which the transformation $e_m(S_n)$ might prove useful (besides being irrelevant to the main thesis of this paper which is primarily one of computational expediency) would be excessively discursive and, in view of possible developments, necessarily incomplete. By way of a footnote however it may be pointed out that the transformation may be applied to establish a criterion for the fitting of certain types of statistical data. The determination of the constants α_i, b_i when fitting tabular data, which is given at regularly spaced intervals in t of magnitude w, to a function of the form

$$\phi(t) = \sum_{i=1}^{h} b_i e^{-\alpha_i t}, \tag{8}$$

when m is given, is already well known [6]. To establish the suitability of (8) as an adequate representation of the data, and further to determine h, it is merely necessary to apply the transformation $e_m(S_n)$ to the data, writing $S_{n+r} = \phi(t + rw)$, when $e_h(S_n) = 0$. (Various refinements, such as partitioning the data in groups of p values, and so on, are obvious and need not be discussed here.) This was one of the first problems upon which the transformation $e_m(S_n)$ [5] was used.

It is the author's hope that by demonstrating the ease with which the various transformations may be effected, their field of application might be widened, and deeper insight thereby obtained into the problems for whose solution the transformations have been used.

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TECHNICAL NOTES AND SHORT PAPERS

Note on the Computation of Certain Highly Oscillatory Integrals

The purpose of this note is to draw attention to the possible use of the Faltung theorem for Fourier transforms as an aid to the computation of highly oscillatory integrals.

If F is the Fourier cosine transform of f, defined by

$$F(u) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(t) \cos ut \, dt$$

and G is the Fourier cosine transform of g, then

$$\int_0^\infty f(t)g(t)\cos ut\,dt = \frac{1}{2}\int_0^\infty F(y)[G(u+y) + G(|u-y|)]dy.$$

Numerical quadrature of the integral on the left will be laborious if u is large. But, if the transforms F, G are known, the integral on the right may prove much more tractable. The essential reason for this is that the large parameter u now appears in the argument of G as an additive constant and not as a multiplicative constant as in the integral on the left.

As an example, consider the integral

$$I = \int_1^\infty (t^2-1)^{-\frac{1}{2}}\, e^{-at^2}\cos bt\,dt$$

for large values of b. Here

$$f(t) = e^{-at^2} \qquad F(u) = \frac{1}{\sqrt{(2a)}}e^{-u^2/4a}$$

$$g(t) = (t^2-1)^{-\frac{1}{2}} \quad \text{for} \quad t > 1 \qquad G(u) = -\sqrt{\frac{\pi}{2}}\,Y_0(u)$$

$$= 0 \qquad \text{for} \quad t \le 1$$

so that

$$I = -\frac{1}{4}\sqrt{\frac{\pi}{a}}\int_0^\infty e^{-y^2/4a}[Y_0(b+y) + Y_0(|b-y|)]dy.$$

This form is, for a wide range of values of a and b, very suitable for computation using the simple trapezoidal rule [1,2]. The method was used very successfully in preparing a table of an integral, closely related to the one above, which occurs in the theory of the wave resistance of ships [UMT **174**, *MTAC*, v. 7, 1953, p. 248].

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On a Cubically Convergent Process for Determining the Zeros of Certain Functions

An iterative procedure of the form $x_{n+1} = f(x_n)$ for the determination of the roots of the equation $\phi(x) = 0$ may be classified as linearly convergent, quadratically convergent, and so on, by using a notation due to Hartree [1]: If $x = X$