

# Mathematical Tables *and other* Aids to Computation

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A Quarterly Journal

Edited by

E. W. CANNON

F. J. MURRAY

C. C. CRAIG

J. TODD

A. ERDÉLYI

D. H. LEHMER, *Chairman*

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V

nos. 33-36

1951

*Published by*

THE NATIONAL RESEARCH COUNCIL

Washington, D. C.



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1946-1949 (Nos. 13-28) \$1.25 for single issue

1950-1951 (Nos. 29-36) \$1.50 for single issue

All payments are to be made to National Academy of Sciences, 2101 Constitution Avenue, Washington, D. C.

Agents for Great Britain and Ireland (subscription 42s, 6d for 1951) Scientific Computing Service, Ltd., 23 Bedford Square, London W.C.1.

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Published quarterly in January, April, July and October by the National Research Council, Prince and Lemon Sts., Lancaster, Pa., and Washington, D. C.

All contributions intended for publication in *Mathematical Tables and Other Aids to Computation*, and all Books for review, should be addressed to D. H. Lehmer, 16556 Chattanooga Place, Pacific Palisades, Calif.

Entered as second-class matter July 29, 1943, at the post office at Lancaster, Pennsylvania, under the Act of August 24, 1912.

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