128[L].—G. JONES & D. UFFORD, *Table of the Functions* $C(P) = K_1(P)/[K_1(P) + K_0(P)]$ *and of* $PC'(P)$. Lithographed manuscript, 4 leaves, available as in UMT **126**.

K_1 and K_0 are the usual Bessel functions. The tables give $C(P)$ and $-PC'(P)$ to 7D for $P = 0(.002).1(.01).3(.02)1(.1)2(.5)10(10)100$, and also for $P = .35(.05).95$.

129[L].—Y. L. LUKE & D. UFFORD, *Tables of the Function* $\bar{K}_0(x) = \int_0^x K_0(t)\,dt$. Lithographed manuscript, 3 leaves, deposited as in UMT **126**.

The table gives 8D values of $\bar{K}_0(x)$ and of the auxiliary functions $A_1(x)$ and $A_2(x)$ defined by

$$K_0(x) = (\ln 2 - \gamma - \ln x)A_1(x) + A_2(x)$$

for $x = 0(.01).5(.05)1$.

130[L].—UNIVERSITY OF TORONTO COMPUTATION CENTRE, *Tables of Spherical Bessel Functions for Semi-imaginary Argument.* Photo copy, 2 leaves deposited in UMT FILE.

The tables give 8S values of the real and imaginary parts, absolute values and arguments of

$$(2x/\pi)^{-\frac{1}{2}}e^{-\pi i/4}J_{n+\frac{1}{2}}(xe^{\pi i/2})$$
$$(2x/\pi)^{-\frac{1}{2}}e^{-\pi i/4}Y_{n+\frac{1}{2}}(xe^{\pi i/2})$$

for $n = 0, 1, 2, 3$; $x = 0(1)10$.

AUTOMATIC COMPUTING MACHINERY

Edited by the Staff of the Machine Development Laboratory of the National Bureau of Standards. Correspondence regarding the Section should be directed to Dr. E. W. CANNON, 415 South Building, National Bureau of Standards, Washington 25, D. C.

Notes on Numerical Analysis—5

Table-Making for Large Arguments. The Exponential Integral

The evaluation of a function defined by a definite integral, for the complete range of argument $-\infty$ to $+\infty$, is usually performed in several stages. For small and moderate values of the argument x the integral is evaluated by means of an ascending series in powers of x, or perhaps by numerical quadrature. For very large values of x, numerical values are obtained by means of an asymptotic series.

The exponential integral, for example, defined by the equations

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} t^{-1}e^{t}\,dt$$

(1)

$$-\mathrm{Ei}(-x) = \int_{x}^{\infty} t^{-1}e^{-t}\,dt$$

has been tabulated by the use of ascending series,

$$\text{(2)}\qquad \begin{aligned} \mathrm{Ei}(x) &= \gamma + \log x + \sum_{n=1}^{\infty} \frac{x^n}{n\cdot n!} \\ -\mathrm{Ei}(-x) &= -\gamma - \log x + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n\cdot n!} \end{aligned}$$

for a range of x up to about 15, and from the asymptotic series

$$\text{(3)}\qquad \begin{aligned} \mathrm{Ei}(x) &\sim x^{-1}e^{x}(1 + 1!x^{-1} + 2!x^{-2} + \cdots) \\ -\mathrm{Ei}(-x) &\sim x^{-1}e^{-x}(1 - 1!x^{-1} + 2!x^{-2} - \cdots) \end{aligned}$$

for larger values of x.

In some cases solutions for moderate values of x are obtained by numerical solution of the differential equation satisfied by the function, though this method does not appear to have been used for the exponential integral.

The use of the ascending series is rather cumbersome for $|x| > 10$, and the asymptotic series may not, without further refinement by the "converging factor" method of AIREY,[1] provide the required number of figures. This note gives a method for extending the tabulation, without any use of asymptotic series, to cover the whole range of x from $-\infty$ to $+\infty$. The method is not restricted to the exponential integral, but the latter provides a useful illustration.

The extension to large values of x is most easily performed by using an argument $z = 1/x$, and it is easy to show that the auxiliary function T, defined by the relation

$$\text{(4)}\qquad T = e^{-x}\mathrm{Ei}(x) - x^{-1} = e^{-1/z}\mathrm{Ei}(z^{-1}) - z$$

satisfies the differential equation

$$\text{(5)}\qquad T'' - (z^{-4} - 2z^{-3})T + z^{-2} = 0$$

where dashes denote differentiations with respect to $z = 1/x$.

We can also show that the Taylor series at $z = 0$ has the form

$$\text{(6)}\qquad T = z^2 + 2!z^3 + \cdots + n!z^{n+1} + \cdots$$

and is divergent for all $z > 0$. In spite of this fact we can use finite-difference equations and numerical integration to solve equation (5).

We assume that values of T are available, from previous computation with the ascending series, at $z = \pm 0.1$ ($x = \pm 10$), and we shall fill in the values for $z = -0.1(.01) + 0.1$ by the use of relaxation methods, producing by this means a table from which both $\mathrm{Ei}(x)$ and $-\mathrm{Ei}(-x)$ can be quickly obtained from the relation

$$\text{(7)}\qquad \mathrm{Ei}(1/z) = e^{1/z}(T + z)$$

For numerical purposes equation (5) is replaced by a set of difference equations to be satisfied at every pivotal point in the range of integration and typified by

$$\text{(8)}\qquad \begin{aligned} T_1 + T_{-1} - \{2 + h^2(z_0^{-4} - 2z_0^{-3})\}T_0 + h^2z_0^{-2} + \Delta(T_0) &= 0 \\ \Delta(T_0) = (-\delta^4/12 + \delta^6/90 - \cdots)\, T_0 & \end{aligned}$$

At $z = 0$, T is zero and no further equation is needed, though it is easy to show from (6) and (8) that the final solution should satisfy the equation

$$T_1 + T_{-1} - 2h^2 + \Delta(T_0) = 0 \text{ at } z = 0.$$

The set of equations (8) is ideal for relaxation or any iterative process, as can be seen from the numerical values of the coefficients of T_0, for $h = 0.01$, which are given in the following table:

z	coeff. of T_0	z	coeff. of T_0
.01	9802	−.01	10202
.02	602	−.02	652
.03	118 + 4/81	−.03	132 + 70/81
.04	37 + 15/16	−.04	44 + 3/16
.05	16 + 2/5	−.05	19 + 3/5
.06	8 + 64/81	−.06	10 + 52/81
.07	5 + 1397/2401	−.07	6 + 1796/2401
.08	4 + 13/256	−.08	4 + 213/256
.09	3 + 1639/6561	−.09	3 + 5239/6561

The relaxation process follows familiar lines. A first approximation $T^{(0)}$ is obtained by neglecting Δ, an approximation to which is computed from the differences of $T^{(0)}$. The "missing" differences near the ends of the range are filled in by the methods of the previous note. (See *MTAC*, v. 5, p. 92–95.)

Two applications of the iterative process, the first of which produced a maximum change in T of 200, the second of 15 in the ninth decimal place, gave the results shown in the table appended.

From these values we can obtain by subtabulation a table from which $\mathrm{Ei}(x)$ and $\mathrm{Ei}(-x)$ can be computed for any $|x| \geq 10$. The only other table known to exist in this range is that of COULSON & DUNCANSON,[2] in which interpolation is by no means trivial.

It is also interesting to consider other numerical methods. The powerful method VII of FOX & GOODWIN,[3] for example, replaces equations (8) by

$$\begin{aligned}(1 - h^2f_1/12)T_1 + (1 - h^2f_{-1}/12)T_{-1} - (2 + 5h^2f_0/6)T_0 \\ + h^2(5g_0/6 + g_1/12 + g_{-1}/12) + \Delta(T_0) = 0 \\ \Delta\,(T_0) = \delta^6/240 - 13\delta^8/15120 + \cdots\end{aligned} \tag{9}$$

where

$$f = z^{-4} - 2z^{-3}, \qquad g = z^{-2}.$$

Only two cycles of the iterative process are now required; but the coefficients of T_1 and T_{-1} are no longer unity, and heavier arithmetic is called for in the relaxation. Equations (9) have been applied as a check, however, and indicate that the last figure given is nowhere in error by more than one unit. For small values of z, furthermore, more significant figures could be obtained trivially, a result of the large coefficient of T_0 in the finite-difference equations.

We also considered the solution by step-by-step methods of the first order equation

$$T' - z^{-2}T + 1 = 0. \tag{10}$$

Method II of Fox & Goodwin[3] produces the recurrence relation

$$\begin{aligned}(1 - \tfrac{1}{2}hz_1^{-2})T_1 = (1 + \tfrac{1}{2}hz_0^{-2})T_0 - h + \Delta = 0 \\ \Delta = (-\delta^3/12 + \delta^5/120 - \delta^7/840 + \cdots)T_{\frac{1}{2}}.\end{aligned} \tag{11}$$

Starting with $T = 0$ at $z = 0$ and integrating outwards, we run into trouble associated with the fact that the coefficient of T_1 passes through zero. Other methods have similar disadvantages, and indeed the presence of the large coefficients of T_0 in equations (8) suggests immediately that step-by-step methods will be difficult, relaxation relatively easy.

If we apply step-by-step methods using Taylor series, we soon run into trouble in working outwards from $z = 0$, owing to rapid growth of the complementary function introduced by rounding-off errors. Since it is, however, possible to work inwards from $z = 0.1$, this method was used as a spot check.

An independent computation has been performed by T. VICKERS, who used asymptotic series and interpolation in existing tables to produce values of the function

$$S = xe^{-x}\mathrm{Ei}(x) = xT + 1$$

in our previous notation. The limitations of the asymptotic series make it difficult to obtain more than seven decimals in S, effectively one less than we obtained in T. Vickers points out, however, that the differences of S converge more rapidly than those of T; in particular linear interpolation in S can be performed with an error of less than 2 units in the fourth decimal place.

Tables of S and T, with facilities for interpolation, are given in Table 1.

L. FOX & J. C. P. MILLER

NBSCL

TABLE 1

x^{-1}	$10^9 T$	δ_m^2	γ^4	S	δ_m^2
−.10	8436666	123964	0	0.9156333	1992
−.09	6936157	129007	0	0.9229316	2107
−.08	5564725	134430	0	0.9304409	2238
−.07	4327803	140290	0	0.9381742	2379
−.06	3231260	146634	1	0.9461457	2533
−.05	2281454	153538	1	0.9543709	2711
−.04	1485301	161068	1	0.9628675	2904
−.03	850351	169333	1	0.9716550	3124
−.02	384890	178446	1	0.9807555	3375
−.01	98058	188554	1	0.9901942	3663
.00	0	199858	1	1.0000000	3994
.01	102063	212593	2	1.0102063	4383
.02	417046	227108	2	1.0208523	4848
.03	959551	243875	3	1.0319850	5414
.04	1746477	263623	4	1.0436619	6124
.05	2797795	287587	7	1.0559559	7080
.06	4137929	318250	10	1.0689655	8467
.07	5798138	358767	11	1.0828305	10387
.08	7819020	409516	6	1.0977378	12551
.09	10250440	465725	−3	1.1138938	14372
.10	13147020	518852	−12	1.1314702	15237

Formulae

$$T = e^{-x}\,\mathrm{Ei}\,(x) - \frac{1}{x}$$

$$S = xe^{-x}\,\mathrm{Ei}\,(x)$$

Interpolation

$$f_n = (1-n)f_0 + nf_1 + E_0{}^2\delta_{m0}{}^2 + E_1{}^2\delta_{m1}{}^2 + M_0{}^4\gamma_0{}^4 + M_1{}^4\gamma_1{}^4$$

(See BAAS *Table II*, Auxiliary Table I, for definitions and values of E and M.)

EDITORIAL NOTE: It is easy to find the value of $T(-x)$ for x positive and large, by use of the Laguerre approximate quadrature. We have

$$T(-x) = e^x\mathrm{Ei}(-x) + x^{-1} = x^{-1} - \int_0^\infty e^{-t}(x+t)^{-1}dt.$$

Now, approximately,

$$\int_0^\infty (x+t)^{-1}e^{-t}dt = \sum_{i=1}^{n} \alpha_i^{(n)}[x_i^{(n)} + x]^{-1} \tag{1}$$

where the $x_i^{(n)}$ are the zeros of the Laguerre polynomial and the $\alpha_i^{(n)}$ are the corresponding Christoffel numbers. These have been tabulated by SALZER & ZUCKER.[4]

The error in the formula (1) above[5] is the value of

$$E_n = \frac{(n!)^2 d^{2n}}{(2n)!dt^{2n}}(x+t)^{-1} = (n!)^2(x+t)^{-2n-1}$$

for some t, $0 < t < \infty$. This error is certainly less than

$$E_n = (n!)^2/x^{2n+1}$$

and since

$$E_{n+1}/E_n = (n+1)^2x^{-2}$$

the best n to use is $n = [x]$ which gives

$$E_n \sim 2\pi e^{-2n}$$

which is about 10^{-8} in the case $x = 10$. The accuracy obtained is therefore comparable with that of the above table for $z = .1$ and $z = .09$ and better when $z \geq .08$.

This method will be as convenient in the evaluation of functions of the same general form as $\mathrm{Ei}(-x)$ even when the differential equation which they satisfy is complicated, or awkward to obtain. It cannot, however, be applied conveniently to the case of negative x.

J. T.

The preparation of this note was sponsored in part by the Office of Air Research, AMC, USAF.

[1] J. R. AIREY, "The converging factor in asymptotic series and the calculation of Bessel, Laguerre and other functions," *Phil. Mag.*, v. 24, 1937, p. 521–552.

[2] A. C. COULSON & W. E. DUNCANSON, "Some new values for the exponential integral," Camb. Phil. Soc., *Proc.*, v. 33, 1942, p. 754–761.

[3] L. FOX & E. T. GOODWIN, "Some new methods for the numerical integration of ordinary differential equations," Camb. Phil. Soc., *Proc.*, v. 45, 1949, p. 373–388.

[4] H. E. SALZER & RUTH ZUCKER, "Table of the zeros and weight factors of the first fifteen Laguerre polynomials," Amer. Math. Soc., *Bull.*, v. 55, 1949, p. 1004–1012.

[5] G. SZEGÖ, *Orthogonal Polynomials*. Amer. Math. Soc., *Coll. Publ.*, v. 23, 1939, p. 369.

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1. ANON., "Standards on electronic computers: definitions of terms, 1950," I.R.E., *Proc.*, v. 39, March 1951, p. 271–277.

2. EDMUND C. BERKELEY, "The uses of automatic computers in financial and accounting operations," *Journal of Accountancy*, v. 90, October 1950, p. 306–311.

The author feels that the needs of business in accounting are beginning to create a demand for the large electronic computers such as are now being used in scientific and engineering problems. He explains, in this article, the fundamental nature of the large computer by comparing it in its essential characteristics with the ordinary desk calculator.

By analyzing the basic requirements of typical accounting problems, he is able to forecast the changes in computer design necessary to meet these requirements. The tangible benefits to business, such as the lower cost of the information produced and quick access to completely assimilated up-to-the-minute information, are forcibly pointed out. Of course, there will be construction and training delays, and it would be wise for business to study some of the problems suggested in the paper so that the machines can go to work as effectively as possible after their construction.

J. Blum

NBSCL

3. Engineering Research Associates, Inc., *High Speed Computing Devices*. McGraw-Hill, 1950, 451 pages.

As stated in the preface, this book is primarily a discussion of the mechanical devices and electrical circuits which can be incorporated into computing machines. Included also are a brief treatment of the concepts of number theory which are pertinent to many electronic digital computing machines now in use or under development, a chapter on numerical analysis listing methods for solving algebraic and differential equations, and several chapters on computing systems.

The book contains three parts: Part I, The Basic Elements of Machine Computation; Part II, Computing Systems; and Part III, Physical Components and Methods. It appears to have been directed mainly at readers with some knowledge of engineering who wish to learn how large-scale digital computing machines function, and it serves this purpose well. The treatment of analogue devices is quite cursory, however, and the modicum of numerical analysis included serves best to justify the definition of component, given in the introduction, as "any physical mechanism or mathematical method which is used as a tool in automatic computation."

Granting the definition of component as given in the book, the classification of the material contained therein which is made in the introduction appeals to this reviewer. This classification of contents is the following:

A. General. Chapter 1, Introduction, Chapter 2, Preliminary Considerations; B. Mathematical Components. Chapter 6, Arithmetic Systems, Chapter 7, Numerical Analysis; C. Physical Components and Methods. Chapter 3, Counters as Elementary Components, Chapter 4, Switches and Gates, Chapter 13, Arithmetic Elements, Chapter 14, Transfer Mediums, Chapter 15, Data Conversion Equipment, Chapter 16, Special Techniques and Equipment for Possible Use in Computing Systems, Chapter 17, Factors Affecting Choice of Equipment; D. Computing Systems. Chapter 5, A Functional Approach to Machine Design, Chapter 8, Desk Calculators, Chapter 9, Punched-card Computing Systems, Chapter 10, Large-scale Digital Computing Systems, Chapter 11, Analog Computing Systems, Chapter 12, The Form of a Digital Computer.

As the chapter titles indicate, the book by the staff of the Engineering Research Associates touches but lightly on control and input-output elements of high-speed computing devices. By way of compensation, however, the arithmetic element of such devices is treated in detail and Chapter 14 on transfer mediums, the largest chapter in the book, provides an excellent description of the physical phenomena basic to the high-speed storage elements in use in large-scale electronic computing machines. To those who are concerned most with electronic computing devices, Chapter 14 is probably the most interesting chapter of the book, covering both storage elements of the dynamic type, like the acoustical delay line and the magnetic drum, and electrostatic storage elements depending, for their action, on the secondary-emission phenomenon. The book, prepared by competent staff members of a company active in the development of large-scale, high-speed computers, has real merit in more than a single chapter, however. In the opinion of this reviewer, it is a valuable addition to the literature on computing devices, which will be most useful to those interested in large-scale electronic digital computers.

E. W. C.

4. S. Gill, "A process for the step-by-step integration of differential equations in an automatic digital computing machine," Camb. Phil. Soc., *Proc.*, v. 47, 1950, p. 96–108.

As the author observes, many of the methods of integrating differential equations which are effective for hand computation have serious disadvantages when the integration is to be performed on a high-speed automatic machine. Among the disadvantages may be listed:

(1) Starting values may be obtained by a process different from that emphasized in the remainder of the integration. The extra orders required to do this on a machine are wasteful of memory space.
(2) Preceding functional values may be required for the calculation at each point. The transferring of values and the "searching" which this necessitates is cumbersome and expensive of orders and time.
(3) Changing the interval may be complicated. Here again, the number of orders required to effect the change may be excessive.

These problems are particularly vexing when we are seeking to solve a large system of differential equations. The author, by an application of processes developed by Kutta, has worked out an extremely effective method, having none of the above disadvantages, by which such systems can be solved. It is straightforward, easily coded and economical of storage space. The paper includes a fairly thorough discussion of the accuracy obtainable in such a scheme.

M. Montalbano

NBSCL

5. D. R. Hartree, "Automatic calculating machines," *Math. Gazette*, v. 34, 1950, p. 241–252, illustrs.

This paper is the text of an address given before the Mathematical Association. Because this discussion was given before a mathematical group, the treatment of automatic calculators in general was limited. The author does touch upon some of the more important aspects of automatic machines with special reference to Babbage's dream of an "Analytical Engine," and the first realization of that dream, the Harvard Mark I Calculator. A short discussion of the first machine using electronic circuits, the ENIAC, with illustrations of the main parts, completes the first part of the paper.

The second part is concerned with a discussion of the EDSAC, which is a serial binary machine with a storage system using ultrasonic acoustic delay units. This machine uses one-address code instructions and serial storage. It has one storage unit for current instructions and another for registering the address from which the current instruction is taken. Normally the contents of the latter are increased by unity for every instruction carried out. Numbers and instructions are received by the machine in coded decimal form by means of teleprinter tape, and output is on a teletypewriter. The numbers are operated on in the machine in the binary system.

The author concludes with a note on programming and coding and gives an example of coding for the EDSAC.

In this reviewer's opinion, the closing comment of the author is worthy of quoting. "These machines have been called 'electronic brains,' which carries the suggestion that they can 'think for themselves,' which they cannot do: they can only carry out, quite blindly, the sequence of instructions which has been thought out for them."

B. F. Handy, Jr.

NBSCL

6. Marshall Kincaid, John M. Alden, & Robert B. Hanna, "Static magnetic memory for low-cost computers," *Electronics*, v. 24, 1950, p. 108–111.

This paper reviews the basic theory and gives several applications for the static magnetic storage units using the special magnetic material Deltamax. The authors feel that these magnetic storage units have application in medium-speed, low-cost computers which could handle many problems encountered in daily operation of business and industry. The applications cited in the paper for use of these devices in computers are as follows:

1. Storage of information. The handling rate varies from zero to 30,000 pulses per second, and the information is not lost in the event of power failure.
2. Transformation of information. The devices transform information slow-speed to high-speed pulse systems and vice versa; they also change parallel-type information to serial-type information and vice versa.
3. A high-speed counter which can count up to 25,000 pulses per second using only simple auxiliary circuitry.

The paper concludes by mentioning some of the production techniques and difficulties encountered in the commercial production of these devices.

M. M. Andrew

NBSMDL

7. B. L. Moore, "Pentode counting or control ring," *Review Sci. Instr.*, v. 21, 1950, p. 337–338.

A counting or control ring circuit using 6AS6 pentodes in a very stable manner is described. The input signal is said to be essentially independent of amplitude and wave shape, and the values of circuit components and supply voltage are not critical. Control voltages may be taken from one or more stable states of the counter, the maximum number being limited essentially by the power required to drive the grid. Combinations of counters may be used to obtain a ring of many stable states.

M. M. ANDREW

NBSMDL

8. ROBERT F. SHAW, "Arithmetic operations in a binary computer," *Review Sci. Instr.*, v. 21, 1950, p. 687–693.

The author gives a summary of the methods used to facilitate the basic arithmetic computations in contemporary binary computers. The use of complements is explained in defining negative numbers for machine use; corrections for multiplication error and round off procedures are discussed. With numerical examples, simply written but thorough, this paper is a good introduction to the logical mechanism of the binary computer.

KARL GOLDBERG

NBSCL

9. M. V. WILKES, "Automatic Computing," *Nature*, v. 166, December 2, 1950, p. 942–944.

Describes a Summer school course in coding for an automatic digital computing machine held at Cambridge in the University Mathematical Laboratory from September 12th through 21st. The course was concerned mainly with the methods used in connection with the EDSAC.

NEWS

Conference on automatic computing machinery and applications.—The conference was jointly sponsored by the Association for Computing Machinery, the Industrial Mathematics Society, and the Advisory Committee for Wayne University Computation Laboratory. There were general sessions on Tuesday morning and evening, March 27th, and two parallel groups of sectional meetings beginning Tuesday afternoon and continuing through Wednesday.

As the conference was held in the industrial center of Detroit, there was considerable interest in the applicability of computing machines to industrial problems. An exhibit of computer equipment was shown Monday evening through Wednesday afternoon. The program for the meeting was as follows:

General Session, Tuesday, March 27	FRANZ L. ALT, President, ACM, *Chairman*
Welcoming address	VICTOR A. RAPPORT, Dean, Wayne University
"Some computing problems in the automotive industry"	PAUL T. NIMS, Chrysler Corporation, Vice-president, Industrial Mathematics Society
"Automatic calculations and their applications"	HOWARD H. AIKEN, Director, Computation Laboratory, Harvard University
Sectional Meeting, Tuesday, March 27, Section A	ALEX L. HAYNES, Ford Motor Co., *Chairman*
"Digital computer research at M.I.T."	ROBERT R. EVERETT, M.I.T.
"The ENIAC—a five year operating survey"	W. BARKLEY FRITZ, BRL, Aberdeen Proving Ground
"Problems solved on B.T.L. Model VI Computer"	ERNEST G. ANDREWS, Bell Telephone Laboratories
"Capabilities and limitations of existing electronic equipment"	IDA R. RHODES, NBS
Sectional Meeting, Tuesday, March 27, Section B	C. C. HURD, IBM, *Chairman*

"Floating decimal calculations on the card programmed electric calculator"	RICHARD H. STARK, Los Alamos Scientific Lab.
"The treatment of systems of differential equations on the SEAC"	JOSEPH H. LEVIN, NBS
"Solution of the non-linear supersonic flow equations on the SEAC"	ETHEL C. MARDEN, NBS, J. CONRAD CROWN, NOL
"Preparation of a theodolite problem for large-scale machines"	BERNARD DIMSDALE, BRL, Aberdeen Proving Ground
Evening Session Dinner, Tuesday, March 27	JOHN R. RICHARDS, Wayne University, Toastmaster
"Electronics for business—luxury or necessity?"	JOHN S. COLEMAN, President, Burroughs Adding Machine Co.
Sectional Meeting, Wednesday Morning, March 28, Section A	ROBERT SCHILLING, General Motors Corporation, President, Industrial Mathematics Society
"The programming of some matrix computations for the Mark II Aiken Relay Calculator"	ALLEN V. HERSHEY, Naval Proving Ground
"The operation of the Fairchild specialized digital computers for certain applications in matrix algebra"	J. J. STONE, Fairchild Engineering and Airplane Corporation
"Matrix algebra programs for the UNIVAC"	H. F. MITCHELL, Jr., Eckert-Mauchly Computer Corporation (Subsidiary, Remington Rand Corp.)
"A UNIVAC program for inventory requirements"	G. M. HOPPER, Eckert-Mauchly Computer Corp. Subsidiary, Remington Rand Corp.)
Sectional Meeting, Wednesday Morning, March 28, Section B	ALBERT C. HALL, Bendix Aviation Corporation, *Chairman*
"Automatic checking features of the Raytheon Digital Computer"	LOUIS FEIN, Raytheon Manufacturing Company
"The external memory of the Raytheon Digital Computer"	KENNETH M. REHLER, Raytheon Manufacturing Company
"The digital reader"	GILBERT W. KING, Arthur D. Little, Inc.
"Preliminary considerations on a magnetic drum controlled computer"	THEODORE SHAPIN, Jr., E. F. MOORE, University of Illinois
"Techniques in the design of digital computers"	R. E. SPRAGUE, Computer Research Corporation
Sectional Meeting, Wednesday Afternoon, March 28, Section A	C. C. BRAMBLE, Naval Proving Ground, *Chairman*
"On the accuracy of Runge-Kutta's method"	MAX LOTKIN, BRL, Aberdeen Proving Ground
"Optimum trajectories"	ARNOLD S. MENGEL, The RAND Corporation
"Reversing digit number system"	GEORGE W. PATTERSON, Burroughs Adding Machine Company
"Analysis of digital computers in control systems"	W. K. LINVILL, J. M. SALZER, M.I.T.
Sectional Meeting, Wednesday Afternoon, March 28, Section B	EDMUND C. BERKELEY, Secretary, Association for Computing Machinery
"The impact of business on computing machinery design"	E. F. COOLEY, Prudential Insurance Company of America
"Plans for tabulating parts of the 1950 population census on an electronic computer"	JAMES L. MCPHERSON, Bureau of the Census
"Engineering applications on the MADDIDA"	GLENN E. HAGEN, Northrop Aircraft, Inc.
"Mathematical techniques in the uses of MADDIDA"	MYRON J. MENDELSON, Northrop Aircraft, Inc.

Institute on Computing Methods and Machines.—On January 26th through 30th inclusive a duplicate meeting was held at the University of California at Berkeley (Jan. 26–27) and at Los Angeles (Jan. 29–30). The purpose of the meeting was to provide a general picture of computer application to problem solutions and a specific picture of the computer facilities available in the University of California. The meeting was co-sponsored by various departments of the universities and by the NBSINA. The following program was presented:

Program	Berkeley	Los Angeles
Morning Session		
(Jan. 26 at Berkeley and Jan. 29 at Los Angeles)		
Chairman	H. A. Schade, U. C., Berkeley	L. Cherry, NAMTC, Point Mugu, Calif.
Welcome and Introduction	M. P. O'Brien, Chairman, Dept. of Eng., U. C., Berkeley	L. M. K. Boelter, Chairman, Dept. of Eng., UCLA
"Current status of digital computer developments"	E. U. Condon, Director, NBS	
"The facilities and program of the Computer Laboratory at Berkeley"	Paul L. Morton, Director, Computer Lab., U. C., Berkeley	
"The California Digital Computer—CALDIC"	David R. Brown, U. C., Berkeley	
Afternoon Session		
Chairman	Jerzy Neyman, Director, Statistical Lab. U. C., Berkeley	Louis A. Pipes, UCLA
"Role of the mathematics department in a university computing center"	Derrick H. Lehmer, U. C., Berkeley	
"Computations relating to some solar system problems"	Leland E. Cunningham, U. C., Berkeley	
"Facilities of the Institute for Numerical Analysis:		
a) problem formulating service	A. S. Cahn, NBSINA	
b) punched card equipment	E. C. Yowell, NBSINA	
c) electronic equipment—SWAC"	Harry D. Huskey, NBSINA	
Inspection of Computer Facilities		
Morning Session		
(Jan. 27 at Berkeley and Jan. 30 at Los Angeles)		
Chairman	Alfredo Banos, Jr., UCLA	William A. Mersman, Ames Aeronautical Lab.
"Recent developments in analog computer techniques"	W. W. Soroka, U. C., Berkeley	
"The network thermal analyzers"	F. W. Schott, UCLA	
"Electronic analog computations"	T. A. Rogers, UCLA	
"Mechanical differential analyzer-problem solution"	Donald Lebell, UCLA	
Inspection of Computer Facilities		

The Institute of Radio Engineers.—At the National Convention of the Institute held in New York on March 19th through March 22nd, two sessions were devoted to computers. The programs were as follows:

Computers I	E. R. Piore, Office of Naval Research, Washington, D. C., *Chairman*
"The Raytheon selection matrix for computer and switching applications"	Kenneth M. Rehler, Raytheon Manufacturing Company, Waltham, Mass.
"Saturable reactors as substitutes for electron tubes in high-speed digital computers"	James G. Miles, Engineering Research Associates, Inc., St. Paul, Minn.
"Ferromagnetic cores for three-dimensional digital storage arrays"	William N. Papian, Servomechanisms Laboratory, MIT, Cambridge, Mass.
"Dependable small-scale digital computer"	J. J. Connolly, Teleregister Corporation, New York, N. Y.
"An asynchronous control for a digital computer"	D. H. Gridley, Naval Research Laboratories, Washington, D. C.
Computers II	J. W. Forrester, MIT, *Chairman*
"A sampling analogue computer"	John Broomall and Leon Riebman, Moore School of Electrical Engineering, Univ. of Penn.
"A time division multiplier for a general-purpose electronic differential analyzer"	R. V. Baum and C. D. Morrill, Goodyear Aircraft Corp., Akron, Ohio
"A high-speed product integrator"	Alan B. MacNee, University of Michigan, Ann Arbor, Michigan
"Plug-in units for digital computation"	G. Glinski and S. Lazecki, Computing Devices of Canada, Ltd., Ottawa, Ontario, Canada
"A five-digit parallel coder tube"	J. V. Harrington and K. N. Wulfsberg, Air Force, Cambridge Research Laboratories, Cambridge, Mass., and G. R. Spencer, Philco Tube Laboratory, Lansdale, Pa.

National Physical Laboratory.—In November 1950 at the National Physical Laboratory a pilot model of the ACE (Automatic Computing Engine) was demonstrated to members of the British Press. The ACE is an electronic digital computer which is being constructed under the aegis of the Department of Scientific and Industrial Research by the staff of Electronics and Mathematics Divisions of the National Physical Laboratory at Teddington, Middlesex, in association with the English Electric Company of Stafford.

Owing to the size and complexity of the ACE itself, it was thought desirable to build first a smaller version known as the ACE pilot model. This is a serial machine operating on numbers in the binary scale, with a pulse repetition rate of one megacycle per second. It uses about 800 tubes in 40 plug-in units mounted on a rack about $12' \times 16'$ and has separate control panel and power supply units. The internal memory is of the mercury delay line type and consists of eight long tanks, each with a delay of 1024μsec. and 10 short tanks, each with a delay of 32μsec. Storage capacity is 264 words, each word being the binary equivalent of a 10-decimal-digit number or an instruction in a three-address code. The time necessary to multiply two 10-decimal-digit numbers is .002 sec. For the input and output sections of the machine, modified Hollerith punched card equipment is used. The demonstration included the extraction of 3rd and 4th roots of 7-decimal-digit numbers, and a program for testing whether any given number less than a million is prime. Numbers, suggested by the audience, were fed into the machine in decimal form by hand switching. If the number was not prime, the lowest factor was indicated in decimal form by means of lights; if it was prime, after three seconds the machine sounded a buzzer. The machine was also used to solve a problem of practical importance—i.e., the tracing of rays through a compound lens. The machine computed in a few minutes, the paths of 20 rays through a compound lens with six refracting surfaces. Such a computation would have taken a human computer two 7-hour days with a desk calculator.

Reeves Instrument Corporation.—Project Cyclone Symposium I on REAC Techniques was held in New York City, March 15–16, 1951 under the sponsorship of the Reeves Instrument Corporation with the approval of the U. S. Navy Special Devices Center.

It was the purpose of Project Cyclone Symposium I on REAC Techniques to provide a means for discussing techniques, applications and engineering difficulties which have arisen among users of REAC equipment, and to stimulate new techniques in the field of analogue computation.

The program consisted of five sessions, under the chairmanship of Stanley Fifer, head of Project Cyclone.

Morning Session, Thursday, March 15:

Welcoming addresses by David T. Bonner, President, Reeves Instrument Corporation, and Paul Staderman, Head, Synthetic Warfare Section, Special Devices Center.

1. "Optimum trajectories," Arnold S. Mengel, The RAND Corporation.
2. "Solution of some partial differential equations on the REAC," G. W. Evans II, Argonne National Laboratory.
3. "Analytical and mechanical methods of solution of differential equations with discontinuous forcing functions," William A. Mersman, National Advisory Committee for Aeronautics, Ames Aeronautical Laboratory.
4. "The solution of polynomial equations on the REAC," Louis Bauer and Stanley Fifer, Project Cyclone.
5. "Computation of multidimensional integrals on the REAC," Leslie C. Merrill, Argonne National Laboratory.

Afternoon Session, Thursday, March 15:

6. "RAND REAC modifications," Arnold S. Mengel and Wesley S. Melahn, The RAND Corporation.
7. "Some modifications for maximum utilization of the REAC," John L. Burnside, North American Aviation, Inc.
8. "The use of the REAC by Minneapolis-Honeywell in the analysis and synthesis of automatic flight control," Remus N. Bretoi and David L. Markusen, Minneapolis-Honeywell Regulator Company.
9. "Slaving of a rotational mount to an analog computer," C. F. Cook, National Bureau of Standards.
10. "Preliminary investigation of suitability of REAC for experimental curve fitting," Donald S. Teague, Jr. and R. D. Gilpin, U. S. Naval Air Missile Test Center, Point Mugu.
11. "The generation of an N dimensional normal distribution by means of analog equipment," Harry H. Goode, William A. Wheatley and George G. den Broeder, Willow Run Research Center, University of Michigan.

Morning Session, Friday, March 16:

12. "The generation and measurement of ultra low frequency random noise," Robert R. Bennett and A. S. Fulton, Hughes Aircraft Company.
13. "Simulation of noise," Harold L. Ehlers and Erwin Vogel, Fairchild Engine and Airplane Corporation.
14. "Frequency analysis by electronic analog methods," William A. McCool, Naval Research Laboratory.
15. "Application of REAC equipment to the solution of problems involving integral operators, I," Paul Brock and Seymour Sherman, Project Cyclone.

Noon Session, Friday, March 16:

Numerical checks; introduction by Paul Brock, Project Cyclone.

Afternoon Session, Friday, March 16:

16. "A REAC solution of a linear proton accelerator design problem," A. H. Miller, presented by Nathanial B. Nichols, University of Minnesota.
17. "Representation of functions of several variables on REAC equipment," Hans Meissinger, Project Cyclone.
18. "A method for solving problems on the REAC by the use of transfer functions without passive networks," Cyrus Beck, Naval Air Experimental Station, Philadelphia.
19. "REAC techniques," W. Frank Richmond, Jr., The Glenn L. Martin Company.
20. "The generation of straight line transfer relationships," Robert R. Bennett, Hughes Aircraft Company.

UNIVAC Acceptance Tests.—On March 23rd the first UNIVAC successfully passed its acceptance tests in the presence of representatives of the Bureau of the Census and the Bureau of Standards. A discussion of these tests will follow a brief description of this new computer.

The UNIVAC system consists of a computer with an elaborate Supervisory Control Console, one to ten Uniservos, one or more Unitypers, and one or more Uniprinters. There is also available a Punch-Card-To-Tape Converter, and a device to perform the reverse conversion is under construction. The computer is a decimal machine containing 1000 words in its 100 mercury delay lines. Each word consists of 12 alphanumeric digits and has room for two one-address commands. The pulse repetition rate of the machine is 2.25 megacycles allowing about 2000 additions or over 450 multiplications to be performed in one second. Each half of the twin Arithmetic Unit checks the operations of the other half and indicates on the Supervisory Console any discrepancy that may occur.

Each Uniservo uses an 8-channel metal tape with a magnetic plating. The tape is initially examined for flaws, which are marked and which the machine is constructed to skip over. These tapes move either forwards or backwards at ten feet per second, so that over 800 words can be delivered to the Memory or recorded from it every second. A reel of tape is over 1500 feet in length, has a capacity of 120,000 words, and may be reused some 700 times. It is completely erasable, moves in either direction, and can be rewound, all at the behest of the coder's commands.

The Unityper's keyboard possesses a number of keys in addition to the conventional alphanumeric characters. One set of these keys serves to prepare commands leading to the desired format in which the final results are to appear; another set calls into play the group of commands which appears on three tracks of the rudimentary memory built into the Unitypers. These commands can be set to facilitate, shorten, and correct the typing of the input information.

The Uniprinter converts the binary-coded information on the tapes into alphanumeric characters at the rate of 11 per second. At the same time it is able to carry out printing instructions stored on the tape such as: carriage return, upper case, ignore (i.e., do not print), etc. It should be added that a fast line printer is under construction which will list information at about twice the speed of the standard punch-card tabulator.

The Card-to-Tape Converter handles six 80-column cards per second, storing the contents of 10,000 cards on a reel of tape and stopping automatically after a preset number of cards have been converted.

The four acceptance tests which the UNIVAC system underwent successfully consisted of:

1. A general test with emphasis on the computer's performance. It was subdivided into three parts. The first, which was repeated 404 times per test, investigated the

reliability of the execution of every one of the computer's commands. The second part required the computer to sort a number of items obtained as a result of the operations in the first part, as well as to solve a rather complicated partial differential equation. In the last part, the tapes of the four Uniservos were run through a difficult routine of a series of movements, delivering and recording information. Two such tests, requiring twenty minutes for a flawless performance, were designated as a unit. Out of the successive nineteen units which the UNIVAC performed during the test period, sixteen were carried out perfectly; in the three units where stops occurred due to the action of the error-detection circuits, the operator was able to rectify them by the mere manipulation of the console buttons.

2. A Uniprinter test requiring a sentence and a numerical table to be printed out in proper form 200 times. This test lasted eight hours, during which five errors occurred, whereas the conditions of the test allowed eight.
3. A Card-to-Tape-Converter test, which was passed satisfactorily. Although required to process a deck of 10,000 very badly punched cards, the Converter made only one mistake while storing this information repeatedly on ten tape reels.
4. A general test, with emphasis on Uniservo performance. Various aspects of tape movements were under scrutiny such as any tendency of the tapes to move out of position during prolonged inactivity, the efficiency of flaw-detection in the magnetic coating of the tapes, the resistance of this coating to a continuous rerunning of the tape under the reading heads for 700 times, the effect that various juxtapositions of tape commands might have on their performance, and several other points of possible weakness. This test lasted ten hours during which six errors were detected by the machine.

The machine will remain on the premises of the Eckert-Mauchly Computer Corporation for about a year, performing computations for the Bureau of the Census and operating 24 hours a day.

OTHER AIDS TO COMPUTATION

Bibliography Z-XVI

10. B. P. Bogert, "A network to represent the inner ear," *Bell Lab. Record*, v. 28, 1950, p. 481–485.

An electrical network consisting of 175 sections is used to simulate the cochlea which is believed to be the frequency sensing portions of the ear. The results of measurements on this simulator are compared for a number of frequencies with the results computed from the theory developed in a paper by Peterson & Bogert.[1] The simplified model upon which a hydrodynamical theory is based consists of two parallel ducts separated by a movable membrane of varying width. Measurements on the electrical network are compared with the numerical solution of the equations of wave propagation in this model, in regard to the amplitude of the response of the membrane at different points along its length to different frequencies. The response patterns vary considerably with the frequency but good agreement was obtained between the network, the theoretical model and certain experimental results for the actual cochlea.

F. J. M.

[1] L. C. Peterson & B. P. Bogert, "A dynamical theory of the cochlea," Acoust, Soc. Amer., *Jn.*, v. 22, 1950, p. 369–381.