It is reasonable to suppose that the number of Mersenne primes less than x, when x is large, is about 2.3 log log x. This conjecture may be shown to be equivalent to the assertion that the probability of $2^p - 1$ being prime, when p is known to be prime and is large, is about $1.6(\log p)/p$, and is perhaps asymptotically $(\log_2 p)/p$. If so, the probability that p_5 is prime is negligible, and we should be able to say with confidence that our original conjecture was the exact opposite of the truth.

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[1] E. Catalan, *Nonu. Corresp. Math.*, v. 2, 1876, p. 96; cf. L. E. Dickson, *History of the theory of numbers*, v. 1, 1934, p. 22, ref. 116.
[2] D. H. Lehmer, *MTAC*, v. 7, 1953, p. 72.

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

55[A, F].—Horace S. Uhler, "*Hamartiexêresis* as applied to tables involving logarithms," Nat. Acad. Sci., *Proc.*, v. 40, 1954, p. 728–731 [1].

Hamartiexêresis appears to be a technical term in theology, meaning the absolute removal of sin.

This paper contains in tabular form, the exponents of the prime factors $(2, 3, \cdots, 997)$ in the product $(1!)(2!)\cdots(1000!)$.

This table was used to check the first thousand entries in the table of F. J. Duarte [2]. Two errors were found:

log 99!: the seventh quartet *should read* 8029 *instead of* 8929.

log 266!: the eighth quartet *should read* 1897 *instead of* 1987.

Later calculations indicate no (non-cancelling) errors in the range from $n = 1001$ to $n = 1200$.

J. T.

[1] See also Nat. Acad. Sci., *Proc.*, v. 41, 1955, p. 183, for errata.
[2] F. J. Duarte, Nouvelles tables de log $n!$ à 33 décimales, depuis $n = 1$ jusqu'à $n = 3000$. Geneva and Paris, 1927.

56[C, D, E, K, L, S].—Cecil Hastings, Jr., Jeanne T. Hayward, & James P. Wong, Jr. *Approximations for Digital Computers.* Princeton University Press, Princeton, N. J., 1955, viii + 201 p., 25 cm. Price $4.00.

This book contains rational approximations of the following functions with approximate precision as indicated (there are several approximations to each function and the approximate precision of each is shown):

$\log_{10} x$, $10^{-\frac{1}{2}} \leq x \leq 10^{\frac{1}{2}}$, 3D, 5D, 6D, 7D; $\varphi(x) = (1 - e^{-x})/x$, $0 \leq x < \infty$, 3D, 4D, 5D; $\arctan x$, $-1 \leq x \leq 1$, 3D, 4D, 5D, 6D, 7D, 8D; $\sin \frac{1}{2}\pi x$, $-1 \leq x \leq 1$, 4S, 6S, 8S; 10^x, $0 \leq x \leq 1$, 4S, 6S, 7S, 9S; $W(x) = e^{-x}/(1 + e^{-x})^2$, $-\infty < x < \infty$, 3D, 4D, 5D; $E^1(x) = e^{-x^2/2}/\sqrt{2\pi}$, $-\infty < x < \infty$, 3D, 3D, 4D; $K(n) = (n - 2n^2 - 2n^3) \ln(1 + 2/n) + (2n + 18n^2 + 16n^3 + 4n^4)(2 + m)^{-2}$, $0 \leq n < \infty$, 3D; $\Gamma(1 + x)$, $0 \leq x \leq 1$, 5D, 5D, 6D, 7D; $\Psi(x) = (\pi/2 - \arcsin x)(1 - x)^{-\frac{1}{2}}$, $0 \leq x \leq 1$, 4D, 5D, 6D, 7D, 8D; $\log_2 x$, $2^{-\frac{1}{2}} \leq x \leq 2^{\frac{1}{2}}$, 5D, 8D, 10D; $\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt, 0 \leq x < \infty$, 5D, 6D, 7D, and by another formula 4D, 4D, 7D; $K(k) = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}}, \varphi \leq k < 1$, 5D, 6D, 8D; $E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \varphi}\, d\varphi, 0 \leq k < 1$, 5D, 6D, 8D; $\ln(1 + x), 0 \leq x \leq 1$, 4D, 5D, 6D, 7D, 8D; $e^{-x}, 0 \leq x < \infty$, 4D, 5D, 6D, 7D; $-Ei(-x) = \int_x^\infty \frac{e^{-t}}{t} dt$, $1 \leq x < \infty$, 4S, 6S, 8S; $x(q)$ where $q = \frac{1}{2\pi} \int_{x(q)}^\infty e^{-t^2/2} dt, 0 < 2 \leq .5$, 3D, 4D; $W(z) = \int_0^\infty \frac{e^{-uz}}{k_1^2(u) + \pi^2 I_1^2(u)} \frac{du}{u}, 0 \leq z < \infty$, 4D, 5D; $P(x) = \int_x^\infty \frac{\sin(t - x)}{t} dt$, $1 \leq x < \infty$, 4D, 5D, 7D; $Q(x) = \int_{\cdot}^\infty \frac{\cos(t - x) dt}{t}, 1 \leq x < \infty$, 4D, 5D, 7D.

The notation above is that of the authors. In particular their $E'(x)$ is denoted as $z(x)$ in FLETCHER, MILLER, and ROSENHEAD, *An Index to Mathematical Tables*. The authors' $\Phi(x)$ is $H(x)$ in the *Index*. The notation with regard to precision is an adaptation (by the reviewer) of notation used for precision of tables; kD means that the absolute value of the difference between function and approximation does not exceed 5×10^{-k}, and kS means that the absolute value of the ratio of this difference to the value of the function does not exceed 5×10^{-k}.

The function $K(n)$ is involved in KLEIN-NISHINA cross section determinations for nuclear scattering. The function $W(z)$ occurs in some aerodynamical problems [1].

Many of these approximations have appeared previously in *MTAC*, v. 7 and v. 8.

The approximations are in a sense equivalent to tables with a minimal number of entries and complicated rational interpolation formulas. The tabular entries are so few in number that all must be used in any interpolation; these approximations are equivalent to the resulting interpolation formula.

The formulas are developed by fitting rational functions of the type chosen and with undetermined coefficients to the function approximated; the coefficients are then determined to give a fit (in terms of the error function chosen to estimate the precision of the approximation) usually at the roots of a Chebyshev polynomial of suitable degree. The error functions are carefully plotted and the goal of the approximation is to "level" the error curves; that is, the coefficients and the form are adjusted, if necessary, to give an error function with the largest number of maxima and minima consistent with the approximation form chosen and with these maximal and minimal values all approximately equal in absolute value. The form chosen is frequently obtained by varying coefficients of TAYLOR's series.

Most readers unfamiliar with the work of the authors will be amazed by the precision obtained with a small number of coefficients.

The following approximations are polynomials not requiring divisions: arctan x (odd powers, three coefficients give 3D, eight coefficients give 8D), sin $\frac{1}{2}\pi x$ (odd powers, three coefficients give 4S, five coefficients give 8S), 10^x, $\Gamma(1 + x)$, ln $(1 + x)$. Other approximations involve divisions.

The early part of the book is devoted to an exposition of the techniques used by the authors in curve fitting. This is copiously illustrated—the ratio of space devoted to displayed illustration to that devoted to text appears to be about 5:1. The portion of this exposition which tells of the more or less subjective art of curve fitting as seen by these masters should be most valuable. Up to date no purely mechanical scheme has seemed to match their approximations, and their development of their method of curve fitting by example is most instructive. Earlier descriptions of the general properties of approximating functions are less impressive, for here summary statements in the form of theorems would appeal to the reviewer more than the narrative style used by the authors. However, bibliographic references at the ends of the chapters alleviate this objection.

Use of these approximations and of these methods by the operators of high speed digital computers and punched card machinery is obvious. These machines have limited storage facilities for tables and incomparably greater computing facility for interpolation or for making the required approximation. Thus, for them these approximations, which are extreme in using complicated interpolation instead of listing more tabular values, are ideal.

It would have been convenient to the operators of many machines to have had the coefficients listed in binary form also. The reviewer also hopes that future editions will contain a short list of iterative schemes commonly used in automatic computation for calculation of square roots, inverses, etc. The book as it stands is an aide which every coder or programmer of an automatic machine will want; with this slight additional information it would become indispensable.

The type is easily read. Numbers of more than seven digits are divided into groups of four digits (the division might have been carried out for all numbers of more than four digits). Each error graph appears clearly on the same page as the approximation.

C. B. T.

[1] G. N. Ward, "The approximate external and internal flow past a quasi-cylindrical tube moving at supersonic speeds," *Quart. J. Mech. and Appl. Math.*, v. 1, 1948, p. 225–265.

57[D, E, L].—W. Flugge, *Four-place Tables of Transcendental Functions.* McGraw-Hill Publishing Co., New York, 1954, 136 p., 23.5 × 15.5 cm. Price $5.00.

Contents. $\sin x$, $\cos x$, $\tan x$, and $\cot x$, $x = 0(.1)90°$, $\cos x$, $\sin x$, $\tan x$, $\cosh x$, and $\sinh x$, $x = 0(.01)10.09$ and $\tanh x$, $x = 0(.01)5.09$. $e^x - 1$ and $1 - e^{-x}$, $x = 0(.001).209$, e^x and e^{-x}, $x = 0(.01)10.09$. $\ln 10^k$, $k = 1(1)10$ and $\ln x$, $x = 1(.01)10.09$. $J_0(x)$, $J_1(x)$, $Y_0(x)$, $Y_1(x)$, $I_0(x)$, $I_1(x)$, $K_0(x)$, and $K_1(x)$, $x = 0(.01)10.09$. ber x, bei x, ber′ x, bei′ x, ker x, kei x, ker′ x and kei′ x, $x = 0(.01)10.09$. Elliptic integrals $F(x,y)$ and $E(x,y)$, $x = 0(2)90°$ and $y = 0(1)90°$. erf x $x = 0(.01)10.09$, $1 -$ erf x $x = 1(.01)3.09$, Fresnel integrals $C(x)$ and $S(x)$ and exponential cos and sin integrals $Ei(-x)$, $\overline{Ei}\,x$, $Ci\,x$ and $Si\,x$ $x = 0(.01)10.09$

and $\Gamma(x)$ $x = 1(.001)2.09$. All to 4D or 4S as appropriate. Also lists of formulas and a few notes concerning sources of more extensive tables and the use of these tables. In particular there are formulas for computing values beyond the range of the tables.

The author does not indicate how the tables were computed or checked, but his bibliographic references indicate that they were ethically extracted from standard tables, to which he refers explicitly. He estimates errors at no more than 0.6 in the decimal following the ones listed except for the Fresnel integrals, where his source material was admittedly weak. The Fresnel integrals are not in the notation of FLETCHER, MILLER, and ROSENHEAD (see 20.6 and 20.62 in their *Index*); standard arguments are obtained by multiplying $\sqrt{\frac{2}{\pi}}$ by the square root of the author's arguments. (These standard arguments are used in both the Table of Fresnel Integrals by A. van WIJNGAARDEN and W. L. SCHEEN, Report R49 of the Mathematical Centre at Amsterdam, RMT **790**, *MTAC*, vol. 4, p. 155–6, and the 1953 Tablit͡sy Integralov Frenelya of the Akad. Nauk SSSR, **40**, *MTAC*, 1955, v. 9, p. 75). A few values tested by the reviewer differed by at most 1 in the fourth decimal from values obtained by interpolation from the Akad. Nauk tables. The author mentions neither the Amsterdam nor the Akad. Nauk tables as possible sources of values.

The author seems to have put together a convenient set of tables for the use of engineers, the trigonometric tables combined with Bessel functions and other selected functions indicating fairly well the tables most likely to be reached for by a recently educated engineer. The tables will probably be popular, although this reviewer feels that in the long run an elementary set of tables (including some statistical tables) plus Jahnke and Emde would provide almost minimal requirements for the expected users; however there are some tables found in Flügge's book and not found in Jahnke and Emde—ker x and kei x, for example.

C. B. T.

58[D].—NBS Applied Mathematics Series No. 43, *Tables of Sines and Cosines for Radian Arguments*. U. S. Gov. Printing Office, Washington, 1955, xi + 278 p., 26.7 × 20 cm. Price $3.00.

Contains sin x and cos x, $x = 0(.001)25.199$, 8D; sin x and cos x, $x = 0(1)100$, 8D; sin x and cos x, $x = n \times 10^{-p}$, $n = 1(1)9$, $p = 1(1)5$, 15D; sin x and cos x, $x = 0(.00001)0.00999$, 12D; conversion tables between radians, degrees, minutes, and seconds; and values of $\frac{1}{2}p(1 - p)$, $p = 0(.001)1$, exact. This is a new edition of table MT4, which was published in 1940 (RMT **81**[D] *MTAC*, v. 1, 1943, p. 14–16. The present table has been extended .2 of a radian to finish a 4th whole cycle in the main table. A misprint in an argument in the early edition is noted, and this has been corrected in the present edition. Evidently no incorrect functional values have been noted, and no changes were reported in this edition. The last table in the book has been changed from a table of $p(1 - p)$ to a table of $\frac{1}{2}p(1 - p)$ over the same range of arguments p.

The printing was designed for accuracy and economy rather than legibility. Most numbers are easily read. However, the publisher notes that the value for cos 16.639 is hard to read in many volumes; this should be cos 16.639 = −0.5970 0260.

C. B. T.

59[F].—I. M. VINOGRADOV, *Elements of Number Theory*. Translated from the fifth edition by SAUL KRAVETZ. Dover Publications, Inc., New York, 1954, 256 p. Paperbound edition $1.75. Clothbound edition $3.00.

This volume contains two kinds of tables:

(1) Tables of indices and powers of least positive primitive roots Modulo p for $p < 100$ (p. 220–225). These tables are equivalent to those in USPENSKY and HEASLET [1], though the format is different. A description of their construction is given on page 112. A comparison with Uspensky and Heaslet reveals the following misprints:

$p = 71$. *For* ind 56 = 43, *read* ind 56 = 42
$p = 97$. *For* ind 11 = 6, *read* ind 11 = 86.

(2) Table of primes <4000 and their smallest primitive roots (p. 226–227). Three errata are in this table:

For $p = 2763$, *read* $p = 2753$.

A comparison of this table against a SWAC calculation determined the following two errors:

$p = 1459$. *For* $g = 5$, *read* $g = 3$
$p = 3631$. *For* $g = 21$, *read* $g = 15$.

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[1] J. V. USPENSKY & M. A. HEASLET, *Elementary Number Theory*, New York and London, 1939, p. 477–480.

60[F].—ALBERT GLODEN, *Table des Solutions de la Congruence* $x^4 + 1 \equiv 0 \pmod{p^2}$ *pour* $4000 < p < 6000$. 2 handwritten pages, 30 × 21.2 cm., deposited in the UMT FILE.

J'ai donné dans mon article "Sur quelques congruences d'ordre superieur," [1], une méthode pour résoudre la congruence

$$x^4 + 1 \equiv 0 \pmod{p^2}.$$

Dans cet article j'ai également publié les deux solutions minima pour $p < 1000$.

La présente Table couvre l'intervalle $4000 < p < 6000$. Elle donne les deux solutions x_1 et x_2 qui sont $< p^2/2$. On sait que les autres solutions $< p^2$ sont $p^2 - x_1$ et $p^2 - x_2$.

Cette Table sera étendue prochainement à toutes les valeurs de $p < 10000$. Je communiquerai mes résultats aux personnes qui en feront la demande.

ALBERT GLODEN

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[1] *Bulletin* de la Société Royale des Sciences de Liège, 1950, p. 429–436.

61[F].—ALBERT GLODEN, *Table des Solutions Minima de la Congruence* $x^4 + 1 \equiv 0 \pmod{p^2}$ *pour* $6000 < p < 10000$. 2 handwritten pages, 26.7 × 20.3 cm., deposited in the UMT FILE.

Celle Table est construite de façon analogue á une Table précédente couvrant l'intervalle $4000 < p < 6000$. Elle donne les deux solutions $x_1, x_2 < p^2/2$. On sait que les autres solutions $< p^2$ sont $p^2 - x_1$ et $p^2 - x_2$.

Je communiquerai mes résultats aux personnes qui en feront la demande.

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62[F].—R. J. PORTER, *List of Irregular Determinants of Exponent* $3n$ [$80000 < -D < 150000$]. Typewritten manuscript of 72 pages, 20.4 × 12.3, on deposit in the UMT FILE.

This list extends to $D = -150000$ the two previous lists [UMT **155**, *MTAC*, v. 7, p. 34] and [UMT **185**, *MTAC*, v. 8, p. 96]. In the entire range up to 150000 there are 5836 D's of which 14 and 56 have exponents of irregularity 6 and 9, respectively. All others have exponents of 3.

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63[G].—TAKAYUKI TAMURA, "Notes on finite semigroups and determination of semigroups of order 4," *Journal of Gakugei*, Tokushima University, Japan, v. 5, 1954, p. 17–27.

On pages 24 to 27 is a table of all 126 distinct semigroups of order 4. (For terminology see OLGA TAUSSKY's review in *MTAC*, v. 8, 1954, p. 231.) Each semigroup is represented by one multiplication table in the form

a	*b*	*c*	*d*
b	*c*	*d*	*a*
c	*d*	*a*	*b*
d	*a*	*b*	*c*

The semigroups are classified into six main categories: unipotent; commutativity-indecomposable; decomposable, 2-2 type; decomposable, 3-1 type; decomposable, 2-1-1 type; commutative, non-uninpotent.

The tables were computed by hand by M. YAMAMURA and T. TAMURA in 1953, and again in 1954, and were submitted for publication in August 1954.

The reviewer used SWAC in April 1955 to verify that Tamura's tables are equivalent (in the sense of isomorphism or anti-isomorphism) to the 126 semigroups computed by the reviewer with SWAC in May and June 1954. (See the review cited above; the SWAC tables are to appear shortly in *Proc. Amer. Math. Soc.*) Thus Tamura's table is undoubtedly complete and without misprints (except in the classifying labels 3.1–4 and 3.1–8).

The creators of this table are to be congratulated on the completeness and accuracy with which they carried out a very long calculation.

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64[H].—NBS Applied Mathematics Series No. 29, *Simultaneous Linear Equations and the Determination of Eigenvalues*. U. S. Gov. Printing Office, Washington, 1953, iv + 126 p., 26.7 × 20 cm. Price $1.50.

This volume consists of a collection of 19 papers representing the majority of reports presented at a Symposium in Los Angeles on August 23–25, 1951. For the most part the papers are short and descriptive giving a general view of results rather than the full mathematical details which are adequately covered in accompanying references. By bringing together various methods for the solution of linear equations and eigenvalue problems the book serves a useful purpose for those who are concerned with the explicit computation of such solutions, especially by means of automatic computing machines.

A list of the papers and a brief synopsis of each follows:

1. Tentative classification of methods and bibliography on solving systems of linear equations, by George E. Forsythe.
2. Simultaneous systems of equations, by A. M. Ostrowski.
3. The geometry of some iterative methods of solving linear systems, by Alston S. Householder.
4. Solutions of linear systems of equations on a relay machine, by Carl-Erik Froeberg.
5. Some special methods of relaxation technique, by Eduard Stiefel.
6. Errors of matrix computations, by Paul S. Dwyer.
7. Rapidly converging iterative methods for solving linear equations, by J. Barkley Rosser.
8. Some problems in aerodynamics and structural engineering related to eigenvalues, by R. A. Frazer.
9. Inclusion theorems for eigenvalues, by H. Wielandt.
10. On general computation methods for eigenvalues and eigenfunctions, by Gaetano Fichera.
11. Variational methods for the approximation and exact computation of eigenvalues, by Alexander Weinstein.
12. Determination of eigenvalues and eigenvectors of matrices, by Magnus R. Hestenes.
13. New results in the perturbation theory of eigenvalue problems, by F. Rellich.
14. Bounds for characteristic roots of matrices, by Alfred Brauer.
15. Matrix inversion and solution of simultaneous linear algebraic equations with the IBM 604 electronic calculating punch, by George W. Petrie, III.
16. Experiments on the inversion of a 16 × 16 matrix, by John Todd.
17. A method of computing eigenvalues and eigenvectors suggested by classical results on symmetric matrices, by Wallace Givens.

18. Computations relating to inverse matrices, by JACK SHERMAN.

19. Results of recent experiments in the analysis of periods carried out in the Istituto Nazionale per le Applicazioni del Calcolo, by GAETANO FICHERA.

Paper 1 classifies known methods of solving linear equations into six main groups, and lists a bibliography of some 500 titles with cross-reference to the methods. Paper 2 deals with nonlinear equations and essentially describes a method of obtaining an estimate of the error of an approximate solution. Paper 3 shows how various standard iterative methods of solution can be regarded as particular aspects of a general geometric method of projections. Paper 4 presents experimental results on inverting matrices of order up to 20 on the Swedish relay machine BARK by the Gauss and Jordan elimination methods. Paper 5 describes a general relaxation method and a special form of it that permits the solution of a system of linear equations of order n in n steps. Paper 6 discusses in a very general way the errors (inherent and computational) in solving linear equations or inverting matrices; it contains a bibliography of 30 papers. Paper 7 presents an orthogonalization method which generates a sequence of trial solutions whose residuals are mutually orthogonal relative to a positive definite matrix; the procedure of paper 5 belongs to this method. Paper 8 describes ways of approximating certain continuous physical systems by discrete systems which are amenable to computation. Paper 9 solves the following problem for a broad class of matrices: let x and y be fixed vectors, and consider the matrices A such that $y = Ax$; characterize those point sets in the complex plane which contain at least one eigenvalue of every matrix A. Paper 10 presents first a method of finite least squares approximation for operators in Hilbert space and second a finite approximation method for differential equations based on the Cauchy-Lipschitz procedure. Paper 11 derives inequalities for eigenvalues of a positive definite completely continuous operator on a subspace Q of a real Hilbert Space H by considering a sequence of intermediate problems corresponding to a chain of diminishing subspaces from H to Q. Paper 12 describes the following three methods for the solution of eigenvalue problems: (i) power method, (ii) gradient method, (iii) an orthogonalization method closely related to that of paper 7. Paper 13 gives two results of E. HEINZ based on an essential inequality of Heinz. Paper 14 extends the standard "circle" bounds for characteristic values to more general "oval" bounds. Paper 15 gives a program for carrying out the standard elimination method. Paper 16 summarizes tentatively the results of applying (i) the method of paper 15 on IBM equipment, (ii) the Monte Carlo method, and (iii) the iteration method $X_{n+1} = X_n(2 - AX_n)$ on the SEAC to a particular 16×16 matrix. Paper 17 uses the method of reducing a symmetric matrix to a triple diagonal form by a sequence of at most $(1/2)(n - 1)(n - 2)$ planar rotations, n being the order of the matrix. Paper 18 gives an algorithm for obtaining the inverse of B from the inverse of A when B differs from A in a column or in a row. Finally, paper 19 outlines an approximate method for finding the periods of an empirically given function on a finite interval.

For the most part the papers employ mathematical methods which are accessible to others than professional mathematicians and for this reason should interest a comparatively large audience to whom the problems treated are of importance. Paper 13, which employs the most advanced techniques of any of the papers, seems out of place in the series because of its purely theoretical concern. For another reason, paper 8 strikes one as exceptional, for it expresses results mainly in physically descriptive terms leaving aside the mathematical problems of explicit calculation. But in the main, the papers bear directly on the apparent objective of the collection.

Various of the papers contain numerical examples but from the point of view of one who wishes to compute solutions, especially with high speed machines, what is required is considerably more experimental evidence to distinguish the computational merits or faults of the proposed methods, particularly for systems of high order. Undoubtedly new information of this kind has been obtained since the time of the Symposium, and it would be of interest to have it available.

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65[H].—NBS Applied Mathematics Series No. 39, *Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues.* U. S. Gov. Printing Office, Washington, 1954, iii + 139 p., 26.7 × 20 cm. Price $2.00.

This volume is a sequel to the one reviewed above and concerns itself with the same general problem. As the term, "Contributions," in the title suggests, the separate papers in the collection are less closely connected than those in the preceding volume, but they are sufficiently united in general purpose to serve as a useful companion to the earlier collection. The papers vary in length from four pages to fifty-four pages and include such topics as practical methods for computation, iterative methods of solution in Hilbert space, numerical experiments with matrices, condition of a matrix, and bounds for the rank and eigenvalues of a matrix. Descriptions of the individual papers now follow.

The first paper, "Practical solution of linear equations and inversion of matrices," (54 p.) by L. FOX describes various methods from the point of view of the computer using a desk calculator, and illustrates the methods by some twenty-six examples with matrices of order 6. It is a very convenient compilation which allows a ready comparison of various techniques. After a brief treatment of iterative methods of successive approximation the author turns to a group of elimination methods including GAUSS elimination, JORDAN elimination, and AITKEN's "below-the-line" scheme. The next group of methods are compact elimination methods, among which are the procedures of DOOLITTLE and CROUT. These elimination methods are related to the triangular decomposition $A = LDU$ (L = lower diagonal unit matrix, U = upper diagonal unit matrix, D = diagonal matrix), and then attention is turned to schemes depending explicitly upon this decomposition. Among these are the procedures of CHOLESKY, DWYER-WAUGH, BANACHIEWICZ, and the author. Following this, orthogonalization methods are described, and finally the complex matrix is singled out for special attention.

The second paper, "Punched-card experiments with accelerated gradient methods for linear equations," (16 p.) by A. I. and G. E. FORSYTHE reports the results of numerical experiments on two matrices of order 6 designed to test some proposed devices to accelerate the method of steepest descent, which often converges too slowly for practical computation. The results were favorable to a procedure which inserts a certain accelerating step periodically in the iteration by steepest descent.

The third paper, "Iterative methods of solving linear problems in Hilbert Space," (34 p.) by R. M. HAYES, is a report on a doctoral dissertation dealing with the solution of $Ax = b$ in Hilbert space for a certain class of linear operators A. After establishing general convergence theorems, the method is specialized for positive definite operators to the gradient method, the conjugate direction method, and the conjugate gradient method; iteration formulas for successive approximations are given and rate of convergence is studied. Finally, the theory is applied to ordinary differential equations, integral equations, and partial differential equations. The investigation is essentially theoretical and is closely related to the work of M. R. HESTENES. It would be valuable to have numerical data available pertinent to the application discussed in the paper.

The fourth and fifth papers are respectively, "Tables of inverses of finite segments of the Hilbert matrix," (4 p.) by I. R. SAVAGE and E. LUKACS, and "The condition of the finite segments of the Hilbert matrix," (8 p.) by JOHN TODD. In the former paper a formula is derived for the elements of the inverse of the matrix $H_n = \|1/(i + j - 1)\|$, $i, j = 1, 2, \cdots, n$, and the inverse H_n^{-1} is tabulated for $n = 2, 3, \cdots, 10$. In the latter paper various measures of the "condition" of a matrix are estimated for H_n, corroborating that H_n is ill-conditioned. Computations of H_n^{-1} by a standard program were carried out on the SEAC and broke down at such low orders as $n = 6$ and 7.

The sixth paper, "Lower bounds for the rank and location of the eigenvalues of a matrix," (14 p.) by K. FAN and A. J. HOFFMAN deals with Problem 1: determination of lower bounds for the rank of an arbitrary matrix $A = (a_{ij})$ of order n; and Problem 2: determination of ρ_i such that every eigenvalue λ of A satisfies at least one of the inequalities $|\lambda - a_{ii}| \leq \rho_i (i = 1, 2, \cdots, n)$. The best-known criterion for the non-singularity of A is $|a_{ii}| > \sum_{j \neq i} |a_{ij}|$. This criterion is used and generalized, as is the known solution $\rho_i = \sum_{j \neq i} |a_{ij}|$ to Problem 2. Problems 1 and 2 are closely related and this is clearly demonstrated in the paper. In the last part of the paper A is taken to be normal and inequalities involving k eigenvalues are established which generalize results of WIELANDT and of WALKER-WESTON for $k = 1$.

The last paper, "Inequalities for the eigenvalues of Hermitian matrices," (9 p.) by K. Fan deals with the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ of an Hermitian matrix H. Sufficient conditions on a set of numbers d_i are established for the inequalities $\lambda_i \leq d_i$ $(i = 1, 2, \cdots, n)$ to hold; the conditions are expressed in terms of $|a_{ii}|$ and $\sum_{j>i} |a_{ij}|^2$ $(i = 1, 2, \cdots, n)$. In the second part of the paper the λ_i are related to the eigenvalues $k_1 \geq k_2 \geq \cdots \geq k_n$ of a block diagonal matrix obtained by replacing by 0 all elements of H except those belonging to principal submatrices; e.g., the inequality $\sum_{i=1}^{h} k_i \leq \sum_{i=1}^{h} \lambda_i$, $(h = 1, 2, \cdots, n)$ is shown to hold.

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66[H, P].—H. F. P. PURDAY, *Linear Equations in Applied Mechanics.* Interscience Publishers, Inc., New York, 1954, xiv + 240 p., 22.1 × 13.9 cm. Price $3.50.

This is an elementary textbook in which the author determinedly faces the problem of obtaining numerical solutions to the equations he sets up. It is filled with examples and illustrations. Theorems and precise mathematical statements are almost completely absent. The book is essentially a set of recipes lucidly illustrated by numerical calculations and applications to mechanics. The author states in his introduction that "this book is intended to explain the easier parts of a cross-section or slice of mathematical study; not so much a selection of isolated topics as a group of related studies. No attempt is made at logical rigour in the modern sense and the main object is to help the reader get acquainted with the main ideas, notations, and results by the easiest means, bearing in mind the final objective which is to obtain substantially correct numerical results to definite problems."

The author's approach may be illustrated by quoting some advice he gives: if an equation has repeated roots make some trifling or negligible alterations to the constants thus separating the roots (like the Belfast constable who dragged a dead horse from Chichester Street to May Street to simplify his report).

The bibliographical references are incomplete. The author states that a considerable body of literature not easily accessible has grown up around the subject of the simultaneous solution of linear algebraic equations. At least as regards the accessibility of the literature, this statement is misleading.

The numerical examples seem to be correctly solved, and it is impressive and encouraging to see an author devote himself so directly to the task of arriving at numerical solutions. The references to mechanics and other engineering applications are lucid, and they help the exposition. Although the methods used are sound, they are not always fully explained. Thus, in solving the heat equation by a difference equation approximation, the author takes his time increment to be small relative to the square of the size of his length increment, but he gives no real explanation of why he does so. In solving systems of algebraic linear equations by relaxation, the author gives no automatic program of relaxation and a careless reader might impute to him an implication that none is known. It seems likely that use of this book in a course in a university in the United States would require that the instructor furnish a great quantity of additional material—possibly in another text book. However, many students may be expected to benefit from the advice and the examples contained in this book—especially if they are made aware of the mathematical background of the methods of calculation used and of the existence of other methods.

The book covers material in linear algebraic equations and matrices, elementary loading problems on beams, elementary finite differences, numerical solution of ordinary differential equations (with no mention of any predictor-corrector process), scalar and tensor fields with examples from mechanics, partial differential equations, and integral equations. No nonlinear problems are mentioned.

C. B. T.

67[K, M, Z].—K. D. TOCHER, "The application of automatic computers to sampling experiments," Roy. Stat. Soc., *Jn.*, v. 16, 1954, p. 39.

This paper describes some of the problems that arise when automatic computers are used for conducting sampling experiments. The generation of random elements is discussed in detail and some methods of producing random variables with common distributions are described. The use of sampling methods to evaluate multivariate integrals is discussed. Finally a programme is devised to conduct a special form of a restricted random walk.

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68[L, M].—G. EASON, B. NOBLE, & I. N. SNEDDON, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," Royal Soc. of London, *Phil. Trans.* Sect. A, Math. and Phys. Sciences, v. 247, 1955, p. 529–551.

Contains the following 11 tables all to 4D and for ρ, $\zeta = 0(.2)2(1)3$, where $\rho = b/a$ and $\zeta = c/a$.

Table 1. $a^2 \int_0^\infty J_0(at)J_0(bt)e^{-ct}tdt.$

Table 2. $a^2 \int_0^\infty J_0(at)J_1(bt)e^{-ct}tdt.$

Table 3. $a^2 \int_0^\infty J_1(at)J_0(bt)e^{-ct}tdt.$

Table 4. $a^2 \int_0^\infty J_1(at)J_1(bt)e^{-ct}tdt.$

Table 5. $a \int_0^\infty J_0(at)J_0(bt)e^{-ct}dt.$

Table 6. $a \int_0^\infty J_1(at)J_0(bt)e^{-ct}dt.$

Table 7. $a \int_0^\infty J_0(at)J_1(bt)e^{-ct}dt.$

Table 8. $a \int_0^\infty J_1(at)J_1(bt)e^{-ct}dt.$

Table 9. $\int_0^\infty J_0(at)J_1(bt)e^{-ct}dt/t.$

Table 10. $\int_0^\infty J_1(at)J_0(bt)e^{-ct}dt/t.$

Table 11. $\int_0^\infty J_1(at)J_1(bt)e^{-ct}dt/t.$

Also, auxiliary tables to facilitate interpolation in the neighborhood of a singular point in each of these tables.

The publication includes an introduction indicating possible uses for the tables, development of formulas upon which the integrations are based, recurrence relations and brief instructions for use outside the range of the tables, the tables themselves, auxiliary tables, and a bibliography.

C. B. T.

69[L].—K. I. McKenzie & M. Rothman, *Table of* $\mathrm{Bi}'(+x)$ *for* $x = 0(0.01)2$ *and* $\mathrm{Bi}'(-x)$ *for* $x = 0(0.01)10$. $i + 12$ mimeographed pages, 33×20.5 cm., deposited in the UMT File.

8D tables with δ_m^2 of the derivative of the Airy integral of the second kind, $Bi(x)$. The derivative was computed from a table of reduced derivatives of $Bi(x)$. Tables of $\log_{10} Bi(x)$ and $Bi'(x)/Bi(x)$ are being prepared by the authors.

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70[P].—Chester Snow, "Formulas for computing capacitance and inductance," National Bureau of Standards Circular 544. U. S. Gov. Printing Office, Washington, 1954, 37 figures, 69 p. Price $.40.

This is a collection of explicit formulas for the computation of the capacitance between conductors having a great variety of geometrical configurations, the inductance of circuits of various shapes, and the electrodynamic forces acting between coils when carrying current. Formulas for skin effect and proximity effect in concentric cables and parallel wires are included. The formulas for the simpler configurations are expressed in terms of elementary functions; in the formulas for more complicated shapes Legendre functions, elliptic functions, and Bessel functions occur.

Section 1 gives formulas for capacitance. Section 2 gives formulas for inductance and electromagnetic force, including a derivation of the general formulas from a vector potential. Section 3 treats frequency effects. Section 4 is a brief section of Legendre functions and in particular it shows the computation of Legendre functions of order $n + 1/2$ (n integer) by means of tables of elliptic integrals. Section 5 contains the derivation of some of the more difficult formulas, especially those derivations involving spheroidal, toroidal or other curvilinear coordinates. Section 6 is a bibliography of 29 numbers.

A. E.

71[Z].—Václav Hruška (editor), Československá Akad. Věd. Laboratoř matematických strojů. Matematické Stroje (Mathematical Machines). Nakladatelství Česk. Akad. Věd, Praha, 1953, 132 p. 25 cm.

The booklet is the first issue, made in 1320 copies and entitled "Mathematical machines," of a new series entitled "Machines for processing information" (Stroje na zpracování informací). The booklet is issued by the Laboratory of

Mathematical Machines of the Czechoslovak Academy of Sciences. Editor of the booklet is Dr. Václav Hruška. A principal man in the Laboratory seems to be Antonín Svoboda. Assistants credited with helping prepare the booklet are K. Bém, V. Černý, H. Jermář, Z. Korvas, K. Krištoufek, J. Marek, J. Oblonský, O. Pokorná, Z. Pokorný, J. Raichl, F. Svoboda, H. Šnelerová, M. Štěrbová, M. Valach, and V. Vyšin. Professor E. Čech is mentioned as a member of the Academy of Sciences interested in the work.

The booklet is 132 pages long, with three page summaries in both Russian and English. There are seven chapters.

"Chapter 1 describes the general character of a modern automatic computer" (from the summary). Basic concepts are defined, and symbols and flow charts are introduced, independent of any particular machine. The Czechoslovak automatic computer SAPO is mentioned next, with a simplified diagram but no mention of the type of memory, speed, or such interesting details.

SAPO is a binary machine, with a memory of 1024 cells (numbered octally) of 32 bits each. The 32nd bit is always a parity check digit. The words are often divided into: one parity bit, 24 significant digits, sign of exponent, five-bit exponent, and sign of number. Sometimes they are in a coded floating decimal form, with six significant decimals.

There are 27 different three-address commands, including division and several logical commands.

Chapters 3, 4, and 5 explain programming and coding, using three examples: computing cosines, geometrical ray tracing, and the Runge-Kutta method for solving differential equations.

Chapters 6 and 7 describe (with pictures) the punched card machinery of the Laboratory, built by the state corporation ARITMA. They use a 90 column card, really 45 double columns. Holes are round, and everything looks like Remington-Rand equipment. There is a calculating punch seemingly comparable with the IBM 602; there does not seem to be any card-programming. An example deals with computing coordinates of points associated with the cross-sections of turbo-compressor blades. The errors of interpolation are discussed.

One feels strongly the lack of any engineering specifications for the machines. I see no way to decide whether SAPO's memory is a magnetic drum, or cathode-ray tube, or other.

It is impossible to determine from the booklet whether SAPO exists only on paper, is being built, or is actually operating; this is a usual situation with reports concerning computing machines.

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72[Z].—T. Pearcey, G. W. Hill, & R. D. Ryan, "The effect of interpretive techniques on functional design of computers," *Australian Journal of Physics*, v. 7, 1954, p. 505.

A number of computer codes were analyzed by counting the relative frequency of the various commands as they occur in complete programs and in interpretive function blocks. The results suggest that computers, suitably designed to use interpretive methods, could store the standard function blocks in a fixed or semi-fixed storage system of rapid access. A relatively small additional amount of rapid-access erasable store would be required and large amounts of problem data would be held in slower-access backing store. A comparison of certain interpretive techniques with direct coding techniques shows interpretive techniques to be roughly twice as expensive in time and half as expensive in storage space as direct techniques. Based on this study, details of a functional machine design are suggested and the advantages of the resulting code system are discussed.

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73[Z].—R. J. König, *Introduction to FLAC Coding.* Air Force Missile Test Center, Patrick Air Force Base, Florida, v + 114 p. 27 cm.

This is primarily an introduction to FLAC programming and contains a certain amount of background information on modern high speed digital computers in addition to specific FLAC coding information. Among its sixteen operations FLAC has certain notable operations such as "decimal-binary conversion," "binary-decimal conversion," "tape advance *n* words," "tape reverse *n* words," "tape hunt," "tally," "read out *n* words," etc.

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74[Z].—Los Alamos Scientific Laboratory, *MANIAC.*

This is a 306 page programmer's manual for the MANIAC. It contains chapters on coding and flow diagrams, binary arithmetic, a simplified discussion of the various computer components, "descriptive" coding and subroutines, and operating procedures. It is liberally illustrated with examples and is another good example of a programming manual.

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75[Z].—*Manual of the SWAC Computing System*, National Bureau of Standards Institute for Numerical Analysis, Los Angeles, California, viii + about 300 p. continuously revised. 29 cm.

This work views the SWAC Computing System from a point midway between that of the engineer and the programmer. Thus it is neither a programming manual, nor an operation and maintenance manual, but includes most of the essential features of both.

The manual is divided into five sections. These consist of (1) an introduction, (2) a detailed description of the "hardware" of the machine itself, (3) a discussion of the order code and of standard coding procedures, and (4 and 5) a discussion of operating and testing procedures and of the service routines used.

Most of the manual is written from an engineering standpoint. For example, the description of the machine itself describes the physical location of each piece of equipment and its use in great detail.

The sections on operating and coding provide enough information to enable one unfamiliar with the SWAC to make use of standard service techniques in programming. With this end in mind, the chief criticisms are:

(1) The information concerned with preparation of data for input and the processing of output data is scattered throughout the manual and hence is difficult to follow.

(2) It would be hard to use the manual as a reference, since there is no detailed index and much of the information is given in descriptive style. (However, this is unfortunately true of most manuals of this type).

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76[Z].—PROGRAMMING RESEARCH GROUP, APPLIED SCIENCE DIVISION, INTERNATIONAL BUSINESS MACHINES CORPORATION, "Specifications for the IBM mathematical FORmula TRANslating system, *FORTRAN*, International Business Machines Corp., New York, 1954. 43 p. 28 cm.

This contains proposed specifications for a code which IBM plans to prepare which will permit the IBM 704 computer to directly accept mathematical problems written in their usual mathematical form. Besides mathematical expressions, there are provisions for accepting input-output formulas (card reading formulas, tape reading formulas, print formulas, etc.), and formulas for all other information necessary in solving a mathematical problem.

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77[Z].—ALBERT H. RUBENSTEIN, Editor, *Coordination, Control, and Financing of Industrial Research*, Proceedings of the Fifth Annual Conference on Industrial Research, June, 1954, with selected papers from the Fourth Conference, June, 1953. King's Crown Press, Columbia University, New York, 1955.

Includes nontechnical chapters on Introduction to Computer Technology by C. B. TOMPKINS, Application of High Speed Computers to Research Problems by RICHARD F. CLIPPINGER, Automatic Data Reduction by G. TRUMAN HUNTER, and Operation of an Industrial Computing Facility by H. R. J. GROSCH, extending through about 30 pages.

C. B. T.

TABLE ERRATA

In this issue references have been made to errata in Review **55** and Review **59.**

244.—T. LAIBLE, "Höhenkarte des Fehlerintegrals," *Zeit. angew. Math. Physik*, 1951, p. 484–487. [*MTAC*, v. 6, 1952, p. 232.]

T. Laible gives the first five complex zeros of the error function $\int_0^z e^{-u^2}du$ which he obtained graphically to 3D. His 2nd and 3rd zeros do not deviate by