

Number of primes  $p \equiv 1 \pmod{3}$  such that

Group	$x_p$ is the largest root	$x_p$ is the middle root	$x_p$ is the smallest root
I	54	28	18
II	41	38	21
III	46	33	21
IV	39	32	29
V	43	29	28
VI	44	38	18
VII	5	3	3
Total	272	201	138
Density	.4452	.3290	.2258

These results would seem to indicate a significant departure from the conjectured densities and a trend toward randomness.

The method of calculation was this: Each root of (2) lies in one of the intervals  $(-2p^{\frac{1}{3}}, -p^{\frac{1}{3}})$ ,  $(-p^{\frac{1}{3}}, +p^{\frac{1}{3}})$ ,  $(p^{\frac{1}{3}}, 2p^{\frac{1}{3}})$  as may be seen directly from the form of (2) with the help of (3). For each relevant  $p$  the expression (1) for  $x_p$  was evaluated, its sign was determined and its square compared to  $p$ . This determined in which of the three intervals just described the  $x_p$  lies. To check that  $x_p$  was indeed a solution of (2), (3) and to determine the precision of the evaluation the expression

$$(4) \quad f(x_p)/f'(x_p)$$

was then calculated. This latter check was performed by first transforming the  $x_p$  into decimal form for tabulation and then retransforming these results back into binary form before evaluating the expression (4). In this manner both the calculation proper and the conversion to decimal form of the results were checked.

The trigonometric expressions appearing in (1) were evaluated by power series. Each angle was reduced mod  $2\pi$  and then mod  $\pi$  until it lay between  $-\pi/2$  and  $+\pi/2$ . Then the cosine of  $\frac{1}{4}$  of this angle was calculated keeping five terms in the series expansion. The "double-angle" formula for cosines was then used twice to obtain the desired cosine.

The calculation involved about 15 million multiplications counting the checking mentioned above. The values of  $p$  were introduced in blocks of 200. The entire calculation was carried out twice to ensure reliability. The authors are indebted to Mrs. ATLE SELBERG who programmed and coded the calculation.

J. VON NEUMANN  
H. H. GOLDSTINE

Institute for Advanced Study  
Princeton, New Jersey

<sup>1</sup> E. E. Kummer, "De residuis cubicis disquisitiones nonnullae analyticae," *Jn. f. d. reine u. angew. Math.*, v. 32, 1846, p. 341-365.

### CORRIGENDA

v. 6, p. 262, *insert* Emch, G. F. 247.

v. 6, p. 265, under Myers *insert* 54.

v. 6, p. 268, under Yowell *insert* 254.

v. 7, p. 31, l. 11 of MTE 218; *for*  $-\log p$  *read*  $\log p$ .