



THE SEAC

## A Guide to Tables on Punched Cards

A list of tables on punched cards, compiled by W. J. ECKERT, was published in the October 1945 issue of *MTAC* [v. 1, p. 433-436]. Since then, the number of laboratories interested in mathematical computations has greatly increased and as a consequence many new and important tables have become available.

Computing techniques, too, have undergone radical changes with the advent of the electronic "C.P.C." of the IBM Corporation and the new electronic multipliers which are accessible to laboratories of moderate size. Then there are the large, high-speed digital machines which employ punched cards; they are continually adding punched card tables of general interest.

In a sense, the need for the more elementary punched card tables, such as square roots, reciprocals, and squares has decreased—at least for those laboratories that have electronic multipliers. In fact, several of the larger laboratories that formerly kept files of elementary tables on punched cards have discarded them. There are, however, a considerable number of smaller laboratories that still need the elementary functions, and even the laboratories of moderate size would prefer a tested, available table of the Riemann Zeta function, say, to generating needed values with equipment currently at their command. In addition, "optimum interval" tables [*MTAC*, v. 1, p. 173-176] have found favor in some quarters. A number of tables of this character have been added since 1945.

When, at the request of the editors of *MTAC*, the present authors undertook to bring the list of punched card tables up-to-date, it soon became apparent that the wealth of new material made it advisable to revise the guide completely.

It was felt that classification of material according to subject matter rather than location would be most helpful to the laboratory that needed a certain table. In this type of classification it would have been difficult to include the file number which a given source attached to a table. Such information is therefore omitted. If a required table is adequately described, even a large and busy laboratory will probably find it quite easy to locate its file. An exception was made in the case of the UMT depository, that with time may acquire much heterogeneous material which might be hard to locate without a precise file number.

Quite often the user of a table may want to know the original source of the keypunched values; whether such values were taken from well-known books, or whether the laboratory computed the table on punched card equipment from first principles. Moreover, one would like to know whether the possessor of a table checked it, or acquired it from another laboratory and did not get around to testing its accuracy. The authors tried to elicit this information. In the case of many tables of the higher mathematical functions, they knew from their own experience where such tables originated. For the elementary tables, the statements of the reporting laboratories were taken at face value, without exhaustive checks. Memory for such information becomes nebulous over the years, and many laboratories probably failed to

get credit for their own work; perhaps a few were credited for efforts they did not make. It is hoped that readers will report any such errors when they notice them.

The tables reported are all "unclassified." Some of the largest laboratories reported only a small fraction of their tables of general interest. Since such laboratories no doubt have good and sufficient reasons for limiting the library they wish to make available to others, the authors made no attempt to list tables which the source did not specifically mention. However, most tables of general interest are eventually published and reported in *MTAC* and private correspondence with the laboratories involved may bring fruitful results in such cases.

The authors acknowledge gratefully the cooperation of the following members of laboratories who submitted lists of their tables: M. ABRAMOWITZ, H. H. AIKEN, P. ARMER, W. D. BELL, J. BELZER, J. M. BOERMEESTER, E. C. BOWER, C. C. BRAMBLE, S. R. BRINKLEY, Jr., B. F. CHEYDLEUR, G. M. CLEMENCE, R. P. COATES, C. F. DAVIS, W. J. ECKERT, B. FERBER, A. D. FRANKLIN, C. C. GOTLIEB, R. W. HAMMING, C. HASTINGS, Jr., R. HENSEL, F. H. HOLLANDER, C. C. HURD, J. P. KELLY, E. C. KENNEDY, W. A. KITTS, 3d, H. KRAFT, D. H. LEHMER, M. LOTKIN, E. LUNDT, H. McALLISTER, A. OPLER, H. POLACHEK, G. W. REITWIESNER, C. V. RUZEK, J. SCHILT, V. SCHOMAKER, J. SHERMAN, C. T. TAI and F. M. VERZUH. Many of them went to considerable trouble in answering correspondence with the authors and in checking the listing of the items reported by them. The difficult task of preparing the typed manuscript from a preliminary card index was performed by Mrs. KATHARINE WARREN. She not only checked the typed data with painstaking care, but also gave much attention to maintaining uniformity in style and format.

To the extent that this guide reflects accurately the tables available, credit is due to the wholehearted cooperation of all the laboratories involved. The authors of this guide alone bear the responsibility for any errors of their commission in reporting the data, and they will appreciate information about any inaccuracies discovered by readers.

The meaning of "sources" and the symbols employed are explained below.

## SOURCES

The source of the original tables from which card entries were key-punched is indicated wherever it is known. A list of numbered "sources" with the addresses at time of publication of this guide is given below. Some of these are laboratories which retain key-punched values. Others, such as (66) and (40), are books of tables or other texts to which reference is made in the guide. Some laboratories, like (64), no longer have the cards credited to them. In such cases, the place where the tables have been deposited is indicated alongside the name. Reference details about tables or texts were sometimes taken from (40) without further verification.

## MEANING OF SYMBOLS

The symbol  $x=0(.01)5(.1)8$ ; 5D or 5S means as usual that the argument ranges from 0 to 5 at intervals of 0.01, from 5 to 8 at intervals of 0.1, and that the entries are given to 5 decimals (5D) or to 5 significant figures (5S).

The symbol [(41)–(9), (7)] in the source column indicates that the entries are based on values in source (41), and that both (9) and (7) key-punched them independently. Similarly [(41), (9)–(9)] means that all key-punching was done by (9); that some entries were taken from (41) and others were computed by (9). The symbol [(9)] implies that computing and key-punching were done by (9).

The laboratories that possess a given punched card table are listed in the column headed "Available at." Some laboratories have stated that they have verified, mathematically or otherwise, the accuracy of the tables. This fact is indicated by the letter C following the source number.

The symbol  $\Delta^k f$  (or merely  $\Delta^k$ ) indicates the  $k^{\text{th}}$  forward difference of the function;  $\delta^{2k} f$  (or  $\delta^{2k}$ ) indicates the central difference of order  $2k$ . Similarly  $\delta^k f$ ; and  $\Delta^k f$ ; will sometimes be abbreviated to  $\delta, \Delta, \delta^k, \Delta^k$ , respectively.

### CLASSIFICATION

The classification followed is that adopted by FLETCHER, MILLER, & ROSENHEAD in *An Index of Mathematical Tables* [source (40)]. However, since the number of tables listed here is not very large, many sub-classifications have been omitted. For example, the trigonometric functions are listed as follows:

- 7.1 Tables with argument in radians
- 7.2 Tables with argument in decimals of a degree
- 7.3 Tables with argument in centesimal measure (grades or parts of the complete circle)
- 7.4 Tables with argument in degrees, minutes, and seconds

On the other hand, sub-headings such as 7.11, 7.12 have usually been omitted.

When many similar tables are listed under one heading, the grouping is according to the number of decimals or significant figures given in the entries. For example: all tables given to 5 decimals or significant figures will be followed by those given to 6 decimals, and so forth.

Some items seemed to belong to classes not specifically listed in (40). The following additional classes have been added:

25. Random numbers and functions corresponding to random arguments
26. Tables relating to aerodynamics. A wide variety of specialized tables, all of them available on punched cards, are listed in the voluminous unclassified reports issued by (54). Mention of the existence of such tables is made, but too much space would be required to include them in this guide. The reader interested in aerodynamic tables may consult the reports distributed by (54). Some of the punched cards are available at (6).
27. Tables relating to number theory
28. Tables relating to astronomy
29. Actuarial tables. Here, too, no specific details are given. Complete details can be obtained from the Society of Actuaries or from the IBM Corp., 590 Madison Ave., New York, N. Y.
30. Tables relating to map projections and geodesy

Some items belong equally well to more than one category. Thus  $2^n$  belongs under (2.3) as a table of powers and under (27) as a power of the prime 2. In this instance the table has been indexed in both categories. In other cases, where a listing is given in (40) the listing is usually included only under the category assigned to the item in (40), even though another choice might have been more desirable from a logical viewpoint.

An effort was made to unify the entries as far as possible. On the other hand, two very similar tables may differ in minor details which might be important to the laboratory operator. For the most extensive table of a type is not necessarily the most desirable one; a compact table may often be preferred to a large table, given to a higher accuracy. For this reason, tables which differed sufficiently in format or other details are listed separately. This is especially true in the case of tables of trigonometric functions, and in the tables of powers. In this connection, it was not always possible to indicate in one item all the data available on one card. For example, some laboratories have the functions  $x^{\frac{1}{2}}$ ,  $1/x$ ,  $x^2$ , and  $x^3$  on one card. On account of classification difficulties, these are listed separately.

In the case of optimum interval tables, the number of cards comprised in the compilation has been indicated, when this information was available to the authors. Unless otherwise indicated, such tables are interpolable linearly.

#### SOURCES

- 1 Bell Telephone Laboratories, Murray Hill Laboratory, Murray Hill, N. J.
- 2 Wm. D. Bell, Telecomputing Corp., 133 East Santa Anita Ave., Burbank, Cal.
- 3 California Inst. of Tech., Cooperative Wind Tunnel, Pasadena 4, Cal.
- 4 Central Statistical Laboratory, Union Carbide and Carbon Corp., Oak Ridge, Tenn.
- 5 Computation Laboratory, Harvard University, Cambridge 38, Mass.
- 6 Consolidated Vultee Aircraft Corp., Ordnance Aerophysics Laboratory, Daingerfield, Texas
- 7 Consolidated Vultee Aircraft Corp., San Diego, Cal.
- 8 Dow Chemical Co., Western Division, P.O. Box 351, Pittsburg, Cal.
- 9 E. C. Bower, Engineering Department, Douglas Aircraft Corp., Santa Monica, Cal.
- 10 General Electric Co., Apparatus Dept., Schenectady 5, N. Y.
- 11 Massachusetts Inst. of Tech., Cambridge, Mass. (Office of Statistical Services)
- 12 North American Aviation Corp., att: Charles F. Davis, Tabulating Unit, International Airport, Los Angeles, Cal.
- 13 Ohio State University, Cryogenic Laboratory, Columbus 10, Ohio
- 14 Rand Corporation, 1500 Fourth St., Santa Monica, Cal.
- 15 The Texas Company, Research Laboratories, Beacon, N. Y.
- 16 University of Toronto, Computation Center, Toronto, Ontario, Canada
- 17 Thos. J. Watson Astronomical Computing Bureau, Columbia Univ., New York, N. Y.
- 18 Watson Scientific Computing Laboratory, Columbia Univ., New York, N. Y.

- 19 Central Computing Service, Univ. of Cal., Berkeley 4, Cal., att: P. L. Morton, Cory Hall
- 20 California Institute of Technology, Gates & Crellin Lab. of Chemistry, Pasadena, Cal.
- 21 Ballistic Research Laboratories, Aberdeen Proving Ground, Aberdeen, Md.
- 22 U. S. Naval Observatory, Washington 25, D. C.
- 23 National Bureau of Standards, Computation Lab., Washington 25, D. C.
- 24 National Bureau of Standards, Inst. for Numerical Analysis, Los Angeles 24, Cal.
- 25 U. S. Dept. of Interior, Bureau of Mines, Pittsburgh 13, Pa.
- 26 U. S. Naval Ordnance Laboratory, White Oak, Silver Spring 19, Md.
- 27 U. S. Naval Proving Ground, Dept. of Computation and Ballistics, Dahlgren, Va.
- 28 U. S. Army Map Service, Corps of Engineers, Washington 16, D. C.
- 30 Society of Actuaries,  $\%$  Service Bureau, IBM Corp., 590 Madison Ave., New York, N. Y.
- 40 FLETCHER, MILLER & ROSENHEAD, *An Index of Mathematical Tables*. London, Scientific Computing Service, Ltd., 23 Bedford Square, W. C. 1, 1946
- 41 *Barlow's Tables of Squares, Cubes, Square Roots, Cube Roots and Reciprocals*. 4th ed., London and New York, 1944
- 42 JOHN TODD, *Table of Arctangents of Rational Numbers*. NBS Applied Math. Series, no. 11, March 1951
- 43 *Marchant Methods*. Marchant Calculating Machine Co., New York, N. Y.
- 44 *Vega's Logarithmic Tables of Numbers and Trigonometric Functions*. New York, D. Van Nostrand Co., 1912
- 45 *Mathematical Tables from Handbook of Chemistry and Physics*. 9th ed., compiled by CHARLES D. HODGMAN, Cleveland, Ohio, Chemical Rubber Co., 1948
- 46 EDWIN CHAPPELL, *Five-Figure Mathematical Tables*. 1915, reprinted by D. Van Nostrand, New York
- 47 H. ANDOYER, *Nouvelles Tables Trigonométriques Fondamentales*, v. 1, 3, Paris, Hermann, 1915, 1918
- 48 G. N. WATSON, *A Treatise on the Theory of Bessel Functions*. 2nd ed., New York, Macmillan, 1944
- 49 L. SCHWARZ, *Luftfahrtforschung*. V. 20, no. 12, p. 341-372, 1944
- 50 K. HAYASHI, *Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen*. Berlin, Springer, 1926
- 51 F. CALLET, *Tables Portatives de Logarithmes*. Paris, F. Didot, 1795 (Tirage, 1819). (sub-tabulated by E. C. BOWER)
- 52 J. J. ÅSTRAND, *Hülftafeln zur leichten und genauen Auflösung des Kepler'schen Problems*. Leipzig, Engelmann, 1890
- 53 J. C. P. MILLER, *Tables of  $I_n(x)$ ,  $K_n(x)$ , and  $Y_n(x)$* . The University Math. Laboratory, Cambridge, England. To be published for the Royal Society (the last of the British Assn. for the Adv. Sc. tables). Typed manuscript made available by author to (23).
- 54 Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Md. *Handbook of Supersonic Aerodynamics*, Jan., 1949. Reports can

be purchased from Supt. of Documents, Gov't Printing Office, Washington, D. C. Vols. 1 and 2 now available.

- 55 SAMUEL HERRICK, *Tables for Rocket and Comet Orbits*. Punched cards available at (24). Functions tabulated:  $C_e(U)$ ,  $S_e(U)$ ,  $X_e(U)$ ;  $C_h(U)$ ,  $S_h(U)$ ,  $X_h(U)$ , where

$$(a) \quad U = E - \sin E; \quad C_e(U) = 1 - \cos E; \quad S_e(U) = \sin E; \quad X_e(U) = E \text{ for } 0 < U \leq 3.15$$

$$(b) \quad U = \sinh F - 1; \quad C_h(U) = \cosh F - 1; \quad S_h(U) = \sinh F; \quad X_h(U) = F \text{ for } 0 < U \leq 300$$

Especially useful for finding the functions corresponding to very small values of  $U$ .

- 56 C. CRANZ, *Lehrbuch der Ballistik*. V. 1, Berlin, Springer, 1925
- 57 Applied Mathematics Panel, National Defense Research Committee, AMP Report 24-1, Sept. 1943, submitted by MERRILL M. FLOOD, Princeton Univ.
- 58 HAROLD T. DAVIS, *Tables of the Higher Mathematical Functions*. V. 2, Principia Press, Bloomington, Indiana, 1935
- 59 R. L. ANDERSON & E. E. HOUSEMAN, *Tables of Orthogonal Polynomial Values Extended to  $n = 104$* . Research Bull. 297, Agricultural Exp. Station, Statistical Section, Iowa State College, Ames, Iowa, 1942
- 60 D. H. LEHMER: (a) Tables of Ramanujan's  $\tau(n)$ , reported in *MTAC*, UMT 101, v. 4, p. 162, July, 1950  
(b) Table of the sums of fifth powers of the divisors of  $n$ ,  $n = 1(1)5000$ , reported in *MTAC*, UMT 114, v. 5, p. 28, Jan., 1951
- 61 National Bureau of Standards, *Tables Relating to Mathieu Functions*. New York, Columbia Univ. Press, 1951
- 62 T. S. KELLEY, *The Kelley Statistical Tables*. New York, Macmillan, 1938
- 63 F. E. FOWLE, *Smithsonian Physical Tables*. 8th rev. ed., Smithsonian Institution, Washington, D. C., 1933
- 64 Dept. of Engineering Research, U.C.L.A. [tables available at (24)]
- 65 H. W. HOLTAPPEL, *Tafels van  $e^x$* . Groningen, Holland, Noordhoff, 1938
- 66 British Assoc. for the Advancement of Science, *Mathematical Tables*, London, England, Cambridge Univ. Press
- 67 Admiralty Computing Service, H. M. Nautical Almanac Office, Herstmonceux Castle, Sussex, England
- 68 J. PETERS, *Seven-Place Values of Trigonometric Functions for every Thousandth of a Degree*. New York, D. Van Nostrand, 1942 ed.
- 69 AMELIA DE LELLA, *Five-Place Table of Natural Trigonometric Functions to Hundredths of a Degree*. New York, John Wiley and Sons, 1934
- 70 U. S. Lake Survey, Military Grid Unit, Dept. of Engineers [cards at (23)]
- 71 ARNOLD N. LOWAN & JACK LADERMAN, "Table of Fourier Coefficients," *Jn. Math. Physics*, v. 22, 1943, p. 136-147
- 72 H. J. GRAY, R. MERWIN & J. G. BRAINERD, Solution of the Mathieu Equation, *Am. Inst. Elec. Engrs.*, v. 67, 1948, p. 429-441
- 73 UMT FILE, (Unpublished Mathematical Tables) % D. H. Lehmer. (When writing for cards, mention UMT reference number.)

- 74 R. M. ROBINSON, (a) *Stencils for the Solution of Systems of Linear Congruences Modulo 2*. See *MTAC*, UMT 120, v. 5, p. 85. (b) *Table of Integer Solutions of  $|y^2 - x^3| < x$* . See *MTAC*, UMT 125, v. 5, p. 162.
- 75 J. BAUSCHINGER & J. PETERS, *Logarithmisch-trigonometrische Tafeln mit acht Dezimalstellen*. V. 1, second ed., Leipzig, Engelmann, 1936
- 76 Royal Observatory at Greenwich, *Nautical Almanac and Astronomical Ephemeris*, London, H. M. Stationery Office, London W. C. 2
- 77 U. S. Navy, Hydrographic Office, *Tables of Computed Altitude and Azimuth*, Publication No. 214, Washington, D. C., U. S. Government Printing Office, 1940
- 78 The University Mathematical Laboratory, University of Cambridge, Cambridge, England
- 79 C. E. VAN ORSTRAND, *Tables of the Exponential Function and of the Circular Sine and Cosine to Radian Argument*. Nat'l Academy of Sciences, 5th memoir, v. 14, Washington, U. S. Government Printing Office, 1921
- 80 H. BRANDENBURG, *Siebenstellige trigonometrische Tafel . . .* Leipzig, Lorentz, 1931
- 81 P. WIJDENES, *Five-place Tables*. Groningen, Noordhoff
- 82 H. M. Nautical Almanac Office (L. J. Comrie), *Planetary Coordinates for the Years 1800 to 1940*. London, H. M. Stationery Office, 1933

Source	Available at	Description of Tables
		<b>2.</b> Powers, real and complex; simple rational functions of powers (Also see 6.19)
[(21)]	(11)C, (16)C, (21)C, (24)	<b>2.1</b> $x^2: x = 0(.0001)0.9999$ ; exact On same card with $x^3, x^4, x^5$ at (24)
[(18)]	(18)C	$x^2: x = 0(.01)2.25$ ; 2D; on same card with $2x^2, x^3, x^4$
[—(3)]	(16), (24)C  (16), (24)C	$x^2, \Delta: x = 1(1)1000$ ; exact; on same card with $\sqrt{x}, \sqrt{10x}, 1/x$ $x^2: x = 1(1)9999 \Delta, \delta^2$ ; exact; on same card with $1/x$ Available at (17)C for $x \geq 1000$ .
[(41)–(27)]	(27)	$x^2: x = 1(1)10,000$ ; exact; on same card with $x^3$ .
[(41)–(18)]	(18)C	$x^2: x = 1(1)9999$ ; 7D; on same card with $1/x, 1/x^2$
[(41)–(21)]	(21) (23)C	$x^2: x = 1(1)12,500$ ; exact $x^2: x = 10(10)10,000$ ; 7S or 8S On same card with $x^k, k = -1, -2, -3, 3, \frac{1}{2}, \frac{3}{2}, \frac{1}{3}$
[(18)]	(18)C	$2x^2: x = 0(.01)2.25$ ; 2D; on same card with $x^2, x^3, x^4$
[(21)]	(16)C, (21), (24)	<b>2.2</b> $x^3: x = 0(.0001)0.9999$ ; exact; on same card with $x^2, x^4, x^5$



<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(18)]	(18)C		$x^3: x = 0(.01)2.25; 2D$ ; on same card with $x^2, 2x^2, x^4$
[(41)-(27)]	(27)		$x^3: x = 1(1)10,000$ ; exact; on same card with $x^2$
[(21)]	(21)		$x^3: x = 1(1)12,500$ ; exact
[(23)]	(11)C, (23)C		$x^3: x = 10(10)10,000$ ; 7S or 8S
[(18)]	(18)C	2.3	$x^4: x = 0(.01)2.25; 2D$ ; on same card with $x^2, 2x^2, x^3$
[(11)]	(11)C		$x^4: x = 1(1)2500$ ; 8S
[(41)-(27)]	(27)		$x^4: x = 0(1)10,000$ ; exact
[(21)]	(16)C, (21), (24)		$x^4, x^5: x = 0(.0001)0.9999$ ; exact
[(16)]	(16)C		$x^6, x^7: x = 0(.0001)0.9999$ ; exact
[(18)]	(18)C		$x^p: x = 0(.001)0.999; p = 2(1)6; 8D$
[(26)]	(26)C		$2^n; n = 1(1)100$ ; exact
[(22)]	(11)C, (22)C	2.4	$1/x, \Delta: x = 1(.001)4(.01)9.99; 6D$
[(18)]	(13), (18)C		$1/x: x = 0.1(\text{opt.int.})3; 6S; 1896$ cards
[(18)]	(10), (18)C		$1/x: x = 0.629(\text{opt.int.})16.15; 6S; 1196$ cards
[(14)]	(14)C		$1/x: x = 1(\text{opt.int.})10.00810; 5S$ or $6S; 577$ cards
	(16)		$1/x: x = 2(.01)6.6; 7D$ ; on same card with $\log_e x$
[(41)-(21)]	(21)		$1/x; A, B; A = \frac{1}{2}(\Delta_i + \Delta_{i-1}), B = \frac{1}{2}\delta^2$ $x = 0.1(.0001)0.9999; 7S$
[-(3)]	(16), (24)C		$1/x, -\Delta: x = 1(1)1000; 7D$ ; on same card with $\sqrt{x}, \sqrt{10x}, x^2$
[(41)-(18)]	(18)C		$1/x: x = 1(1)9999; 7D$ ; on same card with $1/x^2, x^2$
[(41)-(27)]	(27)		$1/x, \Delta(1/x): x = 1(1)10,000$ for $1/x$ ; $x = 1000(1)10,000$ for $\Delta(1/x)$ ; 7S
[(41)-(21)]	(21)		$1/x: x = 1(1)12,500; 7S$
[(41)-(26)]	(23)C, (24)		$1/x, -\Delta: x = 1000(1)2000(2)4000-$ $(5)10,000; 7S$ ; on same card with $\sqrt{x}, \sqrt{10x}$
[(23)]	(23)C		$1/x: x = 10(10)10,000; 7S$ or $8S$
[(41)-(11)]	(11)C		$1/x: x = 1(.1)20$ and $x = 1(1)2500; 8D$
[(23)]	(16), (24)C		$1/x: x = 1(1)9999; \Delta, \delta^2; 10D$ ; on same card with $x^2$
[(17)]	(17)C		$1/x: x = 1000(1)9999; 10D$ ; on same card with $x^2$

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(26)]	(26)		$1/x, \Delta: x = 1000(1)19,999; 10D$
[(41)-(18)]	(18)C	2.5	$1/x^2: x = 1(1)9999; 7D; \text{ on same card with } 1/x, x^2$
[(23)]	(23)C		$x^{-2}, x^{-3}: x = 10(10)10,000; 7S \text{ or } 8S$
[-(3)]	(16), (24)C	2.6	$\sqrt{x}, \sqrt{10x}, \Delta: x = 1(1)1000; 4D$
[(25)]	(25)		$\sqrt{x}: x = 1(\text{opt.int.})100; 5S; 401 \text{ cards}$
	(14)C		$\sqrt{x}: x = 1(\text{opt.int.})100; 5D; 1082 \text{ cards}$
[(43)-(13)]	(13)C, (18)		$\sqrt{x}: x = 1(\text{opt.int.})100; 6D; 332 \text{ cards}$
[(18)]	(18)C		$\sqrt{x}: x = 0(\text{opt.int.})0.9950; 6D; \text{ on same card with } \sqrt{1-x}; 583 \text{ cards}$
[(18)]	(18)C		$\sqrt{x}: x = 0.5(\text{opt.int.})2; 6D; 246 \text{ cards}$
	(24)		$\sqrt{x}: x = 1(\text{opt.int.})99.430; 6D; 181 \text{ cards}$
[(18)]	(18)C		$\sqrt{x}: x = 1(\text{opt.int.})100; 6D; 1793 \text{ cards}$
[(18)]	(18)C		$\sqrt{x}: x = 1(\text{opt.int.})0.0001; \text{ to about } 10D; 242 \text{ cards}$
[(21)]	(23)C, (24)		$\sqrt{x}, \sqrt{10x}, \Delta: x = 1000(1)2000(2)4000-(5)10,000; 6D$
[(41)-(21)]	(21)		$\sqrt{x}, \sqrt{10x}: x = 1(1)12,500; 7D \text{ for } x < 1000, 6D \text{ for } x > 1000$
[(26)]	(23), (26)		$\sqrt{x}, \Delta, \delta^2, \frac{1}{2}(\Delta_0 + \Delta_{-1}): x = 101-(1)10,000; 6D$ Range extended by (23) to include $x = 1(1)100.$
[(26)]	(26)		$\sqrt{x}, \sqrt{10x}, \Delta: x = 1000(1)1995; 2000-(2)4000(5)9995; 6D$
[(26)]	(23)C, (26)		$\sqrt{x}, \Delta: x = 10,000(10)100,000; 6D$
[(23)]	(23)C		$\sqrt{x}, \sqrt{10x}: x = 1000(1)10,000; 8D \text{ in } \sqrt{x}, 7D \text{ in } \sqrt{10x}$
[(2)]	(24)		$\sqrt{x}, \Delta, 1/\sqrt{x}, \Delta: x = 1(1)9999; 6D \text{ in } \sqrt{x}, 8D \text{ in } 1/\sqrt{x}$ [last place inaccurate]; on same card with $1/x, \Delta(1/x)$
[(41)-(27)]	(27)		$1/\sqrt{x}: x = 1(1)1000; 7D$
	(24)		$1/\sqrt{x}: x = 1.000(\text{opt.int.})99.180; 7D; 902 \text{ cards}$
[(41)-(21)]	(21)C		$\sqrt{x}, A, B; A = \frac{1}{2}(\Delta_i + \Delta_{i-1}), B = \frac{1}{2}\delta_i^2: x = 0.0001(.0001)1(.001)9.999; 8D$
[(11)]	(11)C		$\sqrt{x}: x = 0(1)1200; 8D$
[(41)-(27)]	(27)		$\sqrt{x}, \sqrt{10x}, \Delta; x = 1,000(1)10,000; 8S-9S; \text{ also } \sqrt{x}, x = 1(1)1000$
[(23)]	(24)C		$\sqrt{x}, \Delta, \delta^2: x = 1(1)1000; 10S$

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(23)]	(24)C		$\sqrt{x}$ , $\sqrt{10x}$ ; $\Delta$ , $\delta^2$ : $x = 1000(1)10,000$ ; 10S Tables for reducing interferometer data
[(26)]	(26)		$f(x, y) = (\sqrt{(x+1)^2 - y^2} - \sqrt{x^2 - y^2}) / (2x+1)$ $x, y = 1(1)100$ ; $y \leq x$ ; 5D
[(18)]	(18)C		Table of $f = \sqrt{1-x}$ : $x = 0(\text{opt.int.})-$ 0.9950; 6D; 583 cards On same card with $\sqrt{x}$
[(26)]	(26)		$(1-x^2)^{-\frac{1}{2}}$ : $x = 0(.005)0.1(.001)-$ 0.808(.0005)0.9; 4D to 6D $x = 0.9(.0001)0.96-$ (.00001)0.99499; 5D to 7D
[(23)]	(23)C		$(1-x^2)^{-\frac{1}{2}}$ : $x = 0(\text{various})1$ ; 8D
[(26)]	(23)C	2.63	$x^{\frac{1}{2}}$ : $x = 10(10)10,000$ ; 7S or 8S
[(82)-(17)]	(17), (18)		$x^{-\frac{1}{2}}$ : $x = 2(.001)7.5(.01)20$ ; 8D
[(25)]	(25)	2.7	$x^{\frac{1}{2}}$ : $x = 1(\text{opt.int.})1000$ ; 5S; 540 cards
[(26)]	(23)C, (24)		$x^{\frac{1}{2}}$ , $\Delta$ : $x = 1,000(1)2,000(2)4,000(5)-$ 9,999; 6D At (26)C with slight modifications.
[(41)-(21)]	(21)		$x^{\frac{1}{2}}$ , $A$ , $B$ ; $A = \frac{1}{2}(\Delta_i + \Delta_{i-1})$ , $B = \frac{1}{2}\delta^2$ ; $x = 1(.001)9.999$ ; 7D
[(41)-(21)]	(21), (24)		$x^{\frac{1}{2}}$ : $x = 1(1)12,500$ ; 7D for $x < 1000$ ; 6D for $x > 1000$
[(26)]	(23)C		$x^{\frac{1}{2}}$ : $x = 10(10)10,000$ ; 7S or 8S
[(41)-(27)]	(27)		$x^{\frac{1}{2}}$ : $x = 1(1)10,000$ ; also $\Delta$ for $x >$ 1000; 8S
[(23)-(24)]	(16), (18), (23), (24)C		$x^{\frac{1}{2}}$ , $(10x)^{\frac{1}{2}}$ , $(100x)^{\frac{1}{2}}$ : $x = 0(.01)10$ ; 9D or 10D
[(23)-(24)]	(18), (23), (24)C		$x^{\frac{1}{2}}$ , $(10x)^{\frac{1}{2}}$ , $(100x)^{\frac{1}{2}}$ : $x = 0(.01)10$ ; 9D or 10D
[(23)-(24)]	(24)C		$x^{\frac{1}{2}}$ , $(10x)^{\frac{1}{2}}$ , $(100x)^{\frac{1}{2}}$ : $x = 0(.01)10$ ; 15D in $x^{\frac{1}{2}}$ ; 14D in $(10x)^{\frac{1}{2}}$ and $(100x)^{\frac{1}{2}}$
[(23)-(24)]	(24)C		$x^{\frac{1}{2}}$ , $(10x)^{\frac{1}{2}}$ , $(100x)^{\frac{1}{2}}$ : $x = 0(.01)10$ ; 15D
[(18)]	(18)C	2.8	$x^{1/6}$ : $x = 0.1(\text{opt.int.})1$ ; 6D; 284 cards
[(67)-(24)]	(24)C		$x^{\frac{1}{2}}$ , $x^{\frac{1}{3}}$ , $x^{-\frac{1}{2}}$ , $x^{-\frac{1}{3}}$ : $x = 1(.01)10(.1)100$ (1)1000(10)10,000; 10S
[(13)]	(13)C	2.81	$10^{-10^a}10^{az}$ , $a = 0, 1, 2$ : $x = -6.99-$ (.01)0.99; 7D
		2.9	Powers of complex numbers

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(16)]	(16)C		$\sqrt{w^2 + i} = a + ib$ ; $w = 0(.01)17$ ; 8D in $a$ ; 7D in $b$
[(24)]	(24)C		$(x + iy)^n = u_n + iv_n$ ; $x = -20(2)(-10)(1)0$ ; $y = 0(1)20(10)400$ ; $n = 0(1)10$ ; exact
[(10)]	(10)	<b>2.96</b>	$1/z$ ; $z = x + iy$ ; $x, y = -4(.02)4$ ; 5D
		<b>3.11</b>	Factorials
[(41)-(27)]	(27), (28)		$x!$ ; $x = 0(1)100$ ; 8S
[(21)]	(21)		$x!$ and $1/x!$ ; $x = 1(1)1000$ ; 62S; Computed on the ENIAC in connection with determination of $e$
		<b>3.3</b>	Binomial coefficients
[(26)]	(26)		$n = 1(1)100$ ; exact
		<b>4.4</b>	Modified Bernoulli polynomials
[(24)]	(24)C		$b_k(x) = \sum_{n=1}^{\infty} n^{-k} \sin [nx + \frac{1}{2}(k-1)\pi],$ $k \geq 1$ $= [(-1)^{k\frac{1}{2}}(2\pi)^k B_k(\frac{1}{2}x/\pi)]/k!$ , where $B_k$ is the Bernoulli polynomial: $x = \pi y/36$ , $y = 0(1)36$ ; $k = 1(1)11$ ; 17D
		<b>6</b>	Common logarithms and antilogarithms
[(18)]	(18)C	<b>6.1</b>	$\log_{10} x$ ; $x = 1(1)99$ ; 3D
[(23)]	(23)C		$\log_{10} x$ ; $x = 1(1)13,000(10)23,000$ ; 5D
[(45)-(4)]	(4)		$\log_{10} x$ ; $x = 1(.001)9.999$ ; 5D
[(18)]	(10), (18)C		$\log_{10} x$ ; $x = 1(\text{opt.int.})10(10^i)$ ; $i = -1(1)5$ ; 5D; 1463 cards
[(13)]	(13)C, (18)		$\log_{10} x$ ; $x = 1(\text{opt.int.})10$ ; 6D; 568 cards
[(23)]	(23)C		$\log_{10} x, \Delta$ ; $x = 10(10)10,000$ ; 7S or 8S
[(44)-(27)]	(27)		$\log_{10} x$ ; $x = 1000(1)10,000$ ; 7D
[(44)-(21)]	(21)		$\log_{10} x$ ; $x = 1(.0001)9.9999$ ; 7D
[(64)]	(16), (24)		$\log_{10} x$ ; $x = 1000(\text{opt.int.})10,000$ ; 7D; 586 cards
[(64)]	(24)		$\log_{10} x$ ; $x = 0(\text{opt.int.})0.962400$ ; for quadratic interpolation; 7D; 119 cards
[(18)]	(26)		$\log_{10} x$ ; $x = 1(\text{opt.int.})9.9958$ ; 8D
[(18)]	(10), (16), (18)C		$\log_{10} x$ ; $x = 1(\text{opt.int.})10$ ; 8D; 5640 cards

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(64)]	(16), (24)		$\log_{10} x: x = 0(\text{opt.int.})0.998284; 8\text{D}; 707$ cards
[(75)-(2)]	(2)C, (18)C		$\log_{10} x, \Delta: x = 1(.0001)10; 8\text{D}$ . (Prepared at Vega Aircraft Corp. by RAY CRAWFORD, in collaboration W. D. BELL.)
[(75)-(2)]	(2)C, (24)C	<b>6.2</b>	$\log_{10} x, \Delta, 1/\Delta: x = 1(.001)9.999; 8\text{D}$ Radix table
[(18)]	(18)C		$f = 10^{\pm x}; m \cdot 10^n, m = 1(1)9;$ $n = 1(1)4; \text{to about } 7\text{S}; 9$ cards
[(8)]	(8)	<b>6.26</b>	$\log_{10} \left( \frac{1-x}{y-x} \right): x = 0.02(.01)0.99;$ $y = 0(.005)0.05(.01)0.2; 5\text{D}$
[(8)]	(8)	<b>6.4</b>	Antilogarithms $10^x: x = 0(\text{opt.int.})0.999; 5\text{D}; 407$ cards
[(46)-(21)]	(21)		$10^x: x = 0(.0001)1; 5\text{D}$
[(44)-(22)]	(22)C		$10^x: x = 0(.001)0.999; 8\text{D};$ auxiliary functions for interpolation
[(23)-(14)]	(14)C		$10^x, \Delta: x = 0(.00001)1; 8\text{D}$
[(64)]	(24)		$10^x: x = 0(\text{opt.int.})-5.3467875; 9\text{D}; 707$ cards
[(51)-(18)]	(18)C		$10^{-x}: x = 0(.025)1; 10\text{D}$
[(23)]	(23)C, (24)	<b>7.0</b>	$10^x: x = 0(.00001)1; 10\text{D}$ Trigonometric functions. Also see <b>24.2</b> and <b>28</b> .
	(4)	<b>7.1</b>	$\sin x, \cos x: x = 0(.001)6.283; 5\text{D}$
[(18)]	(18)C		$\sin x: x = 0(.001)1.571; 7\text{D}$
[(18)]	(18)C		$\sin x: x = 0^\circ(\text{opt.int.})90^\circ; 8\text{D}; 176$ cards
[(23)-(11)]	(11)C		$\sin x, \cos x: x = 0(.1)20; 8\text{D}$
[(23)-(11)]	(11)C		$\sin x, \cos x: x = 0(.001)1.599; 8\text{D}$
[(26)]	(26)		$\sin x: x = 0(.01)99.99, \Delta; 8\text{D}$
[-(3)]	(16), (23), (24)C (4)		$\sin x, \cos x; \Delta, \delta^2: x = 0(.01)27; 8\text{D}$
			$\sin x, \cos x, \tan x: x = 0.0001(.0001)-1.5709; 9\text{D}$ for $\sin x, \cos x; 8\text{S}$ (or less) for $\tan x$
[(23)-(18)]	(18)C		$\sin x: x = 0(.001)1.999; 9\text{D}$
[(23)-(18)]	(18)C		$\sin x: x = 0(.001)1.571; 9\text{D}$
[(66), (24)-(24)]	(24)C		$\sin x, \cos x, \Delta, \delta^2: x = 0(.0001)1; 11\text{D}$
[(23)-(12)]	(12)C, (24)	<b>7.12</b>	$\tan x, \cot x: x = 0(.0001)2; \text{mostly } 8\text{S}$ on same card with $\tanh x, \coth x$

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(18)]	(10), (13), (18)C	7.2	$y = \tan f: f = 89^{\circ}.942(.001)90^{\circ}; 3D$ <i>Note:</i> This is used by (9) as a critical table for $f = \arctan x$ in the region close to $f = 90^{\circ}$ .
[-(3)]	(24)C		$\sin x, \cos x, \tan x, \cot x, \Delta:$ $x = 0(^{\circ}1)360^{\circ}; 5D$
[(68)-(27)]	(27)		$\sin x, \cos x, \tan x, \cot x:$ $x = 0(^{\circ}001)90^{\circ}; 4S - 7S$
[(69)-(21)]	(21)		$\sin x, \cos x, \tan x: x = 0(^{\circ}01)44^{\circ}99;$ 5D
[(11)]	(11)C		$\tan x: x = 0(^{\circ}1)90^{\circ}$
[(68)-(11)]	(11)C, (24)		$\sin x, \cos x, \sin kx, \delta^2; k = 1(1)9;$ $x = 0(^{\circ}1)360^{\circ}; 7D$
[(68)-(17), (22)]	(10), (13), (16), (17)C, (18), (22)C, (23)C, (24)C		$\sin x, \cos x, \tan x, \Delta: x = 0(^{\circ}01)90^{\circ};$ 7S or 7D [range extended to $180^{\circ}$ at (18); argument also given in deg., min. and sec. at (22)]
[(68)-(21)]	(21)		$\sin x: x = 0(^{\circ}001)89^{\circ}999; 7D$
[(47)-(22)]	(22)C		$\sin x, \cos x, \Delta: x = 0(^{\circ}01)90^{\circ}; 8D$
[(68)-(26)]	(26)		$\sin x, \tan x, \Delta: x = 0(^{\circ}01)89^{\circ}99; 8D$
[(23)]	(23)C		$\sec x, \csc x: x = 0(^{\circ}01)90^{\circ}; 9S$
[(23)-(24)]	(24)		$\sin x, \cos x: x = 0(1^{\circ})360^{\circ}; 15D$
[(23)]	(24)C		$\sin x, \Delta: x = 0(^{\circ}01)90^{\circ}; 15D$
[(23)]	(23)C		$\sin x, \cos x, \delta^2: x = 0(^{\circ}01)90^{\circ}; 15D$
[(23)]	(23)C, (24)C		$\sin x, \cos x, \csc x, \sec x: x = 0(^{\circ}01)45^{\circ};$ 15D in $\sin x$ and $\cos x; 8D$ in $\csc x$ and $\sec x$
[(27)]	(27)	7.3	$\sin x, \cos x: x = 0(^{\circ}01)90^{\circ}; 17S.$ Computed on IBM Relay calculator. Argument in grades or units of complete circle; $\frac{1}{2}\pi = 100^{\circ}; 2\pi = 1^{\circ}$
[(51)-(14)]	(14)C		$\sin x, \cos x; 0.25 - x, \Delta:$ $x = 0(0^{\circ}0001)0.125; 4D; 5D; \text{ and } 6D$
[(81)-(1)]	(1), (4)		$\sin x, \cos x: x = 0(^{\circ}001)1; 5D$ First difference given at (1).
[(18)]	(18)C		$\sin x, \cos x, \Delta^3: x = 0(^{\circ}01)1; 8D$ (third order interpolation)
[(51)-(14)]	(14)C, (24)		$\sin x, \cos x, \Delta, \delta^2; 0.25 - x:$ $x = 0(^{\circ}00001)0.125; 15D$ Rounding error in last place may occasionally be greater than $\frac{1}{2}$ unit. At(14) C, $\Delta^2$ is given in place of $\delta^2$ .

<i>Source</i>	<i>Available at</i>	<i>Description of Tables</i>
(26)	(26)	<b>7.31</b> $\sin x, \cos x: \pm x = 0.001(^{\circ}001)99.999;$ 8D (First differences for functions at intervals of $^{\circ}01$ )
[(47)-(18)]	(18)C	<b>7.35</b> $\sin x, \cos x: x = 0(1^{\circ})50; 20D$ Trigonometric functions; argument in mils
[(26)-(21)]	(21)	<b>7.4</b> $\sin x, \cos x, \tan x, \sec x, \csc x, \cot x,$ $\frac{1}{2} \tan x: x = 0(.1)800$ mils; 5D (with extension to 1600 mils) $\frac{1}{2} \cot x, x = 50.9(.1)800$ mils; 5D Trigonometric functions; argument in degrees, minutes, and seconds
[-(3)]	(24)C	$\sin x, \cos x, \Delta, \delta^2: x = 0^{\circ}(10')360^{\circ}; 5D$
[(47), (28)-(28)]	(28)	$\cos x, \Delta$ per sec.: $x = 0(1')180^{\circ}; 5D,$ 6D
[(47), (28)-(28)]	(28)	$1 - \cos x: x = 90^{\circ}(1')180^{\circ}; 5D$
[-(3)]	(24)C	$\tan x, \cot x; \Delta, \delta^2: x = 0^{\circ}(10')360^{\circ}; 5S$
[-(3)]	(24)C	$\sec x, \csc x; \Delta, \delta^2: x = 0^{\circ}(10')360^{\circ}; 5S$
[-(3)]	(24)C	$\sin x, \cos x, \tan x, \cot x, \sec x, \csc x:$ $x = 0^{\circ}(10')360^{\circ}; 5D$ or 5S
[(45)-(4)]	(4)	$\sin x, \cos x, \tan x, \cot x:$ $x = 0^{\circ}1'(1')89^{\circ}59'; 5D$ or 5S
[(47), (28)-(28)]	(28)	$\tan^2 x: x = 0(1')88^{\circ}; 6D$
[(80)-(17)]	(17)	$\tan x: x = 0(10'')30^{\circ}; 7D$
[(23)]	(23)C	$\cos x, \Delta/60: x = 0^{\circ}(1')90^{\circ}; 8D$
[(47), (28)-(28)]	(28)	$\tan x, \cot x, \Delta$ per sec.: $x = 0(1')90^{\circ};$ 8D
[(47), (28)-(28)]	(28)	$\sin nx, n = 1, 2, 4, 6, 8, \Delta$ per sec.: $x = 0(1')90^{\circ}; 12D$ for $n = 1; 10D$ to 7D for $n \geq 2$ . Also other sets to 4D, 5D, and 6D for some values of $n$ .
[(47), (28)-(28)]	(28)	$\sin^2 x, \cos^2 x: x = 0(1')90^{\circ}; 10D$
[(47)-(28)]	(28)	$\sin x, \cos x, \tan x, \cot x, \Delta$ per sec.: $x = 0(1')90^{\circ}; 15D$
[(47)-(70)]	(23)C, (26), (28)	$\sec x, \csc x: x = 0^{\circ}(1')90^{\circ}; 10D$
[(23)]	(23)C	<b>7.44</b> $\sin y, \cos y, \Delta/60; y = (b/a) \tan x,$ where $a$ and $b$ are the major and minor axes of the ellipse on Clarke's spheroid (1866): $x = 0(1')90^{\circ}; 8D$

<i>Source</i>	<i>Available at</i>	<i>Description of Tables</i>
[(11)]	(11)C	7.63 $\frac{\sin x}{x}, \frac{\cos x}{x}; \frac{d^n}{dx^n} \left( \frac{\sin x}{x} \right);$ $x = 0(.1)20; 8D$
[(23)]	(23)C	7.8 Area and circumference of circle $A = \frac{1}{4}\pi D^2; C = \pi D; A,$ $D = 0(.0001)0.9999; 8D \text{ in } A; 7D$ $\text{in } D$
[(56)-(21)]	(21)C	7.9 (a) $\xi(\theta) = \int_0^\theta \sec^3 t \, dt; 1 - \beta = \xi/b;$ $\theta = 0(1')87^\circ; 4D \text{ to } 7D$ (b) $\sin \theta(\xi), \cos \theta(\xi)$ , where $\theta(\xi)$ is the inverse of $\xi(\theta); \xi = 0(.01)2(.05)50;$ $8D \text{ or } 9D$ (c) $X(\beta, b) = \int_1^\beta \cos \{\theta[b(1-t)]\} \frac{dt}{t}$ (d) $Y(\beta, b) = \int_1^\beta \sin \{\theta[b(1-t)]\} \frac{dt}{t}$ (e) $T(\beta, b)$ $= \int_1^\beta \cos \{\theta[b(1-t)]\} t^{-1} dt$ $X, Y, T: \beta = 0.02(.02)3;$ $b = 0.1(.1)2; 8D$
[(24)]	(24)C	$f(\cos x, \alpha, k)$ $= k \left\{ \frac{\cos x}{\alpha} + \frac{1 - \frac{\cos x}{\alpha}}{(1 + \alpha^2 - 2\alpha \cos x)^{\frac{1}{2}}} \right\};$ $\cos x = -1(.01)1; 5S \text{ or } 6S$ $\alpha = 0.015, 0.016, 0.017, 0.018$ , with $k = 12045.4$ $\alpha = 0.0000425$ , with $k = 0.02480289$
		8.5 Tables with arguments in time. See 28.5
		9 Inverse circular functions.
[(18)]	(18)C	9.10 $\arcsin x: x = 0.005(.01)0.995; 7D$
[(23)-(7)]	(7)C, (24)C	$\arcsin x, \arcsin(x-h), \arcsin(x-2h),$ $\delta_0^2, \delta_{-1}^2$ $x = 0(.0001)0.989(.00001)0.99999;$ $12D$
[(18)]	(18)C	$x_1 = \arcsin f;$ $f = 0(.01)1(-.01)-1(.01)0; 2D$ $x_2 = \arccos f; f = 1(-.01)-1(.01)1;$ $2D$ $x_1, x_2 = 2\pi\lambda, 0 \leq \lambda \leq 1; 2D \text{ in } \lambda$ (critical table of $\sin x, \cos x$ )
[(23)-(18)]	(18)C	$2\pi\lambda = \arcsin x: x = 0.005(.01)0.995;$ $5D \text{ in } \lambda$



<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(68)-(14)]	(14)C		$y = \arccos x$ and $\arccos -x$ ( $y$ in degrees): $x = 0.99899(.00001)1$ ; accuracy to within $0^\circ.01$
[(68)-(14)]	(14)C		$y = \arccos x$ ( $y$ in degrees): $x = 0$ (opt.int.)0.99899; accuracy to within $0^\circ.01$ ; 95 cards
[(18)]	(18)C	<b>9.12</b>	$\arctan x: x = 0$ (opt.int.)100; 6D; 110 cards arctan given in radians
[(23)-(11)]	(11)C		$\arctan x: x = 0$ (.001)7; 6D
[(26)]	(26)		$\arctan x: x = 0$ (.001)1; 10D
[(23)-(24)]	(24)C		$\arctan x, \nabla, \delta^2: x = 0$ (.001)7(.01)-50(.1)300(1)2000(10)10,000; 12D. [ $\nabla$ is the "backward" first difference.]
[(23)-(10)]	(10)		$\arctan x: x = 0$ (.001)1(1)10(10)-100(100)1000; 12D
[(23)-(27)]	(27)		$\arctan x: x = 0$ (1)2000(10)9000; 12D
[(42)-(24)]	(23)C, (24)C		$\arctan m/n, \operatorname{arccot} m/n, m^2 + n^2:$ $m = 1$ (1) $n; n = 1$ (1)100; 12D
[(23)-(24)]	(16), (24)C		$\arctan x: x = 1$ (1)2000; 15D
[(24)]	(24)C		$n \arctan 1 = n \frac{\pi}{4}: n = 0$ (1)100; 20D
[(21)]	(21)	<b>9.15</b>	$y = \arctan x: x = 0$ (.001)3.75(.01)-18.25; ( $y$ in degrees); accuracy to $0^\circ.01$ . [Extension for $x > 18.25$ by special table.]
[(18)]	(10), (13), (18)C		$y = \arctan x: x = 0$ (opt.int.)1000; ( $y$ in degrees) accuracy $0^\circ.001$ ; 350 cards
[(23)-(22)]	(22)C		$y = \arctan x, \Delta: x = 0$ (.001)1000; ( $y$ in degrees); 6D
		<b>9.7</b>	General spherical triangle (see <b>28.4</b> )
		<b>10</b>	Exponential and hyperbolic functions
[(23)-(13)]	(13)C	<b>10.0</b>	$e^x, e^{-x}: x = 0$ (.00001)0.00099; 7D or 8D
[(23)-(13)]	(13)C		$e^x, e^{-x}: x = 0.001$ (.001)0.999; 7D or 8D
[(23)-(13)]	(13)C		$e^x, e^{-x}: x = 1$ (1)15; 8S
[(23)-(4)]	(4)		$e^x, e^{-x}: x = 0$ (.0001)1; 10D for $e^x$ ; 9D for $e^{-x}$
[(45)-(18)]	(18)		$e^{-x}, \Delta: x = 0$ (.01)3(.1)10; 6D
[(23)]	(24)C		$e^{-x}: x = 0$ (.1)10; 10D
	(16), (24)		$e^{-x}: x = 0$ (.001)2.499; 10D

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(65)-(24)]	(16), (24)C		$e^x, e^{-x}; \Delta, \delta^2, \Delta^3: x = 2.5(.001)10; 10D$
[(79)-(4)]	(4)		$e^x, e^{-x}: x = 1(1)100; 10S$
[(23)-(21)]	(21)		$e^x: x = 2.5(.001)5(.01)10; 15D$ for $x \leq 5; 12D$ for $x > 5$
[(27)]	(27)		$e^x: x = -100(.01)100; 15S$ . Computed on IBM Relay Calculator.
[(23)-(14)]	(14)C		$e^x, e^{-x}: x = 0(10^{-6})0.0001; 18D$
[(23)-(24)]	(24)		$e^{-x}: x = 0(.01)2.49; 18D$
[(23)-(14)]	(14)C, (21), (24)		$e^x, \Delta, \Delta^2, \Delta^3, \Delta^4: x = -2.5(.0001)2.5; 18D$ for $x < 1; 15D$ for $x \geq 1$
[(23)-(24)]	(16), (24)		$e^x: x = 0(.001)2.499; 18D$ for $x < 1; 15D$ for $x \geq 1$
[(27)-(26)]	(23), (26)		$e^x, e^{-x}: x = 0(.01)100; 14S$ and $18S$
[(79)-(14)]	(14)C		$e^x, e^{-x}: x = 1(1)100; 19S$
[(14)]	(14)C, (24)	<b>10.33</b>	$(1 - e^{-x})/x: 0 \leq x \leq 13.0369; 6D;$ (pseudo-opt. int.); 792 cards
[(26)]	(26)	<b>10.4</b>	$\sinh x, \cosh x, \Delta, \delta^2: x = 0(.0001)0.01(.001)1(.01)99.99; 8D$
[(23)-(4)]	(4)		$\sinh x, \cosh x: x = 0(.0001)1; 9D$
[(26)]	(26)		$\sinh x, \cosh x: x = 0(.01)16.11; 16D$
[(23)-(12)]	(12)C, (24)		$\tanh x, \coth x: x = 0(.0001)2; \text{mostly } 8S$
	(26)		$\tanh x, \Delta: x = 0(.01)9.9; 8D$
[(18)]	(18)C		$\tanh x: x = 0(.005)6; 9D$
		<b>11</b>	Natural logarithms
[(23)-(21)]	(21), (23)	<b>11.1</b>	$\log_e x: x = 1(1)100,000; 16D$
	(16)	<b>11.2</b>	$\log_e x: x = 2(.01)6.6; 7D$
[(24)]	(24)		$\log_e x, \Delta: x = 1(.001)9.999; 7D$
[(23)-(4)]	(4)		$\log_e x: x = 0(.0001)1; 16D$
[(23)-(21)]	(21)		$\log_e x: x = 0.0001(.0001)10; 16D$
[(18)]	(18)C		$\log_e x$ , (radix table): $0.001 \leq x \leq 9.9999; 5D; 76$ cards
[(18)]	(18)C	<b>11.26</b>	$\log_e (-\log_e x): x = 0(.0001)0.001-.001)0.999(.0001)0.9999; 4D$
[(18)]	(18)C, (23)		$\log_e (-\log_e x); \Delta, \Delta^2: x = 0(.00001)-0.006(.0001)0.08(.001)0.84(.0001)-0.984(.00001)0.99995; 5D$
		<b>11.6</b>	Inverse hyperbolic functions: $\operatorname{arctanh} x = \frac{1}{2} \log_e [(1+x)/(1-x)];$ $\operatorname{arc} \sinh x = \log_e (x + \sqrt{1+x^2})$
[(23)-(18)]	(16), (18)C		$\operatorname{arctanh} x: x = 0(.002)0.5; 6D$ ( $\Delta$ also on cards at 18)

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(50), (14)-(14)]	(14)C		arcsinh $x$ , arc tanh $x$ : $x = 0.00001(.00001)0.001$ ; 20D and $x = 0.001(.0001)0.1(.001)0.999$ ; 7D to 10D
[(50), (14)-(14)]	(14)C		arcsinh $x$ , arccosh $x$ $x = 1(.001)3(.01)10(.1)20(1)50$ ; 7D and 8D
[(10)]	(10)	11.7	Logarithms of complex numbers $f = \log_e z$ , $z = x + iy$ : $x, y = -4(.02)4$ ; 5D
[(14)]	(14)C	11.8	Addition logarithms $\log_e(1 + e^x)$ : $x = 0(\text{opt.int.})13$ ; 5D; 270 cards
		13	Exponential integrals for real and complex arguments; sine and cosine integrals $E_n(x) = \lim_{c \rightarrow +\infty} \left[ x^{n-1} \int_x^{x+c} u^{-n} e^{-u} du \right];$ $c$ real
[(23), (14)-(14)]	(14)C	13.2	$E_1(x)$ : $x = 0.01(.01)10(.1)90$ ; about 9S
[(23)-(16)]	(16)C	13.4	Si( $x$ ), $\Delta$ , $\delta^2$ : $x = 0(.01)99.99$ ; 10D
[(23)-(16)]	(16)C	13.5	Ci( $x$ ), $\Delta$ , $\delta^2$ : $x = 0(.01)99.99$ ; 10D
[(23), (2)-(2)]	(2)	13.52	Modified cosine integral: $-\text{Ci}(x) + \gamma + \log_e x$ $x = 0(.001)10(.01)49.99$ ; 6D
[(23), (24)-(24)]	(24)C	13.8	$E_2(x)$ : $x = 0(.001)2$ ; 7D $x = 2(.001)10$ ; 5S or 6S; $x = 10(.001)16$ ; 5S to 8S
		13.9	Exponential integrals for complex arguments; $z = x + iy$ $E_1(z)$ : $x = 0(.02)4$ ; $y = 0(.02)3(.05)10$ ; 6D $E_1(z) + \log_e z$ : $x, y = 0(.02)1$ ; 6D $e^z E_1(z)$ : $x = 3(.1)10$ ; $y = 0(.05)10$ ; 6D
[(23)]	(23)C		$e^z E_1(z)$ : $-x = 0.5(.5)4.5$ ; $y = 0(.1)4(.5)10$ ; 6D $-x = 5(.5)10$ ; $y = 0(.5)10$ ; 6D $x = -10(1)10$ ; $y = 10(1)20$ ; 6D $\pm x = 11(1)20$ ; $y = 0(1)20$ ; 6D [Branch cut on negative imaginary axis]

Source	Available at	Description of Tables
		Available at (23); only partially punched at time of publication of this guide.
[(24)]	(24)C	$E_1(z)$ : $-x = 0(.1)3.1$ ; $y = 0(.1)3.1$ ; 10D $E_1(z) + \log z$ : $-x = 0(.1)1$ ; $y = 0(.1)1$ ; 10D
		14 Gamma function
[(63), (21)-(21)]	(21)	14.2 $100 + \log_{10} \Gamma(x)$ : $x = 1(.01)16.99$ ; 7D
[(23)]	(23)C	14.6 $\text{Log}_e \Gamma(z)$ , $z = x + iy$ : $x = 9(.1)10$ ; $y = 0(.1)10$ ; 14D or 15D [Extension of range to $x = 0(.1)10$ is in progress at time of publication of this guide.]
		15 Probability functions; Hermite polynomials; moments
		$z(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2);$
		$\alpha(x) = \int_{-x}^x z(t) dt$
[(23)-(14)]	(14)C	15.1 $z(x), \frac{1}{2}\alpha(x)$ : $x = 0(\text{opt.int.})4.751$ ; 5D; 886 cards Prepared for linear interpolation; fixed interval.
[(23)-(14)]	(4), (14)C, (24)	$z(x), \frac{1}{2}\alpha(x), \Delta, \delta^2$ : $x = 0(.001)7.8$ ; 8D At (14)C without differences.
[(23)-(14)]	(14)C	$z(x), \alpha(x)$ : $x = 0(.01)7$ ; about 10D [with auxiliary functions, prepared for third-order interpolation].
[(23)-(14)]	(4), (14)C, (23), (24), (26), (24)	$z(x), \alpha(x), \Delta$ : $x = 0(.001)7.8$ ; 15D At (14)C and (26) without differences.
		$e^{-\frac{1}{2}x^2}$ : $x = 0(.05)6.25$ ; 8D
[(4)]	(4)	15.281 $A = Be^{-x^2}$ ; $B = \int_0^x \exp t^2 dt$ : $x = 0.1(.01)9.99$ ; 10D in A; 8S, 9S, or 10S in B
		15.411 Inverse tables
[(62)-(14)]	(14)C	$x(p)$ , where $p = \frac{1}{2}[1 + \alpha(x)]$ , and $z(x)$ $p = 0.5(.0001)0.9999$ ; 8D
[(23)-(14)]	(14)C, (24)	15.521 $\frac{2}{\sqrt{\pi}} \frac{d^n}{dx^n} (e^{-x^2})$ : $x = 0(.1)5.9$ ; $n = 1(1)15$ ; (24 - n)D

Source	Available at		Description of Tables
[(23)-(14)]	(14)C, (24)	15.523	$\sqrt{2/\pi} \frac{d^n}{dx^n} (e^{-x^2})$ ; $x = 0(.1)8.4$ ; $n = 1(1)16$ ; (24 - n)D
[(14)]	(14)C	15.6	$\frac{d^n}{dx^n} z(x)$ and $\frac{1}{z(x)} \frac{d^n}{dx^n} z(x)$ (Hermite polynomials) $n = 1(1)10$ ; $x = 0(.01)12$ ; about 6S
[(14)]	(14)C		Hermite polynomials: $n = 1(1)10$ , $x = 0(.01)12$ ; exact
[(26)]	(26)	15.7	$m_i(x) = \frac{1}{(i-1)!!\sqrt{2\pi}} \int_0^x t^i$ $\times \exp(-\frac{1}{2}t^2) dt$ [see (40)] $M_i = m_i \left( \frac{1}{M} \right)$ : 5D; $\exp(M_i)$ ; 6D; $\exp(M_i - M_1)$ ; 2D [ $\exp(-M_i) - \exp(-M_{i+1})$ ]/ $\exp(-M_i)$ ; 6D [ $M_i \exp(-M_i)$ $- M_{i+1} \exp(-M_{i+1})$ ]/ [ $\exp(-M_i) - \exp(-M_{i+1})$ ]; 6D $1/M = 0.24(.01)0.99$ ; $i = 1(1)8$
[(14)]	(14)C		$F_m(x) = \frac{1}{2} \left[ \int_0^x (\frac{1}{2}t)^{m-1} e^{-t} dt \right] / \Gamma(\frac{1}{2}m)$ $m = 2(1)10$ ; $x = 0(\text{opt.int.})44$ ; 5D; 7637 cards
		15.8	Probability integrals relating to complex arguments
[(14)]	(14)C, (24)		$\phi_2(x, y) + i\phi_1(x, y)$ $= \int_0^x \exp(iy)z(t)dt$ ; 5D $\phi_3(x, y) = \int_x^\infty [\sin ty]z(t)dt$ ; 5D $x = 0(.1)4.5$ , $y = 0(.1)0.9$ ; $x = 0(.05)2(.1)4.5$ , $y = 1(.1)1.9$ ; $x = 0(.05)3(.1)4.5$ , $y = 2(.1)2.9$ ; $x = 0(.02)1.5(.05)3(.1)4.5$ , $y = 3(.1)3.9$ ; $x = 0(.02)2(.05)3.5(.1)4.5$ , $y = 4(.1)4.9$ ; $x = 0(.02)2.5(.05)3.5(.1)4.5$ , $y = 5(.1)6$ . Altogether 6216 cards.
[(14), (24)]	(14)C	15.9	$q(R, x) = \int_R^\infty t e^{-t(t+x^2)} I_0(tx) dt$ (Offset circle probabilities) $R = 0.1(.1)20$ ; $x = 0(.05)1$ ; 6D
		17-20	Bessel functions

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(5)-(16)]	(16)C	17.1	$J_0(x), J_1(x), \Delta: x = 0(.1)25; 6D$
[(5)-(16)]	(16)C		$J_0(x), J_1(x), \Delta J_0(x): x = 0(.1)99.9; 6D$
[(23)]	(14)C, (24)		$J_0(x), J_1(x): x = 0(.01)10; 10D$ First four advancing differences given along with functions at (14)C.
[(66)-(14)]	(14)C, (24)		$J_0(x), J_1(x): x = 10(.01)25; 10D, \Delta, \Delta^2, \Delta^3, \Delta^4$
[(66)-(11)]	(11)C		$J_0(x), J_1(x): x = 0(.01)25; 10D$
[(5)-(16)]	(16)C		$J_2(x), J_3(x): x = 0(.1)25; 8D$
[(5)]	(5)C		$J_n(x):$ $n = 0(1)15; x = 0(.001)25(.01)99.99;$ 18D for $n \leq 3; 10D$ for $n \geq 4$ $n = 16(1)100; x = 0(.01)99.99; 10D;$ $J_n(100): n = 0(1)100; 10D$ <i>Note:</i> At the time of preparation of this guide, values for $n = 79(1)100$ and $J_n(100)$ were not yet available for distribution.
[(23)-(16)]	(16)C	17.2	$\sqrt{\frac{\pi}{2x}} J_{n+\frac{1}{2}}(x): n = 1, 2; x = 0(.1)16; 8D$
[(66), (16)-(16)]	(16)	17.3	$Y_n(x), \Delta: n = 0(1)3; x = 0.01(.01)1; 6D$ [ $Y_0(x)$ and $Y_1(x)$ checked]
[(66)-(11)]	(11)C		$Y_0(x): x = 0(.01)25; 8D$
[(66)-(14)]	(14)C, (24)		$Y_0(x), Y_1(x): x = 10(.01)25; 8D$
[(66), (16)-(16)]	(16)		$Y_n(x), \Delta: n = 0(1)3; x = 0.5(.1)25; 8D$ [ $Y_0(x)$ and $Y_1(x)$ checked]
[(23)]	(23)C, (24)C, (26)		$Y_0(x), Y_1(x): x = 0(.01)10; 10D$
[(53)-(24)]	(24)		$Y_n(x): x = 0(.1)10; n = 1(1)21; 10S$
[(66)-(24)]	(24)C	18.1	$I_0(x)$ and $I_1(x), \delta^2: x = 0(.001)5; 8D$
[(23)-(14)]	(14)C, (23)C		$I_0(x), I_1(x): x = 0(.01)10; 10D$
[(66), (16)-(16)]	(16)		$I_n(x)$ or $e^{-x}I_n(x): n = 0(1)3;$ $x = 0(.1)20; 8D$ [ $e^{-x}I_n(x)$ for $x > 5$ ; entries for $n = 0, 1$ checked]
[(23)-(14)]	(14)C	18.2	$I_{\frac{1}{2}}(x)$ and $I_{-\frac{1}{2}}(x); x = 0.01(.01)25; 10S$
[(66)-(24)]	(24)C	18.3	$K_0(x), K_1(x)\delta^2$ ; also auxiliary functions for interpolating near the origin: $x = 0(.01)5; 7D$ to $10D$

Source	Available at		Description of Tables
[(66), (16)-(16)]	(16)		$K_n(x)$ or $e^x K_n(x)$ ; $n = 0(1)3$ ; $x = 0.5(.1)20$ ; 8D [ $e^x K_n(x)$ for $x > 5$ ; entries for $n = 0$ and 1 checked]
[(48)-(21)]	(21)	18.4	$e^{-x} I_0(x)$ , $e^{-x} I_1(x)$ , $e^x K_0$ , $e^x K_1$ , $e^x$ ; $x = 0(.02)16$ ; 7D
[(23), (14)-(14)]	(14)C		$e^{-x} I_0(x)$ , $e^{-x} I_1(x)$ ; $0 \leq x \leq 733.4104599$ ; 8D or better; 1996 cards [Quasi-optimum interval, arranged for quadratic interpolation, with modified first and second divided differences.]
[(14)]	(24)C		$e^{-x}$ , $e^{-x} I_0(x)$ , $e^{-x} I_1(x)$ ; $\Delta$ , $\delta^2$ ; $x = 0(.01)2.49$ ; 18D for $e^{-x}$ ; 8D for other functions
[(14)]	(24)		$e^{-x} I_0(x)$ ; $x = 0(.01)36$ ; 8D
[(14)]	(24)		$e^{-x} I_1(x)$ ; $x = 0(.01)72$ ; 8D
[(66)-(24)]	(24)		$e^{-x} I_0(x)$ , $e^{-x} I_1(x)$ , $e^x K_0(x)$ , $e^x K_1(x)$ ; $\delta^2$ ; $x = 5(.01)10(.1)20$ ; 8D
[(66)-(24)]	(24)		$e^{-x} I_0(x)$ ; $x = 2.5(.01)5$ ; 9D
[(24)]	(24)		$e^{-x} I_1(x)$ ; $x = 20(.5)120(1)625$ ; 8D to 10D
[(14)]	(24)		$e^{-x} I_0(x)$ ; $x = 20(1)48$ ; 37.5(2.5)92.5; 75(5)175; 160(10)380, 325(25)800; 10D
[(14)]	(24)		$e^{-x} I_1(x)$ ; $x = 16(1)48$ , 37.5(2.5)92.5; 70(5)170; 130(10)430, 325(25)700; 8D, 9D or 10D
		19.0	Bessel functions for complex argu- ments $Y_0(z)$ and $Y_1(z)$ ; $z = \rho \exp(i\phi)$ ; $\rho = 0(.01)10$ ; $\phi = 0(5^\circ)90^\circ$ ; 10D
[(23)]	(23)C, (24), (26)		$Y_0(ix)$ , $Y_1(ix)$ ; $x = 0(.01)10$ ; 10D
[(23)]	(23)C, (24)C	20.0	Bessel-Clifford functions
[(23)]	(23)C, (24)		$x^{-\frac{1}{2}n} J_n(2\sqrt{x})$ , and $x^{-\frac{1}{2}n} Y_n(2\sqrt{x})$ ; $n = 0, 1$ ; $x = 0(.02)1.5(.05)3(.1)-$ $13(.2)45(.5)115(1)410$ ; 8D or 9D
[(24)]	(24)		$I_0(2\sqrt{x})$ , $[I_1(2\sqrt{x})]/\sqrt{x}$ ; $\delta^2$ ; $K_0(2\sqrt{x})$ , $[K_1(2\sqrt{x})]/\sqrt{x}$ ; $x = 0(.02)1.5(.05)6.2$
[(24)]	(24)		$e^{-2\sqrt{x}} I_0(2\sqrt{x})$ , $[e^{-2\sqrt{x}} I_1(2\sqrt{x})]/\sqrt{x}$ , $e^{-2\sqrt{x}} K_0(2\sqrt{x})$ , $[e^{-2\sqrt{x}} K_1(2\sqrt{x})]/\sqrt{x}$ , $\delta^2$ ; $x = 6.2(.1)13(.2)36(.5)115(1)160(5)-$ 410; 7D to 9D

Source	Available at		Description of Tables
[(49)-(24)]	(24)C	20.8	$\int_0^x J_0(\lambda u) \exp iud u$ $= J_c(\lambda, x) + iJ_s(\lambda, x)$ $\int_0^x Y_0(\lambda u) \exp(iu) du$ $= N_c(\lambda, x) + iN_s(\lambda, x)$ <p>Auxiliary functions:  <math>C_c(\lambda, x) = N_c(\lambda, x)</math>  <math display="block">- \frac{2}{\pi} [\ln(\lambda x)] J_c(\lambda, x)</math>  <math>C_s(\lambda, x) = N_s(\lambda, x)</math>  <math display="block">- \frac{2}{\pi} [\ln(\lambda x)] J_s(\lambda, x)</math>  <math>J_c(\lambda, x), J_s(\lambda, x), N_c(\lambda, x), N_s(\lambda, x)</math>:  <math>x = 0(.02)2(.1)5; \lambda = 0.1(.1)1; 6D</math></p>
		22	Riemann Zeta function; Mathieu functions
[(78)-(16)]	(16)C, (24)	22.1	$(z-1)\zeta(z); z = x + iy; x = 0(.05)2;$ $y = 0(.05)4; 6D$ [computed on the EDSAC]
[(61)-(24)]	(24)	22.2	(a) Characteristic values, $be_r(s)$ and $bo_r(s)$ , associated with even and odd periodic solutions, respectively (period $\pi$ or $2\pi$ ), of Mathieu's equation $y'' + (b - s^2 \cos x)y = 0$ $b = be_r(s), bo_r(s), \delta^2; 0 \leq s \leq 100;$ $r \leq 15; 8D$ (b) Trigonometric coefficients $De_k^{(r)}(s)$ and $Do_k^{(r)}(s)$ associated with periodic solutions, same range of $r$ and $s$ , but at larger intervals; 9D or 9S (c) Joining factors, relating various solutions, $\delta^2$ ; mostly 8S, same range of $s$ and $r$ . For precise definitions see (61).
[(23)]	(23)C, (24)		(d) Periodic solutions $Se_r(s, x)$ and $So_r(s, x); 0 \leq s \leq 100, r = 6(1)15;$ $x = 0(1^\circ)90^\circ; 7D$
[(72)]	(24)		Solutions $g(t)$ and $h(t)$ of $y'' + \epsilon(1 + k \cos t)y = 0;$ $g(0) = h'(0) = 1; h(0) = g'(0) = 0$ $\epsilon = 1(1)10, k = 0.1(.1)1; t = 0(.1)3.1;$ $3.14(.0004)3.1428$



<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
			$\epsilon = 30(10)90,200, 300, 400; k = 0.02;$ $\epsilon = 900, k = 0.01$ $\epsilon = 5, k = 0.12(.02)(.24)$ and $1.1(.1)1.9$ Functions tabulated: $g(t), g'(t), h(t),$ $h'(t); 3D$ to $5D$ [computed on the ENIAC]
		<b>22.73</b>	See <b>24.2</b> , Harmonic analysis
[(13)]	(13)C, (16), (18)C	<b>23.1</b>	4-Pt. Lagrangean interpolation coefficients: $x = 0(.0001)0.5; 8D$ [7D at (18)] [from symmetry, this is equivalent to giving coefficients up to $x = 1$ ]
[(23)-(4)]	(4)C, (24)		6-Pt. Lagrangean interpolation coefficients: $x = 0.001(.001)0.999; 10D$
[(59)-(15)]	(15)C	<b>23.82</b>	Orthogonal polynomials for curve fitting $\xi_0'(x) = \lambda_0; \xi_1'(x) = \lambda_1(x - \bar{x});$ $\bar{x} = \frac{1}{2}(n + 1)$ [Orthogonal with respect to summation from 1 to $n$ .] $\xi_k'(x)$ and $\lambda k: k = 1(1)5; n = 3(1)104;$ exact See <b>23.82</b> of (40); <i>MTAC</i> , v. 1, p. 148-150, (59), and (58), p. 307-327, for uses of these polynomials.
[(13)]	(13)C	<b>24.2</b>	Harmonic analysis $A \sin 2\pi nx, A \cos 2\pi nx$ $n = 1(1)30; x = 0(1)120;$ $A = 1(1)10(10)100(100)900$ $5D$ for $A = 1; 1D$ for others
[(20), (23)-(20)]	(20)		$A \sin 2\pi nx, A \cos 2\pi nx$ $n = 0(1)30; x = 0(.002)0.25;$ $\pm A = 1(1)5; 10(10)50;$ $100(100)500; 3S-5S$ $n = 0(1)20; x = 0\left(\frac{1}{60}\right)\frac{15}{60};$ $\pm A = 1(1)5; 10(10)50; 100, 200,$ $500; 1S-3S$ $n = 0(1)40; x = 0\left(\frac{1}{120}\right)\frac{15}{120};$ $\pm A = 1(1)33; \frac{100}{3} \frac{(100)}{3} \frac{1,000}{3};$ $1S-3S$ $A \sin 2\pi nx: n = 1(1)143;$ $x = 0.0005(.0005)0.25; 3S-5S$

Source	Available at	Description of Tables
[(18)]	(18)C	$f_1 = \cos z; f_2 = \sin z;$ $z = 0.01 mn \pi/20; m = 1(1)600;$ $n = 1(1)200; 6D$ $f_3 = (\sin z)/z;$ same range of $z;$ 5D. Total cards: 40,200 <i>Note:</i> Same range of $z,$ at double the interval in $m$ and $n$ for $f_1$ and $f_2$ and at four times the interval in $m$ and double the interval in $n$ for $f_3,$ to four decimals, on 7500 cards. Additional table of $f_3,$ without $f_1$ and $f_2,$ same range of $z;$ 5D; 13,400 cards
[(18)]	(18)C	$f_1 = A \cos (2\pi mn/k);$ $f_2 = A \sin (2\pi mn/k);$ $m = 0, 1, 2; n = 0, 1, 2; k = 3$ $m = 0, 1, 2, 3, 4; n = 0, 1, 2, 3, 4;$ $k = 5$ $A = 0(.000001)0.000999$ and $-0.499(.001)0.500; 7D$
[(24)]	(24)C	$\sin nx, \cos nx; n = 1(1)31;$ $x = 0(1^\circ)90^\circ; 15D$
[(71)-(12)]	(12)C, (24)	$S(k, n) = \int_0^1 x^k \sin (n\pi x)dx;$ $C(k, n) = \int_0^1 x^k \cos (n\pi x)dx$ $S(k, n)$ and $C(k, n); k = 0(1)10;$ $n = 1(1)100; 10D$
		<b>25</b> Random numbers
[(14)]	(4), (14)C, (23), (24)	<b>25.1</b> Digits of 0(1)9 drawn at random, with equal probability. 50 random digits per card; 20,000 cards
[(14)]	(24)	Random numbers, positive and negative; logs of random numbers, numbered serially 0(1)6999
[(14)]	(4), (14)C, (24)	<b>25.2</b> Random Gaussian deviates, $x$ from $N(0,1).$ Range: $-4.417 \leq x \leq 4.417;$ $x$ to 3D; $x^2$ to 6D 10 deviates per card, 10,000 cards
[(4)]	(4)	<b>25.3</b> Cosines of random angles; 9D Cards 0(1)14,566; random angle $\beta:$ $0 \leq \beta \leq 9.4247$
[(14)]	(14)C	<b>25.4</b> Correlated Gaussian deviates from $N(0, 1)$ $x_{n+1}(\rho) = \rho x_n(\rho) + \sqrt{(1-\rho^2)}\xi_n,$

<i>Source</i>	<i>Available at</i>	<i>Description of Tables</i>	
			where $\xi_n$ is a random Gaussian deviate, $\rho$ is the correlation coefficient, and $x_0(\rho)$ is an uncorrelated Gaussian deviate. $\rho = 0.6, 0.8, 0.9, 0.95, 0.97, 0.98, 0.99, 0.995$ ; $n = 0(1)9999; 3D$
	(6) see (54)	26	Tables relating to aerodynamics A wide range of tables on punched cards, relating to shock waves and other phenomena, for various Mach numbers; "Busemann" coefficients, altitude functions, etc., are available at (6). See (54).
		27	Tables relating to number theory
[(26)]	(26)	27.1	Powers of primes. $2^n$ : $n = 1(1)100$ ; exact; [see also 2.3]
[(66)-(16)]	(16)C		Table of primes $p$ : $p < 10,000$
[(16)]	(16)C	27.2	Powers of primes $p$ for all primes less than 1580 $p^k$ : $k = 1(1)5$ ; exact; also $1 + p^2 + p^4; \sum_{k=0}^N p^k$ ; exact, for $N = 2, 3, 4, 5$
[(21)]	(21)	27.25	Fermat's quotients and extraordinary primes. See description in <i>MTAC</i> , v. 5, p. 84-85, [(73), file 117(F)]. $n, p, p^2, \epsilon, f, r, R$ : $n = 2(1)2962$ [computed on the ENIAC]
	(24)	27.3	The largest prime factor of $x$ : $x = 1(1)4200$
[(60)-(19)]	(73) Ref. 114	27.4	Sums of fifth powers of the divisors of $n$ : $n = 1(1)5000$ ; exact [See (60)b]
[(60)-(19)]	(73) Ref. 101		Ramanujan's function $\tau(n)$ ; [See (60)a] (a) $\tau(n)$ ; $\sum_{n \leq N}  \tau(n) $ ; $\sum_{n \leq N} \{\tau(n)\}^2$ : $n = 1(1)2500$ ; $N = 10(10)2500$ ; exact (b) $\tau(p)$ , $p$ a prime: $1000 < p < 2500$ ; exact (c) $ \tau(p) p^{-11/2}$ ; 6D
[(74)-(19)]	(19)	27.5	Stencils for the solution of systems of linear congruences modulo 2; 1024 cards

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
[(74)-(19)]	(73) Ref. 125	27.6	Table of integer solutions of $ y^2 - x^2  < x$ Range: $x < (10^6)/9$ or $y < (10^9)/27$
		28	Tables relating to astronomy [also see 7.44]
[(18)]	(18)C	28.1	Calendar date to Julian date 1853 (1 mo.) 1950; 92 cards
[(52)-(18)]	(18)C	28.2	[Kepler's equation] $E = M - e \sin E$ ; $e = 0(01)0.38$ ; $M = 0(1^\circ)360^\circ$ ; accuracy: $0^\circ 01$ ; 14,079 cards
[(55)-(24)]	(24)C	28.3	Tables for rocket and comet orbits (Kepler's equation) [See (55) under Sources, for definitions] (a) Elliptic orbits: $C_e(U)$ , $S_e(U)$ , $X_e(U)$ , $\Delta$ , $\delta^2$ : $U = 10^{-3n}u$ ; $n = 1, 2, 3, 4$ , and $n \geq 5: 0.15 \leq u \leq 150$ $n = 0: 0.15 \leq u \leq 3.15$ (b) Hyperbolic orbits: $C_h(U)$ , $S_h(U)$ , $X_h(U)$ , $\Delta$ , $\delta^2$ : $n = 1, 2, 3, 4$ and $n = 5$ : $0.15 \leq u \leq 150$ $n = 0: 0.15 \leq u \leq 300$ mostly 7S and 8S
[(76), (77)-(22)]	(22)C	28.4	General spherical triangles Arguments: sides $a$ , $b$ ; angle $C$ Functions tabulated: $(90 - c)$ (degrees and min.) and $B$ (degrees) Range: $a = 0(1^\circ)89^\circ$ ; $b = 61^\circ(1^\circ)90^\circ$ to $119^\circ$ $C = 0(1^\circ)$ upper limit The upper limits of $b$ and $C$ are such that $c$ covers the range 0 to $90^\circ$ . $\Delta c$ along $b$ .
		28.5	Tables with arguments in time [ $1^s = 15''$ ; $1^m = 15'$ ] $\tan x$ , $\sec x$ : $x = 0(1^s)20^m$ ; 7D $\tan x$ , $\sec x$ : $x = 0(1^s)20^m$ ; 8D
[(17)]	(17)C		
[(17)]	(17)C		
[(22)]	(22)	28.6	Navigation Table H. O. 218 This table tabulates altitudes ( $-5^\circ$ to $+90^\circ$ ) and azimuth (0 to $360^\circ$ ) as functions of declination ( $-30^\circ$ to $+30^\circ$ ), local hour angle (0 to $360^\circ$ ),

<i>Source</i>	<i>Available at</i>		<i>Description of Tables</i>
			and latitude ( $-90^\circ$ to $+90^\circ$ ). Complete information about this and similar punched card tables is available at (22).
	(30)	29	Actuarial tables Tables of "CSO Monetary Values," comprising a wide range of tables relating to annuities, etc., at various interest rates, have been deposited with (30); write for more complete information. Additional tables based upon an interest rate of $2\frac{1}{2}\%$ and the use of continuous functions, prepared by the John Hancock Mutual Life Insurance Company, can be obtained from (30).
		30	Tables relating to map projections and geodesy
[(23)-(28)]	(28)	30.1	Meridional arc; $x = 0(1')90^\circ$ ; length to 0.001 meters
[(28)]	(28)	30.2	Tables for the conversion of latitude and longitude to grid coordinates and vice versa, on the following spheroids: Hayford, Clarke (1886) and (1880), Bessel: all for lat. 0-80 Everest: lat. 0-45 All at 1' intersections
	(28)	30.3	Tables for polar stereographic grid coordinates, north and south polar areas, lat. $79^\circ 30' - 90^\circ$ .
	(28)	30.4	Tables for the universal transverse Mercator projection (for computation of plane coordinates to 1 cm. accuracy) Also see 7.44. For other punched cards relating to map projections write to (28).

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