RECENT MATHEMATICAL TABLES

933[A, K].—M. T. L. Bizley, "A note on the variance-ratio distribution," Institute of Actuaries Students' Soc., *Jn.*, v. 10, 1950, p. 62–64.

A folded page following p. 64 gives a table of the binomial coefficients $\binom{m}{t}$ for $t \leq m = 1(1)20$. The table is used in this case to facilitate the evaluation of

$$\int_u^\infty y^n(y + k)^{-m-1}dy.$$

D. H. L.

934[A, D].—L. Îa Neĭshuler, *Tablit͡sy Perevoda Pri͡amougol'nykh Dekartovykh Koordinat v Poli͡arnye* [Tables for the Transformation of Rectangular Cartesian into Polar Coordinates]. Moscow and Leningrad, Gostekhizdat, 1950, 292 p. 16 × 33.6 cm. Boards, 33.75 roubles.

This is the fourth, and by far the most elaborate, table of Neĭshuler to which we have made reference (see *MTAC*, v. 1, p. 7, for tables of 1930 and 1933; v. 2, p. 203–204, for table of 1945). It was published in November 1950, in an edition of 5000 copies, and seems to have met a real need, since the edition was sold out in less than a year.

It is a table giving the polar coordinates (s, α) corresponding to rectangular coordinates (x, y), where $s = (x^2 + y^2)^{\frac{1}{2}}$, $\alpha = \arctan(y/x)$; x and y are positive integers up to 10000. The main table occupies very large quarto pages 7–290. For using the tables it is generally supposed that y is not less than x. If in a given case x is the greater, the x is called y^* and the angle $(90° - \alpha)$.

The first column on each page is for y, and the first range of values is 1000 to 1090. To this range, beginning on page 7, and ending with the first column on page 12, corresponds a series of ranges for x: 0–10(5), 10–20(15), $\cdots$, 180–200(190), 520–550(535), $\cdots$, 1070–1105(1105), the maximum terminal value of x approximating to 1090—the maximum terminal value of y. The next range of y is 1090–1180, and the last 9910–10000.

Under each of the x-ranges are columns headed s, α, Δ. The values in the s and α columns correspond exactly to the series of x values above in brackets ( ), namely: 5, 15, $\cdots$, 190, $\cdots$, 535, $\cdots$, 1105. All of the values of x in this series except the last (a range boundary-value) are means of the end-values of the x-ranges. Throughout the table the means are almost invariably chosen. The s and α corresponding to other values of x, are got by interpolation—using the values of Δs and $\Delta\alpha$ given in the Δ column. In the range of y chosen as illustration the Δs and $\Delta\alpha$ never have more than 9 entries but on the last pages of the table these run up to 22.

For values of y less than 1000, such for example, as $y = 12$, $x = 9$, the table-entry for $y = 1200$, $x = 900$ would naturally be used. Similarly for other cases of y less than 1000, the final results obtained for s are usually to one place of decimals, and the angle to the nearest tenth of a minute. Illustrative examples are worked out on pages 4–6. No previously published table of this kind compares with it in extent.

The small tables of W. J. Seeley, and of J. C. P. Miller have been reviewed in *MTAC*, v. 2, p. 22–23, and the single-page table of JAHNKE & EMDE, p. 17 of the more elementary section, is well known.

From an entry in *MTAC*, v. 3, p. 340, we learned that among "Tables completed or almost completed," by 1948, were "Cartesian to Polar Conversion Tables," supervised by E. H. NEVILLE: "To give, for integral values of x, y, with $y \leq x \leq 105$, values to 12 figures of r with θ in degrees, and of $\ln r$ with θ in radians."

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935[A].—H. S. UHLER, (a) "Many-figure approximations to $\sqrt{2}$, and distribution of digits in $\sqrt{2}$ and $1/\sqrt{2}$," Nat. Acad. Sci., Washington, *Proc.*, v. 37, 1951, p. 63–67. (b) "Approximations exceeding 1300 decimals for $\sqrt{3}$, $1/\sqrt{3}$, $\sin(\pi/3)$ and distribution of digits in them," Nat. Acad. Sci., Washington, *Proc.*, v. 37, 1951, p. 443–447.

In (a) the value of $\sqrt{2}$ is given to 1544D. In (b) values of $\sqrt{3}$, $1/\sqrt{3}$, and $\sin(\pi/3) = \frac{1}{2}\sqrt{3}$ are given to 1316D. In (a) are given data on the distribution of the digits in $\sqrt{2}$ and $1/\sqrt{2}$, namely, the values of χ^2 and the corresponding probabilities of obtaining such distributions from a normal population [*MTAC*, v. 4, p. 109]. These data are remarkably small for $1/\sqrt{2}$ but reasonably sized for $\sqrt{2}$. Corresponding data are given for the constants in (b). These are all fairly large. A detailed comparison of distributions of digits in $1/\sqrt{2}$ and $1/\sqrt{3}$ is given in a separate table.

Agreement with Coustal's value of $\sqrt{2}$ [*MTAC*, v. 4, p. 144] was exact.

D. H. L.

936[D, F, L]—JOHN TODD. *Table of Arctangents of Rational Numbers*. National Bureau of Standards AMS No. 11, Washington 1951, xii + 105 p. 26 × 19.7 cm. Price $1.50.

This work gives two tables. Table 1 gives for every pair of integers m, n with $0 < m < n \leq 100$ the principal values of $\arctan m/n$ and $\text{arccot}\, m/n$ in radians to 12D. In addition are given $m^2 + n^2$ and the canonical representation of $\arctan m/n$ as a linear combination, with integer coefficients, of "irreducible" arctangents of integers.

For r an integer, $\arctan r$ is called reducible in case it can be expressed as a linear combination of arctangents of integers less than r. Table 2 gives a list of reducible arctangents of integers ≤ 2089 together with the unique reductions in terms of irreducible arctangents. Thus the entry

$$601 \quad 2(1) + 1(24) - 1(25)$$

is a statement of the fact that

$$\arctan 601 = 2 \arctan 1 + \arctan 24 - \arctan 25.$$

For further properties of these reducible arctangents see *MTAC* v. 2, p. 62–63, 147–148, v. 4, p. 82–83, 85.

The table was produced by punched card methods. Besides its theoretical interest the table is very useful in computing the logarithms of complex numbers belonging to a rectangular grid.

D. H. L.

937[F, G].—H. GUPTA. "A generalization of the partition function," Indian Acad. Sci., *Proc.*, v. 17, 1951, p. 231–238.

The author denotes by $v_r(n, m)$ the number of partitions of n into parts not exceeding m, each part k being of k^{r-1} different types.

The generating function of $v_r(n, m)$ is

$$\prod_{k=1}^{m} (1 - x^k)^{-k^{r-1}} = \sum_{n=0}^{\infty} v_r(n, m)x^n.$$

For $r = 1$, $v_r(n, m)$ becomes the familiar function first tabulated by EULER. The author gives a table of $v_2(n, m)$ for $1 \leq m \leq n \leq 50$.

The function may be built up from the recursive relation

$$nv_r(n, m) = \sum_{k=1}^{n} \sigma_r(k, m)v_r(n - k, m)$$

in which $\sigma_r(k, m)$ denotes the sum of the r-th powers of those divisors of k which do not exceed m.

The author states that

$$v_2(n, m) = \exp \{n^{\frac{1}{3}}(C + o(1))\},$$

where $C^3 = 27\zeta(3)/4$ so that $C = 2.009$.

However, $50^{-\frac{1}{3}} \ln v_2(50,50) = 1.700$.

D. H. L.

938[F].—A. KATZ. "Third list of factorization of Fibonacci numbers," *Riveon Lemat.* v. 5, 1951, p. 13.

New factors of U_n or V_n are given as follows:

n	U_n or V_n	Factor
138	V	16561, 1043766587
141	U	108289
147	U	3528
147	V	65269
153	V	13159
165	U	86461
180	V	8641
189	U	38933
189	V	85429

The residual factors are in every case greater than 200000.

D. H. L.

939[F].—ERNST S. SELMER "The Diophantine equation $ax^3 + by^3 + cz^3 = 0$." *Acta Math.*, v. 85, 1951, p. 203–362.

The extensive tables at the close of this paper should be very useful in the further study of cubic diophantine equations. The main table on page 348 lists all equations $x^3 + my^3 + nz^3 = 0$ for $2 \leqq m < n < 50$, stating in nearly every case whether the equation is soluble or insoluble in integers, and if soluble giving an absolutely least integral solution. A second group of tables on pages 349–353 lists the soluble and insoluble equations $ax^3 + by^3 + cz^3 = 0$ with abc cube-free and $\leqq 500$ and a, b, c positive co-prime integers. Finally on page 357 there is a table listing the number of generators and the basic solutions of the equation $X^3 + Y^3 = AZ^3$ with A cube-free and $\leqq 500$. Apparently the only previously published table is one given by FADDEEV[1] for this last equation extending up to $A = 50$.

There are in addition some useful auxiliary tables of cubic residues in pure cubic fields $K(\sqrt[3]{m})$, $m < 50$ and the EISENSTEIN field $K(P)$. In table 3 on page 351, the author has noted that the entry for w when $\lambda = 0$ and $p = 17$ should be $w = 1$ instead of $w = 0$.

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[1] D. K. FADDEEV, "Ob uravnenii [on the equation] $x^3 + y^3 = Az^3$," Akad. Nauk S.S.S.R., Leningrad, Fiz-mat. Inst. imeni V. A. Stekloff, *Trudy*, v. 5, 1934, p. 25–40.

940[H].—A. ZAVROTSKY. "Tablas para la resolucion de las ecuaciones de quinto grado," Acad. de Cien. Fis., Mat. Nat., Caracas, *Boletin*, v. 13, 1950, p. 51–93.

The author reduces the general quintic equation to the form

$$x^5 = px^3 + qx^2 + rx + 1.$$

The tables give the least positive or greatest positive root of this equation according as $p + q + r$ is negative or positive. The coefficients p, q, r, range over all integers not exceeding 10 in absolute value. Values of the roots are given to 5D. Each entry of the table was computed separately by one of a number of iterative methods. For a table on the cubic by the same author, see *MTAC*, v. 2, p. 28–30.

D. H. L.

941[K].—W. E. DEMING, *Some Theory of Sampling*. New York, John Wiley & Sons, 1950. xvii + 602 p., 15.9 × 23.5 cm. $9.00.

Table 1, page 558, Fiducial Factors between s and σ, seems to be new.

The distribution of the standard deviation s in random samples of size n from a normal population of standard deviation σ is given by

$$(1) \qquad f(s)ds = n^{\frac{1}{2}(n-1)}2^{\frac{1}{2}(5-n)}(n-1)^{-1}s^{n-2}\sigma^{1-n}\exp\{-\tfrac{1}{2}ns^2\sigma^{-2}\}ds$$

Writing

$$(2) \qquad P_s = \int_s^\infty f(t)dt$$

it is evident that (2) is an incomplete Γ-function in which s and σ occur only in the form s/σ (let $s/\sigma = t$ in (1)). For each value of P_s and n equation (2) is satisfied by a value of s/σ which is $1/f_{100P_s}$ in Deming's notation. Letting $ns^2/\sigma^2 = u$ in (1), it is evident that u is distributed as χ^2 with $n - 1$ degrees of freedom. Thus

$$P_s = P(\chi^2) = .95, .50,$$

where

$$(3) \qquad \chi^2 = ns^2\sigma^{-2} = n/f_{P_s}^{\,2}.$$

Given n and P_s, a value of χ^2 is found satisfying

$$P(\chi^2) = \int_{\chi^2}^{\infty} f(\chi^2) d\chi^2$$

for $n - 1$ degrees of freedom, and f_{P_s} is immediately given by (3). Tables of f_{95} and f_{50} are useful in finding the .05 and .50 fiducial limits of σ, given s and n. Table 1 gives values of f_{50} and f_{95} to 6D for $n = 2(1)25$.

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942[K].—W. J. DIXON, "Ratios involving extreme values," *Annals of Math. Stat.*, v. 22, 1951, p. 68–78.

Let $x_1, \cdots, x_n$ represent the values of a sample of size n from a normal population arranged in increasing order of magnitude. Let

$$r_{10} = (x_n - x_{n-1})/(x_n - x_1), r_{11} = (x_n - x_{n-1})/(x_n - x_2),$$
$$r_{12} = (x_n - x_{n-1})/(x_n - x_3), r_{20} = (x_n - x_{n-2})/(x_n - x_1),$$
$$r_{21} = (x_n - x_{n-2})/(x_n - x_2), r_{22} = (x_n - x_{n-2})/(x_n - x_3).$$

The six tables of this paper present the values of R which satisfy the relation

$$Pr(r_{ij} > R) = \alpha,$$

for $i = 1, 2; j = 0, 1, 2; n = 2 + i + j$ (1) 30; $\alpha = .005, .01, .02, .05, .10$ (.10) .90, .95.

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943[K].—E. J. GUMBEL & J. A. GREENWOOD, "Table of the asymptotic distribution of the second extreme," *Annals Math. Stat.*, v. 22, 1951, p. 121–124.

Consideration is given to the asymptotic distribution of the next to last and of the second value of a large sample from an initial distribution of the exponential type. Table I enables one to test (using the asymptotic distribution) the hypothesis that a sample came from a completely prescribed population by means of the second largest (penultimate) variate. A transformation is given which enables one to use the table for the second smallest value. Probability values are given to 5D (with a method to obtain more places if desired) with second central differences for $y_2 = -1.95$ (.05) 5.25, 5.35, 5.50, 5.65, 5.90, 6.45, where $y_2 = \frac{1}{2}n(x_2 - u_2)f(u_2)$, in which n is the sample size, $f(x) = F'(x)$ is the initial density function, and u_2 is defined by $F(u_2) = \dfrac{n-2}{n}$. Probability points for y_2 to 5D are given in Table II for .005, .01, .025, .05, .1, .25, .5, .75, .9, .95, .975, .990, .995. Mention is made of applying the tabular values in the construction of a probability paper, but insufficient information is presented to enable one actually to follow the proposal without using the references.

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944[K].—F. J. MASSEY, JR., "The distribution of the maximum deviation between two sample cumulative step functions," *Annals Math. Stat.*, v. 22, 1951, p. 125–128.

Let $x_1, x_2, \cdots, x_n$ and $y_1, y_2, \cdots, y_m$ be the ordered results of two random samples from populations having continuous cumulative distribution functions $F(x)$ and $G(x)$, respectively. Let $S_n(x) = k/n$ where k is the number of observed values of X which are less than or equal to x, and similarly let $S_m'(y) = j/m$ where j is the number of observed values of Y which are less than or equal to y. The statistic $d = \max_x |S_n(x) - S_m'(x)|$ can be used to test the hypothesis $F(x) \equiv G(x)$, where the hypothesis is to be rejected if the observed d is significantly large. The limiting distribution of $d(mn/(m+n))^{\frac{1}{2}}$ has been derived[1] and tabulated.[2] In this paper the author describes a method for obtaining the exact distribution of d for small samples and a short table for equal size samples is included. In Table 1, the probability of $d \leqq k/n$ is given for $n = m$, $k = 1(1)12$, $n = 1(1)40$ usually to 6D.

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[1] N. SMIRNOV, "On the estimation of the discrepancy between empirical curves of distribution for two independent samples," Moscow, Univ., *Bull. Math.*, (*série internationale*), v. 2, 1939, fasc. 2, 16p.
[2] N. SMIRNOV, "Table for estimating the goodness of fit of empirical distributions," *Annals Math. Stat.*, v. 19, 1948, p. 279–281.

945[K].—F. J. MASSEY, JR., "The Kolmogorov-Smirnov test for goodness of fit," Am. Stat. Assn., *Jn.*, v. 46, 1951, p. 68–78.

Let $F_0(x)$ be a continuous population cumulative distribution, $S_N(x)$ the observed cumulative step-function of a sample (i.e., $S_N(x) = k/N$, where k is the number of observations less than or equal to x), then the sampling distribution of d = maximum $|F_0(x) - S_N(x)|$ is known, and is independent of $F_0(x)$. In Table 1, the author gives the prob$\{\max|S_N(x) - F_0(x)| > d_\alpha(N)\} = \alpha$, for $\alpha = .20, .15,. 10, .05$, and $.01$, $N = 1(1)20$ to 3D; and for $N = 25$, 30, and 35 to 2D. For $N > 35$ the limiting values of SMIRNOV[1] apply. In Table 3 the author compares the minimum deviation of actual from assumed population that is detectable with probability .50 by the χ^2 and d tests at the 5 percent and 1 percent levels of significance, with $N = 200(50)1000(100)1500(500)2000$, for values of χ^2 to 4D and values of d to 3D. (The χ^2 portion of this table is from WILLIAMS.[2])

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[1] N. SMIRNOV, "Table for estimating the goodness of fit of empirical distributions," *Annals Math. Stat.*, v. 19, 1948, p. 279–281.
[2] C. A. WILLIAMS JR., "On the choice of the number and width of classes for the Chi-square test for goodness of fit," Amer. Stat. Assn., *Jn.*, v. 45, 1950, p. 77–86.

946[K].—H. S. Sichel, "The estimation of the parameters of a negative binomial distribution with special reference to psychological data," *Psychometrika*, v. 16, 1951, p. 107–127.

The negative binomial distribution is written in the form,

$$f(r) = \frac{\Gamma(r+p)}{\Gamma(p)\,\Gamma(r+1)}\left(\frac{p}{p+m}\right)^p\left(\frac{m}{p+m}\right)^r, \tag{1}$$

in which optimum estimates of p and m are uncorrelated. The moment estimate $\bar{m}$ = mean, is shown to be efficient and the moment estimate, $\bar{p}$ = (mean)^2/(variance–mean), to have efficiency given by

$$\text{Eff. }(\bar{p}) = \left[2\sum_2^\infty \frac{1}{r}\left(\frac{m}{p+m}\right)^{r-2}\frac{\Gamma(r)\,\Gamma(p+2)}{\Gamma(r+p)}\right]^{-1}. \tag{2}$$

These results are essentially those given by Fisher.[1] Figure 1 gives Eff. ($\bar{p}$) for p varying continuously from 0 to 3 and m = .1, .5, 1(1)4, ∞. It is suggested (as also by Fisher) that Eff. ($\bar{p}$) may be fairly accurately estimated by using $\bar{m}$ and $\bar{p}$ in (2) above.

The maximum likelihood estimate, $\hat{p}$, is the solution of

$$\frac{1}{n}\sum_1^n\left[\psi(\hat{p}+r_i-1)-\psi(\hat{p}-1)\right]-\log\left(1+\frac{\hat{m}}{\hat{p}}\right)=0,$$

where $\hat{m} = \bar{m} = \frac{1}{n}\sum r_i$. This equation is given by Haldane[2] as his equation (2.1). The remainder of the paper, except for three examples of applications, is given over to tables useful in solving this equation. Table 1 gives values of $\lambda(r,\hat{p}) = \psi(\hat{p}+r-1) - \psi(\hat{p}-1)$ to 5D for r = 0(1)35 and $\hat{p}$ = .1(.1)3.0. Table 2 gives values of $\psi(\hat{p}-1)$ to 5D for $\hat{p}$ = .1(.1)3.0. The author seems to be unaware of the existence of the numerous earlier tables of the digamma function.[3] Checks with the earlier tables indicate some errors in Table 1; Table 2 is correct.

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[1] R. A. Fisher, "The negative binomial distribution," *Annals of Eugenics*, v. 11, 1941, p. 182–187.

[2] J. B. S. Haldane, "The fitting of binomial distributions," *Annals of Eugenics*, v. 11, 1941, p. 179–181.

[3] See FMR *Index*, p. 202–203.

947[L].—M. Abramowitz, "Tables of the functions $\int_0^\phi \sin^{1/3} x\, dx$ and $(4/3)\sin^{-4/3}\phi \int_0^\phi \sin^{1/3} x\, dx$," N.B.S. *Jn. of Research*, v. 47, 1951, p. 288–290.

The first integral is given to 8D for ϕ = 0°(1°)90°, the second integral to 8D for ϕ = 0°(30′)90° and to 7D for ϕ = 90°(30′)180°.

A. E.

948[L].—B. P. Bogert, "Some roots of an equation involving Bessel functions," *Jn. Math. Phys.*, v. 30, 1951, p. 102–105.

Table I gives 4 to 6 S values of the first root x of

$$(1) \qquad J_0(x)Y_1(kx) - Y_0(x)J_1(kx) = 0$$

together with 4S (in one case 5S) values of

$$y = \frac{2}{\pi}(k - 1)x$$

for $k = 1,1.1,1.2,1.25,1.3, 1.4,1.5,1.59334, 1.6(.1) 2.1,2.45882, 3(1)10, 20$.

Table II gives 4 to 5 S values of the first root of equation (1) for $k = 1$ (.01)2, 2.1, 3(1)10, 20.

Table III gives 4 S values of the first root of

$$(2) \qquad Y_1(x)J_0(kx) - J_1(x)Y_0(kx) = 0$$

for $.05 \leqslant k \leqslant 1$ (irregular interval, 111 values)

In addition to entries listed in *MTAC*, v. 1, p. 222, the author refers to the following related tables:

CARSTEN & MCKERROW—unpublished manuscripts, equation (1) for $k = (1.1)^{1/2}$, 1.06(.02)1.2(.5)1.5,2(1)5; 2–3D.

RENDULIC, *Wasserwirtschaft und Technik*, no. 25–26, 1935, p. 270, equation (1), $k = 0(.1)1(1)7$, 6.25; 3–4 D.

DURANT, equation (2), see *MTAC*, v. 2, p. 172.

A. E.

949[L].—COMPUTATION DEPARTMENT OF THE MATHEMATICAL CENTER, Amsterdam. Interim reports nos. R 53, Int. 1–8. The oscillating wing in a subsonic flow. R 53, Int. 1. (1949), 7 p., 5 Datasheets. R 53, Int. 2. (1949), 21 p. R 53, Int. 3. (1950), 2 p., 9 Datasheets (reproduced on 17 p.). R 53, Int. 4. (1950), 12 p. R 53, Int. 5. (1950), 12 p. R 53, Int. 6. (1951), 2 p., 42 Datasheets. R 53, Int. 7. (1951), 35 p. + 3 p. of corrections. R 52, Int. 8. (1951), 2 p., 18 Datasheets. 21.5 × 32.6 cm.

The Computation Department of the Amsterdam Mathematical Center has carried out voluminous computations on behalf of the (Dutch) National Aeronautical Research Institute. The final report will not be available for general distribution, but a certain amount of unclassified information regarding expansions and methods of computation of certain functions, and a number of working sheets, are released in a series of interim reports. Seven of the eight interim reports carry the same title; R 53, Int. 5. is entitled "Expansions of $B_m^{(n)}$ and a_n into power-series with respect to τ." The first report carries a loose sheet with a list of the 23 members of the Computing Section of the Computation Department. The effort to make accessible to the general computing public as much information as is consistent with security, is praiseworthy and perhaps worthy of imitation.

The numerous expansions and descriptions of computations contained in this report will prove very valuable to future workers in this field. It is impossible to describe them concisely beyond saying that the larger part refers to Mathieu functions, and the lesser part to certain integrals related to Bessel functions. The work on Mathieu functions is a useful supplement to MCLACHLAN's book[1] whose notations the authors follow.

A description of the Datasheets (mostly to 8 D or 8 S) follows.

R 53, Int. 1. Gegevenblad 1. General orientation about parameters. Gegevenblad 2. For $\tau = .1(.05).35$ this gives the corresponding values of $\cos \eta_1 = 1 - 2\tau$ (on the worksheet $\sin \eta_1$ appears, presumably a mistake or a survival from an earlier notation), and then $\sin n\, \eta_1$ for $n = 1(1)27$. Datasheets 3, 3a. Values of $J_n(\beta^2\Omega)$ for $n = 0(1)26$ and $\beta^2\Omega$ from .0175 to 11.2 at varying intervals (59 values). Spotchecking a few values against the Harvard University tables of Bessel functions[2] revealed no discrepancy. Datasheet 4 gives $n^{-1} \sin n\, \eta_1$ for the same values of n and η_1 as in Datasheet 2. Actually, the table is given twice: first n increases, and then n decreases, in the columns. This is to facilitate computation of convolution sums like

$$\sum_{n=2}^{m-1} \frac{\sin n\, \eta_1}{n} \frac{\sin (m - n)\, \eta_1}{m - n}.$$

R 53, Int. 3 gives numerical values of the Fourier coefficients and characteristic values of odd Mathieu functions. Mathieu's equation is written in the form

$$y'' + (a - 2q \cos 2x) = 0, \quad q = k^2$$

and the function

$$se_n(x,q) = \sum_{m=1}^{\infty} B_m^{(n)}(q) \sin mx$$

is that solution of Mathieu's equation which has period 2π, reduces to $\sin nx$ when $q = 0$, and for which

$$\sum_{m=1}^{\infty} [B_m^{(n)}(q)]^2 = 1.$$

The corresponding characteristic values of a are $b_n(q)$. Datasheets 5 to 12 (each comes in two parts) give $B_m^{(n)}(q)$ for $k = .025(.025).15(.05).3$ $(.075).45(.1).65$, .8,1,1.2,1.5,2,3 and $n = 1(1)8$. The range for m varies, $m = 33$ being the highest value that occurs (for $n = 1$ and two values of k). Thus, quite apart from the question of different normalization, there are values in these worksheets which are not covered by the NBSCL Mathieu function tables.[3] Datasheet 13 gives b_n for the same values of k and n as in the previous datasheets, and in addition for $n = 9(1)12$ for a few values of k. Since k rather than q has been taken as a variable, some of the values do not occur in the NBSCL tables (which tabulate $b_n + 2k^2$ as a function of $4k^2$) although they all fall within the range covered there. Spotchecking a few values of b_n with the NBSCL tables did not reveal any discrepancy beyond one unit of the last decimal place (of the Amsterdam table) which is well within the limits of accuracy the authors claim.

R 53, Int. 6. Datasheets 14 to 55 give values of

$$\int_0^{\eta_1} \cos (n\eta) \exp (i\beta^2\, \Omega \cos \eta) d\eta$$

for $n = 0(1)20$, $\tau = \frac{1}{2} - \frac{1}{2} \cos \eta_1 = .1(.05).35$ and for varying ranges of values of $\beta^2\Omega$ between 0 and 4.8. Real and imaginary parts are given on separate sheets.

R 53, Int. 8. Datasheets 60 to 71 gives values of

$$\int_0^x J_0(t) \frac{\cos}{\sin} ct\, dt, \int_0^x Y_0(t) \frac{\cos}{\sin} ct\, dt$$

for $c = .3(.1).8$ and $x = 0(.1)6.1$. The report states that in the heading of the tables $\frac{x}{c}$ should be replaced by cx. These tables exceed both in their range and, partly, in the number of decimals the tables by Schwarz[4] who introduced these functions. Datasheets 72–77 give the real and imaginary parts (both parts on the same sheet) of

$$\int_0^\infty (c + \cosh \xi)^{-1} \exp(-ix \cosh \xi) d\xi$$

for $c = .3(.1).8$ and $x = 0(.1)6.1$.

A. E.

[1] N. W. McLachlan, *Theory and Application of Mathieu Functions*. Oxford University Press, 1947.

[2] Harvard University Computation Laboratory, *Annals* v. 3–14, *Tables of the Bessel Functions of the First Kind of Orders Zero through Hundred Thirty-Five*. Harvard University Press, 1947–1951.

[3] NBSCL, *Tables Relating to Mathieu Functions*, New York, Columbia University Press, 1951.

[4] See *MTAC*, v. 1, 1944, p. 248, 250, 304.

950[L].—Wolfgang Gröbner & Nikolaus Hofreiter, *Integraltafel. Zweiter Teil: Bestimmte Integrale.* Vienna and Innsbruck, Springer-Verlag, 1950, vi, 204 p. 20.7 × 29.8 cm.

The first volume of this work contains indefinite integrals, and it was reviewed in *MTAC*, v. 3, 1949, p. 482. The present second volume lists principally those definite integrals which cannot be evaluated in a simple manner from the indefinite integrals of the first volume. In addition, many integrals which could be computed from the first volume have been included for the convenience of the user. For instance, the indefinite integral

$$\int x^{2m}(x^2 + a^2)^{-n-\frac{1}{2}}dx, \quad n > m \geq 0$$

is given in volume I as a (finite) series. If the limits 0 and ∞ are substituted in that series, one obtains a sum containing binomial coefficients. In volume II the integral (from 0 to ∞) is given in closed form, thus saving the user the labor of summing the series.

Known definite integrals are very numerous, and in order to prevent the book from becoming unwieldly the authors limited themselves to a selection. By introducing variable parameters they often give master formulas from which many integrals can be computed, and do not list too many of the particular cases. All integrals recorded in the table have been carefully checked, and to make assurance doubly sure, each entry is accompanied by a coded instruction telling the user how to verify the result if he wishes to do so.

The general arrangement of the second volume closely resembles that of the first volume, except that there are five sections in volume II instead of the three sections of volume I.

The Introduction contains a list of symbols and notations, a list of references, methods for the evaluation of definite integrals, and general formulas. In the list of references Dirichlet's lectures on definite integrals, Bierens de Haan's tables of integrals, Magnus & Oberhettinger's book on special functions, WATSON's *Bessel Functions*, and WHITTAKER & WATSON's *Modern Analysis* are mentioned, but not the volume of corrections by LINDMAN to Bierens de Haan's tables, nor the integral tables by RYZHIK (*MTAC*, v. 1, 1945, p. 442–443).

Section 1, Rational integrands (21 p.) contains also formulas for, and integrals involving, the classical orthogonal polynomials. Section 2, Algebraic integrands (20 p.) contains also elliptic integrals both in Legendre's normal form and in Weierstrass' canonical form. Section 3, Elementary transcendental integrands (117 p.) contains many integrals which can be evaluated in terms of error functions, Bessel functions, and other higher transcendental functions. There are also special subsections devoted to Euler's dilogarithm and to the exponential integral and related functions. In this section there is a table of Laplace transforms (with a reference to DOETSCH's book of 1937, but without reference to any of the more recent and more extensive tables of Laplace integrals). No table of Fourier transforms is given, but many Fourier integrals occur in various subsections of section 3. The last subsection in this section lists information on limits of definite integrals: Dirichlet's singular integral, the integral occurring in the Riemann-Lebesgue lemma, and Laplace transforms.

The last two sections contain material which has no counterpart in the first volume. Section 4 Euler integrals (18 p.) contains some hypergeometric integrals in addition to integrals which can be evaluated in terms of the gamma function and related functions. Section 5, Bessel functions (20 p.) contains both integrals which can be evaluated in terms of Bessel functions and integrals whose integrands contain Bessel functions.

The book is an offset print from an excellent handwritten copy. In comparison with the first volume, one finds larger letters and explanations written in italic rather than script characters, both features contributing to legibility.

A. E.

951[L].—Y. L. LUKE & M. A. DENGLER, "Tables of the Theodorsen circulation function for generalized motion," *Jn. Aeron. Sci.*, v. 18, 1951, p. 478–483.

The function in question is

$$C(z) = \frac{H_1^{(2)}(z)}{H_1^{(2)}(z) + iH_0^{(2)}(z)} = F(\rho, \theta) + iG(\rho, \theta),$$

where $z = \rho e^{i\theta}$, and $H_n^{(2)}$ is the Hankel function. Tables 1 and 2 give 7D values of F and of $-G$ for $\theta = -5°(5°)30°$ and $\rho = 0(.01).3(.02).5(.05)1(.5)4(1)10$. Table 3 gives 7D values of F and of $-G$ for $\theta = 0$ and $\rho = k = 0(.002).1(.01).3(.02).34(.01).36(.02).44(.01).46(.02).54(.01).56(.02).64(.01).66(.02).74(.01).76(.02).84(.01).86(.02).94(.01).96(.02)1(.1)2(.5)10(10)50,100,\infty$. All tables were computed from standard American and British tables of Bessel functions, and the error is estimated by the authors to be at most 3 units of the seventh decimal.

A. E.

952[L].—NBSCL, *Tables Relating to Mathieu Functions.* Columbia University Press, New York, 1951. xlvii + 278 p. 19.7 × 26.7 cm. Price $8.00.

These tables now make it possible to calculate radiation and scattering from slits, strips and elliptic cylinders with nearly the same facility as has been previously possible for rods and spheres.

Solutions of the Helmholtz equation $\nabla^2\psi + k^2\psi = 0$ may be required in solving the wave equation, the diffusion equation, and, in the limiting case of zero potential, the Schroedinger equation. Factored solutions are possible for the eleven coordinate systems for which the equation separates. For most of these systems one or two of the coordinates are "angle" coordinates, with a finite range of values; the rest are "radial" coordinates, with a range from 0 to ∞ or from $-\infty$ to ∞. To solve most problems of physical interest one needs tables of the eigenfunction solutions for the angle coordinates, with corresponding eigenvalues for the separation constant; one also needs *both* independent solutions for the radial coordinates, for the eigenvalues of the separation constant.

Heretofore adequate tables have been available for only three coordinate systems: rectangular, for which the solutions are trigonometric and hyperbolic functions; circular cylindrical for which the Bessel and Neumann functions are needed;[1] and the spherical, for which tables of spherical harmonics are also required.[2] Other systems of practical interest are the elliptic cylinder coordinates, with solutions tabulated in the tables here reviewed, the parabolic cylinder, the parabolic and the spheroidal coordinates, for which adequate tables are still lacking.

Factors for elliptic cylinder coordinates are solutions of the separated equation

$$y'' + (b + s\cos^2 x)\, y = 0,$$

where the "angle" coordinate corresponds to real values of x from 0 to 2π and the "radial" coordinate to imaginary values, from 0 to $i\infty$. The tables under review provide the foundation for radiation and scattering calculations for s (which is proportional to the square of the ratio between the interfocal distance to the wave-length) from 0 to 100, with enough entries to allow reasonably accurate second-difference interpolation.

Eigenvalues of the separation constant b are given for the first 31 periodic eigenfunctions for the angle coordinate, to 8 decimal places. Corresponding values of the coefficients of the Fourier series expansions of these eigenfunctions are given to allow a 9D accuracy of the resulting Fourier series. These eigenfunctions alternate in symmetry, of course; the first being even about $x = 0$, the next being odd and so on. The series coefficients are normalized so that the value of the even functions, Se_m, are unity and the slopes of the odd functions, So_m, are unity at $x = 0$. This normalization has some advantage in radiation calculations, though the alternative normalization, that the quadratic integral over x from 0 to 2π be 2π, is preferred by some. In the former case the values of the solution for x near $\pi/2$, for s large, may be quite large; in the second case the corresponding values for x near 0 may be vanishingly small. Since the ratio between the coefficients for the two normalizations is tabulated, either one may be computed.

In the case of the radial functions the preference is more clear-cut. Most radiation or scattering problems involve the asymptotic amplitudes of the two independent solutions. Because of the simple relationship between the Fourier series for the eigenfunction and the Bessel-function series for the radial solutions, the former normalization (value or slope unity) results in a simple asymptotic form for the solutions, the latter (integral of the square = 2π) does not. From the coefficients, as tabulated, these simpler forms of the two radial solutions may be computed as functions of $-ix$ by using tables of Bessel and Neumann functions. Values and slopes of the two radial functions at $x = 0$ are given to 8 significant figures in the volume, thus allowing computation of radiation and scattering from degenerate elliptic cylinders (strip or slit) to be carried out without the use of additional Bessel-function tables. Again interpolation is made easy by the type of presentation.

For those with easy access to large computing machines the Tables in this volume will probably allow easy calculation of most radiation or scattering or resonance problems of practical interest. For those having only desk equipment available it would save much labor to have subsidiary tables of the actual angle and radial functions published, though these additional tables would be quite bulky because of the large number of entries required (different values of s, of m and of x all requiring tabulation).

The Mathematical Tables Project is again to be congratulated in carrying out, in so satisfactory a manner, the immense amount of labor required to obtain the entries in this volume. One can hope that the spheriodal functions will eventually be tabulated with similar accuracy and extensiveness.

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[1] See, for example: *Scattering and Radiation from Circular Cylinders and Spheres; Tables of Amplitudes and Phase Angles* [*MTAC*, v. 3, p. 107]. For other tables of Bessel Functions see *MTAC*, v. 1, p. 205–308. The new tables of Bessel Functions [*MTAC*, v. 5, p. 223–224] published by the Harvard Computation Laboratory will be most useful as soon as the corresponding second solutions are published.

[2] See, for example: H. TALLQVIST, "Tafel der 24 ersten Kugelfunktionen $P_n(\cos\theta)$ [*MTAC*, v. 1, p. 4], and also MTP, *Tables of the Associated Legendre Functions* [*MTAC*, v. 1, p. 164–165] and *Tables of Spherical Bessel Functions* [*MTAC*, v. 3, p. 26].

953[L].—MARTHA PETSCHACHER, "Tabelle di Funzioni Ipergeometriche," Rome, Univ., Ist. Naz., Alta Mat., *Rend. Mat. e Appl.*, s. 5, v. 9, 1951, p. 389–420.

For the calculation by the hodograph method of steady motions of a gas in two dimensions, two families of hypergeometric functions are of importance. The first, which is relevant for finding the Legendre potential and position co-ordinates of the gas-flow, is

$$F(\tau) = F(\alpha_\mu, \beta_\mu, \mu + 1; \tau), \tag{1}$$

where F is the hypergeometric series and τ is the variable, μ a parameter, and α_μ, β_μ are determined from

$$\alpha_\mu + \beta_\mu = \mu + 1/(\gamma - 1), \quad \alpha_\mu\beta_\mu = -\tfrac{1}{2}\mu(\mu - 1)/(\gamma - 1), \tag{2}$$

γ being the adiabatic index of the gas. The second, which is relevant for finding the stream function, is

$$F(a_\mu, b_\mu; \mu + 1; \tau), \tag{3}$$

where

$$a_\mu + b_\mu = \mu - 1/(\gamma - 1), \quad a_\mu b_\mu = -\tfrac{1}{2}\mu(\mu + 1)/(\gamma - 1).$$

For physical significance the range of τ is $0 \leq \tau < 1$.

In the present publication the function $F(\tau)$ defined by (1), (2) is tabulated for the case $y = 1.4$, for $\tau = 0(.02)1$ and $\mu = \frac{1}{8}(\frac{1}{8})\frac{7}{8}; \frac{5}{4}(\frac{1}{4})2(\frac{1}{2})7(1)10$, to 6D throughout. A user of these tables is, however, more likely to be interested in significant figures than decimals; the number of significant figures is 6 or 7 for the smaller values of μ, diminishing to 4 (or, for a small part of the table, 3) for the higher μ. The related function

$$Y(\tau) = \tau^{\mu/2}F(\alpha_\mu, \beta_\mu; \mu + 1; \tau)/\Gamma(1 + \mu)$$

also is given; the number of significant figures is in general the same as for $F(\tau)$.

The calculation was made (a) for $0 \leq \tau \leq 0.7$ by numerical solution of the hypergeometric differential equation, (b) for $0.7 \leq \tau < 1$ by expressing (1) in terms of hypergeometric functions of $1 - \tau$ and finding these from their power series; the agreement at $\tau = 0.7$ provided an over-all check. The table is stated to be correct to about half a unit in the last figure, and the reviewer has verified this for a few entries.

The function (3), and its derivatives, have been tabulated by FERGUSON & LIGHTHILL[1] and by VERA HUCKEL,[2] and from these the function (1) can be easily found. But the present table breaks new ground in covering the range $\frac{1}{2} \leq \tau < 1$ and including values of μ which are neither integral nor half-integral. However, for the gas-flow context one requires also the derivative of $F(\tau)$, and it is to be hoped that the author will extract this from her work sheets and publish it.

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[1] D. F. FERGUSON & M. J. LIGHTHILL, "The hodograph transformation in transonic flow," Roy. Soc. London, *Proc.*, v. 192A, 1947, p. 135–142.

[2] V. HUCKEL, *Tables of Hypergeometric Functions for Use in Compressible-Flow Theory.* N. A. C. A. *Technical Note* no. 1716, Washington, 1948.

954[L].—G. PÓLYA & G. SZEGÖ. *Isoperimetric inequalities in mathematical physics*. Annals of Mathematics Studies no. 27. Princeton University Press, 1951. xvi + 279 p. 17.7 × 25.3 cm. Price $3.00.

The tables (p. 247–274) give values for the following ten quantities associated with a plane domain D:

L length of perimeter.

A area.

I moment of inertia of the area with respect to its centroid.

B. Let a be any point of D, and h the distance, from a, of the tangent at any point of the boundary of D. B is the minimum, as a varies over D, of the integral $\int h^{-1}ds$ taken over the boundary of D.

ρ and R radii of the inscribed and circumscribed circles of D.

$\dot{r}$ and $\bar{r}$ maximum inner radius, and outer radius, i.e., the radii of the circles (in the case of the interior, maximal circle) which bound a conformal map of D, or its complement, if the linear magnification at the centre (for D) or at infinity (for the complement of D) is unity.

P torsional rigidity of a cylinder of cross-section D.

Λ principal frequency of a membrane of shape D.

The first 13 tables (p. 251–258) list the values of these constants for the following domains: circle, ellipse, square, rectangle, semi-circle, sector, 3 triangular shapes (equilateral, half of an equilateral, isosceles right-angled triangles), and regular hexagon, together with approximations for narrow ellipses, rectangles, and sectors. Not all tables list all the values. The following 14 tables (p. 259–271) list certain dimensionless combinations (for instance $\dot{r}A^{-1/2}$, $P\Lambda^2A^{-1}$) for some or all of the same domains, together, in some cases, with information about the boundedness or extreme values of the combinations in question. The last two tables (p. 272, 273) list values of B, $\dot{r}$, $\bar{r}$ for some additional domains.

In the main body of the monograph there are some further numerical values, inequalities, and approximations for these and similar quantities, and some corresponding quantities for three-dimensional domains. There is also (on page 22) a 4D numerical table of

$$\frac{(1 + \sin^2\delta)^{\frac{1}{4}}}{\cos(\delta/2)} \text{ and } \frac{j\Gamma[(\pi - \delta)/(2\pi)]}{2\pi^{\frac{1}{2}}\Gamma[(2\pi - \delta)/(2\pi)]}$$

for $\delta^\circ = 0(15)60(5)85(1)90$. Here $j = 2.4048\ldots$ is the first positive zero of the Bessel function $J_0(x)$. This table is reprinted from a paper by one of the authors.[1]

A. E.

[1] G. PÓLYA, "Torsional rigidity, principal frequency, electrostatic capacity and symmetrization," *Quart. Appl. Math.*, v. 6, 1948, p. 267–277.

955[L].—G. W. REITWIESNER. *A Table of the Factorial Numbers and their Reciprocals from* 1! *through* 1000! *to* 20 *Significant Digits*. Ballistic Research Laboratories, Technical Note no. 381. Aberdeen Proving Ground, Md., 1951. Hectographed 20.3 × 26.7 cm.

This table, whose description is quite adequately given in its title is a by-product of the computation of e by the ENIAC [*MTAC*, v. 4, p. 11–15]. A similar table to 62S is on file in the Computing Laboratory of the BRL [*MTAC*, v. 5, p. 195]. This table is an extension of a previous table, Technical Note no. 106, by LOTKIN [*MTAC*, v. 4, p. 15] extending to 200! with 20S. A table of $n!$ for $n = 1(1)1000$ to 16S is described in RMT **956** and UMT 69 [*MTAC*, v. 3, p. 205]. A manuscript table, identical with the one under review, by S. JOHNSON, is mentioned in *MTAC*, v. 3, p. 340.

956[L].—LELIA RICCI, "Tavola di radici di basso modulo di un'equazione interessante la scienza delle costruzioni," *Rivista di Ingegneria*, 1951, no. 2, 8 p.

Numerical tables of the first ten pairs of roots (in case of complex roots, of real and imaginary parts of the roots) of

$$\sin z = \pm kz$$

for $k = .1(.01)1$. Five decimals (claimed reliable) are given for $k = .1(.1)1$, four decimals (of which the last is not reliable) for the other values of k. Table I gives the complex roots, and Table II the real roots. [See also *MTAC*, v. 5, p. 231.]

A. E.

957[L].—H. E. SALZER. *Tables of $n!$ and $\Gamma(n + \frac{1}{2})$ for the First Thousand Values of n.* National Bureau of Standards, AMS 16. Washington 1951. Price 15 cents.

This table brings to publication a manuscript table of $n!$ already reported in *MTAC*, v. 3, p. 205. The values of $n!$ and $\Gamma(n + \frac{1}{2})$ for $n = 0(1)1000$ are given to 16S and 8S, respectively. The manuscript table of $n!$ was checked by comparison with a 24S table prepared by J. BLUM on punched card equipment.

D. H. L.

958[V].—BALLISTIC RESEARCH LABORATORIES, Report no. 757: M. LOTKIN, *Supersonic Flow of Air Around Corners*, May 1951, 20 p., 1 table, 1 diagram.

If μ be the Mach angle and the function: $f(\mu) = k^{-1}\cot^{-1}[k^{-1}\tan\mu]$ be introduced, where $k^2 = (\gamma - 1)/(\gamma + 1)$ and γ is the specific heat ratio, then the relation between the deflection, δ, of a supersonic stream and the initial and final Mach angles, μ_i and μ_f, may be expressed:

$$\delta = f(\mu_f) - f(\mu_i) + \mu_f - \mu_i.$$

Also, the radial and tangential velocity components are:

$$u = c\sin[kf(\mu)]$$
$$v = c\,k\cos[kf(\mu)],$$

where c is the velocity of efflux into a vacuum. The temperature, density, and pressure follow directly from the fact that T is proportional to

$$\left(M^2 + \frac{2}{\gamma - 1}\right)^{-1}$$

and the adiabatic law.

Since δ occurs as the first difference of $F(\mu) = f(\mu) + \mu$, the expansion around a corner δ may be regarded as the continuation of an expansion from $M = 1$ and δ tabulated as a function of μ from $M = 1(\mu = 90°, \delta = 0)$ to $M \to \infty \left(\mu = 0, \delta = \frac{\pi}{2}[k^{-1} - 1]\right)$. The author tabulates $\theta \equiv f(\mu)$, μ, M, q/c (where q is the speed), T/T_0, p/p_0, and ρ/ρ_0 (where the subscript 0 refers to stagnation conditions) for 0°.5 interval of the argument δ. The value $\gamma = 1.405$ is used, angles are given to 0°.001, and the thermodynamic variables are carried to five significant figures.

The author presents an unnecessarily involved form of the above synopsis of the problem, presumably motivating the highly misleading form of the only diagram in the paper which, by its contextual implication, illustrates the physical picture of the general flow around a corner but actually applies only to the case $M = 1$.

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959[V].—I. IMAI & H. HASIMOTO, "Application of the W. K. B. method to the flow of a compressible fluid, II," *Jn. Math. Phys.*, v. 28, 1950, p. 205–214.

Table 1 (p. 21) gives 4 or 5S values of Q and of $Q/qK^{\frac{1}{2}}$ for $q = 0(.01)1$, where

$$K = (1 - q^2)\,(1 - a^2q^2)^{-1/a^2}, \quad a^2 = \frac{\gamma - 1}{\gamma + 1}, \quad \gamma = 1.4,$$

$$\mu = \left(\frac{1 - q^2}{1 - a^2q^2}\right)^{\frac{1}{2}}, \quad Q = \frac{2}{1 + \mu}\left(\frac{1 + a\mu}{1 + a}\right)^{1/a} q(1 - a^2q^2)^{(1-a)/2a}.$$

There are also some tables relating to the physical problem in hand.

A. E.

960[Z].—PAUL S. DWYER, *Linear Computations*. London, Chapman and Hall, New York, John Wiley and Sons, 1951, xii + 344 p. 229 × 14.4 cm. $6.50.

The main purpose of the book is to explain how to get numerical solutions for sets of simultaneous linear equations. Related matters, such as the numerical evaluation of determinants, numerical inversion of matrices, etc., are also treated, but with less detail. Attention is centered on the use of desk calculators.

Quite literally, the reader needs only to know grade school arithmetic and some high school algebra to read the first half of the book. The first three chapters discuss elementary computational matters, such as round off errors, significant digits, etc. Then there are five chapters which deal with a great many variations of the basic scheme of solving simultaneous linear equations by successive elimination of variables. Next come two chapters on determinants. The basic theorems are stated without proof, and the main attention is devoted to methods of numerical computation. A chapter on linear forms and three chapters on matrices follow, with the same pattern. Except for a thorough chapter on the errors that can arise in the above methods, the few remaining chapters seem very sketchy. For example, two related methods are given for finding the characteristic equation of a matrix, but no mention is made of well-known methods for finding the greatest eigenvalue directly. Indeed, the author seems distrustful of all iterative methods, since he dismisses even the best known and most widely used iterative methods with a bare mention.

The author is very liberal with diagrams and numerical illustrations. The explanations are full and painstaking. The book will be a great help to those with little mathematical maturity who must nonetheless personally get numerical solutions of simultaneous linear equations.

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