subtracted from column 2, the matrix

$$A_2 = \begin{bmatrix} -3 & 1 & 3 \\ 10 & 0 & -6 \\ -10 & 2 & 8 \end{bmatrix}$$

becomes the similar matrix

$$T_2 = \begin{bmatrix} -3 & -2 & 3 \\ 10 & 6 & -6 \\ 0 & 0 & 2 \end{bmatrix}$$

Here obviously $(\lambda - 2)$ is a factor and all that remains to do is find the other factor from the matrix

$$\begin{bmatrix} -3 & -2 \\ 10 & 6 \end{bmatrix}$$

This is easily found to be $(\lambda - 2)(\lambda - 1)$.

EDWARD SAIBEL
W. J. BERGER

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This work was supported in part by a contract between Carnegie Institute of Technology and the Department of the Army Ordnance Corps.

[1] FIAT *Review of German Science*, Applied Mathematics. Part I, 1948, p. 31–33.
[2] R. ZURMÜHL, *Matrizen*. Berlin, 1950, p. 316–322.
[3] P. S. DWYER, *Linear Computations*. New York, 1951.
[4] I. M. GEL'FAND, *Lekt͡sii po Lineĭnoĭ Algebre* [*Lectures on Linear Algebra*]. Moscow 1951, p. 233–239.
[5] H. E. FETTIS, "A method for obtaining the characteristic equation of a matrix and computing the associated modal columns," *Quart. Appl. Math.*, v. 8, 1950, p. 206–212.
[6] G. BIRKHOFF & S. MACLANE, *A Survey of Modern Algebra*. New York, 1941.

RECENT MATHEMATICAL TABLES

1122[A,B,C].—P. P. ANDREEV, *Matematicheskie Tablit͡sy* [Mathematical Tables]. Moscow, 1952, 471 p. 12.5 × 19.7 cm. Price 7.75 rubles.

The main table of this work is a table of n^s for $n = 1(1)10000$, $s = -1/2$, 3, 2, 1/2, 1/3. Values are given to 6S only. The values of $(10n)^{1/2}$ are also given for $n > 1000$ while the natural logarithm of n is given for $n \leq 1000$. This part occupies 333 pages, only 30 values of n being devoted to each page. This table is certainly no substitute for BARLOW.

The second part of the volume is devoted to 19 small tables of minor importance including a 6S table of $1/n$ for $n = 1(1)10000$, log n for $n = 1(1)1000$ to 9D, and the binomial coefficients of the first 50 integral powers.

D. H. L.

1123[A].—M. LOTKIN & M. E. YOUNG, *Table of Binomial Coefficients*. Ballistic Research Laboratories *Memo. Report* No. 652. Aberdeen Proving Ground, 1953, 37 p., 21.6 × 27.9 cm. mimeographed from typescript.

The table gives

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \binom{n}{n-r}$$

for non-negative integers r and for $n = 0(1)100$. Because of symmetry the table is for $r \leq (n + 1)/2$. The values of the coefficients are to 20 significant figures. This means that exact values are available for $n \leq 69$ whereas only the first 19 values of $\binom{100}{r}$ are exact.

The table was prepared first in exact form and then abridged by rounding to 20 figures. An unabridged edition would have been of some value to number theorists who are concerned with congruence properties of binomial coefficients. The present table extends the table of PETERS & STEIN[1] from $n = 60$.

D. H. L.

[1] J. PETERS & J. STEIN, *Zehnstellige Logarithmentafel*, Band I. Berlin, 1922. Appendix, p. 69–74.

1124[A, B, C, D, E, F, L].—P. WIJDENES, *Noordhoff's Wiskundige Tafels in* 5 *Decimalen, Mathematical Tables to* 5 *Decimal Places*. Groningen, 1953, viii + 269 p., 23.8 × 17.1 cm. Price 8.75 florins.

This collection of tables contains unusually elaborate tables of the trigonometric functions such as a 5D table of $\cot\alpha$ for angles up to 3 degrees at intervals of one second (taken from RHETICUS' *Canon Sinuum*).

Other tables not usually found in 5 decimal collections include a list of primes below 10000 and their natural logarithms, a factor table to 11197, a 6D table of e^x and e^{-x} for $x = .001(.001)1(.01)4$, and some minor tables of the factorial function, the sine, cosine, and exponential integrals, the error function, and the Bessel functions J_0, J_1, Y_0, Y_1. There are many conversion tables, some with real accuracy. The result is a neat, well-printed, and well-arranged handbook of useful tables.

The introduction and explanations are in 6 languages, including Spanish and Malayan.

D. H. L.

1125[A,B,C,D,F].—TSUNETA YANO, *Kokumin Suhyô* [*People's Tables*]. Tokyo, 1952, iv + 146 p., 12.1 × 17.8 cm. Price 230 yen.

This little handbook of useful tables includes the following:

1. p. 60–79—Multiplication table $A \cdot B = N$ for $A = 1(1)100$ and $B = 1(1)99$.
2. p. 80–81—Table of reciprocals, $100/N$, $N = 1(1)1000$ to 3D.
3. p. 82–83—Table of A^k, $k = 2, 3, 1/2, 1/3$, $A = 1(1)100$.
4. p. 84—Natural sines, cosines, and tangents for every degree of the first quadrant.
5. p. 85–93—Factor table giving the least prime factor of all numbers prime to 10 between 1000 and 10000.
6. p. 94–95—Table of the first nine multiples of primes ≤ 557 except 2,5 arranged so that the final decimal digits are 1, 2, 3, $\cdots$, 9. With each p_n is given $p^2_{n+1} - 1$. This table is intended to be used in testing a given number for small prime factors.

7. p. 96–115—Common logarithms, log N for $N = 1(1)999$ to 15D, log p for primes p between 1000 and 10000 to 7D, Δ; log N for $N = 1(1)150$ to 7D, and log N for $N = 99900(1)99999$ to 15D, Δ, Δ^2.
8. p. 116–120—Antilogarithms to 4D, Cologarithms to 4D, and 6D tables of kM and k/M $(k = 1(1)100)$, $M = \log e$.

The rest of the tables are non-mathematical, being tables of weights and measures, dates of emperors, latitudes and longitudes of Japanese cities and description of 12 wind intensities.

D. H. L.

1126[B,F].—D. R. KAPREKAR, *Cycles of Recurring Decimals* (*From* $N = 3$ *to* 161 *and some other numbers*). Khare Wada, Deolali, India, 1950. Published by the author, vi + 55 + 2 p. 24.3 × 16.7 cm. Price 5 rupees.

This work contains tables of the complete periods in the decimal representation of M/N where the integers M and N are prime to each other and N is prime to 10 and does not exceed 163 (not 161 as implied in the title). Since the decimal representation of M/N is a cyclic permutation of that of many other rational numbers with the same denominator, it is possible to save much space by listing only those decimal representations which are not cyclic permutations of another. Thus for $N = 21$ there are the two cycles written as follows:

1	10	16	13	4	19		2	20	11	5	8	17
0	4	7	6	1	9		0	9	5	2	3	8

The first line gives the various numerators M and that cyclic permutation of the second line which begins with the digit under M gives the period of the decimal representation of M/N. Thus

$$13/21 = .619047619\cdots.$$

In other words, the first line consists of the successive powers of 10 modulo $N = 21$.

The booklet contains reprints of several notes by the author on decimals. It should be useful for pencil computers who dislike long division.

D. H. L.

1127[C].—S. KHRENOV, *Semiznachnye Tablit͡sy Trigonometricheskikh͡ Funkt͡siĭ* [*Seven-place Tables of Trigonometric Functions*]. Moscow-Leningrad, 1951, 415 p., 21.6 × 29.2 cm. Price 33.75 rubles.

These tables give the natural values of the functions. All six functions are given to 7S for every 10 seconds of arc. This main table is preceded by a large 7S table of cotangents and cosecants for every second up to 10 degrees. Besides these two big tables there are 9 minor conversion tables for astronomical applications etc.

The page is large and well set out with ample room for first differences and the usual tables of proportional parts. The printing is somewhat irregular but perfectly legible and the paper is of better quality than one is used to in Russian publications.

Apparently this work is original although the author doesn't actually say so. No details of calculation are mentioned. He states that the tables were compared with those of GIFFORD, ANDOYER, and PETERS.

D. H. L.

1128[F].—K. GOLDBERG, "*A table of Wilson quotients and the third Wilson prime*" London Math. Soc., *Jn.*, v. 28, 1953, p. 252–256.

Wilson's quotient is the integer

$$w_p = [1 + (p-1)!]/p$$

where p is a prime. The author defines a Wilson prime as a prime p for which w_p is divisible by p. That 5 and 13 are Wilson primes has been known for a long time. MATHEWS[1] has asked for a rule to discover other Wilson primes. The author used the SEAC to show that below 10000 there is but one other, namely, $p = 563$. His table gives w_p reduced modulo p for all primes $p < 10000$.

Wilson's quotient occurs in the theory of symmetric functions modulo p, Bernoulli numbers and Fermat's last theorem. The previously published table of BEEGER[2] extended to $p = 300$. The unpublished table of WALL is for $p \leq 5381$ and is described in UMT **150**, *MTAC*, v. 6, p. 238.

D. H. L.

[1] G. B. MATHEWS, *Theory of Numbers*. Cambridge 1892, New York, 1927, p. 318.
[2] N. G. W. H. BEEGER, "On the congruence $(p-1)! \equiv -1 \pmod{p^2}$," *Messenger of Math.*, v. 49, p. 177–178, 1920.

1129[F].—SIGEKATU KURODA, "Über die Zerlegung rationaler Primzahlen in gewissen nicht-abelschen galoisschen Körpern," Math. Soc. Japan, *Jn.*, v. 3, 1951, p. 148–156.

The decomposition of rational prime numbers is studied in certain non-abelian normal fields of degree 2^n. The laws for these decompositions have only been fully explored in the case of abelian fields and no complete extension of class field theory to non-abelian fields has yet been achieved. The fields K^* in question are of the form $K^* = R^*K$, where K is a field $R(i, \sqrt{\mu}, \sqrt{\bar{\mu}})$, where μ is an integer of $R(i)$ without square divisors, neither real nor purely imaginary and $\bar{\mu}$ is the complex conjugate of μ. The field R^* is composed of all quadratic extensions $R(\sqrt{l})$ where l runs through the prime factors of the discriminant of $R(\sqrt{\bar{\mu}\mu})$. Let $\mu = c\alpha$ where c is a positive rational integer and α has no rational divisor and put $\alpha\bar{\alpha} = m$. The decomposition of the primes in K^* is linked up with the value of the biquadratic character symbol $(m/p)_4$.

The case $m = 65$ is discussed in detail and there is a table (p. 156) giving the value of the symbol $(65/p)_4$ for all 69 primes $p \leq 4549$ of the form $4x + 1$ which are quadratic residues of 5 and 13. There are two cases according as $p = x^2 + 65y^2$ or not. In the former case x and y are given.

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1130[F].—V. S. Nanda, "Tables of solid partitions," Nat. Inst. Sci. India, *Proc.*, v. 19, 1953, p. 313–314.

The author tabulates the function $p^{(3)}(n,m)$ which enumerates the number of solid partitions of n in which the smallest part is m. Since

$$p^{(3)}(n,m) = 0 \qquad \text{if } \tfrac{1}{2}n < m < n$$

and

$$p^{(3)}(n,n) = \tfrac{1}{2}n(n+1)$$

it suffices to tabulate the function for $m \leq \frac{1}{2}n$. This is done for $n = 1(1)25$. For $m = 1$ we have

$$p^{(3)}(n,1) = p^{(3)}(n-1),$$

where $p^{(3)}(n)$ is the number of unrestricted solid partitions of n generated by

$$\prod_{r=1}^{\infty} (1 - x^r)^{r(r+1)/2} = \sum_{n=0}^{\infty} p^{(3)}(n)x^n$$
$$= 1 + x + 4x^2 + 10x^3 + \cdots.$$

The reader is referred to a previous paper[1] for details and a small table of $p^{(3)}(n)$ for $n = 5(5)25$.

D. H. L.

[1] V. S. Nanda, "Partition theory and thermodynamics of multidimensional oscillator assemblies," Camb. Phil. Soc., *Proc.*, v. 47, 1951, p. 591–601.

1131[F].—G. Palamà & L. Poletti, "Tavole dei numeri primi dell'intervallo 12012000–12072060," Unione Mat. Ital., *Boll.* s. 3, v. 8, 1953, p. 52–58.

This is a 6 page table giving the 3684 primes between the limits mentioned in the title. This list was prepared by hand using the "Neocribrum" of Poletti.

1132[K].—E. E. Slutskiĭ, *Tablitsy dlia Vychisleniia Nepolnoĭ* Γ*-funktsii i Funktsii Veroiatnosti* χ^2 [*Tables for the Computation of the Incomplete Gamma Function and the probability function of* χ^2]. Edited by A. N. Kolmogorov. Leningrad, 1950, Acad. Nauk USSR, 71 p., 22.5 × 29.0 cm.

In the introduction the author states that the tables are intended for "finding the values of two integrals:

(1) The incomplete Gamma function

$$I(u, p) = \frac{1}{\Gamma(p+1)} \int_0^{u\sqrt{p+1}} x^p e^{-x}\, dx$$

and the probability function of χ^2:

$$(2) \qquad P(\chi^2, n) = \frac{1}{2^{(n-2)/2}\,\Gamma\left(\dfrac{n}{2}\right)} \int_x^{\infty} x^{n-1} e^{-\frac{1}{2}x^2}\, dx.\text{"}$$

He lists three reasons why Karl Pearson's, *Tables of the Incomplete Gamma Function*,[1] do not solve this problem completely. Then he continues:

"To remove the indicated shortcomings of K. Pearson's tables it has accordingly been necessary:

1) to construct tables for large values of n, up to $n = \infty$;
2) to construct satisfactory tables for small values of n, close to zero;
3) to facilitate interpolation to the highest possible degree.

It was of course impossible to meet all these requirements by any single construction. In view of this fact the tables are divided into parts."

The parts and their contents are as follows:

I $(\frac{1}{2}\chi^2)^{-n/2}(1-P(\chi^2, n)$	$\chi^2=0(.05).2(.1)100$	$n=0(.05).2(.1)6$
II (a) $P(\chi^2, n)$	$\chi^2=0(.1)3.2$	$n=0(.05).2(.1)6$
(b) $P(\chi^2, n)$	$\chi^2=3.2(.2)7(.5)10(1)m$ $17\leq m\leq 35$	$n=0(.1).4(.2)6$
III $P(\frac{1}{2}(t+(2n)^{\frac{1}{2}})^2, n)$	$t=-4(.1)4.8$	$n=6(.5)11(1)32$
IV $P(\frac{1}{2}(t+2x^{-1})^2, 2x^{-2})$	$t=-4.5(.1)4.8$	$x=0(.02).22(.01).25$

Table V gives the coefficients in the Everett and Newton interpolation formulas for an interval of .001.

All five tables are given to 5 decimal places except the Newton coefficients which are given to four places.

To facilitate interpolation the tables give the second and fourth central differences with respect to χ^2 and t and the second central difference with respect to n and x. The introduction contains one section discussing methods of interpolating with respect to two variables in general, and another section giving practical instructions for the use of the tables. The latter section gives examples of the recommended interpolation methods.

As an evaluation of these tables, two statements made by Kolmogorov in the preface are probably correct. He says, "E. E. Slut͡skiĭ's tables provide a firm basis for the preparation of every kind of simplified tables intended for lesser accuracy" and "As far as I know, such a comprehensive tabular representation of so complicated a function of two variables appears in mathematical literature for the first time. The labor of the last years of E. E. Slut͡skiĭ's life, published after his death, has therefore an even wider methodological significance: it points the path to further work on the preparation of tables of great accuracy of functions of two variables."

In the reviewer's opinion, these tables do not provide the final answer to the tabulation of the incomplete Gamma function. In the first place, values are given to only five decimal places: this accuracy is insufficient for many present-day computing requirements. A second and more serious deficiency of the tables is that they do not provide for inverse interpolation. The reason for giving only five decimal places is that Slut͡skiĭ computed most of his values by interpolation in Karl Pearson's seven place tables. The reason for not giving an inverse table probably is the computational complexity involved in computing it. It is to be hoped that makers of future tables of probability functions will consider the problem of inverse interpolation.

D. TEICHROEW

NBSINA

[1] K. PEARSON, *Tables of the Incomplete* Γ-*function*. London, 1934.

1133[L].—ADMIRALTY RESEARCH LABORATORY, "A solution of the equation $(y'')^2 = y\,y'$," A.R.L./T.1/Maths 2.7, 16 p.; "Table of $F(\beta,\rho)$," A.R.L./T.2/Maths 2.7, 10 p., Teddington, Middlesex, England.

These are the photostat tables referred to in **RMT** 1041, *MTAC* v. 6, 1952, p. 235–236, where F is denoted by f.

1134[L].—ADMIRALTY RESEARCH LABORATORY, "Tables of $F(x)$ and of $x^{-1}F(x)$," A. R. L./T.4/Maths. 2.7, 8 p., Teddington, England.

"The tables give 4D values of $F(x)$ and of $x^{-1}F(x)$,

$$F(x) = 2x \sum_{n=0}^{\infty} [x^2 + (2n+1)^2]^{-3/2}$$

for $x = 0(.002)1.5(.01)5$, and are not in error by more than .7 of a unit of the last place. Linear interpolation is adequate throughout."

1135[L].—ADMIRALTY RESEARCH LABORATORY, "Tables of $G(x)$," A.R.L./T.5/Maths. 2.7, 5 p., Teddington, Middlesex, England.

$$\text{“}G(x) = 6x^2 \sum_{n=0}^{\infty} [x^2 + (2n+1)^2]^{-5/2}$$

is tabulated here to 4D for $x = 0(.002)1.5(.01)5$, with a maximum uncertainty of about two units of the last place. The table is linear."

1136[L].—ADMIRALTY RESEARCH LABORATORY, "Tables of $f(x)$ and of $xf(x)$," A.R.L./T.6/Maths. 2.7, 4 p., Teddington, Middlesex, England.

"The tables give 4D values of $f(x)$ and of $x\,f(x)$,

$$f(x) = \sum_{n=0}^{\infty} (-)^n(2n+1)[x^2 + (2n+1)^2]^{-5/2}$$

for $x = 0\ (.005)2(.05)5$, and are not in error by more than .7 of a unit of the last place. Both tables are linear."

1137[L].—W. E. BLEICK, *Tables of Associated Sine and Cosine Integral Functions and of Related Complex-Valued Functions.* Technical Report No. 10, U. S. N. Bureau of Ships, Monterey, 1953, 103 p. 20.3 × 26.7 cm. Mimeographed.

The author defines

$$(1) \qquad Sia(x,y) = \int_0^x \frac{t \sin t}{t^2 + y^2}\,dt; \quad Cia(x,y) = \int_\infty^x \frac{t \cos t}{t^2 + y^2}\,dt$$

$$(2) \quad Si(x+iy) = \int_{0+iy}^{x+iy} t^{-1} \sin t\,dt; \quad Ci(x+iy) = \int_{\infty+iy}^{x+iy} t^{-1} \cos t\,dt.$$

In (2), the path of integration is to be taken parallel to the x-axis, and the branch cut is taken on the negative real axis, including the origin. Then it can be shown that

$$Sia(x,y) = \text{Re}Si(x+iy) \cosh y + [\text{Im}Ci(x+iy) - \pi/2] \sinh y$$

$$Cia(x,y) = \text{Re}Ci(x+iy) \cosh y - [\text{Im}Si(x+iy) + \tfrac{1}{2}\,\text{Ei}(y) - \tfrac{1}{2}\,\text{Ei}(-y)] \sinh y,$$

where

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} t^{-1} e^{t}\, dt$$

is the well known exponential integral.

It is to be noted that the author's definition of $Si(z)$ differs from the one usually adopted, and that his function is not an analytic function of $z=x+iy$. However, if we let $f(z)$ denote the usually accepted function $Si(z)$, with the lower limit of the integral in (2) at the origin, then

$$f(z) = Si(z) + \tfrac{1}{2}\,[\mathrm{Ei}(y) - \mathrm{Ei}(-y)]i.$$

Thus $f(z)$ can be obtained rather simply from $Si(z)$; the five functions are identical when z is real.

The following three tables are given:

Table I: $\mathrm{Re}(Ci(x + iy))$, $\mathrm{Im}Ci(x + iy)$
Table II: $\mathrm{Re}Si(x + iy)$, $\mathrm{Im}Si(x + iy)$
Table III: $Sia(x,y)$, $Cia(x,y)$.

All three tables are given for $x,y = 0(.1)3.1$, with the origin excluded. Entries are to 12D, but only 10D are guaranteed.

There is a useful Introduction with some of the basic properties of the functions and asymptotic expansions.

G. Blanch

NBSINA

1138[L].—T. M. Cherry, "Tables and approximate formulae for hypergeometric functions, of high order, occurring in gas-flow theory," R. Soc. London, *Proc.*, v. 217A, 1953, p. 222–234.

If trans-sonic flow is discussed by means of the hodograph method, the functions

$$\chi_\nu(\tau) = \tau^{\frac{1}{2}\nu} F(\nu - a_\nu, \nu - b_\nu; \nu + 1; \tau)$$
$$\psi_\nu(\tau) = \tau^{\frac{1}{2}\nu} F(a_\nu, b_\nu; \nu + 1; \tau)$$

are needed. Here a_ν and b_ν are determined by

$$a_\nu + b_\nu = \nu - \frac{1}{\gamma - 1},\ a_\nu b_\nu = -\frac{\nu(\nu + 1)}{2(\gamma - 1)},\ a_\nu > b_\nu,$$

F is Gauss' hypergeometric series, and the adiabatic exponent, γ, is taken to be 1.4 in this paper.

The point $\tau_s = (\gamma - 1)/(\gamma + 1) = 1/6$ is a transition point of the differential equations satisfied by χ_ν and ψ_ν, and in an interval including this point there is no asymptotic representation in terms of elementary functions. Another difficulty connected with these functions is caused by the fact that they are not defined when ν is a negative integer. For this reason the author introduced certain logarithmic solutions $\chi y_\nu(\tau)$, $\psi y_\nu(\tau)$ which exist for positive integer ν (except ψy for $\nu = 1$), and which are analogous to Bessel functions of the second kind.

The functions $\chi_\nu(\tau)$, $\psi_\nu(\tau)$, $\chi y_\nu(\tau)$, $\psi y_\nu(\tau)$ were investigated in a previous paper by the author;[1] in that paper there are also references to the literature.

Tables 1 and 2 (p. 227–233) of the present paper give numerical values of certain slowly varying auxiliary functions sufficient to calculate $\chi_\nu(\tau)$, $\psi_\nu(\tau)$, $\chi y_\nu(\tau)$, $\psi y_\nu(\tau)$, $\chi_\nu'(\tau)$, $\chi y_\nu'(\tau)$ for $\tau = .08\ (.02).30$, $\nu = 10.5\ (1)30.5$. The entries are given to 6S for $\nu \leq 20.5$, and to 4S, $\nu > 20.5$. These tables supplement earlier tables referred to in *MTAC*, v. 3, p. 522 (RMT **676**) and v. 6, p. 30–31 (RMT **953**). Table 3 (p. 233) gives 7D values of

$$\frac{\Gamma(1 + a_\nu)\Gamma(\nu - b_\nu)2\pi\delta^{2\nu}}{\Gamma(1 + a_\nu - \nu)\Gamma(-b_\nu)\Gamma(1 + \nu)\Gamma(\nu)}$$

for $\nu = 10.5\ (1)\ 30.5$, together with a simple approximation which is good to 8D when $\nu \geq 20$.

If $\nu > 30$, the auxiliary functions of Tables 1 and 2 may be approximated by algebraic functions. The coefficients in these algebraic functions are tabulated in Tables 4 and 5, p. 233–234.

A. E.

[1] T. M. CHERRY, "Asymptotic expansions for the hypergeometric functions occurring in gas-flow theory," R. Soc. London, *Proc.* v. 202A, 1950, p. 507–522.

1139[L].—GEOFFREY KELLER & MARY FENWICK, "Tabulation of the incomplete Fermi-Dirac functions," *Astrophys. Jn.*, v. 117, 1953, p. 437–446.

The function tabulated here is

$$F(\eta,u) = \int_0^u (e^{x-\eta} + 1)^{-1} x^{1/2}\, dx,$$

and the authors give 3, 4, or 5 S values for $\eta = -2\ (.5)\ 10$, $u = 0\ (.2)\ 1\ (.1)\ X_\eta$, where X_η is that value of u past which $F(\eta,u)$ remains constant (to the degree of accuracy chosen). X_η runs from 6.8 (for $\eta = -2$) to 19.5 (for $\eta = 10$). The authors believe that the maximum error in the entries is less than two units in the last place.

The authors give approximations in terms of the probability integral for $\eta < -2$, and in terms of the functions

$$G_0(v) = \int_v^\infty (e^y+1)^{-1}\, dy = \log\,(e^{-v}+1),\quad G_n(v) = C_n \int_0^v (e^y+1)^{-1}\, y^n dy,$$
$$C_1 = 1/2,\ C_2 = 1/8,\ C_3 = 1/16, \cdots,$$

for $\eta > 10$. In the latter case $u \leq \eta$ and $u \geq \eta$ have to be treated separately. A 3-4D table of $G_n(v)$ for $n = 0(1)3$, $v = 0(.1)10$ is appended.

A. E.

1140[L].—H. LOTTRUP KNUDSEN, *Bidrag Til Teorien For Antennesystemer Med Hel Eller Delvis Rotations-symmetri.* I Kommission Hos Teknisk Forlag, Copenhagen, 1953, 228 p.

The author gives tables of $\bar{J}_n(x) = \int_0^x J_n(t)dt$, $n = 0,1,2,\cdots, 8$, $x = 0(.01)10$, 5D and a rough graph of $\bar{J}_n(x)$, $x = 0(.5)10$. $\bar{J}_n(x)$ was computed recursively using values of $\bar{J}_0(x)$ (10D, $x = 0(.01)10$) tabulated by LOWAN & ABRAMOWITZ.[1] A table of $\frac{1}{2}\bar{J}_0(x)$, 7D, $x = 0(.02)1$, is given by WATSON.[2]

In addition, there is a table of integrals of the form $\int_0^x J_n(t) \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} \alpha dt$, $\alpha = t, x - t$.

DONALD RUBIN

NBSCL

[1] A. LOWAN & M. ABRAMOWITZ, "Table of integrals $\int_0^x J_0(t)dt$ and $\int_0^x Y_0(t)dt$," *Jn. Math. Phys.*, v. 22, 1943, p. 1–12.

[2] G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, 2d ed., Cambridge, 1944, p. 752.

1141[L].—E. KREYSZIG, "Der allgemeine Integralkosinus Ci(z, α)," *Acta Math.* v. 89, 1953, p. 107–131.

The generalized sine integral was investigated in an earlier paper[1] by the same author. The present paper contains the corresponding investigation of the generalized cosine integral

$$\mathrm{Ci}(z, \mu) = \int_0^z t^{-\mu} \cos t\, dt \qquad \mathrm{Re}\mu < 1.$$

Table 1 (p. 120) gives 3D values of Ci(x, α) for $x = 0(.2)4(.5)20$, $\alpha = .25, .5, .75$.

Table 2 (p. 121–123) gives 2D or 3S values of the real and imaginary parts of Ci$(x + iy, \alpha)$ for $x = 0(1)20$, $y = 0(1)5$, $\alpha = .25, .5, .75$.

Table 3 (p. 124) gives real and imaginary parts of the first three pairs of simple zeros of Ci(z, α) for $\alpha = .25, .5, .75$.

Table 4 (p. 125) gives those values of $\alpha = \omega(n)$, to 4D, for which a double zero occurs at $x = \left(2n + \frac{3}{2}\right)\pi$, $n = 0(1)10$.

Table 5 (p. 125) gives 5D values of Ci(∞, α) for $\alpha = .001(.001).02(.005).045$.

This paper, as its predecessor, is accompanied by relief diagrams, altitude charts, and a bibliography. Between them, the two papers give an adequate picture of the incomplete gamma function in the complex plane, for the three selected values of α.

A. E.

[1] E. KREYSZIG, "Ueber den allgemeinen Integralsinus Si(z, α)," *Acta Math.*, v. 85, 1951, p. 117–181. *MTAC*, v. 5, p. 156.

1142[L].—MARCEL MAYOT, "Tables de fonctions intervenant dans le calcul des corrections de diffusion dans la photométrie de la lumière du ciel nocturne," *Ann. Astrophysique*, v. 15, 1952, p. 374–382.

The integrals

$$\Phi_i(\tau) = \int_0^{\frac{1}{2}\pi} (1 - \alpha^2 \sin^2 \theta)^{-\frac{1}{2}} e^{-\tau \sec \theta} \sin \theta \cos^i \theta\, d\theta$$

are tabulated to 4D for $i = 0, 1, 2$, $\tau = 0\ (.05).7$, $\alpha = R/(R + H)$, $R = 6370$, $H = 0, 50, 100, 200, 400, 800, \infty$.

Several series expansions of these integrals are obtained, and the method of computation is described.

A. E.

1143[L].—WASAO SIBAGAKI, *Theory and Applications of the Gamma Function, with a Table of the Gamma Function for Complex Arguments Significant to the Sixth Decimal Place.* (Japanese.) 202 p., Tokyo, Iwanami Syoten, 1952. 18.5 × 25.5 cm.

The table gives 6D values of the real and imaginary parts of $\ln \Gamma(x + iy)$ for $x = -10(.2)-6(.1)10.4$, $y = 0(.1)2(.2)10$. For a previous tabulation of $\ln \Gamma(x + iy)$ see RMT **234** (*MTAC*, v. 2, p. 19), for other tables of the gamma function in the complex domain see RMT **855** (*MTAC*, 5, p. 25–26).

A. E.

MATHEMATICAL TABLES—ERRATA

231.—(1) F. CALLET, *Tables Portatives de Logarithmes*, Paris 1795 and many later editions.

(2) F. MASERES, *Scriptores Logarithmici*. London 1796, v. 3, p. 119–123.
(3) H. M. PARKHURST, *Astronomical Tables*, New York 1868, 1889.
(4) J. PETERS & J. STEIN, *Zehnstellige Logarithmentafel*, Band 1, Berlin 1922. *Anhang*. Table 14b, p. 156–161.

The tables referred to are the 61D common logarithms of primes between 100 and 1098 originally calculated by ABRAHAM SHARP. Seven errors occurring in all four tables have been noted by UHLER[1] as the result of an extensive examination of (4). Three of these are last figure errata.

Only five of these seven errata occur in Sharp's table of 1717, his value for log 1097 being correct. [*MTAC*, v. 1, p. 58, v. 7, p. 171, and R. C. ARCHIBALD, *Mathematical Table Makers*, p. 73].

[1] H. S. UHLER, "Omnibus checking of the 61-place table of denary logarithms compiled by Peters and Stein, by Callet and by Parkhurst," National Acad. Sci. *Proc.*, v. 39, 1953, p. 533–537.

232.—RUEL V. CHURCHILL, *Modern Operational Mathematics in Engineering*, 1944, p. 296, eq. 33.

For the Laplace transform pair

$$\frac{s^n}{(s^2 + a^2)^{n+1}} \subset \frac{t^n \sin at}{2^n a\, n!}$$

read

$$\frac{s^n}{(s^2 + a^2)^{n+1}} \subset \sum_{0=\mu}^{n} A_{n,\mu}\, t^{n-\mu} \sin\left(at + \frac{\pi\mu}{2}\right)$$

where

$$A_{n,\mu} = \sum_{\nu=0}^{\mu} \frac{(-1)^\nu a^{-(\mu+1)} (n + \mu - \nu)!}{2^{n+\mu-\nu} (n - \nu)!\, (\mu - \nu)!\, (n - \mu)!\, \nu!}$$

$$A_{n,0} = \frac{1}{2^n n! a}$$

$$A_{n,1} = \frac{-(n - 1)}{2^{n+1} (n - 1)!\, a^2}$$

$$A_{n,2} = \frac{n^2 - 5n + 2}{2^{n+2} (n - 2)!\, 2!\, a^3}$$