TABLE 13

j	K_j
1	$.01384\,34770 - .01075\,48348i$
2	$.01384\,34770 + .01075\,48348i$
3	$.01764\,36945$
4	$.00000\,00000$

permitted 6 coefficients, initially the truncation error practically vanishes. Even after a change of 10 seconds in the independent variable, the truncation error has been reduced by 76 percent relative to the polynomial case.

Tables 9 to 13 for $n = 4$ are analogous to Tables 4 to 8, respectively, for the $n = 6$ case.

P. BROCK
Reeves Instrument Corp.
F. J. MURRAY
Columbia University

The preparation of this paper was assisted by the Office of Naval Research.

RECENT MATHEMATICAL TABLES

992[C].—G. W. SPENCELEY, R. M. SPENCELEY & E. R. EPPERSON, *Smithsonian Logarithmic Tables to Base e and Base 10.* Smithsonian Misc. Collections, v. 118, Washington, 1952, xii + 402 p., 14.6 × 22.9 cm. Price $4.50.

This volume gives both common and natural logarithms of numbers of the forms

$$n, \qquad 1 + n\cdot 10^{-4}, \qquad 1 + n\cdot 10^{-8},$$

where $n = 1(1)10^4$.

The values are given to 23D. In the case of common logarithms the characteristics are omitted. P. xii contains 23D values of the natural logarithms of 10^k, $k = 1(1)10$ as well as log e.

The table is intended to be used with a calculating machine to find logarithms and antilogarithms to accuracy not exceeding 23D by the well-known factorization method. It was computed on desk calculators with the assistance of 4 students. The natural logarithms of integers were built up from the table of WOLFRAM.[1] All common logarithms were found by multiplication by log e. The work was carried to 28D and then rounded to 23D.

These tables prove that it is still possible to produce a hand-set volume of over a million digits from a very small computing organization. The existence of large scale computing units that could calculate the present table in three days does not seem to daunt the authors.

The FMR *Index* lists only one table comparable with the present one: the four-figure radix table of STEINHAUSER[2] to 21D which "contains many errors." The need for the present table is certainly not as great as it was in 1880 with the advent of the modern automatic desk calculator. Having both natural and common logarithms is something of a luxury in the face of present day printing costs. The hand computer who has a need for many very accurate logarithms will be grateful to the Smithsonian Institution for bringing out this new addition to their series of tables.

D. H. L.

[1] The reprint used was that of VEGA [*MTAC*, v. 4, p. 194].
[2] A. STEINHAUSER, *Hilfstafeln zur präcisen Berechnung zwanzigstelliger Logarithmen*, Vienna, 1880.

993[D, E, K, L].—BAASMTC, *Mathematical Tables*, v. 1: *Circular & Hyperbolic Functions, Exponential & Sine & Cosine Integrals, Factorial Function & Allied Functions, Hermitian Probability Functions.* Third ed. Cambridge University Press, for the Royal Society, 1951, xl, 72 p. 20.9 × 28 cm. 18 shillings.

The first edition of these tables was published in 1931; in the review of the second edition, 1946 [*MTAC*, v. 2, p. 122–123] the list of the XVI tables (p. 1–72) was given. Most of the valuable Introduction of the first edition was omitted from the second edition, but it is now with suitable modification restored (p. viii–xxxvii) and elaborated (p. xxxviii–xl) for the third edition. Readers may be reminded that different sections of this Introduction had been prepared by A. J. THOMPSON, L. J. COMRIE, J. HENDERSON, A. LODGE & J. WISHART, J. O. IRWIN, & R. A. FISHER. Thus the four-page Introduction of the second edition has now been dropped.

On p. xxxviii the extension of Table II, Factorial Function, to 18D, and Constants, is reprinted from the second edition. The four new Constants now added are 16D values of $1/\pi$, π^2, $\log \pi$, $\ln (2\pi)^{\frac{1}{2}}$. The remaining entirely new material of the Introduction includes: the values of the first 12 Bernoullian numbers; Corrigenda to the first edition, 1931, the only serious error listed being in sin 47.6 [*MTAC*, v. 2, p. 135]. The serious error in cos 48.6 listed in 1952 for the second edition by S. H. COHN [*MTAC*, v. 6, p. 100] occurs also in the first and third editions. Then follow: Corrigenda in the first, second, and third editions; 9 unit errors in the final digits discovered by J. W. WRENCH, JR.; Corrigenda to the second edition; Addenda.

This third edition, so much more satisfactory than the second, is indeed wholly admirable, except for that serious error (to which we referred), discovered too late for correction.

R. C. ARCHIBALD

Brown University
Providence, R. I.

994[D, R].—B. GOUSSINSKY, *Tables for Checking Traverse Computations with the aid of a Calculating Machine.* [Palestine] 1952, Ministry of Labour, Survey of Israel, ii + 18 p., 14.6 × 23.5 cm. Price 400 pruta.

The table gives 5D values of $\cos x - \sin x$ for $x = 0(1')45°$. Tables of proportional parts are intended for interpolating to 10″.

995[F, G].—F. C. AULUCK, "On some new types of partitions associated with generalized Ferrers graphs," Camb. Phil. Soc., *Proc.*, v. 47, 1951, p. 679–686.

The author tabulates three special partition functions which have complicated verbal definitions. The first two may be defined in terms of the number $p(n)$ of unrestricted partitions of n as follows:

$$P(n) = p(n-1) - p(n-3) + p(n-6) - p(n-10) + \cdots$$
$$Q(n) = P(1)p(n-1) + P(2)p(n-2) + \cdots + P(n)p(0).$$

Here 1, 3, 6, 10, $\cdots$ are the triangular numbers. If we denote by $p_k(n)$ the number of partitions of n into precisely k positive parts, the third partition function may be defined by

$$R(n) = \sum p_k(m)p(n - 2k^2 - k - 2m - 1),$$

the sum extending over all positive integers (m, k). All three functions P, Q, R are tabulated for $n = 1(1)20$.

D. H. L.

996[F, G].—J. RIORDAN, "The arithmetic of ménage numbers," *Duke Math. Jn.*, v. 19, 1952, p. 27–30.

The general ménage number $U_{n,r}$ is the number of arrangements of n married couples seated at a circular table, gentlemen alternating with ladies, in which precisely r husbands are seated next to their own wives. The paper gives a small table of $U_{r+k,r}$ for $r = 0(1)5$ and $k = 0(1)5$. The heading of the table being a little obscure, it may be worth noting that, for example, $U_{5,2} = 40$.

D. H. L.

997[I].—H. E. SALZER, "Formulas for numerical integration of first and second order differential equations in the complex plane," *Jn. Math. Phys.*, v. 29, 1950, p. 207–216.

Let

$$F(z_j) = \int_\alpha^{z_j} f(z)\,dz; \qquad G(z_j) = \int_\beta^{z_j} F(z)\,dz, \tag{1}$$

where $z = x + iy$. If $f(z)$ can be approximated by a polynomial in z of degree $n - 1$, then both $F(z)$ and $G(z)$ can be expressed as follows:

$$F(z_j) \cong F_{j,n} = \sum_{k=1}^{n} a_k{}^j f(z_k) + A'$$

(2)

$$G(z_j) \cong G_{j,n} = \sum_{k=1}^{n} b_k{}^j f(z_k) + A + B.$$

In the above A', A, and B are constants of integration and the points z_k are fixed, convenient points in the complex plane at which $f(z_k)$ is known. The author tabulates the coefficients $a_k{}^j$ and $b_k{}^j$, for $j = 1, 2, \cdots, n$, such that (2) coincides with (1) when $f(z)$ is a polynomial of degree no higher than $n - 1$, for $n = 2, 3, \cdots, 8$. The points z_k were chosen over an L-shaped grid in the complex plane from the following considerations:

(a) The configuration shall be such that it will be convenient to initiate integration in either the x- or y-direction.

(b) The points z_k shall be as close together as possible. Thus for the five-point formula, the choice of the grid was

$$z_0,\ z_0 + h,\ z_0 + 2h,\ z_0 + ih,\ z_0 + (1 + i)h.$$

This grid was selected in preference to one that is symmetric about the 45°-ray even though the choice involved more tedious computations in the evaluation of the coefficients.

(c) The configuration shall permit of simple translations in either the x- or the y-direction.

Since, in stepwise integration of a differential equation, it is often necessary to start with some approximation to $f(z)$ at a point where $f(z)$ is not yet known, the author also provides the coefficients for the corresponding n-point extrapolation formulas for $f(z)$, which are exact when $f(z)$ is a polynomial in z of degree no higher than $n - 1$.

With modern high-speed computing equipment, the evaluation of the constants of integration at each step of the calculation is manageable, and the formulas can be expected to give a better approximation, for a comparable amount of work, than that which would result from breaking up $f(z)$ into real and imaginary components, and then considering the integration problem in real variables x and y.

The paper occupies only 10 pages, but any one who has tried to calculate at least one set of the coefficients will appreciate the prodigious amount of work that this table represents. Because of the author's reputation for accuracy in his many tables of coefficients, the present work can no doubt be used with complete confidence. Those interested in computing methods owe a debt of gratitude to the author for providing a well planned and useful new tool for numerical integration in the complex plane.

GERTRUDE BLANCH

National Bureau of Standards
Los Angeles 24, Calif.

998[K].—NILS BLOMQVIST, "On a measure of dependence between two random variables," *Annals Math. Stat.*, v. 21, 1950, p. 593–600.

The author investigates the statistic, $q' = (n_1 - n_2)/(n_1 + n_2)$, as a measure of dependence between two random variables. With the $x - y$ plane divided into four quadrants by the axes $x = \bar{x}$ and $y = \bar{y}$, n_1 is the number of sample points in the 1st and 3rd quadrants, while n_2 is the number in the 2nd and 4th quadrants. This statistic was suggested earlier by MOSTELLER[1] as a "rough" but more readily computed measure of dependence than the Pearson correlation coefficient. He did not, however, fully investigate its properties.

To facilitate tests of significance based on q' the author derives the necessary formulas and presents, in tabular form, values of $P\{|n_1 - k| \geq \nu\}$ for $\nu = 0(1)15$ and $2k$ for $k = 2(1)25$, where $2k$ is the largest even number contained in n ($n = n_1 - n_2$). Separate tables are given for odd and even values of ν and k. Most entries are given to 3D. Those for smaller probabilities are carried to 4D.

A. C. COHEN, JR.

University of Georgia
Athens, Georgia

[1] F. MOSTELLER, "On some useful 'inefficient' statistics," *Annals Math. Stat.*, v. 17, 1946, p. 377–408.

999[K].—Z. W. BIRNBAUM & F. H. TINGEY, "One-sided confidence contours for probability distribution functions," *Annals Math. Stat.*, v. 22, 1951, p. 592–596.

Let X be a random variable with the continuous probability distribution function, $F(x)$ = Probability $\{X \leq x\}$. An ordered sample $X_1 \leq X_2 \leq \cdots \leq X_n$ of X determines the empirical distribution function, $F_n(x) = 0$, for $x < X_1$; $F_n(x) = \frac{k}{n}$ for $X_k \leq x \leq X_{k+1}$, $k = 1, 2, \cdots, n - 1$; $F_n(x) = 1$ for $X_n \leq x$. The function $F^+_{n,\epsilon}(x) = \min\,[F_n(x) + \epsilon, 1]$, also determined by the sample, is called an upper confidence contour. Let $P_n(\epsilon)$ = Prob. $\{F(x) < F^+_{n,\epsilon}(x)$ for all $x\}$. The authors derive an explicit expression for $P_n(\epsilon)$ and tabulate values of $\epsilon_{n,\alpha}$ for $\alpha = .10, .05, .01, .001$, and $n = 5, 8, 10, 20, 40, 50$ to at least 4D in Table 1. In Table 2 the authors present the values to 4D for the same values of n and α as in Table 1 of $\bar{\epsilon}_{n,\alpha} = (2n \ln \alpha)^{\frac{1}{2}}$ obtained from the asymptotic results of N. SMIRNOV.[1] For $n = 50$ the difference between $\epsilon_{n,\alpha}$ and $\bar{\epsilon}_{n,\alpha}$ is small.

L. A. AROIAN

Hughes Aircraft Company
Culver City, California

[1] N. SMIRNOV, "Sur les écarts de la courbe de distribution empirique," *Rec. Math.* [*Mat. Sbornik*], n.s., v. 6, 1939, p. 3–26.

1000[K].—E. G. CHAMBERS, *Statistical Calculation for Beginners*. Cambridge University Press, London and New York; second ed., 1952 (first ed., 1940). x + 168 p. 14.9 × 22.2 cm. 12 s. 6 d.

The title of this volume may be a bit deceptive, especially to readers of *MTAC*, since it is not concerned with computational methods as such, but is rather another introductory and non-mathematical book on statistical methods for "students of the biological sciences, especially psychology." The details of the calculation of elementary statistical quantities and standard significance tests (there are some for use with qualitative data which not all statisticians would regard as standard) are explained and illustrated and there are further exercises with answers for the reader to do. The student is not assumed to have access to a desk calculator. If this review were addressed to a statistical audience certain inaccuracies of statement should be mentioned though these are not numerous or really serious. Besides standard statistical tables, usually in a shortened form, there are some auxiliary tables for assistance in computation, namely of $[n_1n_2(n_1 + n_2 - 2)/(n_1 + n_2)]^{\frac{1}{2}}$ for n_1 and $n_2 = 10(1)50$ to 2D; $N(N^2 - 1)/6$ for $N = 10(1)69$; $n(n - 1)(2n + 5)$ for $n = 2(1)60$; $t(t - 1)(t - 2)$ for $t = 3(1)50$; $mn(m - 1)(n - 1)/4$ for $m = 2(1)6$ and $n = 3(1)15$; $\binom{n}{2}m(m - 1)/(m - 2)^2$ for $m = 3(1)6$ and $n = 3(1)15$; $\binom{n}{2}\binom{m}{2}(m - 3)/2(m - 2)$ for $m = 4(1)6$ and $n = 2(1)15$; and $m^2(n^3 - n)/12$ for $m = 3(1)6$ and $n = 3(1)15$.

C. C. C.

1001[K].—ARTHUR LINDER, *Statistische Methoden für Naturwissenschafter, Mediziner und Ingenieure*, Verlag Backhäuser, Basel. Second ed., 1951 (first ed., 1945). 238 p. 17.5 × 24.4 cm. 30 Swiss francs.

This book filled a need for a modern introductory book in mathematical statistics, including derivations, in German. Besides standard statistical tables it includes a table for the direct evaluation of the significance levels of regression coefficients. 5, 1, and .1 percentage points for 1(1)6 independent variables and for 1(1)30, 40, 60, 120 degrees of freedom are given to 4S.

C. C. C.

1002[K].—SIGEITI MORIGUTI, "Extremal properties of extreme value distributions," *Annals Math. Stat.*, v. 22, 1951, p. 523–536.

Schwarz's inequality is used to obtain an upper bound for the expectation, a lower bound for the coefficient of variation, and lower and upper bounds for the standard deviation of the extreme value taken from a symmetrical distribution with zero expectation and unit standard deviation. The bounds, given as functions of the sample size n, are actually reached for specific distributions depending upon n. The upper bound for the expectation has been already derived by PLACKETT,[1] by a more complicated method based on the calculus of variations.

Table 1 gives 4D and 5D values for the upper bound of the expectation of the extreme value, the (known) expectation of extreme values for the normal distribution and those for a rectangular distribution for $n = 2(1)20$. Table 2 gives to 4D and 5D the lower bound of the coefficient of variation of the extreme value, the (known) values for the normal and those for the rectangular distribution for $n = 2(1)6$. Seven graphs show the upper bounds for the expectation of the extreme value for $n = 20$, the lower bounds for the coefficient of variation for $n = 10$, the upper and lower bounds for the standard deviation of extreme values for $n = 10$ and the probability and density functions where the bounds are reached. The proofs are unusually simple and clear.

E. J. GUMBEL

New School for Social Research
New York, N. Y.

[1] R. L. PLACKETT, "Limits of the ratio of mean range to standard deviation," *Biometrika*, v. 34, 1947, p. 120–122.

1003[K, L].—P. NATH, "Confluent hypergeometric function," *Sankhyā*, v. 11, 1951, p. 153–166.

The present paper contains 7S tables of

$$M(\alpha, \gamma, x) = \sum_{r=0}^{\infty} \Gamma(\gamma)\Gamma(\alpha + r)x^r/[r!\Gamma(\alpha)\Gamma(\gamma + r)]$$

for $\gamma = 3$, $\alpha = 1(1)40$ and $\gamma = 4$, $\alpha = 1(1)50$; in both cases $x = .02(.02)$-$.1(.1)1(1)10(10)50, 100, 200$. The computation of the tables is described in the introduction and the author states that as he worked throughout with 10S and as the 40 times repeated use of the recurrence relation produced errors in at most the last two figures, the seven significant figures given in the table "can be taken to be all correct."

It is pointed out that in statistics special forms of $M(\alpha, \gamma, x)$ occur in the distribution functions of the F statistic and D^2 statistic and in the error function and the incomplete function.

I. R. SAVAGE

NBSSEL

1004[K].—PAUL R. RIDER, "The distribution of the quotient of ranges in samples from a rectangular population," Amer. Stat. Assoc., *Jn.*, v. 46, 1951, p. 502–507.

Two independent random samples of size m and n from a continuous rectangular population have ranges R_1 and R_2, respectively. The density function of the quotient, $u = R_1/R_2$ is given and upper percentage points for sample sizes not greater than 10 are computed. Tables 2, 3, and 4 give, respectively, the upper 10, 5, and 1 percent points of the distribution of u to 3S for m and $n = 2(1)10$.

LEO KATZ

Michigan State College
East Lansing, Mich.

1005[K].—W. L. STEVENS, "Asymptotic regression," *Biometrics*, v. 7, 1951, p. 247–267.

In fitting a regression curve of the form $y = \alpha + \beta\rho^x$ by least squares, estimates a, b and r are obtained of the three parameters. A covariance matrix can be written in terms of b and of six functions F_{aa}, F_{ab}, F_{ar}, F_{br}, F_{bb}, F_{rr} of r alone, namely, of the components of the inverse of

$$\begin{pmatrix} n & \Sigma r^x & \Sigma x r^{x-1} \\ \Sigma r^x & \Sigma r^{2x} & \Sigma x r^{2x-1} \\ \Sigma x r^{x-1} & \Sigma x r^{2x-1} & \Sigma x^2 r^{2x-2} \end{pmatrix}.$$

For the case where the x values are spaced equally over n values (coded as $0, 1, \cdots, n-1$) values of the functions are tabled to 5D: $n = 5, 6$, $r = .25(.01).65(.005).70$, and $n = 7$, $r = .30(.01).70(.005).75$.

The tables facilitate the numerical problem of approximating values of a, b, r which satisfy the least squares principle. Examples are given.

F. J. MASSEY

University of Oregon
Eugene, Ore.

1006[K].—MARJORIE THOMAS, "Some tests for randomness in plant populations," *Biometrika*, v. 38, 1951, p. 102–111.

Power functions are obtained for three tests designed to detect departure from randomness in the distribution of a plant species over an area. The area is divided into N quadrats, in each of which the probability, P_k, of observing k plants is

$$P_k = \sum_{r=1}^{k} \frac{e^{-m}m^r}{r!} \frac{e^{-r\lambda}(r\lambda)^{k-r}}{(k-r)!}, \qquad k > 0 \text{ and } P_0 = e^{-m},$$

on the assumption that plants are distributed in clusters, the number of clusters per quadrat having a Poisson distribution with mean m and the number of plants in excess of one per cluster having a Poisson distribution with mean λ. The hypothesis of randomness ($\lambda = 0$) is to be tested against alternatives, $\lambda > 0$.

The three tests compared are (i) STEVENS'[1] test based on n, the number of quadrats containing no plants, (ii) $z = \sum(x_i - \bar{x})^2/\bar{x}$, where x_i is the number of plants in the ith quadrat and (iii) $l = -\ln\left[\frac{n_1}{n_0 \ln (n_0/N)}\right]$, the maximum likelihood estimate of λ based on observed frequencies of quadrats with none and one plant. As might be expected, z is the preferred statistic.

Table 1 gives the power of Stevens' test for $\alpha = .03$ to 3D for $\lambda = 0(.2)1.4$ when $N = 25$ exactly for two artificial situations in which the total number of plants observed, S, is taken to be 5 and 10, respectively, and approximately for a more realistic situation where $M = m(1 + \lambda) = .4$. It should be noted that the approximate powers were obtained by experimental sampling, in each case involving 200 random samples of 25.

Table 2 gives the power of z for $\alpha = .05$ to 3D for $\lambda = 0(.1).6$; $N = 100, 200$; $M = 1, 2, 3$. Table 3 gives the power of l for $\alpha = .05$ to 3D for $\lambda = 0(.1)1.4$; $N = 100, 200, 300$; $M = 1, 2, 3$.

LEO KATZ

Michigan State College
East Lansing, Mich.

[1] W. L. STEVENS, "Significance of grouping," *Annals of Eugenics*, v. 8, 1937, p. 57–69.

1007[K].—H. R. VAN DER VAART, "Gebruiksaanwijzing voor de toets van Wilcoxon," Mathematisch Centrum, Statistische Afdeling, *Rapport* S 32 (M 4), 1950.

Let X and Y be independent random variables with continuous cumulative distribution functions $F(x)$ and $G(y)$. Let $x_1, \cdots, x_n$ be a set of n independent observations on X and $y_1, \cdots, y_m$ a set of m independent observations on Y. By u we denote the number of pairs (i, j) with $x_i > y_j$. The quantity u is used in WILCOXON's test[1] for the hypothesis H that X and Y are the same random variables ($F(x) \equiv G(y)$) against the alternative $\bar{H}$ that $F(x) > G(x)$ for all x (Y is stochastically larger than X). Whenever $u \leqq u_0$ the hypothesis H is rejected, u_0 being chosen such that under H the probability $P(u \leqq u_0)$ is about equal to the desired level of significance α. This probability depends only on u_0, n, and m; it will be denoted here by $H(u_0, n, m)$. It satisfies $H(u, m, n) = H(u, n, m)$, $H(mn - u, m, n) = 1 - H(u, m, n)$ and $H(u, m, n) = 0$ for $u < 0$. Hence in tabulating this function only integral values of m, n, and u need to be considered with $1 \leq m \leq n$ and $0 \leq u \leq \frac{mn}{2}$. Complete tables of $P(u \leqq u_0) = H(u_0, m, n)$ have been given by MANN & WHITNEY[2] for $m, n = 1(1)8$. The present report gives complete tables for $m, n = 1(1)10$ to 7D.

Wilcoxon's test is important in comparing different treatments (with "effects" X and Y) when, except for continuity, nothing is known about $F(x)$ and $G(y)$. The research worker without any statistical background can find in this report a very detailed explanation of this test and the relatively easy computations involved. The author also points out how to obtain

$P(u \leqq u_0)$ whenever m and n are large, using the normal distribution as an approximation.

J. H. B. KEMPERMAN

Purdue University
Lafayette, Ind.

[1] FRANK WILCOXON, "Individual comparisons by ranking methods," *Biometrics Bull.*, v. 1, 1945, p. 80–83.
[2] H. B. MANN & D. R. WHITNEY, "On a test of whether one of two random variables is stochastically larger than the other," *Ann. Math. Stat.*, v. 18, 1947, p. 50–60.

1008[K].—J. E. WALSH, "Some bounded significance level properties of the equal-tail sign test," *Annals Math. Stat.*, v. 22, 1951, p. 408–417.

The tables exhibit coefficients for confidence intervals based upon a simple equal-tail procedure (essentially a sign test) for testing hypotheses about the population median. Significance levels are determined by confidence coefficients which depend upon the population conditions.

Two tests are used. Test 1, the simpler, assumes two conditions on the populations: (i) the populations have a common median value, (ii) no population has a discrete amount of probability concentrated at the median. Table 1 shows significance levels for confidence intervals for samples of n assuming conditions (i) and (ii) for $n = 4(1)15$ to 4D.

When conditions (i) and (ii) are not satisfied, the null hypothesis is stated in terms of some function of a set of median values contained in the set of 50% points common to all of the populations. Test 2 applies here and this reduces to Test 1 when the set becomes a single value, thus satisfying condition (i). Test 2 is not exact but rather gives upper and lower bounds to the significance level. These are also given in Table 1. If the populations are assumed to be continuous at the median, the lower bounds of Test 2 are considerably improved. These lower bounds are given in Table 2 for $n = 4(1)15$ to 4D.

T. A. BICKERSTAFF

University of Mississippi
University, Miss.

1009[L].—V. A. DITKIN & P. I. KUZNETŜOV, *Spravochnik po operatŝionnomu Ischislenii͡u Osnovy Teorii i Tablitŝy Formul.* [Handbook of operational calculus. Fundamentals of the theory and tables of formulas.] Moscow and Leningrad, Gostekhizdat, 1951, 256 p. 14.6 × 22.2 cm. Price 7.30 roubles, bound. On pages 98–105 are definitions of higher functions.

In the Heaviside operational calculus we can use the Laplace transform. The transform of a function $f(t)$ is

$$F(p) = p \int_0^\infty f(t)e^{-pt}\,dt,$$

provided $f(t)$ is of exponential order at infinity, and then has the inversion formula

$$f(t) = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} p^{-1}F(p)e^{pt}\,dp.$$

On pages 106–111 are general formulae listing $F(p)$ and corresponding $f(t)$. Then on pages 112–255 are tables of transforms $F(p)$, $f(t)$ arranged under the following 15 function headings: Rational; Irrational; Exponential; Trigonometric, and hyperbolic; Logarithmic, inverse hyperbolic, and inverse trigonometric; Gamma and related; Integral; Confluent hypergeometric; Bessel; Legendre; Elliptic; Theta; Mathieu; Hypergeometric series; various. In all there are 1354 transform pairs.

There is no reference to the elaborate similar lists in (a) J. COSSAR & A. ERDÉLYI, *Dictionary of Laplace Transforms* (1944–1946; see *MTAC*, v. 1, p. 424–425; v. 2, p. 76, 215–216), or in (b) N. W. MCLACHLAN, P. HUMBERT & L. POLI, *Supplément au formulaire pour le calcul symbolique* (1950) although the Second edition of the original *Formulaire* is mentioned.

This work was published in an edition of 10,000 copies.

R. C. ARCHIBALD

Brown University
Providence, R. I.

1010[L].—OTTO EMERSLEBEN, "Numerische Werte des Fehlerintegrals für $\sqrt{n\pi}$," *Zeit. angew. Math. Mech.*, v. 31, 1951, p. 393–394.

$F(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. On p. 393 is a table of $F(\sqrt{n\pi})$, for $n = [1(1)11; 15D]$, for $n = 1$, 17D.

R. C. ARCHIBALD

Brown University
Providence, R. I.

1011[L].—G. B. HAGEN, "Über iterierte Integration von Bessel-Funktionen," *Zeit. angew. Math. Mech.*, v. 32, 1952, p. 27–30.

On p. 29 is a table of $\int_0^x J_0(t)\,dt$, $x = [0(.2)15(1)24, \infty; 4D]$.

This integral has been tabulated to 8D by A. N. LOWAN & M. ABRAMOWITZ for $x = 0(.01)10$ [see *MTAC*, v. 1, p. 154]. From this table it may be verified that in Hagen's table there are 13 fourth-place unit errors for $x = .8$, 1.8, 2.4, 3.6, 4.2, 4.4, 5.2, 5.6, 6.4, 6.6, 7.4, 7.6, 8.6, 8.8; fourth-place 2-unit errors for $x = 4.0$, 4.8, 5.4; fourth-place 3-unit errors for $x = 9.2$, 9.8; fourth-place 60-unit error at $x = 3.2$.

R. C. ARCHIBALD

Brown University
Providence, R. I.

1012[L].—J. M. HAMMERSLEY, "On a certain type of integral associated with circular cylinders," Roy. Soc. London, *Proc.*, v. 210A, 1951, p. 98–110.

Table 1, p. 104. 8D table of

$$k(c) = \frac{16c^2}{3} \{F(-\tfrac{3}{2}, \tfrac{1}{2}; 2; c^{-2}) - 1 + \tfrac{3}{8}c^{-2} - \tfrac{3}{64}c^{-4}\}$$

for $c = 1(1)10$.

Table 2, p. 106, 107. 7D table of

$$g(\xi) = 16\xi^2\{1 - \xi F(-\tfrac{1}{2}, \tfrac{1}{2}; 2; \xi^2)\} = 16\xi^2\{1 - F(-\tfrac{1}{2}, \tfrac{1}{2}; 2; \xi^{-2})\},$$

and

$$h(\xi) = -2\xi + 4\pi^{-1}\xi\{(1 - 4\xi^2)\cos^{-1}\xi + \xi(2\xi^2 + 1)(1 - \xi^2)^{\frac{1}{2}}\}$$

for $\xi = 0(.01)1$, with second central differences (some modified), and in the case of $g(\xi)$ also a few fourth differences.

Table 3, p. 108, 109. 7D table of $g(\xi)$ for $\xi = 1(.01)2(.05)3(.25)10$, with second central differences (some modified) and a few fourth differences.

$k(c)$ has been computed from its expansion in descending powers of c, and $g(\xi)$ from its expression in terms of complete elliptic integrals. The elliptic integrals were partly taken from existing tables,[1] and partly they were computed by the author.

A. E.

[1] A. Fletcher, "A table of elliptic integrals," *Phil. Mag.*, s.7., v. 30, 1940, p. 516–519.

1013[L].—L. Howarth, "The boundary layer in three-dimensional flow," *Phil. Mag.*, s. 7, v. 42, 1951, p. 239–243, 1433–1440.

Part I of this work considers the problem of calculating the boundary layer for a fluid of small viscosity (large Reynolds number) flowing over a surface of quite general curvature. In Part II the work is extended to include the boundary layer near the stagnation point of a three-dimensional body where the stagnation point is a regular point of the body.

Because it is a regular point of the body, a pair of coordinate directions x, y may be found such that, in the immediate neighborhood of the stagnation point, the flow at each axis is only in the direction of that axis; that is, x, y are principal axes. The free stream velocity components U, V in the directions x, y may then be given

$$U = ax; \qquad V = by \qquad (a, b \text{ constants})$$

so that the flow is characterized by the ratio $c = b/a$. Let the corresponding velocity distributions in the boundary layer be

$$u = axf'(z); \qquad v = byg'(z),$$

where

$$z = \xi(a/\nu)^{\frac{1}{2}},$$

ν being the coefficient of kinematic viscosity and ξ the distance normal from the surface of the body. Howarth shows that

$$(f')^2 - ff'' - cgf'' = 1 + f'''$$
$$(g')^2 - gg'' - \frac{1}{c}fg'' = 1 + \frac{1}{c}g'''$$

with boundary conditions

$$f(0) = f'(0) = g(0) = g'(0) = 0$$
$$f' = g' \rightarrow 1 \qquad \text{as} \qquad z \rightarrow \infty.$$

Expanding in powers of c,

$$f = f_0 + cf_1 + c^2f_2,$$
$$g = g_0 + cg_1 + c^2g_2.$$

Howarth solves successively for $f_0'(z)$, $g_0'(z)$, $f_1'(z)$, $g_1'(z)$, $f_2'(z)$, $g_2'(z)$ by numerical methods. Table I gives these results to 3 decimals for $0.0 \leq z \leq 4.0$ at intervals of 0.1. The values of $f'(z)$ and $g'(z)$ are then calculated to 3 decimals for $c = 0.25, 0.50, 0.75, 1.00$ and given in Table II for $0.0 \leq z \leq 3.0$ for intervals of 0.1. The values of $f''(0)$ and $g''(0)$ are required for calculating skin friction and are given in Table III to 3 decimals for $c = 0, 0.25, 0.50, 0.75, 1.00$. Finally in Table IV are given values to 3 decimals of $f(z)$ and $g(z)$ for $0.0 \leq z \leq 4.0$ at intervals of 0.5 in z.

F. E. MARBLE

California Institute of Technology
Pasadena

1014[L].—DAVID MIDDLETON & VIRGINIA JOHNSON, "A tabulation of selected confluent hypergeometric functions," Cruft Lab. *Tech. Report* no. 140, Jan. 5, 1952.

The tables give

$$ {}_1F_1(a; c; -p) = \sum_{n=0}^{\infty} \Gamma(a + n)\Gamma(c)(-p)^n / \{\Gamma(a)\Gamma(c + n)n!\}, $$

generally to 5S, for $p = 0(.25)2(.5)10(10)100$ and $c = 1, 2$, $a = -.5(1)8.5$; $c = 3, 4, 5$, $a = -.5(1)9.5$; $c = 6, 7$, $a = -.5(1)10.5$; $c = 8, 9$, $a = -.5(1)11.5$; $c = 10$, $a = -.5(1)12.5$. A brief account of some properties of the confluent hypergeometric function and of some of its applications in noise problems is included as is a description of the computation. The bibliography of tables is hardly adequate.

A. E.

1015[L].—E. O. POWELL, "A table of the generalized Riemann zeta function in a particular case," *Quart. Jn. Mech. Appl. Math.*, v. 5, 1952, p. 116–123.

Since the Hurwitz zeta function $\zeta(s, a)$ satisfies

$$ \zeta(s, a) - \zeta(s, a + 1) = a^{-s}, $$

a study of the difference equation $f(a) - f(a + 1) = a^{-\frac{1}{2}}$ led the author to a study of $\zeta(\frac{1}{2}, a)$. By means of the equation

$$ (1) \qquad \zeta(\tfrac{1}{2}, 1 + a) = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} \zeta(n + \tfrac{1}{2})(-a)^n, \qquad |a| < 1, $$

the calculation, at least for small a, is reduced to Riemann zeta functions which have been tabulated by GRAM.[1]

The multiplication theorem

$$ \zeta(\tfrac{1}{2}, Ma) = M^{-\frac{1}{2}} \sum_{r=0}^{M-1} \zeta\left(\tfrac{1}{2}, a + \frac{r}{M}\right) $$

then permits calculation for larger values of a, and the difference equation serves as a check. A ten-decimal table of $\zeta(\frac{1}{2}, a)$ is thus obtained for $a = 1.00(.01)2.00(.02)5.00(.05)10.00$ along with modified second central differences.

The author also refers to an unpublished five-decimal table by R. HENS-

MAN[2] which gives $\zeta(s, a)$ for $s = -10(.1)0$, $a = 0(.1)2$, and $(s - 1)\zeta(s, a)$ for $s = 0(.1)1$, $a = 0(.1)2$, obtained from a modification of equation (1).

T. M. APOSTOL

California Institute of Technology
Pasadena 4, California

[1] J. P. GRAM, "Tafeln für die Riemannsche Zetafunktion," K. Danske Vidensk. Selskab, *Skrifter*, s. 8, v. 10, 1925, p. 311–325.
[2] R. HENSMAN, *Tables of the generalized Riemann zeta function*, Report No. T 211, Telecommunications Research Establishment, Ministry of Supply, Great Malvern, Worc. (1948).

1016[L].—L. WEINBERG, "Solutions of some partial differential equations (with tables)," Franklin Institute, *Jn.*, v. 252, 1951, p. 43–62.

The author considers the following differential equations:

a) Laplace's equation $\nabla^2\phi = 0$
b) Poisson's equation $\nabla^2\phi = -K$
c) Diffusion equation $\nabla^2\phi = h^{-2}\partial\phi/\partial t$
d) Wave equation $\nabla^2\phi = c^{-2}\partial^2\phi/\partial t^2$
e) Helmholtz's equation $\nabla^2\Phi + \gamma^2\Phi = 0$, $\gamma^2 =$ constant.

It is pointed out that a)–d) are special cases of Helmholtz's equation. This is solved by the well known method of the separation variables. Φ is represented as a product of three functions U^1, U^2, U^3, where each of those functions depends only on one variable. In the tables (p. 50–62), the expressions for $\nabla\Phi$, $\nabla^2\Phi$, div E, curl E and the three product functions U^1, U^2, U^3, are given (the latter also for the case of Laplace's equation, $\gamma = 0$) for the following system of coordinates: rectangular, circular cylinder, elliptic cylinder, parabolic cylinder, spherical, prolate spheroidal, oblate spheroidal, and parabolic.

F. OBERHETTINGER

American Univ.
Washington, D. C.

1017[L].—J. E. WILKINS, JR., *Some Miscellaneous Mathematical Problems.* U. S. Atomic Energy Commission, Tech. Info. Service NYO–641, Oak Ridge, Tenn., Dec. 19, 1951, 21 mimeographed pages.

The table of Laguerre polynomials described in UMT **135** [*MTAC*, v. 5, p. 232] is reproduced here on p. 15–21.

1018[Q].—ROBERT J. DAVIS, *Table of the Secant of the Zenith Distance for the Latitude of Oak Ridge, Massachusetts* (42°.5 N). Harvard Observatory Mimeograms, Series IV, No. 1, Cambridge, Mass., 1951. vi, 35 p.

This table is for astronomical use, in correcting photoelectric measurements of stellar magnitude for atmospheric extinction. It gives to 3D

$$\sec z = (\sin\phi \sin\delta + \cos\phi \cos\delta \cos t)^{-1}$$

for declination $\delta = -28°(1°)90°$, hour angle $t = 0^h(4^m)12^h$ and latitude $\phi = +42°.5$. Auxiliary tables allow extension to any latitude $+40°$ to $+45°$, and supply corrections arising from curvature of the earth's atmosphere.

JOSEPH ASHBROOK

Yale University
New Haven, Conn.