1089. LOUIS G. WALTERS, "Hidden regenerative loops in electronic analog computers," I. R. E. (Professional Group on Electronic Computers), *Trans.*, v. EC-2, no. 2, 1953, p. 1–4.

The author considers the linear differential equations with constant coefficients that describe a simple electrical network. The characteristic polynomial of the system is of third degree. One formal method of derivation leads to a set of 2 second order equations whose leading matrix is singular. In coding this set for electronic analog computation, it is necessary that the gain of a loop consisting of 2 amplifiers be precisely 1. The fourth order system that results if the loop gain is $1 + \epsilon$ has an extraneous mode which diverges rapidly for positive ϵ and decays rapidly for negative ϵ. The author suggests recasting the equations to avoid the singular matrix or introducing a small negative value of ϵ in the loop.

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1090. BRUCE B. YOUNG, "Advanced time scale analog computer," Franklin Inst., *Jn.*, v. 253, 1952, p. 169–171.

A repeating type of differential analyzer for systems with constant coefficients. Results are displayed on an oscilloscope. Four non-linear elements are available. This device was mentioned also in an anonymous note in the same journal, v. 251, 1951, p. 488.

NOTES

154.—ON A COMPUTATION OF THE CAPACITY OF A CUBE. The electrostatic capacity (transfinite diameter) of a two or three dimensional region is a domain functional to which considerable attention has been paid in the last generation. Although a number of independent approaches are known, the actual computation of the capacity for specific regions is attended by considerable numerical difficulty. The present note reports the result of a computation of the capacity of the unit cube which was recently carried out on SEAC and which employed the purely geometric definitions of FEKETE[1] and PÓLYA & SZEGÖ.[2]

Let M designate a three dimensional region and C its capacity. Then the following two formulas are due to Pólya & Szegö:[2]

$$C = \lim_{n\to\infty} \operatorname*{Max}_{P_i \epsilon M} \binom{n}{2} \Big/ \left[\sum_{i<j\leq n} \frac{1}{d(P_i,P_j)} \right], \tag{1}$$

$$C = \lim_{n\to\infty} \operatorname*{Min}_{(P_k)\epsilon M} \operatorname*{Max}_{P \epsilon M} \left\{ n \Big/ \sum_{k=1}^{n} \frac{1}{d(P,P_k)} \right\}, \tag{2}$$

where $d(P,Q)$ indicates the distance between P and Q.

Formula (2) was employed in the SEAC computation, and the maximizations and minimizations were accomplished by selecting P and P_k from a quasi-random sequence of points lying in the unit cube and monitoring the extreme values. A value $n = 8$ was used. Corresponding to a fixed selection of points $(P_1, \cdots, P_8)$, a total of 50 points P were probed and the maximum value of the bracketed expression in (2) selected. This maximum was then printed out and the maximizing coordinates recorded. A second selection of points $(P_1, \cdots, P_8)$ was then tried. The computation was run on SEAC for approximately 3 hours during which time 250 selections of $(P_1, \cdots, P_8)$ were obtained. This represents a total of 10^5 distances $d(P, P_k)$. The minimax obtained in this way was

$$C = .6835. \tag{3}$$

No *a priori* investigation of the significance of this result has been made, and the computation made no special use of the symmetries of M.

The best known theoretical value for the capacity C of the unit cube has been given recently by W. GROSS.[4] His value is

$$C = .6464 + \epsilon, \ |\epsilon| \leq .032. \tag{4}$$

The agreement between (3) and (4) is surprisingly good, but it is felt, somewhat fortuitous. This method is easily adapted to regions of irregular shape. Formula (1) avoids a minimization at the cost of computing more distances.

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[1] M. FEKETE, "Über die Verteilung der Wurzeln bei gewissen algebraischen Gleichungen mit ganzzahligen Koeffizienten," *Math. Zeit.*, v. 17, 1923, p. 228–249.
[2] G. PÓLYA & G. SZEGÖ, "Über den transfiniten Durchmesser (Kapazitätskonstante) von ebenen und räumlichen Punktmengen," *Jn. für die reine und angew. Math.*, v. 165, 1931, p. 4–49.
[3] G. PÓLYA & G. SZEGÖ, "Isoperimetric inequalities in mathematical physics," *Annals of Math. Studies*, no. 27, Princeton, 1951.
[4] W. GROSS, "Sul calcolo della capacità elettrostatica di un conduttore," *Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat.*, s.8, v. 12, 1952, p. 496–506.

155.—A METHOD OF RADIX CONVERSION. The method of radix conversion presented here is useful mainly for hand computation or for converting from octal to decimal in a decimal computer. The method depends upon the identity:

$$\left(1 + \frac{b-a}{a}\right) \cdots \left(\left(1 + \frac{b-a}{a}\right)\left(\left(1 + \frac{b-a}{a}\right)\left(\left(1 + \frac{b-a}{a}\right) u_n a^n + u_{n-1}a^{n-1}\right) + u_{n-2}a^{n-2}\right) + \cdots + u_1 a\right) + u_0$$
$$\equiv u_n b^n + u_{n-1}b^{n-1} + u_{n-2}b^{n-2} + \cdots + u_1 b + u_0,$$

which can be easily verified. For $n = 2$, for example, the identity is

$$\left(1 + \frac{b-a}{a}\right)\left(\left(1 + \frac{b-a}{a}\right) u_2 a^2 + u_1 a\right) + u_0 = u_2 b^2 + u_1 b + u_0.$$

Let us consider u_i to be the i-th digit of a number of radix b. We now have the right hand member of the identity to evaluate, using the radix a. This is a simple operation since $b - a$ is an integer and a in the denominator indicates a shift. An illustrative example follows:

Let it be desired to convert the octal number 17324172 to decimal

$a = 10$

$b = 8$

$$\frac{b-a}{a} = -.2$$

```
  1 7 3 2 4 1 7 2
- 2
---
  1 5 3
- 3 0
-----
  1 2 3 2
- 2 4 6
-------
  9 8 6 4
- 1 9 7 2
---------
  7 8 9 2 1
- 1 5 7 8 4
-----------
  6 3 1 3 7 7
- 1 2 6 2 7 4
-------------
  5 0 5 1 0 3 2
- 1 0 1 0 2 0 6
---------------
  4 0 4 0 8 2 6
```

Thus 4040826 is the decimal equivalent of the octal integer 17324172.

By this method $n - 1$ extractions and $2(n - 1)$ additions are required to convert the number of n octal digits to its decimal equivalent by the above method. By the other two methods in common use at least $n - 1$ extractions and $5(n - 1)$ additions are required, where 5 is taken as the average value of each digit.

The method is used to good advantage in wiring type 604 IBM computers for octal to decimal conversions. For converting an eight digit octal number it requires only 21 program steps using 21 electronic cycles. The other methods in common use require at least 24 program steps and 141 electronic cycles.

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156.—FACTORS OF FERMAT NUMBERS. On August 14 and 15, the SWAC discovered that $2^{2^{16}} + 1$ is divisible by $825753601 = 1575 \cdot 2^{19} + 1$ and that $2^{2^{10}} + 1$ is divisible by $45592577 = 11131 \cdot 2^{12} + 1$. These are the first factors found of the 16th and 10th Fermat numbers $F_n = 2^{2^n} + 1$. The result concerning F_{16} is of some interest, since it proves that not all numbers of the form

$$2 + 1, 2^2 + 1, 2^{2^2} + 1, 2^{2^{2^2}} + 1, \cdots,$$

are primes.

The composite nature of F_{10} had been revealed on February 4, 1952, when the SWAC, using R. M. ROBINSON's routine, showed that F_{10} did not divide

$3^{2^{1023}} + 1$. The residue found by the SWAC has been checked using the modulus $11131 \cdot 2^{12} + 1$ and found to agree.

The writer's SWAC routine has tested all numbers of the form $D = (2k + 1)2^r + 1$ with $D < 2^{36}$ and $k < 2^{15}$ which are possible divisors of Fermat numbers. This took $3\frac{1}{4}$ hours running time.

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CORRIGENDA

V. 7, p. 114, l. −1, add footnote, A. W. BURKS, H. H. GOLDSTINE, and J. VON NEUMANN, *Preliminary Discussion of the Logical Design of an Electronic Computing Instrument,* Institute for Advanced Study, June 1946.

V. 7, p. 118, l. −7, *for* W. S. MACWILLIAMS *read* W. H. MAGWILLIAMS.

V. 7, p. 168, l. −8, −9, *for* 5 *read* .5.