polygon which contains all the zeros of a polynomial $p(z)$ also contains all the zeros of the derivative $p'(z)$.

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[1] *A Description of the Mark IV Calculator.* Computation Laboratory of Harvard University, *Annals*, v. 28 (In preparation).

[2] W. E. Milne, *Numerical Calculus.* Princeton 1949, p. 53; or D. R. Hartree, *Numerical Analysis.* Oxford 1952, p. 205.

[3] G. Szegö, "Über eine Eigenschaft der Exponentialreihe," Berliner Mathematischen Gesellschaft, *Sitzungsberichte*, 1924, p. 50–64.

[4] J. Berghuis, "Truncated power series," Mathematical Centre, Amsterdam, *Report* R173, 1952.

[5] R. S. Varga, "Semi-infinite and infinite strips free of zeros," Universita e Politecnico di Torino, Seminario Matematico, *Rendiconti*, v. 11, 1951–1952, p. 289.

[6] K. S. K. Iyengar, "A note on the zeros of $\sum_0^n \frac{x^r}{r!}$." *Mathematics Student*, v. 6, 1938, p. 77.

[7] M. Marden, *The Geometry of the Zeros of a Polynomial in a Complex Variable.* American Mathematical Society. New York, 1949.

RECENT MATHEMATICAL TABLES

1094[B].—Marchant Calculating Machine Co. *Table No. 80 of Factors for 8-Place Square Roots*, 1951, 4 p.; *Table No. 81 ··· for 6-Place Roots*, 1952, 4 p.; and *Table No. 82 ··· for 5-Place Roots*, 1952, 2 p. *Publications* of Marchant Calculators, Inc., Oakland, California. 21.5 × 28 cm.

The tables for 5- and 6-place roots resemble a former one for 5-place roots of the same publisher (*MTAC*, v. 1, p. 356). As in the former table the root results from adding a tabular number to the number N of which root is desired, and dividing this sum by an adjacent tabular number. The table for 8-place roots requires two divisions but without need of intermediate copying. Mathematically the table for 8-place roots is equivalent to a similar two-division table reported in *MTAC*, v. 5, p. 180.

Doubtless the most welcome of these tables will be the one for 6-place roots because this number of places is specified by the majority of work sheets for general surveying and military uses. There has not been available a means of obtaining 6-place roots by the use of tables of this type and a single division.

Significant improvement has been made in tables Nos. 81 and 82 as compared with the previous 5-place table (1) by eliminating need of determining which of two arguments is nearer to N, (2) by reducing the number of tabulated divisors—resulting from using a single column of divisors for the entire range of N from 1 to 100 and from altering the range of error from (0 to +5) to (−5 to +5) in units of last place, (3) by using the leeway afforded by integer arguments to reduce the number of digits of the divisors to as few as possible.

The method of selecting argument intervals and the error expression of tables of the single-division type are described in Willers, *Practical Analysis*, Dover edition, Art. 6–21, 1948 (*MTAC*, v. 3, p. 493). Obvious alteration of the error expression is necessary when applying them to these tables because of change of the range of error.

The tables are suitable for use with any of the usual types of desk calculating machines, though the manipulative description accompanying table 80 refers to the Marchant machine. The tables were prepared under the direction of H. T. AVERY who asks that aid rendered by H. J. REYNOLDS be acknowledged.

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1095[B].—VICTOR THÉBAULT, *Les Récréations Mathématiques (Parmi les nombres curieux)*. Paris, Gauthier-Villars, 1952, vi, 299 p., 15.5 × 23.5 cm.

P. 12: Table of the 87 squares formed by the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 ,9, each once and only once.
P. 89–119: Tables of squares of integers 1(1)1000 for bases 2(1)9, 11, 12.
P. 226–230: The tables of squares with (a) 3, (b) 5, (c) 6, (d) 7, (e) 8, (f) 9 digits occurring once and only once.

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1096[C,K].—M. S. BARTLETT, "The statistical significance of odd bits of information," *Biometrika*, v. 39, 1952, p. 228–237.

Table I (p. 230) lists to 4D for $p = 0(.01)1$ the functions $-\ln p$; $-p\ln p$; $p\ln^2 p$; $-p\ln p - (1-p)\ln(1-p)$; and $p(1-p)\ln^2(p/(1-p))$. An example is given using this table to compute an "information" function.

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1097[C].—ALEXANDER JOHN THOMPSON, *Logarithmetica Britannica, being a Standard Table of Logarithms to Twenty Decimal Places. Part II, Numbers 20,000 to 30,000 together with General Introduction*. Cambridge University Press, 1952. vi, [102], xcviii, [xi] p., 21.6 × 27.7 cm., 45 shillings; New York price $8.50.

This is the ninth and final part of the very notable work which Dr. Thompson started 30 years ago, as a publication for KARL PEARSON'S *Tracts for Computers*, Department of Statistics of the University of London, to commemorate the tercentenary of HENRY BRIGGS' publication of *Arithmetica Logarithmica*, 1624. This work of Briggs (1561–1631) contained log N to 14D for $N = 1(1)20000$, $90000(1)101000$, and the square roots of the integers [1(1)200; 11D], with first differences in each case; (see *MTAC*, v. 1, p. 170; v. 2, p., 94,196). On pages lxxviii–lxxxiii of this part II is a list of errors of more than a unit in the values of log N in this Briggsian table. Briggs had practically completed the computation of log N for $N = 20001(1)90000$ when ADRIAN VLACQ published *Arithmetica Logarithmica* (1628) for log N, N = [1(1)100000; 10D], and characterized his volume as the second edition of the work by Briggs. The completion by Vlacq of a 10D table from the earlier 14D tables of Briggs was a comparatively simple matter. But his action in rushing into print with such a publication can scarcely be termed other than reprehensible.

The first part of Dr. Thompson's work, N = 90,000–100,000, appeared in 1924, and this was followed by other parts in 1927, 1928, 1931, 1933, 1934, 1935, 1937. Thus 15 years elapsed after the appearance of 8 parts, before the final part, completing the table for $N = 10000(1)100000$, was published. For brief reviews of parts VI and VII see *MTAC*, RMT **23**, **42**, **65**, v. 1, p. 4, 5, 7, that is, *Scripta Mathematica*, v. 2, 1934, p., 196; v. 3, 1935, p., 192; v. 4, 1936, p., 201.

Since 100 logarithms are displayed on each page, there are in the whole work 900 pages occupied by the fundamental table. To introduce this table there are in the present part 108 pages of preliminary matter. In other parts there was further preliminary matter to which we shall presently refer. Hence in this part are the necessary title-pages and indices for binding the whole work into two volumes. Of the useful factoring Table F (logarithms of $1 + N/10^7$, $1 + N/10^{10}$, and $1 + N/10^{13}$, N = [0(1)1000; 21D]) there are two copies, one for each volume. Reprints of the complete work in two bound volumes are now available at the publisher. For completing sets, copies of the first eight published parts are available at 21 shillings each.

In part I (1934) is a two-page facsimile of the original will of HENRY BRIGGS, the first page in his own handwriting; part III (1937) has four pages with facsimiles of five letters of Briggs, and a Note by Dr. THOMPSON; in part IV (1928) there is a facsimile of the title-page of the work in which Briggs's Treatise on the Northwest Passage to the South Sea was published in 1622; two pages of errors in *Arithmetica Logarithmica*, 1624, are also given; part V (1931) has a facsimile of the title-page of *Arithmetica Logarithmica*; and part VI (1933) has a facsimile of a Briggs' letter to JOHN PELL.

The *Mirifici Logarithmorum Canonis Descriptio* was first published in 1614 by JOHN NAPIER (1550–1617). The second edition in 1619, prepared after Napier's death by his son ROBERT, and Briggs, contained an edited earlier work of Napier: *Mirifici Logarithmorum Constructio, Canonis* in which Napier called logarithms "artificial numbers." See my article, "Napier's *Descriptio and Constructio*," Amer. Math. Soc., *Bull.*, v. 22, 1916, p. 182–187. Part VII (1935) has four pages (three of 6 facsimile pages) illustrating the relation of Henry Briggs to Napier's *Constructio Canonis* with a Note by KARL PEARSON. It is by no means clear how Pearson reasons that the word "logarithm" used by Napier in his work of 1614 "must have been added by Briggs or Robert Napier," because Napier had used a different term in his *Constructio.*

Parts VIII (1927) and IX (1924) contained a number of pages of introductory matter by Thompson and in the latter a "Prefatory Note" by Karl Pearson, and also a facsimile of the title and specimen pages of the excessively rare 1617 tract (16 p.): *Logarithmorum Chilias Prima* of HENRY BRIGGS. This is the earliest publication of logarithms to the base 10; it exhibits the logarithms to 14D of numbers 1(1)1000. Table F was given in part VIII.

Dr. THOMPSON's splendid Introduction to *Logarithmetica Britannica* in part II includes a discussion of: (a) Interpolation with examples (p. xv–xxviii); (b) Method of Construction (p. xxviii–lxii)—here are 12 special tables and a plate illustrating the unique four-bank integrating and differencing machine which Thompson built for carrying through the calculations of this work. He personally set up the type for all table entries by means of a monotype keyboard. This system of type setting involves the use of two entirely separate machines, a keyboard and a typecaster. Thompson used the keyboard to punch holes in a continuous ribbon of specially prepared paper, which then went to the typecaster.

There is a section (p. lvi–lxii) on construction of a table of Anti-logarithms. Shortly after the publication of the table of logarithms had begun, Thompson was urged to undertake a companion volume of anti-logarithms on the same scale. Table 8 (p. lviii) is a single page of such a table $\log N = 0.00(.01)1.00$, corresponding N's being given to 28D.

"Printing and proof-reading" and "Acknowledgments" on p. lxii–lxv are followed by "References" (p. lxv–lxvi) listing works mentioned in the Introduction, and a few other titles.

In the manuscript for the main table the calculations were made to about 24D, the last digit having a possible error of three or four units. In starting computations it was desirable to have a considerable number of values of $\log N$ to a large number of decimal places. For one thing Dr. THOMPSON checked by 63D computation the accuracy of the 61D table of $\log N$, $N = 1(1)100$ and primes to 1097; also 999990(1)1000110, in ABRAHAM SHARP'S *Geometry Improv'd* (1717). The figures in this table are arranged in five-figure groups except the twelfth and last group, which consists of six figures. The following errors in mantissas were found (the error for $N = 751$ was new):

N	Group	*For*	*Read*
103	9	33496	23496
227	12	494656	495656
751	12	287788	287771
839	12	539741	538741
1009	12	382385	382285

No other errors were found except for unit errors in the 61st decimal place. For $N = 127, 149, 293$, the final digits should each be increased by unity.

Thompson had only the CALLET (1795) reprint by Sharp, and found that Callet had an error for $N = 1097$; but here Sharp was correct. In the PETERS & STEIN reprint (1922) all of Callet's errors persist.

Other extensive $\log N$ tables are: (a) A. GRIMPEN'S 84D table (1922) of the prime numbers up to 113; and (b) H. M. PARKHURST'S 100D table (1876) for 96 values of $N \leqslant 109$. See *MTAC*, v. 1, p. 20, 58–59, 121–122.

Appendix (i) of part II contains the first English translation of the earliest published biography of BRIGGS, in THOMAS SMITH'S *Vitae quorundam eruditissimorum et illustrium virorum*, 1707. There is no known portrait of Briggs (see *MTAC*, v. 2, p. 287; v. 3, p. 67).

Appendix (ii) lists the errors in *Arithmetica Logarithmica* (1624), referred to above.

Then follow: Table F (lxxxv–xciv); Table G, antilogarithms of logarithms [0000000(1)0000450; 21D], p. xcv–xcvii; Table H, Short Tables and Constants, p. xcviii; and finally (p. xcix–cv) $\log N$, N = [1(1)1000; 21D].

The nine parts of this work have been published in *Tracts for Computers* prepared in the Department of Statistics, University of London, in the following numbers: XIX, XXII, XXI, XVI, XVII, XVIII, XX, XIV, XI.

Dr. THOMPSON tells us that during the past 30 years no error has been reported for any of nearly 3000000 figures in the eight parts published up to 1937. We tender the author our heartiest felicitations on his truly monumental personal contributions achieved in producing this great work.

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1098[F].—H. J. A. DUPARC, C. G. LEKKERKERKER & W. PEREMANS, *Reduced sequences of integers and pseudo-random numbers*, Math. Centrum, Amsterdam, *Report* ZW1952-002, 15 mimeographed leaves.

This study of the length of the period of a geometric progression or a Fibonacci sequence when the terms are reduced modulo m, contains the following small tables.

P. 3 contains a table of the factors of $10^n - 1$, Euler's totient function of these factors and their least common multiple for $n = 1(1)10$. The corresponding information for $10^n + 1$ is given for $n = 1(1)7$.

P. 4 gives the same information for $2^n \pm 1$ for $n = 1(1)16, 29, 30$.

P. 15 gives the rank of apparition of p in the Fibonacci sequence 0, 1, 1, 2, 3, 5, $\cdots$, for each prime less than 433. This table was taken from the table of JARDEN.[1]

D. H. L.

[1] D. JARDEN, "Table of the ranks of apparition in Fibonacci's sequence," *Riveon Lematematika*, v. 1, no. 3, 1946, p. 54 [*MTAC*, v. 2, p. 343].

1099[F.]—KARL GOLDBERG, "A table of Wilson quotients and the third Wilson prime," London Math. Soc., *Jn.*, v. 28, 1953, p. 252–256.

The Wilson quotients W_p are defined as the non-negative residues modulo p of $[(p + 1)! + 1]/p$, where p is prime and the Wilson primes are solutions of the equation $W_p = 0$. The present table gives the values of W_p for all primes less than 10000, and shows 563 to be the third Wilson prime.

N. G. W. H. BEEGER's table[1] of Wilson Quotients extended to all primes less than 300 and the previously known Wilson primes were 5 and 13.

GOLDBERG states that six months after the completion of this table DONALD WALL [UMT **150**, *MTAC*, v. 6, p. 238] computed a table of Wilson quotients for all primes less than 5000, and that his table checked with this one in every case.

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[1] N. G. W. H. BEEGER, "On the congruence $(p - 1)! \equiv - 1 \pmod{p^2}$," *Messenger Math.*, v. 49, 1920, p. 177–178.

1100[F].—F. GRUENBERGER, *Table of Prime Numbers from* 2 *to* 406253. Numerical Analysis Laboratory, Univ. of Wisconsin, Madison, 1953, 1 "microcard," 7.6 × 12.4 cm. Price, 25 cents.

This little card contains 34320 primes, each prime fully spelled out, as photographed from 65 sheets, calculated and printed with an IBM Card Programmed Calculator.

The condensation achieved is remarkable. Twenty cards of this size would accommodate the 665000 primes in LEHMER's list[1] covering the first 10 million numbers.

The card requires a hand lens of power $\geqslant 5$. The time required to enter the table is nearly the same as for a standard table, the time spent in locating the appropriate region of the card being comparable with that spent in turning pages.

The list was compared at 520 places (every 66th prime) with that of Lehmer.

D. H. L.

[1] D. N. LEHMER, *List of Prime Numbers from* 1 *to* 10006721, Washington, 1914.

1101[F].—GIUSEPPE PALAMÀ & L. POLETTI, "Tavola dei numeri primi dell'intervallo 12 012 000 − 12 072 060," Unione Matematica Italiana, *Bollettino*, s. 3, v. 8, 1953, p. 52–58.

With references to the D. N. LEHMER list of primes up to 10 006 721 (1914); and to the list of N. G. W. H. BEEGER, L. POLETTI, A. GLODEN & R. J. PORTER, from 10 006 741–10 999 997 (1951, *MTAC*, v. 6, p. 81–82); the authors add 3684 primes before 12 072 060.

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1102[F].—G. RICCI, "La differenza di numeri primi consecutivi," Univ., Politec., Torino, *Rend. Sem. Mat.*, v. 11, 1952, p. 149–200.

This topical history contains the following tables and graphs.

Let $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $\cdots$ be the primes in increasing order and let the difference between consecutive primes be denoted by

$$d(n) = p_{n+1} - p_n.$$

Let $A_{2h}(x)$ denote the number of primes $p \leqslant x$ for which $p + 2h$ is also a prime. Let $B_{2h}(x)$ denote the number of those primes $p_n \leqslant x$ for which $d(n) \leqslant 2h$.

The main table (p. 152–155) gives the values of

$$p_n,\ d(n),\ A_{2h}(p_n),\ (h = 1(1)7);\quad B_{2h}(p_n),\ (h = 1(1)5)$$

for $n = 1(1)170$. The function $d(n)$ is graphed on p. 157 and compared with ln n. The functions

$$A_{28}(x),\ (h = 1(1)7)$$

and the function

$$A(x) = d(n) \quad (p_n \leqslant x < p_{n+1})$$

are graphed on p. 158–9 for $0 < x < 1020$.

WESTERN's[1] table of those primes $p_n < 10^7$ whose difference $d(n)$ exceeds that of all smaller primes is reprinted.

D. H. L.

[1] A. E. WESTERN, "Note on the magnitude of the difference between successive primes," London Math. Soc., *Jn.*, v. 9, 1934, p. 276–278.

1103[I,L].—NBS *Tables of Chebyshev Polynomials* $S_n(x)$ *and* $C_n(x)$. Applied Math. Ser. No. 9. U.S. Government Printing Office, Washington, D. C., 1952. xxix + 161 p., 20.5 × 27 cm. $1.75.

"The Chebyshev polynomials are of use in many mathematical investigations. Although direct numerical tabulation is fairly easy to avoid—for example, by double or multiple use of ordinary trigonometrical tables—the present tables are welcome because they will remove the necessity for these roundabout methods, which are often irritating" (from the Foreword by J. C. P. MILLER).

The polynomials tabulated in this volume are

$$C_n(x) = 2\cos n\,\theta$$

$$S_n(x) = \frac{\sin\,(n+1)\,\theta}{\sin\theta}$$

where

$$x = 2\cos\theta.$$

Other notations are

$$T_n(x) = \frac{1}{2}\,C_n(2x),\; T_n{}^*(x) = \frac{1}{2}\,C_n(4x-2)$$

$$U_n(x) = S_n(2x), \qquad U_n{}^*(x) = S_n(4x-2).$$

Explicit expressions are given for $C_n(x)$, $S_n(x)$, $T_n(x)$, $U_n(x)$ for $n = 0\ (1)\ 12$; and for $T_n{}^*(x)$ for $n = 0\ (1)\ 20$. The expansions of x^k in the $T_n(x)$, and in the $T_n{}^*(x)$, polynomials are also given for $k = 0\ (1)\ 12$.

The principal tables give 12D values of $S_n(x)$ and $C_n(x)$ for $n = 2\ (1)\ 12$, $x = 0\ (.001)\ 2$.

The Introduction, by CORNELIUS LÁNCZOS, describes the basic properties of Chebyshev polynomials, their applications to expansions, curve fitting, solution of linear differential equations with rational coefficients; it gives an account of the computation of the tables and instructions for their use, and also a list of references.

The computations were carried out under the technical direction of A. N. LOWAN.

A preliminary MS of these tables was described by LOWAN in *MTAC*, v. 1, p. 125 (UMT **11**). Other tables of Chebyshev polynomials are referred to in *MTAC*, v. 1, p. 385 (RMT **185**), v. 2, p. 256 (RMT **371**), v. 2, p. 262–263 (RMT **381**), v. 2, p. 266 (RMT **383**), v. 3, p. 97 (RMT **495**), v. 3, p. 119 (MTE **126**), v. 3, p. 120–121 (UMT **68**).

"The tabulation of these polynomials—easy to calculate, and easy to sidetrack at the cost of some inconvenience—is long overdue. The Computation Laboratory staff is to be congratulated on the removal, at last, of this source of inconvenience, and in doing so, on the addition of yet one more table to its already magnificent series" (from the Foreword).

A. E.

1104[I].—H. E. SALZER, "Formulas for numerical differentiation in the complex plane," *Jn. Math. Phys.*, v. 31, 1952, p. 155–169.

In an earlier paper[1] the author gave coefficients for numerical integration in the complex plane of polynomials of degree no higher than eight, based on configurations of the grid points chosen so as to be convenient for initiating a computation and "as close together as possible." Thus let the Lagrange polynomial of degree $n - 1$ be defined by

$$f(z) = \sum_{k=1}^{n} P_k(z) f(z_k) \tag{1}$$

where $f(z_k)$ is the given value of $f(z)$ at $z_k = x_k + iy_k$. The points are chosen over a *square* grid, namely

$$z_j = z_0 + jh$$

where j is a complex integer. As an example, for a polynomial of degree four the selected grid-points are

$$z_0, z_0 + h, z_0 + 2h, z_0 + ih, z_0 + (i + 1)h.$$

The s-th derivative of $f(z)$ at any grid point z_j can be expressed by

$$f^{(s)}(z_j) = \textstyle\sum_k P_k^{(s)}(z_j) f(z_k). \tag{2}$$

Letting $z = z_0 + Ph$, it is possible to write

$$h^s f^{(s)}(z_j) = \textstyle\sum_k M_k^{(s)}(j) f_k / M(s). \tag{3}$$

where $f_k = f(z_k)$, and $M_k^{(s)}(j)$ and $M(s)$ are complex integers, independent of f and h. The author tabulates the exact coefficients of f_k for all grid points, in the derivatives of all orders, for polynomials of degree 2, 3, $\cdots$, 9. Methods of computing and checking are described in the paper. The work will add to the author's reputation for supplying accurate and well planned tables of coefficients.

GERTRUDE BLANCH

NBSINA

[1] H. E. SALZER, "Formulas for numerical integration of first and second order differential equations in the complex plane, *Jn. Math. Phys.*, v. 29, 1950, p. 207–216.

1105[K].—E. P. KING, "The operating characteristic of the control chart for sample means," *Annals Math. Stat.*, v. 23, 1952, p. 384–395.

This paper extends the theory of the SHEWHART control chart by deriving expressions for the chances that the $\bar{X}$ chart or chart for sample means will show control for both of the cases of known and unknown standard deviation. The null-hypothesis is that samples are drawn from the usual normal process, $N(\mu, \sigma^2)$, where μ and σ are fixed but unknown. The alternative hypothesis considered is that shifts in the process mean from time to time can be represented by $N(\mu, \theta^2\sigma^2)$, i.e., the process mean itself is a random variable with this normal distribution.

Suppose we consider m samples of n each which are selected at random from rational categories of a process and we plot the $\bar{X}$ and R (sample range) charts. Let $\beta_0(k, \theta, m, n)$ denote the chance that all m sample means will fall within $\pm k\sigma n^{-\frac{1}{2}}$ of the grand mean, where the standard deviation σ is known, and let $\beta(k, \theta, m, n)$ represent the chance that the $\bar{X}$ chart will show control when σ, in effect, is estimated from the m sample ranges on the chart for ranges, i.e., the process itself to date. Of course, it is usual practice to take $k = 3$ and the computed tables of the paper are based on this value. The Tables I–VI (p. 390–392) give values to 2D for the operating characteristics or the chances, β, that the $\bar{X}$ chart will show control for several practical cases: $\beta_0(3, \theta, m, n)$ for $\theta = 0(.5)3$; $n = 2, 5, 10$, $m = 2, 3, 4$; $\beta(3, \theta, m, n)$ for $\theta = 0(.5)3$; $n = 2, 5, 10, m = 2$, and $n = 5, 10, m = 3, 4$.

For $m \geq 4$ bounds for $\beta_0(3, \theta, m, n)$ and $\beta(3, \theta, m, n)$ are given in Tables VII and VIII (p. 393, 394) to 2D for the cases: $\theta = 0(.5)3$; $n = 5, 10$; $m = 5, 10$; $\theta = 0, .75, 1, 1.25, 1.5$; $n = 5, 10$; $m = 15, 20$; $\theta = 0, .25, .75, 1$; $n = 5, 10$; $m = 25$.

It is of interest to note that the problem treated in this paper is somewhat related to that of testing for "outlying" observations, since the chance that the $\bar{X}$ chart will show control is also the chance that the largest and smallest sample means will both lie between control limits.

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1106[K].—W. H. KRUSKAL & W. A. WALLIS, "Use of ranks in one-criterion variance analysis," Amer. Stat., Assn., *Jn.*, v. 47, 1952, p. 583–621.

Consider three samples, of sizes n_1, n_2, and n_3 respectively, arranged together in order of size. Assign scores to the $N = n_1 + n_2 + n_3$ individuals according to their ranks in the combined sample. That is, assign the score 1 to the smallest of the N, the score 2 to the next smallest, etc. Let $H = \dfrac{N-1}{N} \sum_{i=1}^{3} \dfrac{n_i[R_i - \frac{1}{2}(N+1)]^2}{(N^2-1)/12}$, where R_i is the mean rank score of the ith sample.

Under the assumption that the three samples are random selections from a continuous population Table 6.1 (p. 614–617) of this paper gives, for n_1, n_2, $n_3 \leq 5$, exact probabilities (and three approximations to the probabilities) all to 3D, that H will equal or exceed certain selected values. The selected values are chosen so that the probabilities will be close to 10, 5, and 1 per cent.

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1107[K].—L. E. MOSES, "A two-sample test," *Psychometrika*, v. 17, 1952, p. 239–247.

Samples of size m and n are drawn from populations A and B, respectively, and the combined samples are arranged in increasing order. To test the hypothesis that $A = B$ against the alternative that B is more widely dispersed than A, the author proposes the statistic s_h^*, defined as one more than the difference in the ranks of the h-th largest and h-th smallest observation in the first sample. He tables (p. 244, 245) the tail of the distribution of s_h^* to 3D for $h = 3, 4, 5, 8$, $m = 4h$, and 5 suitably selected values of n in each case.

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1108[K].—P. J. RIJKOORT, "A generalization of Wilcoxon's test," Nederlandsche Akademie van Wetenschappen, *Proceedings*, v. 55, series A, 1952, p. 394–404.

Let x_{ij} be the j-th observation in the i-th sample, where $i = 1, \cdots, k$ and $j = 1, \cdots, n_i$. Let r_{ij} be the rank of x_{ij} in the combined sample of $n = \sum n_i$ observations. The author proposes testing the hypothesis that all of the samples come from distributions with the same mean value by the use of the statistic

$$S = \sum (s_i - n_i \bar{r})^2$$

where $s_i = \sum r_{ij}$ and $\bar{r} = (n + 1)/2$.

The tables (p. 400–402) give to 3 and 4D (sometimes more) the exact cumulative distribution function of S: for $k = 3$ and each $n_i = 2, 3,$ or 4; $k = 4$ and each $n_i = 2$; $k = 5$ and each $n_i = 2$, for all values of S, and for $k = 3$ and each $n_i = 5$; $k = 4$ and each $n_i = 3$, for large values of S.

Methods of approximating the distribution function of S by the chi square distribution and the analysis of variance distribution are presented. Using these methods a table (p. 402) is given showing the approximate five per cent points of S for all combinations of $k = 3(1)10$ and $n_i = 2(1)10$.

The statistic S is a linear function of the statistic H proposed by KRUSKAL & WALLIS.[1] These authors give tables of the five and one per cent points for H for $k = 3$ and all possible combinations of $n_i \leq 5$.

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[1] W. H. KRUSKAL & W. A. WALLIS, "Use of ranks in one-criterion variance analysis," Amer. Stat. Assn., *Jn.*, v. 47, 1952, p. 583–621. (RMT 1106.]

1109[K].—COLIN WHITE, "The use of ranks in a test of significance for comparing two treatments," *Biometrics*, v. 8, 1952, p. 33–41.

This paper presents tables for use of the WILCOXON procedure[1] for comparing two treatments when the numbers of individuals in the two groups are not necessarily equal. Let n_1 be the number of individuals in the group for which we compute the rank total T while n_2 is the number of individuals in the other group. Without loss of generality, n_1 can always be taken less than or equal to n_2. The ranks allotted are $1, 2, \cdots, (n_1 + n_2)$. The null hypothesis tested is that both treatments had the same effect; that is, that T represents the sum of n_1 ranks drawn at random from the finite universe $1, 2, \cdots, (n_1 + n_2)$. Let the integer T_α (notation of reviewer) have the property that $\Pr(T \leq T_\alpha) \leq \alpha$ and $\Pr(T \leq T_\alpha + 1) > \alpha$. Then also $\Pr[T \geq n_1(n_1 + n_2 + 1) - T_\alpha] \leq \alpha$ and $\Pr[T \geq n_1(n_1 + n_2 + 1) - T_\alpha - 1] > \alpha$. Tables of T_α (p. 37–39) are presented for $\alpha = 5\%, 1\%, .1\%$ and all n_1, n_2 up to $n_1 + n_2 \leq 30$.

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[1] F. WILCOXON, "Individual comparisons by ranking methods," *Biometrics*, v. 1, 1945, p. 80–83.

1110[L].—LE CENTRE NATIONAL D'ÉTUDES DES TÉLÉCOMMUNICATIONS, *Tables des fonctions de Legendre associées*. Editions de la Revue d'Optique, Paris, 1952. xxi + 292 p., 21 × 30.5 cm.

Legendre functions whose degree is not an integer, arise in potential and wave problems relating to cones. Some computations of such functions have been reviewed previously (*MTAC*, v. 5, p. 152–153; v. 6, p. 98–99; see also RMT **1120**) but no systematic tabulation seems to exist, apart from the case when the degree is an integer or half of an odd integer. Thus the present tables are a pioneering effort in a field which is becoming more and more important.

The function tabulated in this volume is

$$P_n^m(\cos\theta) = (-\sin\theta)^m \frac{d^m P_n(\cos\theta)}{d(\cos\theta)^m}$$

where m is a non-negative integer, θ is real, and n is real.

The introductory material includes a Preface by J. COULOMB; an Introduction by L. ROBIN; an account of the method of computation, checks, accuracy by P. LE GALL; a diagram facilitating the use of the recurrence formulas; the level curves of $P_n^0(\cos\theta) = \text{const.}, \cdots, P_n^5(\cos\theta) = \text{const.}$ in the region $0 \leq n \leq 10$, $0° \leq \theta \leq 90°$ of the n, θ plane; and a detailed page index.

The tables give values of $P_n^m(\cos\theta)$, to various degrees of accuracy, for $m = 0\ (1)\ 5, n = -.5\ (.1)\ 10, \theta° = 0°\ (1°)\ 90°$. A second volume (in preparation) will extend the tables to $\theta = 180°$.

A. E.

1111[L].—P. C. CLEMMOW & CARA M. MUNFORD, "A table of $\sqrt{(\frac{1}{2}\pi)}\, e^{\frac{1}{2}i\pi\rho^2} \int_\rho^\infty e^{-\frac{1}{2}i\pi\lambda^2} d\lambda$ for complex values of ρ." R. Soc. of London, *Phil. Trans.*, v. 245A, 1952, p. 189–211.

Tables of the error function for complex variable were long overdue. They are needed in many wave propagation, and in some other problems. The computation of this function was discussed in *MTAC*, v. 5, p. 67–70.

The function chosen for tabulation in this memoir is

$$G(\rho) = \sqrt{(\tfrac{1}{2}\pi)}\, e^{\frac{1}{2}i\pi\rho^2} \int_\rho^\infty e^{-\frac{1}{2}i\pi\lambda^2} d\lambda.$$

For large ρ with $0 \leq \arg\rho \leq \frac{1}{4}\pi$ it is asymptotically represented by

$$\frac{1}{i\sqrt{(2\pi)}\rho}\left[1 - \frac{1}{i\pi\rho^2} + \frac{1.3}{(i\pi\rho^2)^2} - \cdots\right]$$

and

$$G(0) = \tfrac{1}{2}\pi^{\frac{1}{2}} e^{-\pi i/4}.$$

The table gives 4D values of the real and the imaginary parts of $G(\rho)$ for $\rho = re^{i\theta}$ where $r = 0\ (.01)\ .8$, $\theta^0 = 0°\ (1°)\ 45°$.

A. E.

1112[L].—H. M. DAGGETT, JR., "The Shedlovsky extrapolation function," Amer. Chem. Soc., *Jn.*, v. 73, 1951, p. 4977.

4D tables of

$$\{\tfrac{1}{2}z + [1 + (\tfrac{1}{2}z)^2]^{\frac{1}{2}}\}^2$$

for $z = 0(.001).209$.

A. E.

1113[L].—A. A. DORODNITSYN, "Asimptoticheskie zakony raspredeleniia sobstvennykh znachenii dlia nekotorykh osobykh vidov differentsialnykh uravnenii vtorogo poriadka." [Asymptotic laws of distribution of the characteristic values for certain special forms of differential equations of the second order.] *Uspekhi Matem. Nauk* (N.S.) v. 7, no. 6, 1952, p. 3–96.

Two short tables (p. 95) give numerical values to 4–6 S of

$$\zeta(s, a) = \sum_{n=0}^{\infty} (n + a)^{-s}$$

Table I. $s = 2(2)8$, $a = 3(1/12)4$.

Table II. $s = 2(2)8$, $a = 3(.05)4$.

A. E.

1114[L].—H. GORTLER, "Zur laminaren Grenzschicht am schiebenden Zylinder. Teil I." *Arch. Math.*, v. 3, 1952, p. 216–231.

The functions $F_0, \cdots, F_{222}$ are solutions of the differential equations

$$F_0'' + f_1F_0' = 0$$
$$F_2'' + f_1F_2' - 2f_1'F_2 = -12f_3F_0'$$
$$F_4'' + f_1F_4' - 4f_1'F_4 = -30g_5F_0'$$
$$F_{22}'' + f_1F_{22}' - 4f_1'F_{22} = -30h_5F_0' - 12f_3F_2' + 8f_3'F_2$$
$$F_6'' + f_1F_6' - 6f_1'F_6 = -56g_7F_0'$$
$$F_{24}'' + f_1F_{24}' - 6f_1'F_{24} = -56h_7F_0' - 30g_5F_2' + 12g_5'F_2 - 12f_3F_4' + 16f_3'F_4$$
$$F_{222}'' + f_1F_{222}' - 6f_1'F_{222} = -56k_7F_0' - 30h_5F_2' + 12h_5'F_2 - 12f_3F_{22}' + 16f_3'F_{22}.$$

All F's vanish at 0, $F_0 = 1$, and all the other F's vanish at ∞. The f_i, g_i, h_i, k_i are the functions appearing in the integration of the boundary layer equations according to BLASIUS[1] and HOWARTH.[2]

The present paper gives 3D or 4D tables and diagrams of $F_0(\eta)$, $F_0'(\eta)$, $\cdots$, $F_{222}(\eta)$, $F_{222}'(\eta)$ for $\eta = 0(.1)5.4$ (except for $F_{24}, \cdots, F_{222}'$ when $\eta \leq 5.2$). The values have been computed by numerical integration of the differential equations. The necessary values of $f_i, \cdots, k_i$ were taken from the tables by ULRICH,[3] and subtabulated where necessary. The values of F_0 and F_0' were compared with those given by COOKE[4] and SCHLICHTING;[5] discrepancies in the last decimal were noted.

A. E.

[1] H. BLASIUS, "Grenzschichten in Flüssigkeiten mit kleiner Reibung," *Zschr. f. Math. u. Phys.*, v. 56, 1907, p. 1–37.

[2] L. HOWARTH, *Steady flow in the Boundary Layer near the Surface of a Stream.* ARC *Report* No. 1632, 1934.

[3] A. ULRICH, "Die ebene laminare Reibungschicht an einem Zylinder," *Arch. d. Math.*, v. 2, 1949, p. 37–41; *MTAC*, v. 4, 1950, p. 96–97.
[4] J. C. COOKE, "The boundary layer of a class of infinite yawed cylinders," Cambridge Phil. Soc., *Proc.*, v. 46, 1950, p. 645–648.
[5] H. SCHLICHTING, *Grenzschicht-Theorie.* G. Braun, Karlsruhe, 1951.

1115[L].—J. L. LUBKIN & Y. L. LUKE, "Frequencies of longitudinal vibration for a slender rod of variable section," *Jn. Appl. Mech.*, v. 20, 1953, p. 173–177.

The frequency equation is

$$[J_1(\alpha)Y_1(\beta) - Y_1(\alpha)J_1(\beta)]\sin\gamma + (\epsilon/|\epsilon|)[J_1(\alpha)Y_0(\beta) - Y_1(\alpha)J_0(\beta)]\cos\gamma = 0$$

where $\alpha = (1 + \epsilon)\beta$, $\beta = \nu_n L_2/|\epsilon|$, $\gamma = \nu_n L_1$ and $\nu_1, \nu_2 \cdots$ are proportional to the frequencies. The dimensionless parameters ϵ and $\lambda = L_1/(L_1 + L_2)$ are used, and table 1 gives 5 D values of $\nu_n(L_1 + L_2)$ for $n = 1\ (1)\ 5$, $\epsilon = -1\ (1/3)\ 1$, $\lambda = 0\ (.125)\ 1$. "Seven or eight decimals are carried in the computations, sufficient to insure that only occasional entries, if any, are in error by as much as one unit in the fifth place."

Table 2 gives some auxiliary quantities, useful in interpolating in the ϵ-direction.

A. E.

1116[L].—D. MANTERFIELD, J. D. CRESSWELL, & H. HERNE, "The quick-immersion thermo-couple for liquid steel," Iron and Steel Institute, *Jn.*, v, 172, 1952, p. 387–402.

Table I, p. 396, gives 4D values of the first six roots x of the equation

$$J_0(kx)Y_1(x) - Y_0(kx)J_1(x) = 0$$

for $k = (1.1)^{\frac{1}{2}}\ 1.06(.02)\ 1.1(.05)\ 1.3(.1)\ 1.5,\ 2(1),\ 5.$

Table II gives the corresponding 4D values of

$$\frac{2J_1(x)J_0(kx)}{x\{[J_0(kx)]^2 - [J_1(x)]^2\}}$$

for $k = (1.1)^{\frac{1}{2}},\ 1.15(.05)\ 1.3(.1)\ 1.5,\ 2(1)\ 5.$

The tables were computed by H. CARSTEN and N. MCKERROW; they are referred to in FMR *Index*, section 17.812, p. 268.

A. E.

1117[L].—NBS, Applied Mathematics Series No. 13, *Tables for the Analysis of Beta Spectra.* U. S. Government Printing Office, Washington, D. C., 1952. iii + 61 p., 20 × 26.5 cm. $0.35.

These tables were designed to assist in the theoretical analysis of beta-ray spectra.

The principal tables give values of the "Fermi function"

$$f(Z, \eta) = \eta^{2+2S}\, e^{\pm\pi\delta}|\Gamma(1 + S + i\delta)|^2$$

where Z is the atomic number, η the momentum of the electron,

$$\epsilon = (1 + \eta^2)^{\frac{1}{2}}, \beta = \eta/\epsilon, \gamma = Z/137, S = (1 - \gamma^2)^{\frac{1}{2}} - 1, \delta = \gamma/\beta.$$

Other abbreviations used in this pamphlet are

$$T = 510.91(\epsilon - 1), B\rho = 1704.3\eta.$$

A brief introduction gives definitions and indicates the method of computations, and is followed by sections on beta spectra and their analysis, and a bibliography.

There is a set of 6 auxiliary tables.

Table 1 (p. 15–16). Values of η, ϵ, β, T[4S or 5S) for $B\rho = 0(100)20000$ Gauss cm.

Table 2 (p. 17). 5S values of ϵ, η, β, $B\rho$ for $T = 0$ (10) 60 (20) 200 (50) 1000(100) 5000 (200) 10000 kev.

Table 3 (p. 18). Values of the "nonrelativistic" electrostatic effect factor.

Table 4 (p. 18). Illustrating the importance of the relativistic effect for $Z = 0, 10$ (20) 70, 100 and $\eta = 0, .2, .6, 1, 2, 4, 7$.

Table 5 (p. 18). Values of the ratio

$$\left| \frac{\Gamma(1 + S + i\delta)}{\Gamma(1 + i\delta)} \right| (\delta^2 + \tfrac{1}{4})^{-S}$$

for the same values of Z, η as in Table 4 except that $\eta = 0$ is omitted.

Table 6 (p. 18, 19). Values of $\frac{1}{5.109} \frac{\partial}{\partial\sqrt{1 + \eta^2}} \frac{\sqrt{1 + \eta^2}}{\eta} f(Z, \eta)$ to 2 or 3S and of $\frac{2\pi\gamma}{5.109\eta^3}$ to 4D (two separate tables) for $Z = 0, 10, 20$ (20) 100, $\eta =$ 0 (.2) 1 (1) 6.

Note that the "Fermi-function" $f(Z, \eta)$ is not identical with the "Fermi-Dirac function" $F_n(\eta)$.

The principal table (p. 21–61) gives values of $f(Z, \eta)$ for $Z = 1$ (1) 100, $\eta = 0$ (.05) 1 (.1) 7. Each Z has a column to itself going over two adjacent pages. The column heading gives values of Z, γ, S, $\varphi(Z)$, and each row (for a given η, T, ϵ, $B\rho$ all of which are stated) gives the values of $f(Z, \eta)$ indicated as β^- and β^+ for the upper and lower signs in the exponential respectively. At the foot of each column constants A, B, C are listed, and computation from the approximate formula

$$f(Z, \eta) \approx \eta^{2+2S}\left(A + \frac{B}{\eta} + \frac{C}{\eta^2}\right)$$

is recommended for $\eta = 0$.

On p. 10, and again on p. 21, $\varphi(Z)$ is defined as

$$(4\pi mcR/h)^{2S}[\Gamma(3)/\Gamma(3 + 2S)]^2(1 + S/2),$$

and this definition was used in the auxiliary tables. In a mimeographed correction sheet (which should accompany each copy) it is pointed out that the values listed as $\varphi(Z)$ on p. 22–61 of the principal tables are actually values of

$$\left(\frac{2 + 2S}{3 + 2S}\right)^2 \varphi(Z).$$

This sheet corrects the value for $Z = 100$, and lists 5S values of $\varphi(Z)$ for $Z = 1$ (1) 100.

A. E.

1118[L].—NBS, Applied Mathematics Series No. 25, *Tables of Bessel functions* $Y_0(x)$, $Y_1(x)$, $K_0(x)$, $K_1(x)$, $0 \leq x \leq 1$. U. S. Government Printing Office, Washington, D. C., 1952. ix + 60 p., 20 × 26.5 cm. $0.40.

This is a reprint of *Applied Mathematics Series*, No. 1 and was reviewed in *MTAC*, v. 3, p. 187–188.

A. E.

1119[L].—NBS, Applied Mathematics Series, No. 28, *Tables of Bessel-Clifford functions of orders zero and one*. U. S. Government Printing Office, Washington, D. C., 1953. ix + 72 p., 20 × 26.5 cm. $0.45.

"Although the Bessel-Clifford functions are obtainable from existing tables of Bessel functions, it was felt that they warranted tabulation because it is generally necessary to enter the existing tables with an irrational argument. Furthermore, the Bessel-Clifford functions arise as solutions of a class of differential equations occurring in various branches of applied physics, and they are therefore of importance in themselves." "The present volume carries the tabulation of the functions of orders zero and one up to a point where the asymptotic expansions can be used conveniently." (From the introduction.)

Table I. $J_0(2\sqrt{x})$ for $x = 0$ (.02) 1.5 (.05) 3 (.1) 13 (.2) 45 (.5) 115 (1) 410, 8D. $J_1(2\sqrt{x})/\sqrt{x}$ for $x = 0$ (.02) 1.5 (.05) 3 (.1) 13 (.2) 45 (.5) 115 (1) 125, 8D, $x = 125$ (1) 410, 9D.

Table II. $Y_n(2\sqrt{x})(\sqrt{x})^{-n}$, same values of n and x as in Table I.

Table III. $I_0(2\sqrt{x})$ for $x = 0$ (.02) 1, 8D, $x = 1$ (.02) 1.5 (.05) 6.2, 7D. $I_1(2\sqrt{x})/\sqrt{x}$ for $x = 0$ (.02) 1.5 (.05) 6.2, 7D.

Table IV. $e^{-2\sqrt{x}}\, I_0(2\sqrt{x})$ for $x = 6.2$ (.1) 13 (.2) 36 (.5) 115, 8D, $x = 115$ (1) 160 (5) 410, 9D. $e^{-2\sqrt{x}}\, I_1(2\sqrt{x})/\sqrt{x}$ for $x = 6.2$ (.1) 13 (.2) 36 (.5) 65, 8D, $x = 65$ (.5) 115 (1) 160 (5) 410, 9D.

Table V. $K_0(2\sqrt{x})$ for $x = 0$ (.02) 1.5 (.05) 2.5, 8D, $x = 2.5$ (.05) 6.,2 9D. $K_1(2\sqrt{x})/\sqrt{x}$ for $x = 0$ (.02) .04, 6D, $x = .06$ (.02) .28, 7D, $x = .30$ (.02) 1.5 (.05) 2.5, 8D, $x = 2.5$ (.05) 6.2, 9D.

Table VI. $e^{2\sqrt{x}}\, K_0(2\sqrt{x})$ for $x = 6.2$ (.1) 13 (.2) 36 (.5) 115 (1) 160 (5) 410, 8D. $e^{2\sqrt{x}}\, K_1(2\sqrt{x})/\sqrt{x}$ for $x = 6.2$ (.1) 13 (.2) 36 (.5) 40, 8D, $x = 40$ (.5) 115 (1) 160 (5) 410, 9D.

Except in the neighborhood of singularities, second central differences, sometimes modified, are given in all six tables.

Auxiliary tables. Interpolation coefficients for Everett's formula.

The Introduction (by M. ABRAMOWITZ) gives the mathematical properties of Bessel-Clifford functions, describes (mathematical) applications, interpolation, the method of computation, and accuracy, and lists references.

A preliminary tabulation of these functions was reported in *MTAC*, v. 3, p. 107.

A. E.

1120[L].—K. M. Siegel, J. W. Crispin, R. E. Kleinman, & H. E. Hunter, "The zeros of $P_{n_i}{}^{(1)}(x_0)$ of non-integral degree." *Jn. Math. Phys.*, v. 31, 1952, p. 170–179.

Let $P_n^{(1)}(x)$ be the associated Legendre function of order unity and non-integral degree. This paper is concerned with determination of n_i and $\int_{x0}^{1} [P_{n_i}^{(1)}(x)]^2\, dx$ such that $P_{n_i}^{(1)}(x) = 0$. The evaluation of n_i has previously been considered (*MTAC*, v. 5, p. 152–153; v. 6, p. 98–99). The idea of the paper is to expand $P_{n_i}^{(1)}(x)$, $n_i = n + Z_n$, in a Taylor series and calculate Z_n by inversion. The formula for Z_n depends on Legendre and associated Legendre functions and a certain sum. The latter is finite if n is an integer or an odd multiple of 1/2, and for these choices of n computation is facilitated as many tables exist for the former elements. The series expansion is also used to evaluate the above intergral.

The theory is illustrated for $x_0 = \cos 165°$. In Table 3, values of $P_n(x_0)$ and $P_{n_i}^{(1)}(x_0)/\sqrt{1 - x_0{}^2}$ are tabulated to 10*S* for $n = 1\ (1)\ 20$. Values are also given for $n = -0.5(1.0)20.5$; the number of significant figures varies from 5 to 11. These were calculated using series expansions and the authors state that existing tables were used as checks. The entries for n an odd multiple of 1/2 are mostly new. For n an integer, many of the entries are already available (cf. FMR *Index*, for example).

In Table 4, the first 19 values of n_i and $\int_{x0}^{1} [P_{n_i}^{(1)}(x)]^2\, dx$ are given. Three terms of the Taylor series are used. In each case computations are presented for both n an integer and an odd multiple of 1/2. No estimates of the error are given, but a procedure is defined to indicate preferred value according to the choice of n. The authors are interested in a speedy means to obtain entries within engineering accuracy, and the procedure outlined seems to fulfill this need.

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1121[L].—Friedrich Tölke, *Praktische Funktionenlehre. Erster Band. Elementare und elementare transzendente Funktionen.* Springer-Verlag. Berlin, Göttingen, Heidelberg. 1950. xii + 440 p., 20 × 27.5 cm.

The first edition of this volume appeared in 1943 and was not accessible to the reviewer. The present second edition is said to be considerably enlarged. Further volumes, on theta- and elliptic functions, hypergeometric, Bessel, and Legendre functions are in preparation. The whole work will present the more important functions in a manner suitable for their application in the engineering sciences.

The present volume contains a number of numerical tables. They are designed for use by engineers rather than professional computers. There are instructions for using these tables, but no sources or indications of the method of their computation, or of their accuracy.

Table 1 (p. 168–207), $2\pi x$, $\ln 2\pi x$, $e^{2\pi x}$, $e^{-2\pi x}$, $\sinh 2\pi x$, $\cosh 2\pi x$, $\tanh 2\pi x$, $\coth 2\pi x$, amp $2\pi x$, $\sin 2\pi x$, $\cos 2\pi x$, $\tan 2\pi x$, $\cot 2\pi x$, $\sin^* 2\pi x =$

$\sin(2\pi x - \frac{1}{4}\pi)$, $\cos^* 2\pi x$, $\tan^* 2\pi x$, $\cot^* 2\pi x$, to 4 or 5D, mostly with first differences, for $x = 0\ (.001)\ 1$.

Table 2 (p. 208–247). $\frac{1}{2}\pi x$, $e^{\frac{1}{2}\pi x}$, $e^{-\frac{1}{2}\pi x}$, $\sin\frac{1}{2}\pi x$, $\cos\frac{1}{2}\pi x$ to 4 or 5S, for $x = 0\ (.01)\ 40$.

Table 3 (p. 248–257). $\mathrm{Ei}(x)$, $\mathrm{Ei}(-x)$, $\mathrm{Shi}(x)$, $\mathrm{Chi}(x)$, $\mathrm{Si}(x)$, $\mathrm{Ci}(x)$ to 3–4S, with differences, for $x = 0\ (.01)\ 5$. Here

$$\mathrm{Ei}(\pm x) = \ln|x| + \sum_{n=1}^{\infty} \frac{(\pm x)^n}{n \cdot n!} = \mathrm{Chi}(x) \pm \mathrm{Shi}(x)$$

$$\mathrm{Si}(x) = -i\mathrm{Shi}(ix), \quad \mathrm{Ci}(x) = \mathrm{Chi}(ix).$$

Auxiliary tables on a folding inset to p. 258. $\frac{1}{2}\pi x$, $e^{\frac{1}{2}\pi x}$, $e^{-\frac{1}{2}\pi x}$, $\sin\frac{1}{2}\pi x$, $\cos\frac{1}{2}\pi x$, 3–5S, $x = .001\ (.001)\ .01$. $2\pi x$, $e^{2\pi x}$, $e^{-2\pi x}$, $x = 0\ (1)\ 10$. e^x, e^{-x}, $x = 1\ (1)\ 50$.
$\frac{m\pi x}{4}, \frac{mx}{4\pi}, \frac{k\pi x}{3}, \frac{\pi x}{3k}$, 5D, $m = 1\ (1)\ 8$, $k = 1\ (1)\ 4$.
$m!$, $1/m!$, $m = 2\ (1)\ 10$. $m!/(m-n)!$, $n = 1\ (1)\ m$, $m = 1\ (1)\ 10$. $\binom{n}{m}$, $m = 1\ (1)\ n, n = 1\ (1)\ 15$. Some useful numerical data.

Functions which occur in diffusion problems.

$$F_0(x, y) = \sum_{k=-\infty}^{\infty} e^{-k^2\pi x} \cos 2k\pi y,$$

5D, $x = 0(.01).05(.05).25(.25)1.5(.5)2.5, 4$, $y = 0(.01).5$ (p. 268–9).

$$F_1(x, y) = \sum_{k=1}^{\infty} (k\pi)^{-1} e^{-k^2\pi x} \sin 2k\pi y,$$

4D, $x = 0(.01).05(.05).5(.1)1(.2)2, 2.4, 2.8$, $y = 0(.01).5$ (p. 271–2).

$$F_2(x, y) = \frac{x}{4\pi} + \sum_{k=1}^{\infty} (-1)^k (2\pi^2 k^2)^{-1} [1 - e^{-k^2\pi x}] \cos 2k\pi y,$$

4D, $x = 0(.01).05(.05).5(.1)1(.2)2$, $y = 0(.01).5$ (p. 275–6).

$$F_3(x, y) = \sum_{k=1}^{\infty} (\tfrac{1}{2}\pi^3 k^3)^{-1} [1 - e^{-k^2\pi x}] \sin 2k\pi y,$$

4D, $x = 0(.0025).0125(.0125).125(.025).25(.05).5(.1)1(.25)2.5$, $y = 0(.01).5$ (p. 282–4).

Legendre polynomials $P_n(x)$ $n = 0\ (1)\ 10$, 4D, their derivatives $P_n'(x)$, $n = 3\ (1)\ 6$, 4D, and their integrals

$$P_{n,k}(x) = \int_0^x \cdots \int_0^x P_n(x)(dx)^k, \quad n = 0(1)6, \quad k = 1, 2, \text{5D},$$

for

$$x = 0(.001)1 \quad \text{(p. 372–440)}.$$

The volume also contains tables of integral formulas for indefinite integrals (p. 69–156).

A. E.