REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[F].—A. GLODEN, *Table des factorisation des Nombres* $N^4 + 1$ *dans l'intervalle* $10000 < N \leq 20000$. Manuscript of 84 leaves deposited in the UMT FILE.

This is an extension of earlier tables by the same author. Many of the entries have been completely factored. All unknown factors lie beyond 800 000. The author is preparing a table for the range $20000 < N \leq 30000$. [For previous tables of this kind see *MTAC*, v. 2, p. 211, 252, 300; v. 3, p. 21, 118–9, 486; v. 4, p. 224; v. 5, p. 28, 1334; v. 6, p. 102; v. 7, p. 33–4; v. 8, p. 166.]

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2[F].—RUDOLPH ONDREJKA, *List of the First 17 Perfect Numbers.* Two typewritten pages deposited in the UMT FILE.

Decimal values of the first seventeen perfect numbers, having the following respective numbers of digits: 1, 2, 3, 4, 8, 10, 12, 19, 37, 54, 65, 77, 314, 366, 770, 1327, 1373.

Computation of the first twelve perfect numbers was done by the author with the use of his table of 2^n, $n = 1(2)411$. The last five were computed by H. S. UHLER and were checked by the present author. The present list is believed by the author to be error-free.

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3[F].—R. J. PORTER, *A List of Groups and Series to serve for computations of Irregular Negative Determinants of Exponent* $3n$. 274 typewritten pages deposited in the UMT FILE.

This is very closely related to the author's UMT **155** [*MTAC*, v. 7, p. 34] and UMT **185** [*MTAC*, v. 8, p. 96–7].

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4[K].—D. V. LINDLEY & J. C. P. MILLER, *Cambridge Elementary Statistical Tables.* Cambridge University Press, London, 1953, 35 p., 21.9 × 27.9 cm. Price $1.00. (paper)

"This set of tables is concerned only with the commoner and more familiar and elementary of the many statistical functions and tests of significance now available.—The more familiar statistical tests are either based directly on the normal distribution or, in the case of the t, χ^2 and F tests, they are derived therefrom. Percentage points for these tests are provided in the tables, mainly for significance levels 5%, 1%, and 0.1% in both one-sided and two-sided tests.—Tables of the more common transformations (of the data), square root, logarithm, inverse circular and hyperbolic root-sines, together with that for the correlation coefficient, have been included." (From the Preface.)

A list of the tables follows:

1, 2. Cumulative normal $\Phi(x)$ to 4D with differences for $x = 0(.01)3(.1)4$ with normal ordinates to 4D for $x = 0(.1)4$, special values of x for $\Phi(x) = .001(.001).03(.002).05(.05).50$ and several special values.

3. Percentiles 87.5, 95, 97.5, 99, 99.5, 99.9, 99.95 of the t-distribution for degrees of freedom 1(1)10, 12, 15, 24, 30, 40, 60, 120, ∞ to 2D.

4. $z = \tanh^{-1} r$ for $r = 0(.02).8(.01).94(.001)1$, to 3D with first differences. This is a transformation of the correlation coefficient.

5. Percentiles .5, 1, 2.5, 5, 90, 95, 97.5, 99, 99.5, 99.9 of the χ^2 distribution for degrees of freedom 1(1)30(10)100 to 3 or 4S.

6. Conversion of range to standard deviation for sample size $n = 2(1)13$ to 4D. (Ratio of expected values.)

7. Percentiles 95, 97.5, 99, 99.9 of the F-distribution for degrees of freedom $\nu_1 = 1(1)8, 10, 12, 24, \infty$ and degrees of freedom $\nu_2 = 1(1)30(2)40, 60, 120, \infty$ to 3 or 4S (in general 2D).

8. 2000 random digits.

9. Various functions.

For $n = 0(1)100$: n^2, $\sqrt{n}$ to 4D, $1/n$ to 5D, $1/\sqrt{n}$ to 5D.

For $x = 0(.01)1$: $\sin^{-1}\sqrt{x}$, $\sinh^{-1}\sqrt{x}$, $\sinh^{-1}\sqrt{10x}$, $\sinh^{-1}\sqrt{100x}$ all to 3D with first differences.

For $x = 0(.01)10$: x^2 exact; $\sqrt{x}$, $\sqrt{10x}$, $1/x$, $1/\sqrt{x}$, $1/\sqrt{10x}$ each to 4S when the first significant digit is >2 and to 5 figures when <2, with first differences; $\log x$ to 4D with first differences.

For $\log t = 0(.001)1$: t to 4D for $t < 2$ and to 3D for $t > 2$ with first differences.

10. $\log n!$ for $n = 0(1)300$ to 4D.

The tables are arranged in a convenient format with notes on interpolation and asymptotic expressions for values beyond the given tables. A simple but not exact description of the tables is that this book is an abbreviated form of the FISHER & YATES Tables,[1] since that book contains, among other tables, most of those listed above. The principal area in which the book under review is more complete is in Table 7 and part of Table 9. Tables 3, 5, and 7 are based on tables from *Biometrika* but contain some additional values. Values which are reported here that do not appear either in the *Biometrika* tables or those of FISHER & YATES are those for $\nu_2 = 32(2)38$ for all percentiles and those for $\nu_1 = 7$ and 10 for percentile 99.9. A number of differences of a single unit in the final place were noted in the 99.9% F-table between the FISHER & YATES Tables[1] and the tables under review.

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[1] R. A. FISHER & FRANK YATES, *Statistical Tables for Biological, Agricultural, and Medical Research*. London & Edinburgh, 4th ed., 1953.

5[K].—H. G. ROMIG, *50–100 Binomial Tables.* New York, John Wiley & Sons, 1953, xxvii + 172 p., 18.7 × 22.4 cm., $4.00.

These useful tables supplement the National Bureau of Standards Tables[1] which give individual terms and partial sums of terms of $(q + p)^n$ to 6D for $p = .01(.01)50$ and $n = 2(1)49$. Here the same quantities are given also to 6D for $n = 50(5)100$. The arrangement differs from that of the previous tables by giving all the entries for each n, p pair in adjacent columns, values of individual terms in one and cumulative sums in the other.

Dr. ROMIG provides very adequate explanation and illustrative examples to assure proper use of the *Tables.* The author also includes exact interpolation formulas determined by proper transformations of the interpolation formulas for the incomplete beta-functions. These formulas may be used for those probability determinations for intermediate p and n values for which ordinary linear interpolation is not satisfactory. In addition, there is a satisfying list of references which more completely cover the theory of interpolation.

These tables will find many uses in numerous areas of statistical analysis. Especially will they be useful in the area of statistical quality control where more refined evaluations of the binomial probabilities are necessary to determine the protection afforded by proposed sampling procedures in process control techniques as well as in acceptance plans.

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[1] NBSCL, *Tables of the Binomial Probability Distribution.* AMS no. 6, Washington, 1950. [*MTAC*, v. 4, p. 208–209.]

6[K].—W. L. STEVENS, "Tables of the angular transformation," *Biometrika*, v. 40, 1953, p. 70–73.

The following form of the angular transformation

$$\theta = 50 - \lambda \arcsin (1 - 2p), \quad \lambda = \sqrt{1000}$$

has been tabulated by the author in order to provide "a table similar in accuracy to that of the table of probits given by FISHER and YATES."[1] The author suggests that the present form of the transformation has the following advantages: (i) θ ranges from 0.327 to 99.673 and therefore has almost the maximum possible accuracy for any given number of significant figures; (ii) the weight is given by the extremely simple expression $n/1000$, where n is the number of observations; (iii) like the probit function, complementary values of the function correspond to complementary values of the argument $p = 50\%$ giving $\theta = 50$.

Three tables are presented, each to 3D. Table 1 gives θ for $100p = 0(0.1)50$ while for $100p > 50$ entry is made for $100(1 - p)$ and the transformed value is equal to 100 minus the tabular value. Proportional parts are given for linear interpolation. For small percentages Table 2 gives θ for $100p = 0(0.01)2.0$, with proportional parts for linear interpolation when $.05 < 100p < .2$. For $100p < 98$, θ is determined as above by subtracting the tabular value obtained in Table 2 for $100(1 - p)$ from 100. For $100p < .05$ the formula $\theta = 0.327 + 6.325\sqrt{p}$ may be used. Table 3 provides the values for proper fractions whose denominations are less than or equal to 30.

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[1] R. A. FISHER & F. YATES, *Statistical Tables for Biological, Agricultural and Medical Research.* London & Edinburgh, 1938.

7[K].—A. C. COHEN, JR. & JOHN WOODWARD, "Tables of Pearson-Lee-Fisher functions of singly truncated normal distributions," *Biometrics*, v. 9, 1953, p. 489–497.

A normal variable, x, with frequency function,

$$\phi(x) = (2\pi)^{-\frac{1}{2}}\sigma^{-1} \exp (x - m)^2/2\sigma^2,$$

for all real x, truncated (below) at x_0', gives the truncated variable, x', with frequency function, $f(x') = \phi(x')/I_0(x')$, $x_0' \leq x'$, where $I_0(x') = \int_{x_0'}^{\infty} \phi(x)dx$. It is required to estimate m and σ on the basis of a sample of n observations on $x = x' - x_0'$.

Pearson and Lee[1] gave estimates based on the first two observed moments. FISHER[2] showed their estimates to be maximum likelihood estimates. The authors estimate $\xi = \dfrac{x_0' - m}{\sigma}$ and σ by the relations (equivalent to the earlier ones)

$$(1) \qquad \frac{n\sum x^2}{2(\sum x)^2} = \frac{1}{2}\left[\frac{1}{z - \xi}\right]\left[\frac{1}{z - \xi} - \xi\right] = g(\xi),$$

and

$$(2) \qquad \sigma \equiv \frac{\sum x}{n}\left[\frac{1}{z - \xi}\right] = \frac{\sum x}{n} h(\xi),$$

where $z = \phi(\xi)/I_0(\xi)$. To facilitate computations they give tables of $h(\xi)$ and $g(\xi)$ to 8D except for the largest values of ξ (where 7D and 6D are given) for $\xi = -4.(.1) -2.5(.01).5(.1)3$. The authors suggest using (1) to estimate $\hat{\xi}$ and (2) with $\hat{\xi}$ from (1) to estimate $\hat{\sigma}$.

The variances of the estimates and the correlation coefficient between the estimates are given by

$$\text{var}(\hat{\xi}) = \frac{\sigma^2}{n}\frac{1 - z(z - \xi)}{[1 - z(z - \xi)][2 - \xi(z - \xi)] - [z - \xi]^2} = \frac{\sigma^2}{n} W'(\xi),$$

$$\text{var}(\hat{\sigma}) = \frac{1}{n}\frac{2 - \xi(z - \xi)}{[1 - z(z - \xi)][2 - \xi(z - \xi)] - [z - \xi]^2} = \frac{1}{n} w'(\xi),$$

and

$$\rho(\hat{\xi}, \hat{\sigma}) = \frac{z - \xi}{\sqrt{[1 - z(z - \xi)][2 - \xi(z - \xi)]}}.$$

Tables of $W'(\xi)$ and $w'(\xi)$ are given to 6D and, of $\rho(\xi, \sigma)$ to 4D for $\xi = -3.(.1)2$.

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[1] KARL PEARSON and ALICE LEE, "On the generalized probable error in multiple normal correlation," *Biometrika*, v. 6, 1908, p. 59–68.
[2] R. A. FISHER in *BAAS Math. Tables*, v. I, London, 1931, p. xxvi–xxxv.

8[K].—P. V. K. IYER & A. S. P. RAO, "Theory of the probability distribution of runs in a sequence of observations," Indian Soc. Agricultural Stat., *Jn.*, v. 5, 1953, p. 29–77.

This paper investigates the theory of runs in which the succession of observations may be equal, ascending or descending. As such it augments the existing work on runs in which the equality assumption was not allowed. The purpose of the paper is to investigate the distribution of the number of ascending, descending and stationary runs in a sequence of n observations. Both the infinite case, where a particular value has a given probability of occurrence, and the finite case, where one knows the number of times a given value has occurred, is considered. For each of the three types of runs, the various related configurations are tabulated along with their probability of occurrence in the sequence and the number possible. No actual distributions are attempted in the paper. Variances and covariances for k kinds of elements with equal probabilities of occurrence are tabulated to 4D for $K = 2(1)5$, 10 and $n = 30, 40, 50, 75, 100$ for ascending, stationary, and descending runs as well as for the total number of runs. The author also lists the algebraic expressions for the covariances of runs of lengths p and q for $p = 1(1)4$, $q = p(1)5$, except for $p = 4$ only $q = 4$ is given; of runs of length p and q or more for $p, q = 1(1)4$; and of runs of lengths p or more and q or more for $p = 1, 2, 3$, $q = p + 1(1)4$. For junctions the actual distributions are given for values of $n = 4(1)7$.

Mention is made of the possibility of using the results for testing randomness but the actual discussion of such applications are to be given in a separate article.

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9[K].—S. H. ABDEL-ATY, "Tables of generalized k-statistics," *Biometrika*, v. 41, 1954, p. 253–260.

The author gives a complete table of order 12 for expressing the sample $k_{rs\ldots}$ statistics of TUKEY[1] in terms of the augmented monomial symmetric functions of DAVID & KENDALL[2] and vice versa. This is a very considerable extension of the table of WISHART,[3] which were through order 6, since results for lower orders are at once obtainable by a simple rule for the deletion of subscripts.

C. C. C.

[1] J. W. TUKEY, "Some sampling simplified," Amer. Stat. Assn., *Jn.*, v. 45, 1950, p. 501–519.
[2] F. W. DAVID & M. G. KENDALL, "Tables of symmetric functions—part I," *Biometrika*, v. 36, 1949, p. 431–449. [*MTAC*, v. 4, p. 146.]
[3] J. WISHART, "Moment coefficients of the k-statistics in samples from a finite population," *Biometrika*, v. 39, 1952, p. 1–13. [*MTAC*, v. 7, p. 97.]

10[K].—J. H. CADWELL, "The statistical treatment of mean deviation," *Biometrika*, v. 41, 1954, p. 12–18.

The author wants to obtain properties of the mean deviation m, which are analogous to the well-known properties of the standard deviation σ of a normal distribution. To this end, the distribution of the quotient m/σ is approximated by the χ^2 distribution. He matches the first two moments of the two distributions with a small discrepancy for the third moment. Let $\bar{m}(k, n)$ be the average of k mean deviations, each for samples of size n from a normal population with standard deviation σ. Then it is shown that $c[\bar{m}(k, n)/\sigma]^{1.8}$ has approximately the χ^2 distribution with v degrees of freedom. Table 1 gives the values c to 4S and v to 1D, the expected values of $\bar{m}(k, n)/\sigma$ to 4D and the variances of $\bar{m}(1, n)/\sigma$ to 5D as functions of k and n for $k = 1(1)10$, $n = 4(1)10$. Table 2 gives the same values for $k = 1(1)5$, $n = 10(5)50$. Table 3 gives the lower and upper 2.5 percent and 5 percent points of the probability function of $\bar{m}(k, n)$ for $k = 1(1)10$ and $n = 4(1)10$ to 3D. The values are exact for $k = 1$, and for other values the error will not exceed .003. For values of k beyond 10 a normal approximation can be used.

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11[K].—H. WEILER, "A new type of control chart limits for means, ranges, and sequential runs," Amer. Stat. Assn., *Jn.*, v. 49, 1954, p. 298–314.

In the usual theory of quality control charts the probability of a sample point falling outside the control limits, when the system is in control (a Type I error) is kept constant regardless of the number of items involved at each sample point. In the author's theory the average number of false alarms (Type I error) is a fixed percentage of the number of items tested, independent of the sample size. Given the variable X, normally distributed with known mean, m, and standard deviation, σ, let p be the probability that a random sample of n causes a false alarm at the upper limit, let a be the average number of articles tested before the false alarm is raised; then $a = n/p$. In Table I the author lists the values of B and $B/\sqrt{n}$ to 2D, for $a = 5000$, $n = 3(1)6, 8, 10$, since the upper control limit is given by $m + B\sigma/\sqrt{n}$. The lower control limit is given by $m - B\sigma/\sqrt{n}$. In Table II are listed the values of $B/\sqrt{n}$ to 3S for $n = 3(1)10(5)50$ and $a = 1000(1000)5000$. Suppose the population mean m changes to $m + k\sigma$, $k > 0$. For a given n and B, the average number of items tested $A(n)$ is a function of k. $A(n)$ is plotted against k for $a = 2000$, $n = 5, 10, 20, 50$ (Chart I), and for $a = 5000$, same values of n (Chart II). From these charts the author concludes that if large values of k are expected, $k > 1.6$ say, then small samples, e.g., $n = 5$, should be used; while if small values of k are expected, $k < 1$ say, large samples of n, e.g., $n = 10$ or 20, are more economical. In Chart III $A(n)$ is plotted against k for fixed sample size $n = 10$, $a = 1000, 2000, 3000, 5000$. This chart indicates that small values of a are useful only for the detection of small changes of the population mean.

Similarly for the control chart of the range in Table III are given to 2D the values of $W_1\sigma \leq R \leq W_2\sigma$, the control limits for the sample range R, for

$n = 3(1)10$, $a = 1000(1000)5000$. In Chart IV $A(n)$ is graphed for $n = 5, 10$, $1.0 \leq k \leq 2.4$, for the range. In Chart V $A(n)$ is graphed for $n = 10$, $a = 1000, 2000, 5000$ for the range. For the range it is assumed that σ changes from σ to $\sigma' = K\sigma$, $K > 1$.

The author next turns to the use of runs for controlling the mean, where λ = length of run. In Chart VI is graphed the value of $A(n)$ for $0 \leq k \leq 1.4$, $n = 4, 10, 20$ and $a = 4000$, $\lambda = 2$, and in Chart VII $A(n)$ is graphed for the same values of n and a but $\lambda = 3$. In Chart VIII $A(n)$ is graphed $0 \leq k \leq 1.3$ for $\lambda = 1$, $n = 20$, and for $\lambda = 3$, $n = 4$. In the range $0 < k \leq 1$, the use of $\lambda = 1$, $n = 20$ is superior from the power sense to $\lambda = 3$, $n = 4$.

The use of runs for the range charts reduces rather than improves the power of the chart and hence no charts are given. Considerable use is made of the author's two previous papers.[1, 2]

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[1] H. Weiler, "On the most economical sample size for controlling the mean of a population," *Ann. Math. Stat.*, v. 23, 1953, p. 247–254.
[2] H. Weiler, "The use of runs to control the mean in quality control," Amer. Stat. Assn., *Jn.*, v. 48, 1953, p. 816–825.

12[K].—S. Rosenbaum, "Tables for a nonparametric test of location," *Annals Math. Stat.*, v. 25, 1954, p. 146–150.

If two independent random samples of sizes n and m are drawn from a continuous statistical population, the probability that exactly i points of the sample of m will exceed the greatest value of the sample of n is: $Q_i = n\binom{m}{i}B(n + m - i, i + 1)$, where B is the complete Beta function. We can fix a probability level ϵ and determine a value s_0 such that:

$$\sum_{i=0}^{s_0-1} Q_i \leqq \epsilon < \sum_{i=0}^{s_0} Q_i.$$

This paper presents tables of $s = s_0 + 1$ for $\epsilon = .99, .95$ and $m, n = 1(1)50$. These results can be used to test whether two samples came from the same population. The argument is identical if the number of values of the sample of m which are less than the smallest value of the sample of n is considered.

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13[K].—H. Ruben, "On the moments of order statistics in samples from normal populations," *Biometrika*, v. 41, 1954, p. 200–227.

Many statisticians have treated the problem of evaluating the moments of order statistics in small samples drawn from normal populations. The results have been fragmentary, partly because no one has developed a systematic approach to the problem. The author gives a systematic approach in this paper. The method involves showing from a geometrical point of view that the moments of normal order statistics as well as the moment generating function of any order statistic are closely related to the volumes of members of a class of hyperspherical simplices. These volumes involve calculation of a function $\bar{u}_\beta(x)$ (see the paper for details). Table 1 gives values of this function for $x = 2(1)12$ and $\beta = 1(1)49$ to 8 to 10D. These values are used to compute Table 2, which gives the first ten moments of the extreme members (smallest or largest values) in samples of size n, $1 \leq n \leq 50$ to 9 or 10S. Table 3 gives the second, third, and fourth moments about the mean of extremes in samples of size $1 \leq n \leq 50$, as well as the standard deviations, together with β_1 and $\beta_2 - 3$ to 7 or 8D.

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14[K].—T. J. TERPSTRA, "The exact probability distribution of the T statistic for testing against trend and its normal approximation," K. Ned. Akad. van Wetensch, *Proc.*, s. A, v. 56, 1953, p. 433–437.

Let x_{ih}, $i = 1, \cdots, l, h = 1, \cdots, n_i$, be l random samples of the random variables x_i which the null hypothesis states to have identical frequency distributions. Let $\bar{U}_{ij}$, WILCOXON's statistic,[1] be the number of pairs (h, k), $h \leq n_i$, $k \leq n_j$, with $i < j$ and $x_{i,h} < x_{j,k}$; $i, j = 1, \cdots, l, h, k = 1, \cdots, n_i$; then $T = \sum \sum_{i<j} \bar{U}_{ij}$, a generalization of the T defined by MANN & WHITNEY,[2] was studied by the author[3] as a test against the alternate hypothesis of an upward term in the x_i. In the present paper, in Table 1 the author gives the exact distribution of T to 3D for $n_1 \leq n_2 \leq n_3 \leq 5$, and in Table 2 gives the .005, .01, .025, .05 and 0.1 significance levels for T (the smallest value of T for which the probability of its being exceeded is no greater than the relevant significance level) for the same values of the n's. The values are also given for the normal approximation which are such that they indicate that for $n_i \geq 5$ this approximation is good.

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[1] F. WILCOXON, "Individual comparisons by ranking methods," *Biometrics Bull.*, v. 1, 1945, p. 80–83.

[2] H. B. MANN & D. R. WHITNEY, "On a test of whether one of two random variables is stochastically larger than the other," *Ann. Math. Stat.*, v. 18, 1947, p. 50–60.

[3] T. J. TERPSTRA, "The asymptotic normality and consistency of Kendall's test against trend, when ties are present in one ranking," K. Ned. Akad. van Wetensch., *Proc.* s. A, v. 55, 1952, p. 327–333.

15[K].—H. O. HARTLEY & H. A. DAVID, "Universal bounds for mean range and extreme observation," *Annals Math. Stat.*, v. 25, 1954, p. 85–99.

The authors extend the theory of universal upper and lower bounds for $E(w_n)$, and universal upper bounds for $E(x_n)$, where x_n is the standardized extreme variate and w_n the standardized range of a sample of n. Table I gives the upper bound of $E(x_n)$ to 4D for any population for $n = 2(1)20$. Also given for comparison are previously known values for symmetric populations. Table II gives the universal lower bound for $E(w_n)$ over distributions with finite range $-X \leq x \leq X$. For $n = 2(2)12$, the bound is given to 3D for $X = 1(1)5$; for $n = 12(2)20$, it is given to 3D for $p = .95(.01).99$, where $p = X^2/(1 + X^2)$. Specific values are easily computed from equation (58). As $X \to \infty$, $E(w_n) \to 0$ in agreement with previous results. It is shown that universal upper bounds previously computed for the case of symmetric populations with infinite range are also applicable to the non-symmetric case, and to the case of finite range unless $X < X_n$, where X_n is the limit of the range for the population which maximizes $E(w_n)$. Values of X_n to 3D are tabulated for $n = 2(1)20$, and the algebraic form for the bound when $X < X_n$ is given.

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16[K].—R. E. BECHHOFER, "A single-sample multiple decision procedure for ranking means of normal populations with known variances," *Annals Math. Stat.*, v. 25, 1954, p. 16–39.

Let $X_1, \cdots, X_k$ be independent normal random variables of unit variance. Table I gives to 4D the value of d such that $\gamma = \Pr\{X_1, \cdots, X_{k-t} < X_{k-t+1} + d, \cdots, X_k + d\}$ for $\gamma = .05(.05).8(.02).9(.01).99, .999, .9995$ and for $k = 2(1)10$, $t = 1(1)[k/2]$ as well as 10 other pairs (k, t). Table II gives to 4D the value of d such that $\gamma = \Pr\{X_1 < X_2 + d < X_3 + 2d\}$ for $\gamma = .2(.05).8(.02).9(.01).99$. The tables enable one to compute the numbers of observations needed from normal populations of known variance in order to have confidence γ in certain statements about the order of the population means.

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17[K].—R. E. BECHHOFER & MILTON SOBEL, "A single-sample multiple decision procedure for ranking variances of normal populations," *Annals Math. Stat.*, v. 25, 1954, p. 273–289.

Let U, V, W, X be independent chi-square random variables, each with n degrees of freedom. The paper provides 5D tables of $\Pr\{U < \theta V\}$, $\Pr\{U < \theta V, \theta W\}$, $P\{U, V < \theta W\}$, $\Pr\{U < \theta V < \theta^2 W\}$ and $\Pr\{U < \theta V, \theta W, \theta X\}$, for $n = 1(1)20$ and $\theta = 1.2(.2)2.2$. The tables provide the confidence coefficients of certain statements about the order of the variances of normal populations.

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18[K].—C. W. DUNNETT & MILTON SOBEL, "A bivariate generalization of Student's t-distribution with tables for certain special cases," *Biometrika*, v. 41, 1954, p. 153–169.

The authors consider the simultaneous distribution of two variates, $t_1 = z_1/s$ and $t_2 = z_2/s$. The z_i follow a normal bivariate distribution with zero means, the same variance σ^2, and correlation ρ. The variance σ^2 is assumed independently estimated by s^2 with n degrees of freedom. The probability integral is:

$$\text{Prob}\ \{t_1 \leq h; t_2 \leq h\} = P.$$

Tables of P and h are presented for $n = 1(1)30(3)60(15)120, 150, 300, 600, \infty$ and $\rho = .5$ and $-.5$. P is given to 5D for $h = 0(.25)2.50$ and 3.00, plus some additional values for larger h when n is small. h is given to 3D for $P = .50, .75, .90, .95$ and $.99$.

An asymptotic expansion is derived for P and h;

$$P = \sum_{i=0}^{4} A_i/n^i \quad \text{and} \quad h = \sum_{i=0}^{4} B_i/n^i.$$

Values of the A_i and B_i to 6D are presented for the same values of h and P mentioned above.

This distribution has applications in certain multiple decision problems.

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19[K].—J. M. SENGUPTA, "Significance level of $\sum x^2/(\sum x)^2$ based on Student's distribution," *Sankhyā*, v. 12, 1953, p. 363.

In testing whether the mean of a normal population sampled is equal to a given value μ, one ordinarily applies Student's "t," i.e., $t = \sqrt{n}(\bar{y} - \mu)/s$, where $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ and $s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$ is the usual unbiased estimate of population variance. The author of the paper reviewed has noted that if one puts $x_i = y_i - \mu$, i.e., considers deviations about the hypothetical mean tested, then

$$\frac{\sum x^2}{(\sum x)^2} = \frac{t^2 + N - 1}{Nt^2} \tag{1}$$

and hence percentage points for the right-hand member of (1) can be obtained from percentage or significance levels of Student's "t." A table of such percentage points would therefore simplify computations for the test of a hypothetical mean and will in fact be very useful and time-saving when many t-tests are to be carried out. The presented table contains the 5% and 1% significance levels to 4D for $\frac{t^2 + N - 1}{Nt^2}$ for sample sizes $N = 2(1)30(10)60, \infty$. The user of the table should note that significant values of $(t^2 + N - 1)/Nt^2$ are *less than* those tabulated.

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20[K].—J. C. SPITZ, "Matching in psychology," (Dutch, English Summary), *Statistica*, v. 7, 1953, p. 23–40.

Let p_r be the probability of exactly r matches in a random matching of two similar decks of 3 distinct cards. Then, $p_0 = \frac{1}{3}$, $p_1 = \frac{1}{2}$, $p_2 = 0$, $p_3 = \frac{1}{6}$. In a series of n such random matchings, let $R = r_1 + \cdots + r_n$ be the total number of matches $(0 \leq R \leq 3n)$ and let $P_{n,a} = \Pr(R \geq a)$. Obviously,

$$P_{n,a} = \tfrac{1}{3}P_{n-1,a} + \tfrac{1}{2}P_{n-1,a-1} + \tfrac{1}{6}P_{n-1,a-3} \tag{1}$$

from which the value $P_{n,a}$ is tabulated to 3D for $n = 1(1)30$ and $a = 0(1)3n$. If in an actual experiment the number $R = a$ of matches is such that $P_{n,a} \leq .05$ (say), one rejects the hypothesis that the matchings were random.

For large n, by the central limit theorem, R is approximately normal. Now, R has a mean $\mu = n$, a variance $\sigma^2 = n$ and a skewness $\gamma_1 = 1/\sqrt{n}$. Thus, for large n, $t = (R - \frac{1}{2} - n)/\sqrt{n}$ is about $N(0, 1)$. For $n = 30$, the resulting approximation to $P_{n,a}$ appears to be fairly good. An even better approximation is obtained by replacing t by a type III variable with $\mu = \sigma = 1$, $\gamma_1 = 1/\sqrt{n}$; then $P_{n,a}$ can be readily computed from SALVOSA's tables.[1]

The reviewer would expect a good approximation to $P_{n,a}$ by replacing R by a Poisson variable with parameter n which has $\mu = \sigma^2 = n$, $\gamma_1 = 1/\sqrt{n}$.

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[1] L. R. SALVOSA, "Tables of Pearson's type III function," *Ann. Math. Stat.*, v. 1, 1930, following p. 198.

21[K].—J. W. WHITFIELD, "The distribution of the difference in total rank value for two particular objects in m rankings of n objects," *British Jn. of Stat. Psychology*, v. 7, 1954, p. 45–49.

Let r_{ij} $(i = 1, \cdots, m;\ j = 1, \cdots, n)$ be m rankings of n individuals, where the total ranks assigned to two particular individuals a and b are of interest. The author considers the statistic $d = \sum_{i=1}^{m} (r_{ia} - r_{ib})$, the difference in total rank values, and obtains the one-sided cumulative distribution function $\frac{1}{2}P[|d| \geq k]$ on the assumption of randomness to 5D for $n = 2, 3$, $n = 3(1)8$; $m = 4$, $n = 3(1)7$; $m = 5$, $n = 3(1)5$; $m = 6$, $n = 3, 4$; $m = 7, 8$, $n = 3$. The first four moments of d and a normal approximation are also given.

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22[K].—WM. R. THOMPSON, *Tables of the Four Variable N- and* Psi-*Functions.* 9 + 88 typewritten pages (ozalided), deposited in the UMT FILE.

Let $n = r + s$, $n' = r' + s'$ and define

$$N(r, s, r', s') \equiv \binom{n + n' + 2}{n + 1} \psi(r, s, r', s')$$
$$\equiv \sum_{t=0}^{t \leq s, r'} \binom{r + r' + 1}{r' - t} \binom{s + s' + 1}{s - t},$$

where $\binom{k}{m}$ is the binomial coefficient.

These tables give exact values of N for appropriate positive integral arguments such that $3 \leq n \leq 20$, $2 \leq n' \leq 20$, $n \geq n'$ and 7S approximations (in some cases exact) of $10^7\psi$ for appropriate non-negative integral arguments such that $1 \leq n \leq 20$, $0 \leq n' \leq 20$, $n \geq n'$.

The word appropriate here refers to the fact that a user of the table may take advantage of identities which result from permuting the arguments in certain ways. A nine-page explanation of the tables and their method of construction is included.

The tables include, as subheadings in each "cell" of the table, parenthesized values of n, n', and $D = \binom{n + n' + 2}{n + 1}$. The author believes the tables to be error-free.

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23[L].—NBS Applied Mathematics Series, No. 32, *Table of Sine and Cosine Integrals for arguments from 10 to 100.* U. S. Government Printing Office, Washington, D. C., 1954, xvi + 187 p., 20.5 × 27 cm. Price $2.25.

The first edition[1] of this volume appeared in 1942. In the present second edition the bibliography has been brought up to date, the table of $p(1 - p)$ has been replaced by a table of $p(1 - p)/2$, the table of E_2 and F_2 has been added.

The principal table (180 p.) gives 10D values of

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt, \quad \mathrm{Ci}(x) = \int_\infty^x \frac{\cos t}{t}\,dt$$

with second central differences for $x = 10(.01)100$. Interpolation by EVERETT's formula gives accuracy to within 1.2 units of the tenth decimal place.

Auxiliary tables: $n\pi/2$ for $n = 1(1)100$; 15D. $p(1 - p)/2$ for $p = 0(.001)1$; exact values. The EVERETT coefficients $E_2(p)$ and $F_2(p)$ for $p = 0(.001)1$; 7D.

In the Introduction A. N. LOWAN gives the fundamental formulas for these functions, the method of computation and the preparation and checking of the manuscript; and describes both direct and inverse interpolation and their accuracy. A bibliography of tables, references to applications, and a list of texts and handbooks is also given.

There are graphs of Si(x) and Ci(x), a Preface by A. V. ASTIN, and a Foreword by J. A. STRATTON.

A. E.

[1] NYMTP, "Table of sine and cosine integrals for arguments from 10 to 100." New York, 1942.

24[L].—S. RUSHTON, "On the confluent hypergeometric function $M(\alpha, \gamma, x)$," *Sankhyā*, v. 13, 1954, p. 369–376.

S. RUSHTON & E. D. LANG, "Tables of the confluent hypergeometric function." *Sankhyā*, v. 13, 1954, p. 377–411.

The second of these papers gives 7S values of

$$M(\alpha, \gamma, x) = \sum_{j=0}^{\infty} \frac{\Gamma(\gamma)\Gamma(\alpha + j)}{\Gamma(\alpha)\Gamma(\gamma + j)} \frac{x^j}{j!}$$

for $\gamma = .5(.5)3.5, 4.5$; $x = .02(.02).1(.1)1(1)10(10)50, 100, 200$, and an extensive range of integer and half-integer values of α. This range varies between $0 \leq \alpha \leq 25$ and $0 \leq \alpha \leq 50$, and the interval is .5 or 1.

In the first paper the author describes the principal properties of the confluent hypergeometric function, its application to facilitate certain sequential tests of composite hypotheses, and the construction of the tables. When $\alpha = \gamma$ or $\gamma + 1$, the confluent hypergeometric function can be expressed in terms of the exponential function, and from here the recurrence relations enable the computer to proceed to other values of α, as long as $\alpha - \gamma$ is an integer. In other cases, the power series expansion was used for $x < 5$, and the asymptotic expansion, improved by Airey's converging factor, for $x \geq 5$.

Companion tables[1] for $\gamma = 3, 4$ were reviewed in RMT **1003** (*MTAC*, v. 6, 1952, p. 155–156).

A. E.

[1] P. Nath, "Confluent hypergeometric function," *Sankhyā*, v. 11, 1951, p. 153–166.

25[L].—Milton Abramowitz & Philip Rabinowitz, "Evaluation of Coulomb wave functions along the transition line," *Phys. Rev.* (2), v. 96, 1954, p. 77–79.

Work on Coulomb wave functions at the NBS Computation Laboratory has been reviewed in RMT **1091** [*MTAC*, v. 7, p. 101–102], UMT **186** [*MTAC*, v. 8, p. 97], and RMT **1249** [*MTAC*, v. 8, p. 224]. In continuation of this work, the authors derive asymptotic expansions of F_0, F_0', G_0, G_0' for $\rho = 2\eta$, in descending powers of $\beta = (\frac{2}{3}\eta)^{\frac{1}{3}}$. They also indicate the computation of F_0 and G_0 for ρ near 2η by means of Taylor series.

The paper contains 7D tables of F_0, F_0', G_0, G_0' for $\rho = 2\eta = 0(.5)20(2)50$. The tabular values were computed to 9D on the SEAC of the NBS by means of a program prepared by C. E. Fröberg and based on numerical evaluation of integral representations. The 7D given in the tables are stated to be correct to within one unit of the last place, and five-point Lagrangian interpolation will yield full accuracy for $\rho \geq 3$.

In a companion paper,[1] expansions of Coulomb wave functions (for any L) are obtained for the region $0 < \rho < 2\rho_1 = 2\eta + 2[\eta^2 + L(L+1)]^{\frac{1}{2}}$.

A. E.

[1] Milton Abramowitz & H. A. Antosiewicz, "Coulomb wave functions in the transition region," *Phys. Rev.* (2), v. 96, 1954, p. 75–77.

26[L].—J. Clunie, "On Bose-Einstein functions," Phys. Soc. *Proc.* Sect. A, v. 67, 1954, p. 632–636.

The author provides formulas useful for the computation of the function

$$G_k(\eta) = \int_0^\infty \frac{x^k dx}{e^{x-\eta} - 1} \tag{1}$$

(the Cauchy Principal Value of the integral is to be taken when $\eta > 0$): these formulas include asymptotic expansions for large $|\eta|$ and power series expansions for small $|\eta|$, and the formula

$$G_k(\eta) = \sum_{r=0}^{n-1} 2^{-kr} F_k(2^r\eta) + 2^{-kn} G_k(2^n\eta) \tag{2}$$

where F_k is the Fermi-Dirac function.[1]

4D numerical tables of $G_{\frac{1}{2}}(\eta)$ are given for $\eta = -3(.2) - .6(.1).6(.2)20$: outside this range the asymptotic formulas are valid. The values were computed from (2) and available tables[1] of $F_{\frac{1}{2}}$, they were checked by differencing and, in the interval $-.5(.1).5$, also by independent computation from the power series; and it is stated that the error in the present tables is less than 1 unit in the fourth decimal place.

The power series expansion of G_k and graphs of multiples of $G_{-\frac{1}{2}}$, $G_{\frac{1}{2}}$, $G_{\frac{3}{2}}$ were given by ROBINSON.[2]

A. E.

[1] J. McDOUGALL & E. C. STONER, Roy. Soc. *Phil. Trans.*, v. 237A, 1938, p. 67–104.
[2] J. E. ROBINSON, *Phys. Rev.*, s. 2, v. 83, 1951, p. 678–679.

27[L].—JAMES G. BERRY, *Tables of Some Functions Related to the Legendre Functions $P_n^{-m}(x)$ and $Q_n(x)$ when n is a complex number.* Two copies, each 28 pages of copied typescript, deposited in the UMT FILE.

Tables of $\left(\dfrac{1-x}{1+x}\right)^{-m/2} P_n^{-m}(x)$ and of $\left(\dfrac{1-x}{1+x}\right)^{-m/2} \dfrac{d}{dx}[P_n^{-m}(x)]$, for $m = 0(1)20$, and of $Q_n(x)$ and $\dfrac{d}{dx}[Q_n(x)]$, where $P_n^{-m}(x)$ is the associated Legendre function of the first kind and $Q_n(x)$ is the Legendre function of the second kind, for $n = -0.5 \pm 10.24595735 \pm i(10.18477501)$. The range of x is $.000(.032).960(.002).998$, and $x = .999875$.

Real and imaginary parts of the two functions are in most cases given to 8S, although only spot checks were made and the author admits some instances in which the error is ± 5 in 7th S. Computation was done by a modification of the Runge-Kutta method on MIDAC, in connection with the author's Ph.D. Dissertation in Engineering Mechanics.

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28[L].—J. BERGHUIS, *A Table of Some Integrals.* R 245, *Mathematisch Centrum, Amsterdam.* Eight page mimeographed typescript, deposited in the UMT FILE.

This table contains

$$\text{8D values of } f_n(x) \equiv \int_0^x v^n \tan v \, dv, \quad n = 1(1)5, \quad x = 0(0.05)1.50$$

$$\text{8D values of } g_n(x) \equiv \int_0^x v^n \cot v \, dv, \quad n = 1(1)5, \quad x = 0(0.05)2.50$$

$$\text{7D values of } F_n(x) \equiv \int_0^x v^n \tanh v \, dv, \quad n = 1(1)4, \quad x = 0(0.02)1.98$$

$$\text{7D values of } G_n(x) \equiv \int_0^x v^n \coth v \, dv, \quad n = 1(1)4, \quad x = 0(0.02)1.98.$$

The error is stated to be 10^{-8} in the last 3 functions, 3×10^{-8} in $f_n(x)$. $F_n(x)$ and $G_n(x)$ were computed by the ARRA; the other functions were hand computed and checked by differencing on a National Accounting machine class 31.

29[L].—Staff of the Computation Department of Mathematisch Centrum, Amsterdam, *Table of Polylogarithms, Report R24, Part I: Numerical Values.* 53 mimeographed pages deposited in the UMT FILE.

In three tables are given 10D values of $F_n(z)$, defined by $\sum_{h=1}^{\infty} h^{-n}z^h$ for $|z| < 1$ and for other values of z by analytic continuation, called polylogarithms of order n and argument z. In all 3 tables, $n = 1(1)12$.

Table I: $z = x; x = -1(0.01)1$

Table II: $z = ix; x = 0(0.01)1$

Table III: $z = e^{i\pi\alpha/2}; \alpha = 0(0.01)2$.

The maximum error is stated to be 10^{-10}.

30[S].—ANN T. NELMS, "Graphs of the Compton energy-angle relationship and the Klein-Nishina formula from 10 Kev to 500 Mev.," National Bureau of Standards Circular 542, 1953, iv + 89 p.

This Circular contains eight principal graphs, most of them with a number of subsidiary graphs on a larger scale to give increased accuracy.

Fig. I. Scattered photon energy versus angle,

$$h\nu = \frac{h\nu_0}{1 + \alpha_0(1 - \cos\theta)}, \quad \alpha_0 = \frac{h\nu_0}{mc^2}.$$

Each curve gives $h\nu$ as a function of θ, for a constant initial photon energy $h\nu_0$.

Fig. II. Recoil energy versus angle,

$$T = \frac{2\alpha_0 h\nu_0}{1 + 2\alpha_0 + (1 + \alpha_0)^2 \tan^2\psi},$$

T as a function of ψ, for constant $h\nu_0$.

Fig. III. Photon wave length distribution,

$$f(\lambda_0, \lambda) = \frac{3}{8}\left(\frac{\lambda_0}{\lambda}\right)^2 \left[\frac{\lambda_0}{\lambda} + \frac{\lambda}{\lambda_0} - 2(\lambda - \lambda_0) + (\lambda - \lambda_0)^2\right]$$

as function of λ, for constant λ_0.

Fig. IV. Photon angular distribution, $(2\pi)^{-1}\sigma_0 f$ as function of θ, where $h\nu = mc^2/\lambda$ and θ are connected as in Fig. I. Each curve is plotted for constant $h\nu_0$. The last subsidiary graph gives $h\nu_0$ as a function of the angle at which $\sigma_0 f$ is minimum.

Fig. V. Electron angular distribution,

$$\frac{\sigma_0 f(\lambda_0, \lambda)}{2\pi} \frac{(1 + \alpha_0)^2(1 - \cos\theta)^2}{\cos^3\psi}, \quad \text{where} \quad \tan\psi = \frac{1}{1 + \alpha_0}\left(\frac{2\alpha_0\alpha}{\alpha_0 - \alpha} - 1\right)^{\frac{1}{2}},$$

as a function of ψ, for constant $h\nu_0$.

Fig. VI. Photon energy distribution,

$$\frac{\sigma_0\lambda^2}{mc^2} f(\lambda_0, \lambda)$$

as a function of $h\nu$, for constant $h\nu_0$.

Fig. VII. Electron energy distribution, same quantity as in VI as a function of T, for constant $h\nu_0$.

Fig. VIIIa. Total Compton cross section and effective cross section as functions of $h\nu_0$, Fig. VIIIb. Fraction of incident energy absorbed as function of $h\nu_0$.

It is stated that all calculations and curves are accurate to 1 per cent, and that the subsidiary graphs are such that interpolated values can be obtained in general to 2 per cent accuracy.

The circular is one of a series of surveys and tabulations of information on radiation physics.

A. E.

TABLE ERRATA

242.—R. S. Burington, *Handbook of Mathematical Tables and Formulas.* 3rd Edition. Handbook Publishers, Inc., Sandusky, Ohio, 1953.

I have recently checked, by differentiation, all of the indefinite integrals in this edition of the *Handbook.* The following errors were discovered. They are also present in the 2nd edition.

P. 68, no. 146. In the next to the bottom line of the page,
for $(m + np + n)$, *read* $(m + np + n)a$.

P. 71, no. 177. In the tan^{-1} form, insert the restriction: $a > 0$, $c < 0$.
In the $tanh^{-1}$ form, insert the restriction: $a > 0$, $c > 0$.

P. 71, no. 178. *For* $+\dfrac{(ad - bc)^2}{8ac}$, *read* $-\dfrac{(ad - bc)^2}{8ac}$.

P. 73, no. 195. Insert the restriction: $b > 0$.

P. 75, no. 225. Insert the restriction: $b > 0$.

P. 75, no. 226. The numerator, -1, of the coefficient of $\sin^{-1} U$ should be replaced by: Sgn $(d\cos ax - c\sin ax)$, where Sgn $z = 1$, for $z > 0$, $= -1$ for $z < 0$, and $=0$ for $z = 0$.
The expression for U should read:

$$U \equiv \left[\frac{c^2 + d^2 + b(c\cos ax + d\sin ax)}{\sqrt{c^2 + d^2}\,|b + c\cos ax + d\sin ax|}\right].$$

The final restriction, $-\pi < ax < \pi$, is unnecessary.

P. 78, no. 258. The restriction should read: $b > 0$, $b > c$, $\cos ax > 0$.