UNIVAC input-output system:	
The Uniservo-tape reader and recorder	H. F. WELSH and H. LUKOFF, Eckert-Mauchly Div., Remington Rand, Inc.
Input devices	L. D. WILSON and E. ROGGENSTEIN, Eckert-Mauchly Div., Remington Rand, Inc.
Output devices	E. MASTERSON and L. D. WILSON, Eckert-Mauchly Div., Remington Rand, Inc.
Afternoon session	J. H. HOWARD, *Chairman*, Burroughs Adding Machine Co.
RAYDAC input-output system:	
RAYDAC input-output systems	W. GRAY and K. RAHLER, Raytheon
System design	L. FERN, Raytheon
Operating experience with RAYDAC	F. DEAN, Raytheon
IBM 701 input-output system:	
Type 701 input-output organization	L. D. STEVENS, IBM
Type 726 magnetic tape reader and recorder	W. S. BUSLIK, IBM
Magnetic tape techniques and performance	H. W. NORDYKE, IBM
Friday, December 12, 1952	
Morning session	J. C. MCPHERSON, *Chairman*, IBM
Survey of analog-to-digital data converters	H. E. BURKE, Consolidated Engineering Corp.
Survey of mechanical type printers	J. HOSKEN, A. D. Little, Inc.
Survey of nonmechanical type printers	Lt. R. J. ROSSHEIM, GWU
Anelex printer	L. ROSEN, Anderson-Nichols Co.
Eastman printer	R. G. THOMPSON, Eastman Kodak Co.
Afternoon session	J. G. BRAINERD, *Chairman*, Moore School of Electrical Engineering, Univ. of Penn.
Ferranti input-output equipment	B. W. POLLARD, Ferranti Ltd., London
Garment tag equipment	O. G. HESSLER, Sears Roebuck and Co.
Numerically controlled machine tool	W. PEASE, MIT
Summary and forecast	S. N. ALEXANDER, NBS
Informal discussion	

OTHER AIDS TO COMPUTATION

BIBLIOGRAPHY Z

1025. LEONILDA ALTMAN, "An analog computer as an aid to shower theory calculations," *Rev. Sci. Instruments*, v. 23, 1952, p. 382.

A. c. resolvers are used to solve certain trigonometrical equations.

F. J. M.

1026. S. BOSWORTH, "A new analog computer," *Electronics*, v. 24, no. 8, Aug. 1951, p. 216–224.

This article describes "an inexpensive desk-size electronic differential analyzer" suitable for differential equations with constant coefficients. The device contains 20 d.c. amplifiers, 23 ten-turn potentiometers, and 8 integrating capacitors. The problem is set up on terminal boards which are plugged into the device.

F. J. M.

1027. F. E. Brooks, Jr. & H. W. Smith, "A computer for correlation functions," *Rev. Sci. Instruments*, v. 23, 1952, p. 121–126.

A continuous computer is described for calculating $\int_0^{t_1}(F(t)G(t+\tau) - \bar{F}G(t+\tau))dt$ for a given value of τ, where $F(t)$ and $G(t)$ are given arbitrary functions of t and $\bar{F}$ is the average value of F on the given interval of integration. The correlation function is computed from a number of such expressions. $F(t)$ and $G(t)$ are recorded on magnetic tapes in the form of pulse length modulation of a rectangular wave with a repetition rate of five cycles per second. The mathematical operation required is performed by a multiplier integrator. The integration is performed by a condenser which is fed by a multiplying unit whose output comes through a gate, which is controlled to be off or on by the pulses for one factor. The input to this gate is the result of demodulating the signal for the other factor and obtaining an amplitude modulated signal. The variable τ is introduced by varying the relative position of the two tapes. It is planned to use this in the study of the fading of radio signals due to various factors and other atmospheric studies. An example involving thermistors is given.

F. J. M.

1028. F. B. Daniels, "A planimetric method of harmonic analysis," *Rev. Sci. Instruments*, v. 23, 1952, p. 369–370.

This article gives a formula in terms of the coefficients of the Fourier series of $f(x)$ for an integral which is readily evaluated by a planimeter. Let $\varphi_k(x) = (-1)^j$ if $j/2k\pi \leq x < (j+1)/2k\pi$. The integral obtained by the planimeter is $\int_0^{2\pi}\varphi_k f dx$. This can be considered as an approximation to a_k, the coefficient of cos kx in the Fourier series for $f(x)$. A phase shift yields an approximation to b_k, the coefficient of sin kx.

F. J. M.

1029. E. A. Goldberg, "Step multiplier in guided missile computer," *Electronics*, v. 24, no. 8, August 1951, p. 120–124.

A set of relays can be used to obtain a conductance whose value is determined by a binary coded number $.\alpha_1\alpha_2 \cdots \alpha_n$. Thus if $\alpha_j = 1$, the j'th relay contact will introduce a conductance with value 2^{-j} units. Now suppose the state of the j'th relay is determined by the $(n-j)$'th stage of a binary counter. Apply a fixed voltage to this conductance and compare the resulting current with another current whose value corresponds to a mathematical variable x. The difference controls gates across the input of the binary counter, so that if there is a significant discrepancy between these currents, an appropriate gate is opened and counting pulses reach the counter causing it to count up or down until the value of the conductance corresponds to the value of the variable x. The same binary counter can be used to control any number of such relay conductances and thus any number of conductances with the value of a specified mathematical variable x can be produced. If now, voltages corresponding to other variables, $y_1, \cdots, y_r$, are applied to these conductances, currents with values $xy_1, \cdots, xy_r$ may be obtained.

The actual conductances are T networks and 11 stages are used. The counting pulses are at a rate of 1 kilocycle and the relay conductances involve a specially developed relay with a response time of approximately 100 microseconds. The circuit for the binary counter is also discussed.

These multipliers were developed as part of "Project Typhoon" for the U. S. Navy. They are used in the Typhoon guided missile simulator where multipliers are needed with a precision higher than that available from servo multipliers. The article also discusses various features of this computer, which is essentially a large electronic differential analyzer using 445 stabilized d.c. amplifiers. These can be used for either integration or summation. There are 19 computing servo units and 36 multiplications can be made by the above described stepmultipliers (involving the 9 distinct x factors).

F. J. M.

1030. A. E. Hastings & J. E. Meade, "A device for computing correlation functions," *Rev. Sci. Instruments*, v. 23, 1952, p. 347–349.

This device produces the integral $\int_{-T}^{T} f(t)f(t+\tau)dt$. The function $f(t)$ is recorded on a magnetic tape by amplitude modulation and read by two heads whose separation corresponds to τ. One head signal is used to produce a current, the other a voltage and the result is obtained from a wattmeter.

F. J. M.

1031. Institute of Gas Technology, "Gas distribution system calculator," *Rev. Sci. Instruments*, v. 23, 1952, p. 61.

A photograph of a special purpose network analog computer and a short description are given.

F. J. M.

1032. R. D. Mindlin & M. G. Salvadori, "Analogies," *Handbook of Experimental Stress Analysis*. John Wiley & Son, New York, 1950, Chapter 16, p. 700–827.

When the behavior of two physical systems can be described in identical mathematical form, they are said to be analogous. If experiments and measurements can be made with greater ease and accuracy on one of the systems, then it may be used as a computing device for the solution of complex problems arising in the more intractable system. In this article, the authors present a comprehensive and detailed survey of those analogies between elasticity and other physical phenomena which have been used in the quantitative determination of states of stress in elastic bodies. Each analogy is treated carefully, deriving first the mathematical equations governing both the original physical system and the analogous system. The corresponding quantities in the two systems and the required scale factors are then readily recognized. Methods of physically realizing and measuring quantities in the analogous system are then described, including a discussion of the overall accuracy obtainable. An excellent bibliography containing sixty-seven entries is appended.

The earliest use of analogies in elasticity theory was apparently in the determination of stresses in a cylindrical bar bent by a terminal load or twisted by terminal couples. At a sufficient distance from the end of the bar, the problem may be reduced to the solution of Laplace's or Poisson's equation with suitable boundary conditions. Therefore, any of the other physical phenomena satisfying these equations may be used as analogies. PRANDTL, in 1903, suggested the use of a homogeneous membrane, under uniform tension per unit of length and subjected to a normal pressure per unit of surface area, the deflection of which satisfies Poisson's equation, provided the slope of the membrane relative to a plane is everywhere small. This analogy has been extensively employed and a full description is given of methods of its physical realization, of fixing its boundaries so that the boundary conditions are analogous to those of the original problem and methods of measuring deflections of the membrane accurately. Soap films have been used primarily, although the use of rubber and the separation surface of two immiscible liquids are described. The steady-state electrical potential in a thin plate of constant thickness, which satisfies Laplace's equation, has also been utilized.

The use in analogies of continuous physical systems such as the membrane or thin conducting plate, as opposed to lumped parameter systems, has the advantage that the analogous system satisfies the same system of equations as the original system, whereas the lumped parameter system will satisfy only a finite difference approximation to those equations. However, the use of continuous systems requires usually the construction of new apparatus (molds, etc.) for the solution of each problem and delicate experimental technique for accurate measurements. For these reasons, it appears to the reviewer, that a lumped D. C. network would represent a more flexible analogy to the torsion problem than the membranes or conducting plates heretofore used.

Analogies for the solution of many other problems in elasticity are discussed. Among them are electrical analogies for the torsion of bars of varying circular section and analogies between problems in elasticity which are difficult to study experimentally and other elasticity problems which are more susceptible to direct measurement. Thus an analogy exists between the stresses produced by a two dimensional steady-state temperature distribution and the stresses produced by dislocations, and the latter problem is easier to treat experimentally.

Considerable space is devoted to a lucid exposition of the A. C. electric network analogue devised by G. KRON for the general case of static plane stress. A lumped parameter system, utilizing inductances and capacitances, it satisfies a finite difference approximation to the equations governing the continuous system. Its great generality is very attractive. However, the analogy requires that the inductances and capacitances have no resistive losses. In an actual realization of the analogue described in this article, resistive losses resulted in large errors.

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1033. REEVES INSTRUMENT CORPORATION. *Project Cyclone Symposium II on Simulation and Computing Techniques*. New York City, April 28–May 2, 1952, 237 p.

The conference reviewed below was the second symposium on REAC techniques sponsored by the Reeves Instrument Corp. The first symposium was held in 1951 [*MTAC*, v. 5, p. 250–253]. Project Cyclone at Reeves is sponsored by the U. S. Navy Bureau of Aeronautics and the U. S. Navy Special Devices Center. The functions of Project Cyclone are: "(1) the development and test operation of a guided missile simulator . . . ; (2) the operation of a computer laboratory where classified problems in other fields may be studied and analyzed; (3) the investigation of new applications for Cyclone equipment; (4) theoretical research to ascertain and improve the reliability and accuracy of computer solutions; and (5) the coordination of the experiences of individual computer groups . . . ."

The 24 papers presented in this publication can be roughly classified into four types. (a) Papers dealing with circuitry, techniques and auxiliary equipment to improve the performance of REAC equipment; (b) Papers dealing with the application of REAC equipment to various physical problems encountered by the various users of the equipment; (c) Papers dealing with computers other than REAC; (d) Papers dealing with basic theory and new equipment.

(*a*) *Circuitry*

Paper No. 10, "Some REAC techniques employed at the David Taylor Model Basin" by L. Pode discusses techniques for generating input functions and the use of the REAC as an harmonic analyzer in problems that arose at the DTMB. Paper No. 11, "Simulation studies of a relay servomechanism" by N. P. Tomlinson considers methods for the analog study of a non-linear relay (that is, sampling) servomechanism. Paper No. 13, "Precision in high-speed electronic differential analyzers" by H. Bell, Jr. & V. C. Rideout is concerned with solving linear differential equations with variable coefficients and non-linear equations by analog computers and a comparison of the accuracy of solutions compared with digital solution. Paper No. 15, "Solution of linear differential equations with time-varying coefficients by the electronic differential analyzer" by C. E. Howe, R. M. Howe & L. L. Rauch discusses techniques for solving linear differential equations on the REAC. Examples include Bessel's equation, Legendre's equation and the deflection equation $[EI(x)y'']'' - \mu(x)\lambda^2 y = 0$ of simple beam theory. Paper No. 19 is entitled "Automatic REAC operation for statistical studies" by R. R. Bennett & A. S. Fulton. Determination of the response of a non-linear system to statistical inputs can most conveniently be carried out experimentally, that is, on an analog computer. Methods for changing a Gaussian distribution by passing the noise through a function generator are discussed. Paper No. 21, "Checking analogue computer solutions" by W. F. Richmond, Jr. & B. D. Loveman considers procedures to verify analog computer solutions of linear and non-linear differential equations. Linear equations are treated by frequency (steady state response) or selected root method (transient response). For non-linear systems "dynamic substitution" is used. It consists in breaking-up the time interval into small pieces and substituting the solution on each small sub-interval into the original differential equation. Paper No. 24, "Modifications and additions to the REAC" by J. W. Follin, Jr., G. F. Emch & F. M. Walters discusses circuitry details and auxiliary equipment for improving the operation of the REAC. These include a low frequency noise generator, a programmer for turning the REAC and recorder on and off, air conditioning the racks, power supply changes, circuit changes in amplifiers and techniques of interconnection of machines. Paper No. 25, "REAC servo response" by A. H. MILLER discusses methods for introducing variable coefficients and non-linearities into REAC equipment by the use of servos. A study of the dynamic characteristics of servos for use in the above connection is made. Paper No. 26 is "Applications of differential relays to solution of REAC problems" by L. M. WARSHAWSKY & W. BRAUN. A differential relay is a single-pole double-throw relay at the output of an operational amplifier. Its uses include providing a time-controlled series of switching operations and a simulation of "dead" zone where the operating limits are variable. Paper No. 27, "Techniques applied to the design and test of linear filters" by G. NESTOR is concerned with problems met in building and testing a Wiener filter. Paper No. 28, "The role of diodes in an electronic differential analyzer" by C. D. MORRILL & R. V. BAUM discusses the use of diodes to simulate certain types of non-linearities.

(*b*) *Applications*

Paper No. 7: "REAC solution of problems in structural dynamics" by C. W. BRENNER. In aircraft design one can no longer consider the air-frame as a rigid body. Many components designed only for maximum static loads fail in service due to transient loading. This paper considers a simplified associated problem of a simply-supported beam struck by a pressure wave (a) Normal to the beam, (b) Parallel to the beam. For case (a) there arises the differential equation $\phi_r'' + k^2r^4\phi_r = e^{-t/t_1}(1 - t/t_1)$, $r = 1, 2, \cdots$, where ϕ_r is the normalized displacement of the rth mode of vibration and k is a parameter. A similar equation holds for case (b). These equations are solved on the REAC. A discussion of the circuitry details and the troubles that were encountered together with their solutions, is presented.

Paper No. 8: "The use of an analog computer and feedback theory for the solution of structural problems in the static case" by G. J. MARTIN & L. M. LEGATSKI. Structural frameworks (pinned frames, statically indeterminate structures) give rise to certain systems of linear algebraic equations of the form $\sum_{j=1}^{n} a_{ij}\theta_j = M_i$, $i = 1, \cdots, m$, where the θ's are angles representing the rotations of various members of the structures and the M's are moments. These equations bear great similarity to usual mesh equations in electrical network theory and are solved on the REAC using a network analog.

Paper No. 9: "Application of the electronic differential analyzer to eigenvalue problems" by G. M. CORCOS, R. M. HOWE, L. L. RAUCH & J. R. SELLARS. Determination of eigenvalues for Sturm-Liouville systems, fourth order equations and partial differential equations. The partial differential equations are reduced to ordinary differential equations by separation of variables. Since, in general, there are two-point boundary conditions, the method is the familiar technique of satisfying the initial conditions at one point and varying the solution until the boundary conditions at the second point are satisfied. The authors' experience leads them to conclude that one or two end conditions are easy to handle, three end conditions give considerable trouble and four or more end conditions are apt to be hopeless. Four examples are considered.

Paper No. 12: "Use of the REAC as a curve fitting device" by C. H. MURPHY. In certain problems in aerodynamics it is necessary to determine the coefficients of a differential equation from discrete points on its solution curve. The use of the REAC in this connection is described, based on certain linearized differential equations of yawing motion.

Paper No. 14: "An application of analogue computers to problems of statistical analysis" by H. H. LANING, JR. and R. H. BATTIN. The response $x(t)$ of a linear system to a forcing function $f(t)$ is given by $x(t) = \int_{-\infty}^{t} W(t, \tau) f(\tau) d\tau$ where $W(t, \tau)$ is the impulsive response (Green's function, weighting function) of the system. A method for computing this quantity is discussed using the concept of the adjoint system.

Paper No. 23: "Solution of partial integral-differential equations of electron dynamics using analogue computers with storage devices" by C. C. WANG. In certain problems of electron dynamics there arise simultaneous partial integral differential equations involving two independent variables. How to adapt such problems to REAC analog computers is discussed.

(*c*) *Non-REAC computers*

Paper No. 16: "JAINCOMP computers and their application to simulation problems" by D. H. JACOBS. Description of the uses of the JAcobs-INstrumentCOMPany digital computer. It is a compact ($17 \times 21 \times 30$ inches), light (110 lbs.) electronic digital computer. The operations of the machine are very rapid (adds two 40 binary digits in eight microsec). Paper No. 17: "The decimal digital differential analyzer CRC-105 as a tool for simulation and checking analogue computer solutions" by E. WEISS. Description of the DDDA and its advantages and disadvantages as compared with electronic analogue computers and mechanical computers. It operates on a decimal number system, has 60 integrators and a maximum accuracy of six digits plus sign in each integrator. There are twelve input and twelve output channels.

Paper No. 18: "Problems encountered in the operation of the M. I. T. flight simulator" by W. W. SEIFERT & H. JACOBS, JR. The title accurately describes the paper. The simulator is located at the DACL (Dynamic Analysis and Control Laboratory) at M. I. T. Paper No. 22: "Analogue computation of blade designs" by D. B. BREEDON, M. M. MATTHEWS & E. L. HARDER. Calculation of natural frequencies and stresses in vibrating beams by the Westinghouse analog computer (Anacom) and checks made on a digital machine (IBM Calculating Punch, Type 602-A).

(*d*) *Theory*

Paper No. 20: "Mathematical error analysis for continuous computers" by F. J. MURRAY. This paper presents certain aspects of the work done by the author and the reviewer on the mathematical foundations of an error theory applicable to mathematical machines. The complete results will appear elsewhere.

Paper No. 29: "A high accuracy time division multiplier" by E. A. GOLDBERG. If a train of rectangular pulses is generated, then the DC component is proportional to the amplitude of the pulses multiplied by the duty cycle. This principle is not new. However, by ingenious circuitry including the use of a feed-back system for establishing accurate timing and using a current switch essentially independent of tube characteristics, an accurate multiplier is obtained which should be useful in electronic analog equipment. Using precision resistors, stable voltage sources and steep switching pulses an accuracy of about $\pm$ 0.01% of full scale (75 volts) for a maximum prf of 2000 cps is obtainable.

Paper No. 30: "An AM-FM electronic analog multiplier" by W. A. McCool. A signal having both amplitude modulation and frequency modulation is applied to an (AM) detector and a (FM) discriminator. If it is desired to multiply the two quantities X and Y, then the output of the discriminator can be made to be $X = Y_0 \Delta f$, and if the output of the detector is forced to match the X input plus the reference voltage Y_0, then its output is $Z' = (Y_0 + Y)\Delta f$ and $Z = Z' - X = Y_0^{-1}XY$. (The frequency deviation Δf is the same for both detectors.) This principle has been used before. By the use of feed-back high gain amplifiers, tube characteristics have only a secondary effect. A mathematical analysis is made of the circuit showing the dependence of Z on the circuit parameters. An experimental model is in the process of being developed at the Naval Research Laboratory.

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1034. Denman Shaw & R. N. Weltmann, "A logarithmic counting-rate recorder," *Rev. Sci. Instruments*, v. 23, 1952, p. 528–531.

This device uses a spiral cut on a cylinder to record the logarithm of the time for a specified number of counts from a Geiger counter and scaler.

F. J. M.

1035. S. Softky & J. Jungerman, "Electrolytic tank measurements in three dimensions," *Rev. Sci. Instruments*, v. 23, 1952, p. 306–307.

The authors indicate the techniques necessary to obtain good results in the three dimensional case and obtain in a specific example, an accuracy of .2% in the potential and 2% in the field measured.

F. J. M.

1036. A. W. Vance & C. C. Shumard," Superregulated power supplies," *Electronics*, v. 24, no. 12, Dec. 1951, p. 109–113.

This article describes the power supplies used in the R.C.A. Project Typhoon guided missile simulator, built for the U. S. Navy.

F. J. M.

1037. G. L. Weissler, A. W. Einarsson & J. D. McClelland, "A semi-automatic graphical computer with use in spectroscopy," *Rev. Sci. Instruments*, v. 23, 1952, p. 209–212.

The computer is a simple but ingenious device for graphing a prescribed function φ of a function $f(x)$ where $f(x)$ is given by a graph. The graph of $y = f(x)$ appears on a sheet of paper which passes over a line parallel to the y axis. Above this line is a transparent plate which moves parallel to the x axis. On this plate, one has a graph $x = \varphi(y)$ and there is a recorder moving synchronously with the f graph which records the x motion of the plate. The operator moves the plate so that the $y = f(x)$ and $x = \varphi(y)$ graphs have the same ordinate y on the above-mentioned line and hence the desired output $\varphi(f(x))$ is obtained from the recorder. The article contains details of the construction and of the application mentioned in the title.

F. J. M.

NOTES

147.—Stability of Difference Relations in the Solution of Ordinary Differential Equations. In a recent communication, J. Todd[1] demonstrated the danger of replacing a differential equation, for computational purposes, by a difference equation of higher order. H. Rutishauser[2] has since given some general criteria for determining the stability of difference approximations to ordinary differential equations. In the present note, some standard step-by-step methods of integrating ordinary linear differential equations are examined for stability.

Let a linear differential equation be replaced by a finite difference approximation of order p (i.e., one involving $p + 1$ tabular values). Then the nth tabular entry is calculated from

$$y_n + A_1 y_{n-1} + A_2 y_{n-2} + \cdots + A_p y_{n-p} = 0, \tag{1}$$

where $A_1, A_2, \cdots, A_p$ are functions of x and of the interval length h. Now suppose the errors existing in the entries $y_{n-p}, y_{n-p+1}, \cdots, y_{n-1}$ are ϵ_{n-p}, $\epsilon_{n-p+1}, \cdots, \epsilon_{n-1}$ respectively, then the consequent error in y_n is ϵ_n where

$$\epsilon_n + A_1\epsilon_{n-1} + A_2\epsilon_{n-2} + \cdots + A_p\epsilon_{n-p} = 0. \tag{2}$$

Consider also for convenience that the above errors result entirely from errors in the initial values $y_1, y_2, \cdots, y_p$. Then the general error given by equation (2) is

$$\epsilon_n = a_1\lambda_1{}^n + a_2\lambda_2{}^n + \cdots + a_p\lambda_p{}^n, \qquad (n > p),$$

where $a_1, a_2, \cdots, a_n$ are constants and $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the roots of the auxiliary equation

$$\lambda^p + A_1\lambda^{p-1} + A_2\lambda^{p-2} + \cdots + A_p = 0. \tag{3}$$

The condition for stability is that all the roots of equation (3) lie inside or on the unit circle.

Todd[1] considered the differential equation

$$y'' = -y,$$

and its fourth order central difference replacement

$$y_n - 16y_{n-1} + (30 - 12h^2)y_{n-2} - 16y_{n-3} + y_{n-4} = 0.$$

As h approaches zero, the roots of the corresponding auxiliary equation tend to 1, 1, $7 - \sqrt{48}$, and $7 + \sqrt{48}$, the last root quoted being responsible for the instability found by Todd. The fourth order backward difference