whether anything doubtful has crept in, what still remains to be desired, etc. I recommend this method to you for imitation. You will hardly ever again eliminate directly, at least not when you have more than 2 unknowns. The indirect procedure can be done while half asleep, or while thinking about other things.[11]

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[1] The preparation of this translation was sponsored (in part) by the Office of Naval Research. The author wishes to acknowledge the assistance of Drs. Fritz John and Olga Taussky-Todd.

[2] See R. V. Southwell, *Relaxation Methods in Engineering Science*. Oxford, 1940, and L. Fox, "A short account of relaxation methods," *Quart. Jour. Mech. Appl. Math.*, v. 1, 1948, p. 253–280.

[3] Ludwig Seidel, "Über ein Verfahren, die Gleichungen, auf welche die Methode der kleinsten Quadrate führt, sowie lineare Gleichungen überhaupt, durch successive Annäherung aufzulösen," *Akad. Wiss., Munich, mat.-nat. Abt. Abhandlungen*, v. 11, 1874, p. 81–108. Although Seidel's process is frequently called the Gauss-Seidel process, I know of no place where Gauss mentions it.

[4] The letter is in Gauss's *Werke*, v. 9, 1903, p. 278–281. It is probably the letter referred to in the footnote, p. 257, of E. T. Whittaker and G. Robinson, *The Calculus of Observations*, 1st edit., London and Glasgow, 1924, as a source of their example of relaxation (p. 257–8).

[5] Words within brackets are translations of inserts by L. Krüger, who prepared volume 9 of Gauss's *Werke*.

[6] 'Indirect elimination' was Gauss's term for his iterative process of solving the normal equations. It later came to denote any iterative process for solving linear equations.

[7] The symmetric treatment of the unknowns is an essential idea in this and other letters. Here Gauss mentions only its advantage as a device which sets up a column-sum check to detect errors. In later letters (cf., e.g., Gauss to Gerling, 19 January 1840, *Werke*, vol. 9, p. 250–3) Gauss is convinced that the symmetric treatment of all unknowns yields normal equations whose iterative solution converges significantly faster.

The trick is later described by: Christian Ludwig Gerling (recipient of the letter), *Die Ausgleichungsrechnung der practischen Geometrie*. Hamburg and Gotha, 1843 (p. 157–8, p. 163, p. 386, p. 390); by R. Dedekind, "Gauss in seiner Vorlesung über die Methode der kleinsten Quadrate," *Festschrift zur Feier des 150-jährigen Bestehen der königlichen Gesellschaft der Wissenschaften zu Göttingen*. Berlin, 1901 (pp. 45–59) and *Gesammelte Mathematische Werke*. V. 2, 1931, p. 293–306; and by R. Zurmühl, *Matrizen*, Berlin, 1950, p. 280–282. (Zurmühl is wrong, however, in stating that the trick will improve the convergence for all badly conditioned systems of equations.)

For some discussion of when and why the trick may be expected to improve the convergence of iterative processes for solving linear equations, see George E. Forsythe and Theodore S. Motzkin, "An extension of Gauss's transformation for improving the condition of systems of linear equations," multilithed typescript at the National Bureau of Standards, Los Angeles.

[8] Gauss is here using a method which relaxers recommend: liquidating "that residual . . . which requires the largest 'displacement' " (Fox, *op. cit.*, p. 256).

[9] Study of the table shows the algorithm to be precisely the relaxation method described by Fox (*op. cit.*, p. 255–6). In comparing Gauss's and Fox's presentations, we note that Gauss uses the method mentioned in note 8, and that he "liquidates the residuals" only approximately at each stage, as recommended by Fox (*op. cit.*, p. 257–8) to save unnecessary arithmetic. Gauss does not, however, propose "under-relaxation" or "over-relaxation," as Fox does. On the other hand, Gauss's trick mentioned in note 7 is not mentioned by Fox, although it is extremely helpful in many common problems.

[10] Gauss wrote, "Fast jeden Abend mache ich eine neue Auflage des Tableaus, wo immer leicht nachzuhelfen ist." It is not clear what he meant.

[11] Here one must bear in mind Gauss's gift for mental arithmetic!

QUERIES

49. First Use of the Term Haversine and the First Table of Haversines.—The editors of the great Oxford *New English Dictionary* (*NED*) endeavoured to bring together quotations exhibiting the first use in English (so far as known) of each word. In the case of Haversine the quotation is from F. G. D. BEDFORD (1838–1913), *The Sailor's Pocket Book . . .*, second edition, London, 1875, p. 381. A reference can, however, be given to a work published 40 years earlier. In the *Dictionary of National Biography*, v. 10, 1887, J. K. LAUGHTON'S biography of JAMES INMAN (1776–1859), the following sentences occur: "In 1821 appeared his well-known book *Navigation and Nautical Astronomy for the Use of British Seamen*, with accompanying tables. In the third edition (1835) he introduced a new trigonometrical function, the half-versine, or haversine, the logarithms of which were added in the tables." Hence here is a new reference, to replace that of Bedford, for the editors of *NED*. The earliest table of haversines which I have found is the rather elaborate one in JAMES ANDREWS', *Astronomical and Nautical Tables*, London, 1805 (see *MTAC*, v. 1, p. 422). Can any reader report earlier use of the word haversine, or an earlier table of the function? In what Libraries may the 1835 editions of Inman's works (*Navigation*, and *Tables*) be found?

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CORRIGENDA

V. 3, p. 497, l. −5, *for* b^m *read* b^{km}.
V. 4, p. 124, l. −2, *for* 96–97 *read* 496–497.
V. 5, p. 25, 854, l. 7, *for* 0(.001)25, *read* 0(.001)16(.01)25.