Use of the above values of S_m in (1) for $n = 42$ gave $S_{42} = 3.1229\ 14703$, from which $42 \times S_{42} = 131.16241\ 75$, agreeing with the true value of $131.16244\ 63$ to within 3 units in the 8th significant figure. But the same formula (1) for the very next or 11th zero was so accurate as to give an answer of $33.77582\ 019$, which agrees with the true value of $33.77582\ 021$ to 2 units in the 10th significant figure.

HERBERT E. SALZER

Convair
San Diego 12, California

The work reported was done while the author worked at the Diamond Ordnance Fuze Laboratories, Washington, D. C.

1. H. E. SALZER, "A simple method for summing certain slowly convergent series," *J. Math. Phys.*, v. 33, 1955, p. 356-359.

2. H. E. SALZER & R. ZUCKER, "Table of the zeros and weight factors of the first fifteen Laguerre polynomials," *Amer. Math. Soc., Bull.*, v. 55, 1949, p. 1004-1012.

3. BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, *Mathematical Tables*, v. VI, *Bessel Functions*, Part I, Cambridge Univ. Press, 1950, p. 171.

4. R. B. HORGAN, "Table of coefficients for the partial summation of series" [see note below, this issue].

TECHNICAL NOTES AND SHORT PAPERS

Table of Coefficients for the Partial Summation of Series

This note supplies a table of the coefficients $A_m(n)$ as described by Herbert E. Salzer [1]. The computation of the table was undertaken at the suggestion of the referee of Dr. Salzer's paper and the Chairman of the Editorial Committee. Responsibility for its correctness does not fall upon Dr. Salzer.

For $m = 4(1)10$ and $n = 11(1)50(5)100(10)200(50)500(100)1000$, the table lists

$$(1) \quad A_m(n) = \frac{m^6 \prod_{j=4}^{10} (n-j)}{n^6(n-m) \prod' (m-j)}$$

to fifteen decimals. ($j = m$ is absent from $\prod'$.)

The computation was performed on the SWAC machine by means of double precision routines. The seven values of

$$(2) \quad \frac{m^6}{\prod' (m-j)}$$

were stored as constants in the computing routine. For each n , the seven values

of $\frac{\prod_{j=4}^{10} (n-j)}{n^6(n-m)}$ were obtained and then multiplied by (2), resulting in (1).

The results were converted to 25-decimal numbers and punched on cards by SWAC. They were listed on an International Business Machines Corporation

Coefficients for Partial Summation of Series

m	n	$A_m(n)$			m	n	$A_m(n)$			
10	11	3.95131	75103	76442	10	110	962.93875	62720	10955	
10	12	9.37714	33470	50754	10	120	993.71112	77220	50754	
10	13	17.40280	17267	72284	10	130	1020.37531	93880	26333	
10	14	27.89016	48122	80597	10	140	1043.69507	12500	74375	
10	15	40.55967	07818	93004	10	150	1064.25768	24362	13992	
10	16	55.07469	17724	60937	10	160	1082.52165	05527	49635	
10	17	71.09249	48572	90725	10	170	1098.84984	32878	63827	
10	18	88.29178	59179	09984	10	180	1113.53301	56593	11194	
10	19	106.38550	90735	78621	10	190	1126.80676	59738	37326	
10	20	125.12500	00000	00000	10	200	1138.86395	35000	00000	
10	21	144.29940	23246	07871	10	250	1185.65176	32368	64000	
10	22	163.73271	93362	23815	10	300	1217.71174	10754	45816	
10	23	183.27988	01917	55393	10	350	1241.04485	65364	43149	
10	24	202.82258	71270	57613	10	400	1258.78463	67099	60937	
10	25	222.26534	40000	00000	10	450	1272.72579	35099	77590	
10	26	241.53185	36532	10640	10	500	1283.96982	11957	76000	
10	27	260.56185	16990	72116	10	600	1300.98878	46558	64197	
10	28	279.30837	96929	85066	10	700	1313.25830	08705	89636	
10	29	297.73546	82433	50869	10	800	1322.52269	32061	92016	
10	30	315.81618	65569	27298	10	900	1329.76539	82437	63654	
10	31	333.53101	10110	96866	10	1000	1335.58299	93247	84000	
10	32	350.86646	67606	35376						
10	33	367.81400	03486	21887	9	11	—	6.29967	63870	95900
10	34	384.36904	62365	94911	9	12	—	19.93359	37500	00000
10	35	400.53025	52507	88361	9	13	—	41.61853	05861	49151
10	36	416.29885	77283	27321	9	14	—	71.14548	99744	15422
10	37	431.67813	84464	44154	9	15	—	107.77536	00000	00000
10	38	446.67300	41679	95069	9	16	—	150.52602	48184	20411
10	39	461.28962	78468	17233	9	17	—	198.35269	94371	30558
10	40	475.53515	62500	00000	9	18	—	250.25000	00000	00000
					9	19	—	305.30315	51688	87156
10	41	489.41747	00281	06694	9	20	—	362.70848	25000	00000
10	42	502.94498	71587	40688						
10	43	516.12650	22606	36098	9	21	—	421.77640	26893	55627
10	44	528.97105	55761	27495	9	22	—	481.92524	36128	36364
10	45	541.48782	64943	80373	9	23	—	542.67075	27929	25775
10	46	553.68604	73746	33594	9	24	—	603.61413	57421	87500
10	47	565.57493	41604	04691	9	25	—	664.43015	63289	60000
10	48	577.16363	08793	01430	9	26	—	724.85607	43754	31056
10	49	588.46116	56213	25944	9	27	—	784.68175	58299	03978
10	50	599.47641	60000	00000	9	28	—	843.74104	51653	32896
					9	29	—	901.90435	93786	73385
10	55	650.61572	25181	63361	9	30	—	959.07240	00000	00000
10	60	695.91126	54320	98766						
10	65	736.22631	59781	13077	9	31	—	1015.17085	48575	58614
10	70	772.28678	52680	43077	9	32	—	1070.14595	76934	57602
10	75	804.69824	40384	08780	9	33	—	1123.96078	09158	13794
10	80	833.96507	26318	35937	9	34	—	1176.59214	89338	88081
10	85	860.50810	76723	17788	9	35	—	1228.02807	33503	89718
10	90	884.67972	36193	66967	9	36	—	1278.26562	50000	00000
10	95	906.77629	08297	96555	9	37	—	1327.30917	05359	61070
10	100	927.04830	40000	00000	9	38	—	1375.16891	25983	20775
					9	39	—	1421.85968	24527	34301
					9	40	—	1467.39994	24218	75000

Coefficients for Partial Summation of Series—Continued

m	n	$A_m(n)$			m	n	$A_m(n)$		
9	41	-1511.81096	25685	15978	8	21	480.11474	74070	79307
9	42	-1555.11614	20836	55620	8	22	551.84849	97129	65006
9	43	-1597.34045	08412	69187	8	23	624.59437	18688	37833
9	44	-1638.50997	07506	36726	8	24	697.83950	61728	39506
9	45	-1678.65152	00000	00000	8	25	771.16137	79968	00000
9	46	-1717.79234	61570	18721	8	26	844.21501	65875	63335
9	47	-1755.95987	64710	78889	8	27	916.72132	33268	10422
9	48	-1793.18151	56936	64551	8	28	988.45671	44642	11340
9	49	-1829.48448	33659	34024	8	29	1059.24411	79689	38882
9	50	-1864.89568	38758	40000	8	30	1128.94525	10288	06583
9	55	-2029.48358	59023	76491	8	31	1197.45405	67567	51638
9	60	-2175.50458	12500	00000	8	32	1264.69116	21093	75000
9	65	-2305.64429	22251	61592	8	33	1330.59921	96419	70076
9	70	-2422.17951	01945	61789	8	34	1395.13900	50920	20658
9	75	-2527.02059	71046	40000	8	35	1458.28615	60064	25893
9	80	-2621.76559	87197	87598	8	36	1520.02845	09475	18163
9	85	-2707.75237	05561	56671	8	37	1580.36354	27443	58135
9	90	-2786.10416	00000	00000	8	38	1639.29707	20646	08589
9	95	-2857.76779	50603	22496	8	39	1696.84109	89674	22084
9	100	-2923.54503	26601	60000	8	40	1753.01280	00000	00000
9	110	-3040.07011	23086	36282	8	41	1807.83338	68884	03656
9	120	-3140.04712	95703	12500	8	42	1861.32721	00763	42383
9	130	-3226.72629	48747	29867	8	43	1913.52101	64360	86679
9	140	-3302.56973	16104	15303	8	44	1964.44333	55667	68517
9	150	-3369.47333	53881	60000	8	45	2014.12397	33509	45825
9	160	-3428.91887	18030	69115	8	46	2062.59359	49896	58217
9	170	-3482.08015	64228	31147	8	47	2109.88338	26923	72969
9	180	-3529.89848	75000	00000	8	48	2156.02475	56584	36215
9	190	-3573.13712	52882	55523	8	49	2201.04914	20334	77265
9	200	-3612.42124	64694	90000	8	50	2244.98779	42272	00000
9	250	-3764.93651	67517	07996	8	55	2449.46022	07567	22460
9	300	-3869.50853	94386	40000	8	60	2631.18696	29629	62962
9	350	-3945.64787	09362	78951	8	65	2793.38213	83334	62128
9	400	-4003.55307	59989	80411	8	70	2938.79536	02189	56387
9	450	-4049.06957	97094	40000	8	75	3069.74844	88518	58437
9	500	-4085.78689	38768	77630	8	80	3188.19287	50000	00000
9	600	-4141.37340	72762	97500	8	85	3295.76840	32449	16669
9	700	-4181.45574	72446	93906	8	90	3393.85582	63679	31718
9	800	-4211.72539	76294	68478	8	95	3483.62188	59619	52758
9	900	-4235.39225	03513	60000	8	100	3566.05655	77728	00000
9	1000	-4254.40401	06764	13169	8	110	3712.18554	88848	53527
8	11	5.17907	08872	00610	8	120	3837.65559	17695	47326
8	12	18.43621	39917	69547	8	130	3946.50395	00506	44225
8	13	41.05836	05027	66942	8	140	4041.79455	66490	15291
8	14	73.11239	36455	04849	8	150	4125.89019	99969	97532
8	15	113.91936	79012	34567	8	160	4200.63980	27343	75000
8	16	162.42187	50000	00000	8	170	4267.50953	06496	85560
8	17	217.42549	46718	12227	8	180	4327.67614	58449	76206
8	18	277.74194	31319	75139	8	190	4382.09455	34093	57983
8	19	342.26578	09298	96923	8	200	4431.54741	58592	00000
8	20	410.00960	00000	00000					

Coefficients for Partial Summation of Series—Continued

m	n	$A_m(n)$			m	n	$A_m(n)$			
8	250	4623.64208	66077	36013	7	55	-1435.20542	13476	13771	
8	300	4755.44125	32133	39918	7	60	-1544.77857	48456	79012	
8	350	4851.44900	77387	00876	7	65	-1642.72273	85062	05653	
8	400	4924.48880	83248	00000	7	70	-1730.64320	00000	00000	
8	450	4981.91638	17457	12963	7	75	-1809.90480	62755	46775	
8	500	5028.25129	74297	95635	7	80	-1881.66054	19464	11133	
8	600	5098.41330	25147	25926	7	85	-1946.88304	53757	79143	
8	700	5149.01707	31826	52370	7	90	-2006.39392	39343	59599	
8	800	5187.23858	37385	75000	7	95	-2060.88921	05422	61924	
8	900	5217.12642	87430	44780	7	100	2110.96075	96896	00000	
8	1000	5241.13788	92728	76238	7	110	-2199.78216	96436	08095	
7	11	—	2.32434	27688	91391	7	120	-2276.10677	01222	77950
7	12	—	8.82569	23010	97393	7	130	-2342.36362	83059	88491
7	13	—	20.47422	22035	30323	7	140	-2400.39978	75000	00000
7	14	—	37.50000	00000	00000	7	150	-2451.64185	89274	60082
7	15	—	59.64755	88477	36626	7	160	-2497.20764	05074	59641
7	16	—	86.39309	88311	76757	7	170	-2537.98449	49786	28544
7	17	—	117.09545	29845	15548	7	180	-2574.68529	23793	51611
7	18	—	151.09004	10393	62789	7	190	-2607.88896	80666	75167
7	19	—	187.74223	13549	61766	7	200	-2638.07036	27369	00000
7	20	—	226.47432	50000	00000					
7	21	—	266.77640	60356	65295	7	250	-2755.37272	67763	71609
7	22	—	308.20785	11549	98331	7	300	-2835.91432	77459	17147
7	23	—	350.39378	76510	47240	7	350	-2894.61302	33856	00000
7	24	—	393.01911	02832	43312	7	400	-2939.28518	84210	26758
7	25	—	435.82159	09376	00000	7	450	-2974.41821	34306	25530
7	26	—	478.58494	40075	21326	7	500	-3002.77099	76467	46726
7	27	—	521.13230	19392	50302	7	600	-3045.71390	96231	66357
7	28	—	563.32031	25000	00000	7	700	-3076.69358	81384	00000
7	29	—	605.03392	90580	70673	7	800	-3100.09692	66855	50437
7	30	—	646.18188	75171	46776	7	900	-3118.39992	63337	66844
7	31	—	686.69282	35027	79364	7	1000	-3133.10582	56734	31977
7	32	—	726.51196	54834	27048	6	11	.55305	80092	92369
7	33	—	765.59833	42471	88752	6	12	2.18750	00000	00000
7	34	—	803.92238	08122	51640	6	13	5.21964	71830	56135
7	35	—	841.46400	00000	00000	6	14	9.75932	64711	13226
7	36	—	878.21086	35412	96212	6	15	15.76960	00000	00000
7	37	—	914.15702	35815	42751	6	16	23.12608	33740	23438
7	38	—	949.30174	41845	75904	6	17	31.66123	64733	16761
7	39	—	983.64852	46062	22385	6	18	41.19341	56378	60082
7	40	—	1017.20428	35937	50000	6	19	51.54427	01561	90718
7	41	—	1049.97867	87479	04978	6	20	62.54820	00000	00000
7	42	—	1081.98353	90946	50205	6	21	74.05676	20634	25954
7	43	—	1113.23239	25151	28900	6	22	85.94002	97251	97157
7	44	—	1143.74007	26455	01622	6	23	98.08621	69173	04425
7	45	—	1173.52239	23543	72358	6	24	110.40039	06250	00000
7	46	—	1202.59587	30014	44941	6	25	122.80277	23776	00000
7	47	—	1230.97752	04371	92854	6	26	135.22692	19685	30348
7	48	—	1258.68464	01727	41214	6	27	147.61796	69991	58892
7	49	—	1285.73468	53776	91268	6	28	159.93096	25453	68001
7	50	—	1312.14513	23904	00000	6	29	172.12941	79049	17686
					6	30	184.18400	00000	00000	

Coefficients for Partial Summation of Series—Continued

m	n	$A_m(n)$			m	n	$A_m(n)$		
6	31	196.07140	79066	45065	5	11	—	.06173	93360 99633
6	32	207.77340	53134	91822	5	12	—	.25117	34825 10288
6	33	219.27599	44478	34424	5	13	—	.61181	72482 06838
6	34	230.56871	42727	58785	5	14	—	1.16209	02005 11692
6	35	241.64404	64092	34249	5	15	—	1.90123	45679 01234
6	36	252.49691	35802	46915	5	16	—	2.81631	94656 37208
6	37	263.12425	70025	71157	5	17	—	3.88787	08125 08086
6	38	273.52468	08323	13461	5	18	—	5.09375	68798 79423
6	39	283.69815	33762	78200	5	19	—	6.41162	66629 16569
6	40	293.64575	62500	00000	5	20	—	7.82031	25000 00000
6	41	303.36947	39702	45024	5	21	—	9.30054	74154 53241
6	42	312.87201	76117	09408	5	22	—	10.83525	34854 85399
6	43	322.15667	71429	39398	5	23	—	12.40957	52213 16771
6	44	331.22719	78991	97374	5	24	—	14.01077	08212 77007
6	45	340.08767	73662	55144	5	25	—	15.62803	20000 00000
6	46	348.74247	90551	97216	5	26	—	17.25227	52609 43617
6	47	357.19616	07596	15901	5	27	—	18.87592	95975 18009
6	48	365.45341	49169	92187	5	28	—	20.49273	43796 48361
6	49	373.51901	91578	77238	5	29	—	22.09755	42836 86197
6	50	381.39779	54304	00000	5	30	—	23.68621	39917 69546
6	55	418.15736	37983	67655	5	31	—	25.25535	29972 34498
6	60	450.95050	00000	00000	5	32	—	26.80229	95442 15202
6	65	480.30905	71792	66882	5	33	—	28.32496	20804 18426
6	70	506.69735	98143	63062	5	34	—	29.82173	63459 42708
6	75	530.51306	14784	00000	5	35	—	31.29142	61914 67842
6	80	552.09389	39208	98439	5	36	—	32.73317	63133 16060
6	85	571.72594	37405	64761	5	37	—	34.14641	52482 05055
6	90	589.65167	40740	74074	5	38	—	35.53080	71497 26880
6	95	606.07704	65934	56291	5	39	—	36.88621	10502 51009
6	100	621.17759	20896	00000	5	40	—	38.21264	64843 75000
6	110	647.98371	07590	42449	5	41	—	39.51026	45074 77363
6	120	671.03743	43750	00000	5	42	—	40.77932	32831 41136
6	130	691.06399	69553	38402	5	43	—	42.02016	75360 22183
6	140	708.61598	97483	19153	5	44	—	43.23321	12730 48883
6	150	724.12092	71296	00000	5	45	—	44.41892	32671 17140
6	160	737.91423	88858	79517	5	46	—	45.57781	48753 50937
6	170	750.26251	15381	25483	5	47	—	46.71042	98302 11995
6	180	761.38011	93415	63786	5	48	—	47.81733	56978 49101
6	190	771.44130	69448	01395	5	49	—	48.89911	67455 22113
6	200	780.58909	45944	00000	5	50	—	49.95636	80000 00000
6	250	816.16379	99839	54330	5	55	—	54.89570	15874 70033
6	300	840.60878	09984	00000	5	60	—	59.31061	92129 62962
6	350	858.43361	34170	86880	5	65	—	63.26944	90293 69093
6	400	872.00420	72601	23437	5	70	—	66.83251	02635 80650
6	450	882.68005	51919	14403	5	75	—	70.05185	60658 43621
6	500	891.29754	13687	64007	5	80	—	72.97194	38552 85645
6	600	904.35278	57266	00000	5	85	—	75.63059	54008 87306
6	700	913.77345	18790	75909	5	90	—	78.05997	56134 73555
6	800	920.89153	96405	57838	5	95	—	80.28748	40838 88236
6	900	926.45915	49364	14814	5	100	—	82.33652	70000 00000
6	1000	930.93306	05482	71897					

Coefficients for Partial Summation of Series—Continued

m	n	$A_m(n)$	m	n	$A_m(n)$
5	110	— 85.97667 46671 43836	4	41	1.67957 49371 42942
5	120	— 89.10996 52576 83898	4	42	1.73478 96146 54486
5	130	— 91.83377 87449 22369	4	43	1.78881 50527 58094
5	140	— 94.22247 17100 76158	4	44	1.84166 56270 93845
5	150	— 96.33366 95308 64198	4	45	1.89335 84099 08155
5	160	— 98.21264 97477 29302	4	46	1.94391 26143 25570
5	170	— 99.89544 02988 96711	4	47	1.99334 91260 90177
5	180	— 101.41104 24975 44413	4	48	2.04169 01095 25035
5	190	— 102.78304 95989 64891	4	49	2.08895 86764 66958
5	200	— 104.03084 19062 50000	4	50	2.13517 86086 40000
5	250	— 108.88638 64197 12000	4	55	2.35140 17642 06820
5	300	— 112.22555 24084 36215	4	60	2.54504 69135 80248
5	350	— 114.66175 30495 62682	4	65	2.71896 82698 94251
5	400	— 116.51724 88093 87207	4	70	2.87571 51567 79913
5	450	— 117.97739 09714 30507	4	75	3.01750 50773 63183
5	500	— 119.15629 02246 08000	4	80	3.14624 29687 50000
5	600	— 120.94275 99181 13426	4	85	3.26355 66750 23902
5	700	— 122.23222 49461 38513	4	90	3.37083 54864 60396
5	800	— 123.20671 31642 56095	4	95	3.46926 63012 67056
5	900	— 123.96905 07440 93888	4	100	3.55986 54873 60000
5	1000	— 124.58170 56530 97000	4	110	3.72094 07034 81281
4	11	.00231 20852 17500	4	120	3.85971 10836 76269
4	12	.00960 21947 87380	4	130	3.98043 55316 31768
4	13	.02376 06252 90953	4	140	4.08637 31730 82645
4	14	.04569 52460 28441	4	150	4.18005 42836 72757
4	15	.07551 47325 10288	4	160	4.26346 98852 53906
4	16	.11279 29687 50000	4	170	4.33820 62246 81532
4	17	.15679 72317 34479	4	180	4.40554 15310 44839
4	18	.20665 32314 97005	4	190	4.46651 66258 98876
4	19	.26145 30270 99227	4	200	4.52198 71590 40000
4	20	.32032 00000 00000	4	250	4.73798 01192 37264
4	21	.38244 43453 61032	4	300	4.88664 43058 07626
4	22	.44709 94789 34115	4	350	4.99516 96792 10730
4	23	.51364 66873 92137	4	400	5.07786 09345 10000
4	24	.58153 29218 10700	4	450	5.14295 36638 91081
4	25	.65028 48921 60000	4	500	5.19552 17538 96715
4	26	.71950 14345 91672	4	600	5.27520 39203 87161
4	27	.78884 53416 30860	4	700	5.33273 43969 14505
4	28	.85803 53424 16850	4	800	5.37622 08939 50156
4	29	.92683 86032 22821	4	900	5.41024 55059 97484
4	30	.99506 39231 82442	4	1000	5.43759 28570 10013
4	31	1.06255 56830 78909			
4	32	1.12918 85375 97656			
4	33	1.19486 28049 94586			
4	34	1.25950 04907 08074			
4	35	1.32304 18754 09055			
4	36	1.38544 25985 19872			
4	37	1.44667 11723 35430			
4	38	1.50670 68677 06442			
4	39	1.56553 79186 90180			
4	40	1.62316 00000 00000			

model 402 accounting machine, and the sum checks

$$(3) \quad \sum_m A_m(n) \stackrel{?}{=} 1$$

were totalled and listed concurrently. Inspecting the sum checks and spot-checking by hand-computation suggest that the results $A_m(n)$ are accurate to 14 decimals.

The $A_m(n)$ were rounded on cards to 15 decimals, using an International Business Machines Corporation model 075 sorter for machine inspection of the 16th decimal, and an International Business Machines Corporation model 513 reproducer for subsequent insertion of the corrected 15th decimal. It is believed that the error in $A_m(n)$ is $\leq 1 \times 10^{-15}$.

R. B. HORGAN

University of California
Los Angeles, California

1. HERBERT E. SALZER, "Formulas for the partial summation of series" [see p. 149 this issue].

Polynomial Approximations to Some Modified Bessel Functions

The following approximations were determined by equating tabular values of the functions to polynomials of degree n at $n + 1$ points and solving the equations for the $n + 1$ coefficients. In the region from 0 to x_r , the fit points were spaced at equal intervals in x^2 ; in the region from x_r to ∞ , they were spaced at equal intervals in $1/x$. Both n and x_r were determined by trial and error with the following requirements.

- (1) To yield 7 or 8 accurate significant figures over the full range of positive x .
- (2) To make the polynomial degrees approximately the same in the two regions 0 to x_r and x_r to ∞ .
- (3) To keep the magnitudes of the polynomial coefficients, the definitions of the polynomials, and the regions of validity balanced to the extent that a given number of significant figures in the coefficients would give the same number of significant figures in the polynomials over the entire region of validity.

In order to determine the maximum errors, the polynomials were evaluated and compared (directly or after appropriate multiplications and additions) with values given in [1]. Differences between calculated and tabular values were plotted for

$$x = .0[.1]5[.2]7.6[.4]10[1]20.$$

To determine maximum error estimates in formulas (3) and (4), it was necessary to compute I_0 and I_1 at higher values of x from asymptotic expansions. Local maxima in these two formulas occur approximately at midpoints between fit points, decreasing gradually as x is decreased. The lowest is about $.1 \times 10^{-7}$ at $x = 4.1$ and the highest, about 2×10^{-7} at $x = 60$.

These approximations may be considered a sequel to a similar set for J_0 , Y_0 , J_1 , and Y_1 , which was published earlier [2].