

142[L].—K. HIGA, *Table of*  $\int_0^{\infty} u^{-1} \exp \{ - (\lambda u + u^{-2}) \} du$ . One page type-written manuscript. Deposited in the UMT FILE.

The table is for  $\lambda = .01, .012(.004).2(.1)1(.5)10$ . The values are given to 3S.

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143[L].—Y. L. LUKE. *Tables of an Incomplete Bessel Function*. 13 pages photostat of manuscript tables. Deposited in the UMT FILE.

The tables refer to the function

$$j_n(\mu, \theta) = \int_0^{\theta} \exp \{ i\mu \cos \phi \} \cos n\phi d\phi.$$

Values are given to 9D for

$$\begin{aligned} n &= 0, 1, 2 \\ \cos \theta &= -.2(.1).9 \\ &= 49\omega/51, \omega = 0(.04).52. \end{aligned}$$

There are also auxiliary tables. The tables are intended to be applied to aerodynamic flutter calculations with Mach number .7.

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## AUTOMATIC COMPUTING MACHINERY

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### TECHNICAL DEVELOPMENTS

## Fundamental Concepts of the Digital Differential Analyzer Method of Computation

**Introduction.**—Two fundamentally different approaches have been developed in using machines as aids to calculating. These have come to be known as analog and digital approaches. There have been many definitions given for the two systems but the most common ones differentiate between the use of physical quantities and numbers to perform the required automatic calculations.

In solving problems where addition, subtraction, division and multiplication are clearly indicated by the numerical nature of the problem and the data, a digital machine for computation is appropriate.

When problems have involved calculus methods, as, for instance, in the solution of differential equations, the analog computer has often been used,

as the process of integration seems, psychologically at least, to be more aptly handled by analog devices. These devices have been mechanical or electronic integrators. However, the actual process of integration, if one considers the numerical basis for its origin is, in a sense, a numerical additive process. Thus, digital computers "integrate" by successive additions.

By bridging the gap between these two approaches, a new series of instruments for computation is possible.

The method of computation used in a digital differential analyzer resulted from the adoption of a new point of view. Considering the operations involved in the solution of differential equations, it is possible, with this new approach, to obtain many of the advantages of a digital computer and also the essential advantages of an analog differential analyzer. The result is a different type of digital "logic" from that used in the general purpose digital computers.

The advantages gained by the new method in solving ordinary differential equations of any type are:

1. Ease of preparing problems—arising from the use of analog differential analyzer methods instead of numerical methods in the coding process.
2. Increase in computation speed over equivalent general purpose digital computer approaches and equality in speed to some analog methods.
3. Increase in accuracy over analog differential-analyzer procedures.
4. Repeatability and ease of error analysis inherent in the digital method.
5. Small size—In certain embodiments the digital differential analyzer can be much smaller, have fewer tubes and components, weigh and cost less than analog differential analyzers or any of the general purpose digital computers. This is particularly true when the number of integrators needed to solve the equations becomes large.

**Review of Analog Differential Analyzer Theory.**—There are two different ways of explaining the digital differential analyzer method. The first is a qualitative explanation which follows the analog viewpoint and points out the first advantage. The second is a quantitative numerical explanation which shows the error analysis possibilities and the successive-additions method of integration which actually takes place.

To appreciate the first explanation, it is necessary to review the principles of the analog differential analyzer. In solving an equation such as,

$$(1) \quad \frac{d^2w}{dt^2} - w \frac{dw}{dt} - wt = 0$$

the analog differential analyzer represents the variables  $w$ ,  $t$ ,  $\frac{dw}{dt}$  etc., by mechanical rotations of shafts or by variations of voltages in electronic circuits. The rates of shaft rotations or of changes in voltages are always proportional to the rates of change of the variables.

Integration is accomplished by a mechanical wheel and disc integrator or an operational amplifier used as an integrator. Other mathematical operations such as multiplication and addition of variables in the equation are performed either by integrators or other devices, mechanical and electronic.

Integrators and other units are interconnected in such a manner as to produce an analog of the differential equation. The set is "driven" by a

single shaft or voltage representing the independent variable ( $t$  of Equation 1). The  $w$  or dependent variable shaft or voltage varies in accordance with the actual solution to the equation as  $t$  varies. For a given set of initial conditions, a solution to Equation 1,  $w = f(t)$ , is produced as either a graph or a set of tabular values of  $w$  as a function of  $t$ .

The integrator is the key to the machine's operation, all other units being straightforward in comparison. It may be regarded as a "black box" with two inputs and one output shown schematically in Figure 1.

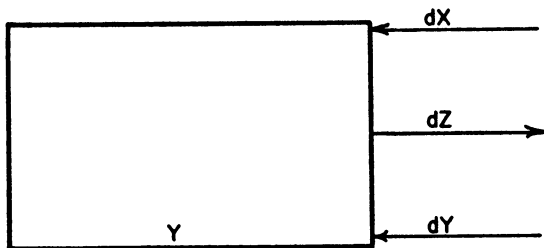


FIG. 1.

The inputs<sup>1</sup>  $dx$  and  $dy$  are the rates of change of some  $x$  and  $y$  variables in an equation as represented physically by shafts rotating or voltages changing. The differential notation is used because the same  $dt$  is inherently used throughout the machine. The inputs and outputs are related by the integrator Equation 2.

$$(2) \quad dz = KYdx,$$

where  $Y = S dy$ . The constant  $K$  is determined by the physical properties of the integrator.

In the mechanical integrator the  $dy$  input causes a worm gear to move a small disc across the surface of a large wheel (see Figure 2) such that its distance from the center of the wheel is  $Y$ . The  $dx$  input turns the wheel and friction causes the disc to rotate at a speed  $dz$ .

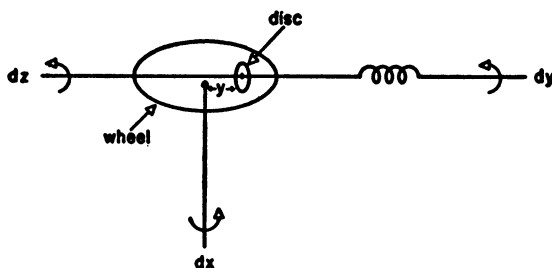


FIG. 2.

In the electronic integrator the  $dy$  input is a varying voltage, the  $dx$  input is always time and the  $z$  voltage is produced by using the integrating characteristics of capacitors in connection with a feedback amplifier to

produce linearity. It should be noted that to interconnect integrators it is necessary that the inputs and outputs all be of the same form.

**Qualitative Explanation of the Digital Integrator.**—The digital integrator is the heart of the new type of computer, the digital differential analyzer, and may be visualized as a black box with the same schematic (Figure 1) and the same equation relating its inputs and output (Equation 2). The  $dx$ ,  $dy$ , and  $dz$  variables are represented by pulse rates, i.e., the rates of occurrence of streams of electronic pulses entering or leaving the integrator. As stated before the equation relating these pulse rates is still Equation 2, and the inputs and output are of the same form.

Without describing the nature of such an integrator, it can be seen that the two properties above will allow these black boxes to be intercoupled, in the same manner as were the analog differential analyzer integrators, to

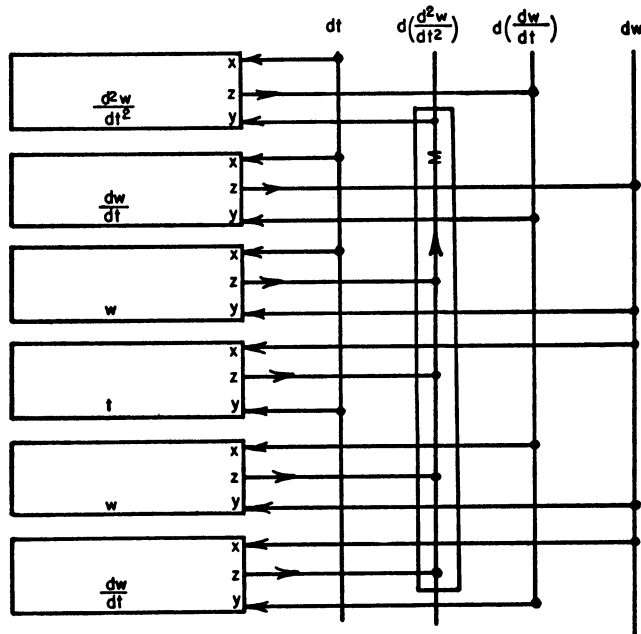


FIG. 3.

solve ordinary differential equations. The same techniques of intercoupling integrators to perform various operations such as multiplication, scaling function generation, division, etc., can be used. A set of digital integrators intercoupled by wires carrying pulse streams can be "driven" by a pulse source representing the independent variable. A solution is produced as a set of tabular values and a graph can be produced. The same schematic or connection diagram can be used for both the digital and the analog differential analyzers. Only the  $K$  in Equation 2 changes. Such a schematic is illustrated in Figure 3 for the solution of Equation 1.

Two distinct advantages of the digital over the analog integrator in addition to increased accuracy should be noted at this point. It will be

observed that Figure 3 contains no adders. The terms

$$w dt, t dw, \left(\frac{dw}{dt}\right) dw, \text{ and } w d\left(\frac{dw}{dt}\right)$$

created by the lower four integrators would in the analog machines have to be added in extra units, known as adders, to form  $d\left(w\frac{dw}{dt} + wt\right) = d\left(\frac{d^2w}{dt^2}\right)$  to be fed back into the "Y" input of the upper integrator. The addition is indicated on the diagram by a box with  $\Sigma$  sign.

Since the outputs of these integrators in the digital case are pulse streams, they may be mixed together directly and sent into the same input, provided, of course, that the pulses do not coincide in time. In several embodiments of the digital differential analyzer this is the case. If time coincidence does occur, it is only necessary to delay one pulse with respect to another.

The other advantage is the superior ability of the digital integrator to receive the output of another integrator at its "dx" input. This greatly facilitates multiplication, division and the solution of non-linear equations without the use of special devices.

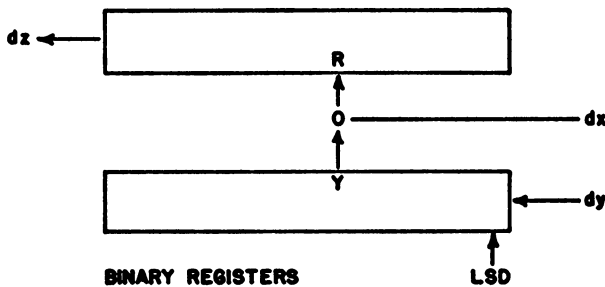


FIG. 4.

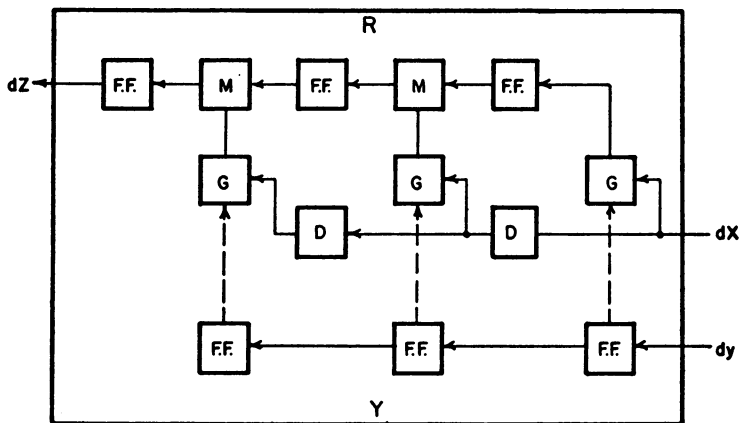
**Embodiments of the Digital Integrator.**—The "contents" of the black box digital integrator may take many physical forms and still have the external properties described. All of the forms so far devised have one common property. Two numbers appear within the box and may be designated as a coupled pair. These two numbers, always labelled *Y* and *R*, may appear in any one of several physical forms. Two specific examples are: Numbers stored in vacuum tube registers made up of two-stable-state devices, and numbers appearing in pulse form on a cathode ray tube screen. The numbers may be of any length and in any number base system. For convenience they will usually be represented in this paper schematically as appearing in two registers as binary numbers. (See Figure 4.) Some other methods of storing the *Y* and *R* numbers are: relays, mechanical registers as on desk calculators, magnetic tapes or drums, and mercury or other delay lines. Each of the storage media must be capable of changing the numbers digitally by the receipt of information at the inputs to the box.

In the integrator diagram in Figure 4, the *Y* register acts as a counter

when receiving  $dy$  pulses and in a sense integrates the  $dy$  pulse rate to produce the number  $Y$ . The  $dx$  pulses are treated as instructions to transfer in an additive manner the number  $Y$  into the  $R$  register without removing  $Y$  from the  $Y$  register. If the  $R$  register contained some previous  $R$  before the transfer, it contains  $R + Y$  after the transfer.

The  $R$  register will of course overflow after a certain number of transfers. Each time it does so, a pulse is transmitted from the integrator as a  $dz$  pulse. The  $Y$  and  $R$  registers have the same length and capacity and, if binary registers are used, the capacity is  $2^N$ , where  $N$  is the number of binary stages in each register.

By qualitative analysis of the relations between the variables, it may be seen that Equation 2,  $dz = KY dx$ , does hold for the integrator, where  $K = 1/2^N$  and  $Y$  is regarded as an integer, provided that  $Y$  remains constant. In other words if  $Y = 1$  the output rate  $dz$  will be  $1/2^N dx$ , since it requires



Transfer method.

FIG. 5.

$2^N$  additions of  $Y$  to  $R$  to cause an overflow. If  $Y = 2^N$ , or the register is filled to capacity, an  $R$  overflow or  $dz$  pulse will occur for every  $dx$  pulse. In this case,  $dz = dx$ . In general,  $dz$  is certainly proportional to  $dx$  for a constant  $Y$  and is proportional to  $Y/2^N$  for constant  $dx$ .

Two fundamentally different ways exist (as well as combinations of the two) to cause the transfer of  $Y$  to  $R$  to take place. In the one described above, called the transfer method, a single pulse at the  $dx$  input caused the entire  $Y$  number to be added into  $R$ . In the second system a large number of  $dx$  pulses are required to transfer  $Y$  to  $R$ . The  $Y$  number may change during this transfer process so that the number of stages required in the register for the same accuracy is larger. This method is called the sieve method.

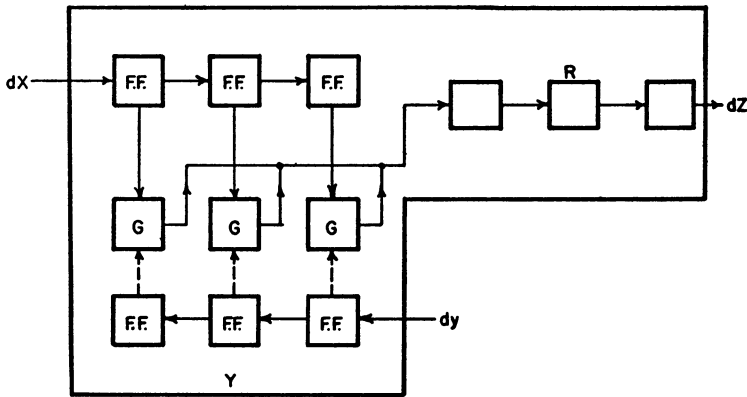
Figures 5 and 6 illustrate electronic methods of causing the two types of transfer. There are, of course, many other electronic ways of effecting the transfer.

In Figure 5 the additive transfer of  $Y$  into  $R$  by a single  $dx$  pulse is accomplished by transmitting the  $dx$  pulse into successive gates and delays. The

gates are controlled (dashed lines) by the  $Y$  register flip-flops, or two-state devices. If a flip-flop is in its "1" state (binary digit one at that digit position), its gate is "open" and the  $dx$  pulse passes up to one of the  $R$  register flip-flops causing it to trigger. If the  $Y$  flip-flop is in its "0" state (binary digit zero), the gate is closed and the pulse does not pass through.

If an  $R$  flip-flop triggers from "1" to "0" it transmits a "carry" pulse to the next flip-flop to the left. The  $dx$  pulse in the meantime is being delayed through  $D$ , and if it passes the next gate it will arrive at the pulse mixer  $M$  non-coincident with the carry pulse from the  $R$  flip-flop. Any carries from the left  $R$  flip-flop represent overflow pulses and are transmitted to the  $dz$  output.

The sieve method of Figure 6 requires  $2^N dx$  pulses to transfer  $Y$  into  $R$ . A third register is used to distribute the  $dx$  pulses through the gates controlled by the  $Y$  flip-flops in such a manner that when the outputs from each gate are mixed they are non-coincident and can be accumulated in  $R$ . The entire operation resembles the action of sieving the  $dx$  pulses through the gates.



Sieve method.

FIG. 6.

Carry pulses are taken from the third register flip-flops at different times to feed to the gates and to trigger the next flip-flop to the right. This means that each set of pulses reaching the gates as the operation proceeds from left to right is anticoincident with all preceding sets and equal to half the adjacent left hand set. The effect of  $2^N$  ( $2^3$  in case shown)  $dx$  pulses for constant  $Y$  will be to transfer  $Y$  into  $R$ .

When other storage methods are used, the electronic operations change. For instance with magnetic drum storage, the two numbers are stored in two parallel channels, and the digits of  $Y$  and  $R$  appear at magnetic read heads one digit at a time in serial fashion. The addition of  $Y$  to  $R$  is then that of time-serial binary addition. The same thing would be true of any serial or delay type of number storage.

**Quantitative Explanation of the Digital Integrator.**—The second or quantitative explanation of the digital differential analyzer method will be covered rigorously in another paper. Briefly, the successive addition process of multiplying an ordinate of a curve,  $Y \equiv f(x)$  (see Figure 7), by a  $\Delta x$

increment and adding the resulting products to get the area under a curve is really being carried out in an integrator.

If the  $R$  register were of unlimited length and each  $dx$  pulse were assumed to have a value of 1, then the successive additions of  $Y$  to  $R$  would produce a number similar to the sum of the area of the rectangles under the curve of Figure 7 (assuming the  $Y$  values to be correct). Since the  $R$  register is broken off and  $dz$  pulses transmitted, it can be seen the sum of these  $dz$  pulses will be in error from the rectangular areas by the remainder  $R$  in the  $R$  register. The total error consists of this roundoff error plus the truncation error difference between the true curve and the rectangles. Automatic corrections of various electronic types can be and have been made for both of these errors. In the future mathematical paper it will be demonstrated that the total truncation and roundoff error for many equations will not exceed the two least significant binary digits of a  $Y$  register.

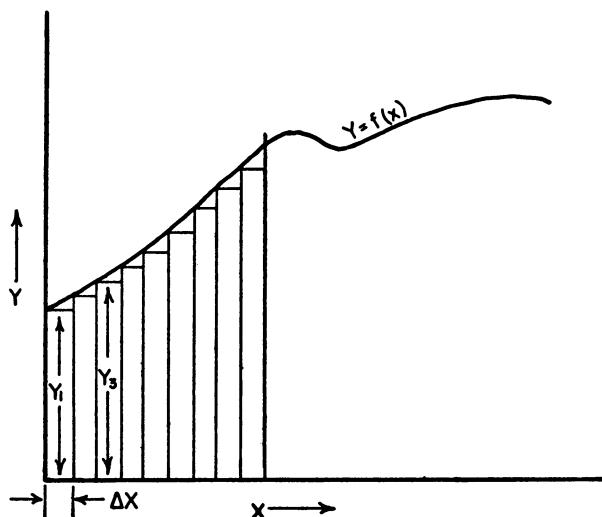


FIG. 7.

**Further Advantages of the Digital Integrator.**—In the foregoing discussion “black boxes” comprising digital integrators which have two fundamental properties were described. Ordinarily, to obtain 50 “black boxes” it would be necessary to use 50 times the equipment required for one box. This is certainly true in case of the mechanical and electronic integrators. However, where the integrators consist merely of paired numbers operating on each other in accordance with the methods already described, it becomes possible to time share operational circuits among all of the number pairs if they are stored in a serial or delay type of memory and the integrators are strung out in a line timewise.

One set of electronic circuits can operate on all integrators in sequence, or rather on all paired numbers in sequence. An integrator, as such, does not really exist when such a system is used. In a scheme like this it is easily seen that no coincidence problems exist, since no two integrator outputs



occur simultaneously. It will also be seen that the amount of equipment does not increase linearly with the number of integrators and that beyond a certain point the digital differential analyzer is smaller and involves less components than the analog machine.

The problems of handling the signs of the variables and their derivatives and of the scales or scale factors for a problem will also be covered in detail in a future paper. They are similar to analog sign and scale problems except that special provisions must be made for handling signs, and scaling takes place digitally.

The writer wishes to express thanks to the following men who furnished the ideas for much of the material this article covered: D. E. ECKDAHL, H. H. SARKISSIAN, I. S. REED, C. ISBORN, W. DOBBINS, F. G. STEELE, B. T. WILSON, J. DONAN, J. MATLAGO, and A. E. WOLFE.

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<sup>1</sup> V. BUSH and others use the  $x$ ,  $y$ , and  $z = y dx$  notation for inputs and outputs.

#### BIBLIOGRAPHY Z-XVIII

1. ANON., "Computing machines," *Mechanical Engineering*, v. 73, Apr. 1951, p. 325-327.

The MADDIDA (Magnetic Drum Digital Differential Analyzer), a new small electronic computer built by Northrup Aircraft, Inc., is briefly described, including some of the main specifications and the uses for this type of computer. The machine is capable of solving many types of differential equations or sets of such equations. The machine is being manufactured for general use in science and industry, at a relatively low cost. It is a 29 binary digit machine and adds binary digits at a rate of 100,000 per second. A big advantage of the machine is that differential equations can be solved without reducing them first to difference equations.

A survey of the Federal Computer Program is also included in the article and a list of many of the analog and digital computers either being used or being constructed at various institutions in the United States.

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2. ANON., "High-speed analog to digital converter," *Review Sci. Instr.*, v. 22, July 1951, p. 544.

Expository article.

3. S. GILL, "The diagnosis of mistakes in programmes on the EDSAC," *R. Soc., London, Proc.*, v. 206A, 1951, p. 538-554.

This paper discusses in detail two methods which have been used to diagnose errors in programming on the EDSAC. The "Blocking Order" routine prints the contents of any given location whenever the blocking order is obeyed, and then returns the control to the main routine immediately after the blocking order. This is useful in investigating arithmetical failures.

The "step by step" routine prints operation symbols as they occur during the run of the problem, thereby giving a compact representation of the progress of the computation. This is of use in investigating order failures. The general ideas of these routines are readily adaptable to other high-speed computers; indeed on a four address machine these routines can be made much more simple and flexible.

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4. OFFICE OF NAVAL RESEARCH, *Digital Computer Newsletter*, v. 3. no. 3, Oct. 1951, 5 pages.

The present status of the following digital computer projects is treated briefly in this number:

1. The Circle Computer
2. Naval Proving Ground Calculators
3. Aberdeen Proving Ground Computers
4. Electronic Computer Corporation Computers
5. The ORDVAC
6. The SEAC
7. The SWAC
8. The Raytheon Computer
9. The University of Toronto Electronic Computer UTEC
10. The Ferranti Computer at Manchester University, England

#### Component Developments

1. The Computer Research Corporation
  2. Physical Research Laboratories Computer Development
5. A. G. RATZ & V. G. SMITH, "A method of gating for parallel computers," *AIEE, Trans.*, v. 70, 1951, p. 424.

The problem is to set a slave register into the same state as a master register, where each register is a row of Eccles-Jordan flip-flop circuits and both are operated at the same supply voltages. A pair of diodes connect each master flip-flop to its corresponding slave. The cathodes of the diodes connect to the plates of the master, and the plates of the diodes connect to the opposite grids of the slave. The diodes are normally nonconducting, but transfer is effected by a pulse which lowers the plate supply voltage of the master register so that the lower voltage plate of a master flip-flop forces down the voltage of the opposite grid of its slave through conduction of the connecting diode.

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6. J. R. TILLMAN, "Transition of an Eccles-Jordan circuit," *Wireless Engineer*, v. 28, Apr. 1951, p. 101-110.

Waveforms of plate voltage during transition are computed for a conventional low-tube bistable circuit without "speed-up" capacitors, triggered on the control grids. Both linear and parabolic plate current characteristics

are considered. Particular attention is given to initial conditions near the threshold of instability and to triggering pulses so short that the loop gain is raised just above unity so that positive feedback can complete the transition. Twenty millimicroseconds can suffice for triggering; however, reliability and practical tolerances may require more stable initial conditions and stronger triggering.

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7. H. C. TUTTLE, "Machine brains dissected at computing conference," *Steel*, v. 128, Apr. 2, 1951, p. 112-114.

This article discusses in a non-technical way some of the uses to which digital computing machines may be put in solving the many practical problems in industry, especially in the design of equipment. Some papers are mentioned in the article which were given at a conference on automatic computing machinery at Wayne University on Mar. 27-28, 1951. These papers apply mainly to the business man's interest.

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8. M. V. WILKES, D. J. WHEELER, & S. GILL, *The Preparation of Programs for an Electronic Digital Computer*. 167 p. Addison-Wesley Press, Inc., Cambridge, Mass., 1951. 22.9 × 15.2 cm. Price \$6.00.

Although this book is almost exclusively devoted to programming for the EDSAC (Electronic Delay Storage Automatic Computer) at the University of Cambridge, it contains a great many items of interest for persons engaged in programming for other machines of similar type. It is of particular interest to persons responsible for putting such high-speed digital computers into business on a production basis. Emphasis is placed, in this book, on minimizing coding time and reducing the incidence of errors by the use of an adequate library of prefabricated subroutines. Such subroutines may be used as building blocks for constructing almost the complete program. By thus economizing on programming time it is possible to exploit to full advantage the capabilities of a limited programming staff, and it becomes feasible for the digital computer to handle problems requiring relatively small amounts of operating time.

The book opens with a Preface by the authors and a Foreword by D. R. HARTREE. The text is divided into three main parts, Part I dealing with general matters of coding for EDSAC, and the remaining two parts with specific details of EDSAC subroutines. There are in addition five appendices, the first of which gives the keyboard code, the corresponding teleprinter symbols, the code as punched on tape, and the numerical interpretations of the keyboard code symbols. The remaining four appendices give further coding details. Throughout the text the explanations are buoyed by means of numerous detailed examples and illustrations. There is also a bibliography of publications on EDSAC at the end of Chapter 2.

The opening chapter briefly discusses large scale automatic digital computing machinery in general terms, and the various kinds of codes, but quickly settles down to a description of the EDSAC, its "single address" order code, and the use of this code.<sup>1</sup>

The second chapter is fundamental and deals with the process by which information is read from tape and placed in the store (memory). This input process is accomplished with the aid of the "initial orders." These are a sequence of built-in instructions, activated when the start button is pressed, which direct the input process. A detailed listing of these initial orders together with accompanying explanation is given in Appendix B. The input process may be further controlled or modified in two ways: (a) by means of "code letters" (used to terminate orders), and (b) by the use of so called "control combinations," i.e., punched groups of symbols appropriately interspersed among the orders on the input tape. These two kinds of indications are detected and interpreted by the initial orders; when properly chosen and placed, they serve to make the input process fully flexible. In particular, the use of this system makes it possible to code subroutines without regard to how they are to be integrated into the complete routine nor where they are to be placed in the store, and to file them in the library in the form of short lengths of tape. Then, when needed, these subroutines are copied mechanically onto the program tape, and by means of the system of initial orders, code letters, and control combinations, they are automatically integrated during input into the complete routine. The control combinations in most common use are given in the text. A further list is given in Appendix C.

Chapter 3 presents a brief explanation of the method used on EDSAC for entering and leaving a subroutine, together with methods for inserting parameters into subroutines.

Chapter 4 contains a general description of the EDSAC library of subroutines. In particular, "assembly subroutines" are described for putting the various components of a complete program together in the store. These are designed to relieve the programmer of the mechanical tasks of deciding where the master routine and each subroutine are to go in the store, and of inserting the necessary orders for linking the components together. This chapter also describes four library subroutines for integrating ordinary differential equations (not necessarily linear), subroutines for operating on complex numbers, for floating point operations, and others.

In the fifth chapter, entitled "Pitfalls," are discussed some of the more common types of errors that arise in program preparation and ways of reducing or eliminating them. Every effort is made to eradicate errors before a problem goes on the machine. However, the authors observe that rarely does a program work right the first time it is tried, so that efficient code checking techniques on the machine become necessary. Some of these are described.

In Chapter 6 is presented a description of the auxiliary EDSAC equipment for tape punching, editing, duplicating, and comparing. Brief mention is made of EDSAC controls, of the operating organization, and of the method of storage of library subroutines.

Chapter 7, which concludes Part I presents detailed coding examples.

Part II itemizes the specifications for the library of subroutines. More precisely, corresponding to each subroutine in the library, this chapter records the vital statistics such as type of subroutine, total number of

storage locations occupied, an explanation of what the subroutine does, etc.

Finally, Part III gives the detailed programs for a selected list of sub-routines.

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<sup>1</sup> In line 5 of page 6 the order code symbol should read LD instead of LF.

## NEWS

**Commonwealth Scientific and Industrial Research Organization.**—About 200 persons from Universities throughout Australia, various Divisions and Sections of C.S.I.R.O., other government bodies including the Department of Supply, and from certain industrial and commercial firms gathered in Sydney for a Conference on Automatic Computing which was held in the Electrical Engineering Department of the University of Sydney on the 7th, 8th, and 9th of August. The conference was arranged by the Commonwealth Scientific and Industrial Research Organization which has been interested in this field of research for some years now, both because of the importance of mathematical analysis in scientific work and also because its Mathematical Instruments Section and the Computing Group in its Radiophysics Division have been developing automatic computing machines. The conference coincided with the presence in Australia of Professor D. R. HARTREE of Cambridge, who gave stimulating leadership to much of the business of the conference and set the discussion against a background of world progress which has been due in no small measure to his own work in this field.

The conference was opened by Professor JOHN MADSEN, who indicated that it would take place in two sessions, the first of which was intended primarily to emphasize the application of computing aids to industrial, commercial, and research problems. The second session would deal with the more detailed problems of numerical methods and programming and also with some engineering developments in computing equipment.

A general introduction to automatic calculating machines by Hartree was followed by a lecture and demonstration by D. M. MYERS and W. R. BLUNDEN on the C.S.I.R.O. Differential Analyser. This instrument was completed several months ago and contains ten integrators of the disc, ball, and cylinder type. Unit construction has been adopted throughout, the interconnection of units being made through step-by-step electrical transmission, providing great simplicity in setting up equations.

The afternoon sitting was opened by Hartree who explained the basic operations that take place in a high-speed automatic digital machine and the manner of controlling the machine by sequentially stored instructions in "one address" form. This was followed by an account and demonstration of the C.S.I.R.O. Mark I Electronic Digital Computer by T. PEARCEY and M. BEARD. This machine, which is now coming into service in the Radiophysics Division, uses mercury delay lines for its main store and has a capacity of 1,024 words of 20 binary digits and a pulse repetition frequency of 333 Kc/s. A magnetic drum auxiliary store is being developed for the machine.

In the first part of Session II, Hartree and Pearcey explained in some detail the problems associated with the organization of calculations for automatic machines. This led to a discussion of programming, i.e., the compilation of sets of instructions to deal with a calculation; and also to the manner in which a mathematical problem is reduced to a form suitable for programming. An account of programming for punched card and certain types of desk machines was also included. The second part of this session started with a discussion led by Pearcey on the interaction between programming and machine design and was followed by brief accounts of some new devices, including magnetic control circuits, magnetic drum storage, and electron beam tubes for decimal counting and binary switching by B. F. C. COOPER, D. L. HOLLWAY, D. M. MYERS, and C. B. SPEEDY.

Short accounts of analogue computing by Myers and of digital-analogue conversions by W. R. BLUNDEN were given, and also an electrical analogue machine for solving polynomials, recently constructed at Adelaide University by W. G. FORTE and G. A. ROSE, was described. The conference concluded with a general discussion. Demonstrations of the C.S.I.R.O. machines and of various desk and punched card machines which were exhibited by various accounting machine firms were carried on concurrently with the conference.

**NBSINA.**—On August 23–25, 1951, at the Institute for Numerical Analysis, a Symposium on Simultaneous Linear Equations and the Determination of Eigenvalues was held under the auspices of the National Bureau of Standards in cooperation with the Office of Naval Research. This was one of a series of symposia which the Bureau is holding as part of its scientific program for the year 1951 in marking the fiftieth anniversary of its establishment. The program was as follows:

Thursday, August 23, 1951

General session

Classification of methods for solving linear equations and inverting matrices

Some problems in aerodynamics and structural engineering related to eigenvalues

The geometry of some iterative methods of solving linear systems

Session on Linear Equations and Inversion of Matrices

Solutions of simultaneous systems of equations

Solutions of linear systems of equations on a relay machine

Some special methods of relaxation technique

Errors of matrix computations

Friday, August 24, 1951

Session on determination of eigenvalues.

Inclusion theorems for eigenvalues

On a general computation method for eigenvalues

Variational methods for the approximation and exact computation of eigenvalues

Session on determination of eigenvalues

Iterative methods for finding eigenvalues and eigenvectors

New results in the perturbation theory of eigenvalue problems

Determination of eigenvalues and eigenfunctions

Bounds for characteristic roots of matrices

Saturday, August 25, 1951

General Discussion of Problems

Registration

J. H. CURTISS, *Chairman*, NBS

G. FORSYTHE, NBS

R. A. FRAZER, National Physical Laboratory, Teddington, Middlesex, England

A. S. HOUSEHOLDER, Oak Ridge National Laboratory

J. TODD, *Chairman*, NBS

A. OSTROWSKI, Universität Basle, Basle, Switzerland

C. E. FRÖBERG, Lund, Sweden

E. STIEFEL, Zurich, Switzerland

P. S. DWYER, University of Michigan

J. B. ROSSER, *Chairman*, Cornell University

H. WIELANDT, Tübingen, Germany

G. FICHERA, Trieste, Italy

A. WEINSTEIN, University of Maryland

A. W. TUCKER, *Chairman*, Princeton University

M. R. HESTENES, NBS and UCLA

F. RELICH, Göttingen, Germany

H. H. GOLDSTINE, IAS

A. T. BRAUER, University of North Carolina

F. J. MURRAY, *Chairman*, Columbia University