; i.e., 82547.355 would be considered as 8.2547355. A suitable decimal setup then would be: counting register, at left of 8; product register, at left of 16; and keyboard, at left of 8; set No. 9 tab key. This setting accommodates at least 9-figure N's and roots to 9 digits as well as intermediate amounts during the solution; an 11th dial in the counting register is not required.

T. W. Simpson

66 Alvarado Road
Berkeley, Calif.

21. R. Roscoe, "Mechanical models for the representation of visco-elastic properties," *Brit. Jn. Appl. Phys.*, v. 1, 1950, p. 171–173.

The author establishes the existence of two canonical forms for the networks of springs and dashpots used to represent visco-elastic properties. One form is parallel, the other is essentially a series arrangement of parallel pairs of springs and dashboards with a simple series addition. The author's discussion is based on the analogy with resistor-capacity circuits for which the corresponding result is known. (Cf. Guillemin, "Communication Networks," v. 2, Chap. V, p. 184–221, John Wiley and Son, New York, 1947.)

F. J. M.

22. L. H. Wilson & A. J. Miles, "Application of the membrane analogy to the solution of heat induction problems," *Jn. Appl. Phys.*, v. 21, 1950, p. 532–535.

The authors describe the application of the membrane analogy to the solution of heat conduction problems. Mathematical justification is obvious from the identity of the equations and is dealt with only briefly. In order to carry out the technique, a membrane is created by using a soap film. The deviation of distance of various parts of the membrane from a reference point is proportional to the deviation of temperature in an equally shaped plane from the temperature of the corresponding reference point. The membrane is stretched across appropriate boundaries; for example, if heat flow through a hollow cylinder is to be studied, the two surfaces of which are at different temperatures, an annular membrane must be stretched between two cylindrical bodies serving as boundaries and placed at different levels according to the different temperatures of the outside and inside surface of the cylinder. The authors show two examples of comparison of determinations carried out by this method against theoretical curves. In one case the check is very good and seems to be better than 1%. The other example shows deviations which are, in part, as large as 10%.

Whereas the technique undoubtedly has some merits, the following serious limitations are not discussed and not even mentioned in the paper. First, in many thermal problems, the surface temperatures are not given; instead the temperature of the ambient is known and a boundary conductance applies, which may or may not be introduced as a function of the surface temperature. Application of such boundary conductances seems impossible with the present technique. Secondly, in many steady-state problems two or more materials of different conductivity enter the picture. Application of the membrane technique to such cases does not seem possible. The authors mention the desirability of investigating the membrane analogy in the realm of transient-heat conduction problems. There is no indication either on a mathematical or experimental ground that such approach is possible.

V. Paschkis

Columbia University
New York 27, N. Y.

NOTES

127. An Iterative Process.—In a recent paper Hartree[1] calls attention to the need for the development of a general method of obtaining iteration formulas of second and higher orders for the solution of any "algebraic" (i.e., not differential) equation. If $y = Y$ is the true solution of $F(y) = 0$, and $y_n = Y + \eta_n$ is an approximation to Y, the formula $y_{n+1} = G(y_n)$ is an iterative process. If $\eta_{n+1} = O(\eta_n{}^r)$, Hartree calls the process r-th order. The number of correct figures in y_{n+1} is approximately r times the number in y_n, so it is obvious why a high-order process is desired.

The Newton-Raphson process, based on

$$G(y) = y - F(y)/F'(y)$$

is second order, as Hartree points out. In effect, this process draws a tangent to the curve $Z = F(y)$ at $y = y_n$, and takes y_{n+1} at the intersection of this tangent with the y-axis. The primary source of error is the curvature of F, and any operation that reduces the curvature will improve the convergence of the iteration. If we write

$$H(y) = F(y)\{F'(y)\}^{-\frac{1}{2}}$$

we find

$$H''(y) = -\tfrac{1}{2}F(y)\{F_0''(y)(F'(y))^{-3/2}\}'.$$

The function $H(y)$ consequently has the same roots as F, and zero curvature at each of them. If we apply the Newton-Raphson process to it we get

$$G(y) = y - 2FF'/(2F'^2 - FF''),$$

and it is easily verified that this process is third-order.

As an example, if $F(y) = y^2 - a$ we get

$$y_{n+1} = y_n(y_n{}^2 + 3a)/(3y_n{}^2 + a)$$

as a third-order process for computing $a^{\frac{1}{2}}$. For $a = 10$, $y_0 = 3$, we find $y_1 = 3.16216$, $y_2 = 3.162277660168341$, which is in error by 4 units in the fifteenth figure.

William B. Jordan

General Electric Co.
Schenectady, N. Y.

[1] D. R. Hartree, "Notes on iterative processes," Camb. Phil. Soc., *Proc.*, v. 45, 1949, p. 230.