REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

106[C, D].—JUAN GARCIA, "Nuevas Tablas de Logaritmos," *Las Ciencias*, Madrid, Spain, v. 19, 1954, p. 567–592.

A new, five-place "triple entry" table of 5D logarithms of numbers and of circular functions is described with the aid of representative excerpts from the table. Instructions for its use are in the form of varied and illustrative examples.

With regard to the logarithmic tabulation of the angles in the intervals (0°, 3°) and (87°, 90°), the table provides a direct and simple interpolation scheme for such angles; the arrangement is "triple entry."

Uniformity in the methods of tabulation and interpolation for the angles throughout the interval (0°, 90°) is attained. The table is compact, and its entries can be read simply and efficiently.

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107[F].—D. N. LEHMER, *Factor Table for the First Ten Millions Containing the Smallest Factor of Every Number not Divisible by 2, 3, 5, or 7 between the Limits 0 and 10017000*, Hafner Publishing Co., New York, 1956, 9 p. in double col. + 476 p., 43 cm. × 30 cm., oblong. Price $22.50. Originally issued as Carnegie Institution of Washington Publication No. 105, 1909.

[F].—D. N. LEHMER, *List of Prime Numbers from 1 to 10006721*, Hafner Publishing Co., New York, 1956, 8 p. in double cols. + 133 p., 43 cm × 30 cm., oblong. Price $15.00. Originally issued as Carnegie Institution of Washington Publication No. 165, 1914.

The availability of these two monumental tables is a cause for rejoicing among all devotees of the theory of numbers—and only a very few of us are not. The author begins his preparatory note to the first of these volumes, "Factors," with the sentence: "The value of a factor table depends chiefly on its freedom from errors." Time has certainly proved the value of this book for no errors have been reported in it; only two serious errors have been found in the companion volume, "Primes." They are listed here.

p. 11, col. 13, line 1, *for* 8151 *read* 8051.
p. 14, col. 30, line 55, *for* 51 *read* 47.
p. 99, col. 20, heading, *for* 224 *read* 724.
p. 119, col. 25, heading, *for* 83 *read* 883.

In the review copy the first of these errors has been corrected while the second has been marked but not corrected. The first three were noted by D. H. Lehmer [1]. The last was pointed out by E. G. H. Comfort (Ripon College, Wisconsin).

Professor Comfort has also pointed out that the description of the Kulik tables in the "Primes" need to be revised in the light of the note by S. A. Joffe [2]. He also noted that he received a copy which had pages 68 and 69 replaced by 28 and 29 and pages 72 and 73 replaced by 48 and 49. The review copy appears faultless in this respect.

Both books have been reproduced by photographic methods from copies of the original edition—which were themselves reproduced photographically from typescript. The original editions were some of the earliest major tables to be produced in this manner.

For those who are not familiar with the tables we give brief descriptions. "Factors" gives the least prime factor of each $n \leq 10017000$, provided n is not divisible by 2, 3, 5, 7. The most convenient way of using it is to find q, r such that $n = 210q + r$, $0 \leq r < 210$ (using a desk calculator, for instance). Then, turning to the page given by the first three digits of q and the line given by the last two digits, we read in the column with heading r, the least prime factor of n, or a dash indicating that n is prime. If r is not among the column headings, a factor 2, 3, 5, or 7 is present and must be removed before using the table. Repeated entry into the table gives the complete factorization of n. By means of this a much greater range can be covered than the other modern tables BAAS [3] which give complete factorization.

In "Primes" we find a listing, 5000 to a page, of all the primes up to 10006721.

Both volumes contain careful introductions which describe the construction of the tables and their checking and printing. In "Primes" there are tables giving comparisons of $\pi(x)$, the number of primes $\leq x$, with various approximate formulae. In "Factors" there is a list of errors discovered in earlier tables.

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1. D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, Bull. National Research Council 105, 1941, p. 161.
2. S. A. JOFFE, Editorial Note in connection with KULIK's *Magnus Canon Divisorum . . .*, UMT **48**, *MTAC*, v. 2, 1946, p. 139–140.
3. J. PETERS, A. LODGE, E. J. TERNOUTH, & E. GIFFORD, *Factor Table giving the Complete Decomposition of all Numbers less than 100,000*. (British Association for the Advancement of Science, *Mathematical Tables*, v. 5.) London, BAAS, 1935.
4. J. KAVÁN, *Factor Tables giving the Complete Decomposition into Prime Factors of All Numbers up to 256,000 . . .*, Macmillan, London, 1937. [RMT **196**, *MTAC*, v. 1, 1945, p. 420–421.]

108[F, L].—D. H. LEHMER, "Extended computation of the Riemann zeta-function," *Mathematika*, v. 3, 1956, p. 102–108.

This paper describes the methods used by the author in the computation of the first 25,000 zeros of the Riemann zeta-function $\zeta(s)$ which was carried out on SWAC. All these zeros have Re $s = \frac{1}{2}$.

A table of the coefficients of 4 polynomials entering in the calculation is included, and there is a table of the number of failures of Gram's law divided into 3 types and tabulated in 15 sets of 1000 roots each, beginning with root number 9892, which overlaps the previous run [1].

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1. D. H. LEHMER, "On the roots of the Riemann zeta-function," *Acta Mathematica*, v. 95, 1956, p. 291–298 [*MTAC*, Review **52**, v. 11, 1957, p. 107–8].

109[F].—ALBERT GLODEN, *Table de factorisation des nombres* $N^4 + 1$ *dans l'intervalle* $3000 < N \leq 6000$, published by the author, rue Jean Jaurès, 11, Luxembourg, 25 p., 30 cm., mimeographed. Price 125 francs belges. A copy deposited in the UMT FILE.

This table is a revision of UMT **108** [1] with all unknown factors $> 8.10^5$. The author intends to extend the tables [2] of solutions of $x^4 + 1 \equiv 0 \pmod p$ to 10^6. The UMT files also contain an earlier revision of UMT **108** published in 1952.

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1. A. GLODEN, *Table de factorisation des nombres* $N^4 + 1$ *dans l'intervalle* 3001–6000. [UMT **108**, *MTAC*, v. 4, 1950, p. 224.]
2. S. HOPPENOT, *Table des Solutions de la Congruence* $x^4 \equiv -1$ *(mod N) pour* $100000 < N < 200000$, Brussels, Librairie du "Sphinx," 1935. [RMT **48**, *MTAC*, v. 1, 1943, p. 6.]
3. A. GLODEN, "Table des solutions de la congruence $X^4 + 1 \equiv 0 \pmod p$ pour $2.10^5 < p < 3.10^5$," *Mathematica* (Rumania), v. 21, 1945. [RMT **280**, *MTAC*, v. 2, 1946, p. 71–72.]
4. ALBERT DELFIELD, "Table des solutions de la congruence $X^4 + 1 \equiv 0 \pmod p$ pour $300000 < p < 350000$," Institut Grand-ducal Luxembourg, Section des Sciences, *Archives*, v. 16, 1946. [RMT **346**, *MTAC*, v. 2, 1947, p. 210–211.]
5. ALBERT GLODEN, *Table des solutions de la congruence* $x^4 + 1 \equiv 0$ *(mod p) pour* $350.000 < p < 500.000$, Luxembourg, author, 11 rue Jean Jaurès, and Paris, Centre de Documentation Universitaire, 1946. [RMT **410**, *MTAC*, v. 2, 1947, p. 300–301.]
6. ALBERT GLODEN, *Table des Solutions de la Congruence* $x^4 + 1 \equiv 0$ *(mod p) pour* $500\,000 > p < 600\,000$, Luxembourg, author, rue Jean Jaurès 11, 1947. [RMT **491**, *MTAC*, v. 3, 1948, p. 96.]
7. A. GLODEN, *Solutions of* $x^4 + 1 \equiv 0$ *(mod p) for* $600000 < p < 800000$ [RMT **1169**, *MTAC*, v. 8, 1954, p. 77.]

110[F].—JOHN LEECH, "Table of groups of 4, 5, 6, 7 primes from 50 to 100 17000," 1 sheet (photostat), 34 × 23 cm., deposited in UMT FILE.

The table gives $15n$ where four primes between 50 and 100 17000 are generated by adding and subtracting 2 and 4 from $15n$. Those values for which additional primes are generated by addition and subtraction of 8 and of these the values which give additional primes by addition and subtraction of 14 and then of 16 are suitably marked. Double fours, primes of the form $210n \pm (11, 13, 17, 19)$ are also marked.

From author's remarks

111[F].—R. M. ROBINSON, "Table of factors of numbers one unit larger than small multiples of powers of two," Los Angeles, 1957, 312 pages listed on an IBM 402 from SWAC punched card output. The table and the punched cards have been deposited in the UMT FILE.

The main table tells whether any number of the form $N = k \cdot 2^n + 1$ is prime or composite for odd $k < 100$ and all $n < 512$. If the least factor of N is less than 10^4 it is listed. For $k = 1, 3, 5, 7$ the bounds on n are larger, namely 2272, 1280, 1536, 1280, and for $n < 2272$ least factors to 10^5 are listed. Each prime found was tested as a possible factor of any Fermat number $F_m = 2^{2^m} + 1$.

In addition to listing the 14 new factors of Fermat numbers previously reported by Robinson [1], the table lists $95 \cdot 2^{61} + 1$ as a factor of F_{58}. This brings to 30 the number of known composite F_m. There are 118 primes greater than 2^{250} in the table, but only the three already mentioned [1] are greater than 2^{508}.

There is a list of the 42 Cullen numbers $n \cdot 2^n + 1$ with $n < 10^3$ for which Cunningham [2] did not find a small factor. These were all tested, and only the smallest, $141 \cdot 2^{141} + 1$, is a prime.

Robinson has presented this wealth of information in a very careful, pleasing, and useful format which is described in the four-page introduction. This was achieved despite the poor type font and limitations of the IBM 402.

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1. R. M. ROBINSON, "Factors of Fermat numbers," *MTAC*, v. 11, 1957, p. 21–22.
2. A. CUNNINGHAM & H. J. WOODALL, "Factorisation of $Q = (2^q \mp q)$ and $(q \,.\, 2^q \mp 1)$," *Messenger Math.*, v. 47, 1917, p. 1–38.

112[F].—EMMA LEHMER, "On the location of Gauss sums," *MTAC*, 1956, v. 10, p. 194–202.

The generalized Gauss sum of order k is defined by $S_k = \sum_{m=0}^{p-1} \exp(2\pi i m^k/p)$, $p = kf + 1$ and prime.

It is known that $-(k-1)\sqrt{p} \le S_k \le (k-1)\sqrt{p}$. The author investigates the distribution of S_k for $k = 3, 4, 5, 7$ with respect to various partitions of the interval $(-(k-1)\sqrt{p}, (k-1)\sqrt{p})$. The research was undertaken to test Kummer's conjecture which states that S_3 falls in the intervals $(-2\sqrt{p}, -\sqrt{p})$, $(-\sqrt{p}, \sqrt{p})$, $(\sqrt{p}, 2\sqrt{p})$ with frequencies of 1 to 2 to 3. It was found that for the first 1000 primes of the form $6n + 1$, these ratios became 3 to 4 to 5, tending to disprove the conjecture.

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113[F].—R. J. PORTER, *Irregular Negative Determinants of Exponent 3n with their Critical Classes. Part II, from* $-D = 50{,}000$ *to* $100{,}000$. 203 typewritten pages, 25.5 × 10 cm., deposited in UMT FILE.

This table is a supplement to *Table of Irregular Negative Determinants of Exponent 3n up to* $-D = 50{,}000$, which were previously deposited in the UMT FILE [*MTAC*, Review **3**, v. 9, 1955, p. 26 and *MTAC*, Review **84**, v. 9, 1955, Part I, p. 198].

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114[G, X].—D. H. LEHMER, "On certain character matrices, "*Pacific J. Math.*, v. 6, 1956, p. 491–499.

Two classes of matrices M, for which det M, $\lambda_i(M)$, $(M^{-1})_{ij}$ (if existing) and $(M^k)_{ij}$ can be given explicitly, are described. (Here $\lambda_i(M)$ denotes the characteristic roots of M.) The matrices are of order $p - 1$, where p is an odd prime, and they are defined in terms of the Legendre symbol

$$\chi(n) = \left(\frac{n}{p}\right) = \begin{cases} 0 \text{ if } p \text{ divides } n \\ -1 \text{ if the congruence } x^2 \equiv n \pmod{p} \text{ is impossible.} \\ +1 \text{ otherwise} \end{cases}$$

The general element of a matrix in the first class is

$$M_{ij} = a + b\chi(i) + c\chi(j) + d\chi(ij)$$

for any a, b, c, d. This is singular except for $p = 3$. The general element of a matrix in the second class is

$$M_{ij} = c + \chi(\alpha + i + j)$$

where c is any constant and α any integer.

These matrices are, incidentally, useful for the evaluation of programs for inverting and finding the characteristic roots of matrices on computers.

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115[L].—S. L. BELOUSOV, *Tablit͡sy normirovannyk͡h prisoedinennyk͡h polinomov Lez͡handra* (*Tables of normalized associated Legendre polynomials*). Akad. Nauk SSSR, Energetic͡heskiĭ Institut im G. M. Krz͡hiz͡hanovskogo (Academy of Sciences of the USSR, Krz͡hiz͡hanovskiĭ Institute of Power). Moscow, Press of the Academy of Sciences, 1956. 380 p., 27 cm. Price 23.70 rubles.

These are tables of the associated functions $\bar{P}_n^m$ (cos θ) defined by

$$\bar{P}_n^m (\cos \theta) = \sqrt{\frac{2n+1}{2} \frac{(n-m)!}{(n+m)!}} P_n^m (\cos \theta),$$

the normalization resulting from this definition being such that

$$\int_{-1}^{+1} [\bar{P}_n^m(x)]^2 dx = 1.$$

The functions tabulated are thus identical in every way with those whose behavior is exhibited graphically up to $n = 8$ in four diagrams in the second and subsequent editions of Jahnke and Emde [1]. Except for the difference of symbolism, they are also those denoted by F_n^m (cos θ) in Section 16 of FMR *Index*.

The arguments in the Belousov tables are $m = 0(1)36$, $n = m(1)56$, $\theta = 0(2°.5)90°$. Each pair of values of m, n belongs to one column of 37 entries corresponding to the 37 values of θ. The function values are to 6*D*, without differences. Nine figures were kept in the computations.

The author refers not only to several well-known tables, but also to tables by I͡a. M. K͡heĭfet͡s [2], which are stated to give values of associated functions for $m = 1(1)12$, $n = m(1)20$, and each 5° of θ. On page 15 are given nine corrections (two of them taken from K͡heĭfet͡s) to numerical values contained in R. and L. Egersdörfer [3].

A. F.

1. E. JAHNKE & F. EMDE, *Funktionentafeln mit Formeln und Kurven*, second edition, Leipzig and Berlin, 1933, p. 178–179; also with various page numbers in later editions.

2. I͡A. M. K͡HEĬFET͡S, *Tablit͡sy normirovannyk͡h prisoedinennyk͡h polinomov Lez͡handra* (*Tables of normalized associated Legendre polynomials*), Moscow, Gidrometeoizdat, 1950.

3. R. and L. EGERSDÖRFER, *Formeln und Tabellen der zugeordneten Kugelfunktionen 1. Art von n = 1 bis n = 20. I. Teil: Formeln.* Reichamt für Wetterdienst, *Wissenschaftliche Abhandlungen*, Bd. I, Nt. 6, Berlin, 1936.

116[L].—George C. Clark & Stuart W. Churchill, *Tables of Legendre Polynomials* P_n (cos θ) *for* $n = 0(1)80$ *and* $\theta = 0°(1°)180°$. Engineering Research Institute Publications, University of Michigan Press, Ann Arbor, Mich., 1957, ix + 92 p., 28 cm. Price $4.50.

This is a straightforward table of the ordinary Legendre polynomials $P_n(\cos\theta)$. As the values P_0 $(\cos\theta) = 1$ and P_n $(\cos 0) = 1$ are not listed, the tabular arguments are $n = 1(1)80$ and $\theta = 1°(1°)180°$, the limit 180° rather than the usual 90° having been chosen for convenience in application. The function values are to $6D$, and are believed accurate to the last place given, since they were calculated to $11D$ on the Michigan Digital Automatic Computer (MIDAC). No differences are provided. Cosines required in the computations were taken from the NBS 15-place tables.

The table, whose special virtue is the high upper limit of n, was computed in connection with a research program on the engineering applications of light-scattering which has been in progress in the Department of Chemical and Metallurgical Engineering of the University of Michigan for the past ten years.

A. F.

117[L].—Harold K. Crowder & George C. Francis, *Tables of Spherical Bessel Functions and Ordinary Bessel Functions of Order Half an Odd Integer of the First and Second Kinds*. Ballistic Research Laboratories, Memorandum Report No. 1027, Aberdeen Proving Ground, Maryland, 1956, 86 p., multilithed, $8\frac{1}{2}'' \times 11''$. Two copies deposited in UMT File.

This gives tables of the functions

$$j_n(x) = \sqrt{\frac{\pi}{2x}}\, J_{n+\frac{1}{2}}(x), \qquad y_n(x) = \sqrt{\frac{\pi}{2x}}\, Y_{n+\frac{1}{2}}(x)$$

and

$$J_{n+\frac{1}{2}}(x), \qquad Y_{n+\frac{1}{2}}(x)$$

for $x = 1(1)50$, $n = 0(1)N$, in all four cases, where N is such that

$$Y_{n+\frac{1}{2}}(x) < 10^{10} < Y_{n+\frac{3}{2}}(x).$$

Some typical values of N are

x	1	10	20	30	40	50
N	10	30	45	59	72	84.

Values are given to 9 decimals for $n < x$ and to 7 significant figures for $n \geq x$.

These tables only partially overlap earlier tables from which the functions can be derived. We mention a few, extracted from material to appear in the projected second edition of the FMR *Index of Mathematical Tables* [1].

Bessel Functions $J_{n+\frac{1}{2}}(x)$

12 decimals, $n = \mu + \frac{1}{2}(1)M + \frac{1}{2}$, $x = 1(1)20$,

with $| J_{\mu-\frac{1}{2}}(x) | > 1$, $| J_{m+\frac{3}{2}}(x) | < \frac{1}{2}\cdot 10^{-12}$

for each x, in the BAAS *Report* for 1925, page 221; this table was prepared by J. R. Airey [2].

$$6 \text{ decimals}, \; n = -6\tfrac{1}{2}(1)34\tfrac{1}{2}, \; x = 1(1)20$$
$$n = -6\tfrac{1}{2}(1) + 6\tfrac{1}{2}, \; x = 21(1)30.$$

This table is by E. C. J. von Lommel [3], and was partially reproduced (up to $n = 18\frac{1}{2}$) in G. N. Watson's *Treatise on Bessel Functions* [4]. It was similarly reproduced, to 4 decimals only, in the various editions of *Funktionentafeln mit Formeln und Kurven* by E. Jahnke and F. Emde [5].

A more recent tabulation by K. Reitz [6] gives 5 decimals or figures for

$$n = -30\tfrac{1}{2}(1) + 31\tfrac{1}{2}, \qquad x = 0(0{\cdot}2)20.$$

Stokes's Functions

$$j_n(x) = \sqrt{\frac{\pi}{2x}}\, J_{n+\frac{1}{2}}(x), \qquad y_n(x) = \sqrt{\frac{\pi}{2x}}\, Y_{n+\frac{1}{2}}(x) = (-)^{n+1} j_{-n-1}(x).$$

The two volumes produced by the National Bureau of Standards Computation Laboratory [7] give 8- to 10-figure values, with central second or second and fourth differences for interpolation for $n = -22(1) + 21$ with

$$x = 0(.01)10(.1)25,$$

in general, and for $n = -31(1) - 23$, $22(1)30$ with $x = 10(.1)25$. The auxiliary function $\Lambda_m(x) = (\frac{1}{2}x)^m m! J_m(x)$ is also tabulated for

$$m = \frac{61}{2}(1) - \frac{29}{2}, \qquad \frac{1}{2}(1)\frac{61}{2}, \qquad x = 0(.1)25$$

with from 7 figures to 9 decimals, with central x-wise differences.

Riccati Bessel Functions

$$xj_n(x), \qquad (-)^n xy_n(x) = -xj_{-n-1}(x).$$

Here we mention only the book of tables by R. O. Gumprecht and C. M. Sliepcevich [8]. This gives values of the first function to 6 decimals and of the second to 6 or 7 figures, in each case with the first derivatives, for

$$n = 0(1)n + d, \qquad x = 1(1)6(2)10(5)100(10)200(50)400$$

where d increases fairly steadily from 4 at $x = 1$ to 20 at $x = 400$. There is also another inconvenient table giving $xj_n(x)$ and its derivative only for very miscellaneous arguments x from $1{\cdot}2$ to 640, and for $n = 0(1)N$, where N varies erratically from 6 near $x = 2$ to 436 at $n = 640$, usually $\frac{2}{3}x < N < x + 10$ approximately.

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1. A. Fletcher, J. C. P. Miller, & L. Rosenhead, *An Index of Mathematical Tables*, Scientific Computing Service Ltd., London, 1946.
2. BAAS, *Mathematical Tables Report*, "Bessel functions of half-odd integral order," Cambridge University Press, 1925, p. 221–233.
3. E. C. J. von Lommel, Munich Abhandlungen, v. 15, 1886, p. 529–664.

4. G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, New York, 1922.
5. EUGENE JAHNKE & FRITZ EMDE, *Funktionentafeln mit Formeln und Kurven*, Teubner, Leipzig and Berlin, 1909 and 1948.
6. K. REITZ, Institut fur Praktische Mathematik, Darmstadt, Reports, "Tabellierung Besselscher Funktionen," 4. Bericht, p. 2–9; 6. Bericht, p. 2–31; 9 Bericht, p. 2–37, 1945.
7. NATIONAL BUREAU OF STANDARDS COMPUTATION LABORATORY, *Tables of Spherical Bessel Functions*, v. I and II, Columbia University Press, New York, 1947.
8. R. O. GUMPRECHT & C. M. SLIEPCEVICH, *Tables of Riccati Bessel Functions for Large Arguments and Orders*, Engineering Research Institute, University of Michigan, Ann Arbor, 1951.

118[L].—FRANK E. HARRIS, "Tables of the exponential integral Ei(x)," *MTAC*, v. 11, 1957, p. 9–16.

This paper gives basic values of high precision for use in molecular structure calculations, with special reference to the region $4 < |x| < 50$ in which $|x|$ is inconveniently large for the Taylor series and too small for the asymptotic formula.

Table 1. Values of $-\mathrm{Ei}(-x)$ to 18S and of $-e^x\mathrm{Ei}(-x)$ to 19D for $x = 1(1)4(.4)8(1)50$.

Table 2. Values of $\mathrm{Ei}(x)$ to 18S and of $e^{-x}\mathrm{Ei}(x)$ to 19D for the same values of x as in Table 1.

Table 3. Tables of the functions

$$R_n(h) = n!\left[1 - \left(1 + h + \frac{h^2}{2!} + \cdots + \frac{h^n}{n!}\right)e^{-h}\right]$$

which occur in a special interpolation formula. Values of $R_n(h)$ are listed for $h = .01(.01).05, .1(.1).5, 1$ to as many decimal places and up to as large values of n as are necessary.

Table 4. Constants, namely Euler's constant γ and the modulus $\log_{10}e$ to 24D, and $e^{\pm h}$ to 25D for the same values of h as in Table 3.

A. F.

119[L, S].—R. B. DINGLE, D. ARNDT, & S. K. ROY, "The integrals $A_p(x) = (p!)^{-1}\int_0^\infty \epsilon^p(\epsilon + x)^{-1}e^{-\epsilon}d\epsilon$ and $B_p(x) = (p!)^{-1}\int_0^\infty \epsilon^p(\epsilon + x)^{-2}e^{-\epsilon}d\epsilon$ and their tabulation," *Appl. Sci. Research B*, v. 6, 1956, p. 144–154.

[L, S]R. B. DINGLE, D. ARNDT, & S. K. ROY, "The integrals

$$C_p(x) = (p!)^{-1}\int_0^\infty \epsilon^p(\epsilon^2 + x^2)^{-1}e^{-\epsilon}d\epsilon$$

and

$$D_p(x) = (p!)^{-1}\int_0^\infty \epsilon^p(\epsilon^2 + x^2)^{-2}e^{-\epsilon}d\epsilon$$

and their tabulation," *Appl. Sci. Research B.*, v. 6, 1956, p. 155–164.

The integrals are tabulated as follows: $\mathfrak{A}_p(x)$ for $p = -.5(.5)4$, $\mathfrak{B}_p(x)$ for $p = 0(.5)4$, $\mathfrak{C}_p(x)$ for $p = -.5(.5)5$, $\mathfrak{D}_p(x)$ for $p = 0(.5)6.5$ all for

$$x = 0(.1)1(.2)2(.5)10(1)20$$

to 4S except for $\mathfrak{D}_p$ when $x \geq 18$, where 3S are given. It is pointed out that values of $\mathfrak{A}_p$ and $\mathfrak{B}_p$ for imaginary arguments can be obtained from the present tables using relations such as $\mathfrak{A}_p(ix) = (p + 1)\mathfrak{C}_{p+1}(x) - ix\mathfrak{C}_p(x)$. A table for $\mathfrak{A}_p(-x)$ is promised for later publication. Functions are given to facilitate interpolation between orders p for small x. No discussion of interpolation with respect to x is given.

The second paper includes a table of the Fresnel integrals

$$C(x) = \tfrac{1}{2}\int_0^x J_{\frac{1}{2}}(t)dt \text{ and } S(x) = \tfrac{1}{2}\int_0^x J_{\frac{3}{2}}(t)dt \text{ to 12D for } x = 0(1)20.$$

This table was prepared using the representations of $C(x)$ and $S(x)$ as sums of Bessel functions of half-integral order which are tabulated in BAAS [1]. The authors point out an error in this table caused by transposition of digits:

for $J_{39/2}(14)$ *read* 0.00437 04731 08.

The reviewer has checked this value.

The tables for $\mathfrak{A}_p$ and $\mathfrak{B}_p$ were computed, using desk machines, by use of recurrence relations such as $p\mathfrak{A}_p + x\mathfrak{A}_{p-1} = 1$ and $\mathfrak{B}_p = x^{-1}\{1 - (p + x)\mathfrak{A}_p\}$ using initial values obtained from $\mathfrak{A}_0(x) = -e^x\text{Ei}(-x)$ and

$$\mathfrak{A}_{-\frac{1}{2}}(x) = (2/x)^{\frac{1}{2}}F[(2x)^{\frac{1}{2}}]$$

where $F(t)$ is the ratio of the tail area of the normal curve to its bounding ordinate at t, which has been tabulated (BAAS [2]). The tables for $\mathfrak{C}_p$ and $\mathfrak{D}_p$ can be obtained similarly from

$$p(p - 1)\mathfrak{C}_p + x^2\mathfrak{C}_{p-2} = 1 \qquad \text{and} \qquad \mathfrak{D}_p = \tfrac{1}{2}x^{-2}\{(p + 1)\mathfrak{C}_{p+1} - (p - 1)\mathfrak{C}_p\}$$

and expressions for $\mathfrak{C}_0$ and $\mathfrak{C}_1$ in terms of $\text{Si}(x) = \int_\infty^x t^{-1} \sin t\, dt$ and $\text{Ci}(x) = \int_\infty^x t^{-1} \cos t\, dt$ and for $\mathfrak{C}_{\pm\frac{1}{2}}(x)$ in terms of $C(x)$ and $S(x)$. The necessity for care in the use of the recurrence relations is pointed out. The authors state, in a letter to the reviewer, that the results were monitored by calculations of asymptotic representations of $\mathfrak{A}_p$ and $\mathfrak{C}_p$ for large p.

The reviewer checked a substantial portion of the tables for $\mathfrak{A}_p$ and $\mathfrak{B}_p$ using SEAC. It is possible to compute $\mathfrak{A}_p$, for integral p, by use of a Laguerre quadrature formula and, for half-integral p, by use of a Hermite quadrature formula after a trivial change of variable. We can handle $\mathfrak{B}_p$, $\mathfrak{C}_p$, and $\mathfrak{D}_p$ analogously using decompositions into complex partial fractions. The error E in the use of these formulas can be expressed in terms of the divided differences of functions such as $y^p/(y + x)$. For example, for $\mathfrak{A}_p(p \geq 0)$, one has

$$|E| < x^p(m!)^2 \Big/ \left[\prod_{i=1}^{m} (x + x_i)\right]^2 (x + \xi), \qquad 0 < \xi < \infty$$

where x_i is the i-th zero of the Laguerre polynomial of degree m. Thus, e.g., for $x = 2, m = 15, p = 4$ a crude estimate of the error is 4×10^{-8}. As a sample of the accuracy actually obtained, agreement with the present tables was achieved for $x \geq .7$ in the cases $p = 0$ and $p = 1$.

In addition to the tables, the papers treat the following topics in connection with the functions $\mathfrak{A}_p$, $\mathfrak{B}_p$, $\mathfrak{C}_p$ and $\mathfrak{D}_p$: physical applications (in particular, their origin in the theory of elemental semiconductors), recurrence relations, related differential equations, relationships to other special functions, and asymptotic expansions.

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1. BAAS, *Mathematical Tables Report*, Cambridge University Press, 1925, p. 221–233.
2. BAAS, *Mathematical Tables*, v. 7, *The Probability Integral*, W. F. Sheppard, Cambridge University Press, 1939.

120[L, S].—M. DANK & S. W. BARBER, "The specific heat function for a two-dimensional continuum. Numerical values of

$$\frac{C_2}{C_\infty} = \frac{6}{x^2}\int_0^x \frac{\xi^2 d\xi}{e^\xi - 1} - \frac{2x}{e^x - 1}."$$

[*MTAC*, v. 9, 1955, p. 191–194]

This function was computed by using a power series for the range $0 \leq x \leq 2.0$ and by using a different expansion for the range $2.0 \leq x \leq 16.0$. The maximum error using seven terms of the power series is less than 0.5×10^{-5} and for the second expansion it is approximately 2×10^{-6}. The table lists values $x = (0.1)9.4$ and $10(.5)16$, 5D.

A. H. T.

121[M].—F. OBERHETTINGER, *Tabellen zur Fourier Transformation*, Springer-Verlag, Berlin, 1957, x + 213 p., 24 cm. Price DM 35.80.

This collection of Fourier transforms contains 109 pages of Fourier cosine transforms, 91 pages of Foureir sine transforms, and 6 pages of exponential Fourier transforms. Also given are concise definitions of the function symbols used, a table of errata, and a short bibliography. The book appears to be the largest collection of Fourier transforms published to date. The author states in the preface that a considerable portion of the approximately 1800 correspondences collected in the volume is new. Fourier transforms of elementary functions occupy about two-fifths of the space; the remainder is taken up by transforms of higher transcendental functions, with the Bessel functions and their associates controlling a strong minority.

Glancing over the book one cannot help being impressed by the enormous wealth of formulas stacked up in the book and by the astounding formal dexterity which Professor Oberhettinger must have commanded in deriving some of them. On the other hand one also notices some shortcomings. There is no indication of the sources of individual formulas, conditions for validity are given in many cases for real values of the parameters only; there is no system of numbering the formulas that could be used for reference purposes; and the bibliography refers only to a few well-known titles. However, one must not forget that all these matters are of secondary interest to most of the actual users of such tables and that the absence of a complicated scholarly apparatus makes for easier reading.

The existence of some trivial printing errors (on p. 210 and 213) does not increase the confidence in the absence of non-trivial errors. With this reservation, the printing of the book is excellent; by its neat and clean type it is distinguished from the ugly appearance of the Varitype process used in a recent American counterpart. Both author and publisher are to be congratulated on this excellent work.

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122[M].—C. F. LINDMAN, *Examen des Nouvelles Tables d'Intégrales Définies de M. Bierens de Haan*, G. E. Stechert and Co., New York, 1944, 231 p., 27 cm. Price $5.00.

According to the preface, Mr. Lindman has inspected Bierens des Haan's famous tables page by page, checking the integrals against such original sources as were available to him and recomputing a considerable number of the integrals. He collected the results of his inspection in a weighty memoir, which was submitted to the Royal Swedish Academy of Sciences and published in 1891. The volume under review is an unaltered reproduction of this memoir. Although many of Lindman's comments consist in obvious restrictions on the parameters of an integral to insure convergence, there is a surprisingly large number of non-trivial corrections. It appears that the majority of the 486 tables which make up Bierens de Haan's collection contain iaulty results.

In our review of the 1957 Hafner reprint of the *Nouvelles Tables d'Intégrales Définies* [1] we pointed out that the extent and nature of the corrections made in that edition (as well as in an earlier 1939 reprint) were not evident. Several spot checks now reveal that the results of Lindman's *Examen* have not been incorporated. Thus the work under review is an indispensable companion volume also to the modern editions of Bierens de Haan's Tables (An earlier review of Lindman's *Examen* appeared in *MTAC* [2]).

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1. D. BIERNES DE HAAN, *Nouvelles Tables d'Intégrales Définies*, Hafner Publishing Co., New York, 1957. [Review **60**, *MTAC*, v. 11, 1957, p. 111.]
2. CHRISTIAN FREDRIK LINDMAN (1816–1901), *Examen des Nouvelles Tables d'Intégrales Définies de M. Bierens de Haan*, Amsterdam, [sic] 1867. (K. Svenska Vetenskaps Akad., *Handlingar*, v. 24, no. 5, Stockholm, 1891.) New York, G. E. Stechert & Co., 1944. [Review **167**, *MTAC*, v. 1, 1944, p. 321–322.]

123[P].—LEONARD PODE, *Tables for Computing the Equilibrium Configuration of a Flexible Cable in a Uniform Stream*, Report 687, The David W. Taylor Model Basin, Washington, D. C., 1951, 31 + 192 p. of tables, 27 cm. This report will be available from the Photoduplication Service, Library of Congress, Washington 25, D. C.

[P.]—LEONARD PODE & LOUIS ROSENTHAL, *Cable Function Tables for Small Critical Angles*, Supplement to Report 687, The David W. Taylor Model Basin, Washington, D. C., 1955, 105 p., 27 cm.

(A) The equations describing the tension in and the configuration of a flexible cable immersed in a uniform steady stream and lying entirely in a plane are derived. They are

$$\ln \tau = \int_{\varphi_0}^{\varphi} \frac{f \dfrac{\cos \varphi}{| \cos \varphi |} + w \sin \varphi}{q(\varphi)} \, d\varphi,$$

$$\sigma = \int_{\varphi_0}^{\varphi} \frac{\tau}{q(\varphi)} \, d\varphi,$$

$$\xi = \int_{\varphi_0}^{\varphi} \frac{\tau \cos \varphi}{q(\varphi)} \, d\varphi,$$

$$\eta = \int_{\varphi_0}^{\varphi} \frac{\tau \sin \varphi}{q(\varphi)} \, d\varphi,$$

where

$$q(\varphi) = -\sin \varphi \, | \sin \varphi | + w \cos \varphi.$$

Here ϕ_0 is the value of ϕ at the point chosen as the origin of the coordinate system, F is the drag per unit length of cable when the cable is parallel to the stream, R is the drag per unit length of the cable when the cable is normal to the stream, $f = F/R$, T_0 is the tension in the cable at the point chosen as origin of the coordinate system, T is the tension in the cable at the point x, y, $\tau = T/T_0$, $\xi = Rx/T_0$, and $\eta = Ry/T_0$.

These functions called the cable functions are tabulated to four decimal places in this report.

The values of the parameter f for which the cable functions have been evaluated are 0.01, 0.02 and 0.03. Another important parameter entering into the evaluation of these functions is ϕ_c, the critical angle of the cable, the value of the angle ϕ obtained when the cable is freely trailed in the stream and the angle for $q(\phi) = 0$. The values $\phi_c = 0°(5)$ to 85° are covered in the tables. These were computed by use of Simpson's rule for the evaluation of integrals. The relative errors in the functions tabulated are said to be less than 0.001 percent.

(B) The cable function tables for small critical angles reported herein are supplementary to those described above. The supplementary tables provide tables for critical angles in the range from 0 to 10 degrees in increments of one degree. More closely spaced intervals of the independent variable in the vicinity of the critical angle ϕ_c are also provided. Numerical integrations were made using the formula

$$\int_{x_{n-2h}}^{x_n} y dx = \frac{h}{3} [y_n + 4y_{n-1} + y_{n-2}] - \frac{h}{90} [y_n - 4y_{n-1} + 6y_{n-2} - 4y_{n-3} + y_{n-4}].$$

The authors state that "It is believed that the maximum error in any of the tabulated values is never greater than one unit in the least significant figure."

A. H. T.

124[P, K, X, Z].—E. F. Beckenbach, Editor, *Modern Mathematics for the Engineer*, McGraw-Hill, New York, 1956, xx + 514 p., 23 cm. Price $7.50.

This collection of essays is based on a series of lectures organized in the Extension Division of University of California at Los Angeles and given at that

University and repeated elsewhere. Those who did not hear the speakers will be grateful to them, and the editor, E. F. Beckenbach, for providing this volume. Engineers are some of the greatest customers of the automatic computers and much of the present volume is relevant in the field of this journal, and all our readers can profit from reading those essays.

The essays are divided into three parts: (I) Mathematical Models, (II) Probabilistic Problems, and (III) Computational Considerations. The contributors and titles are: R. Weller, Introduction; S. Lefschetz, Linear and nonlinear oscillations; R. Bellman, Equilibrium analysis: the stability theory of Poincaré and Liapunov; J. W. Green, Exterior ballistics; M. R. Hestenes, Elements of the calculus of variations; R. Courant, Hyperbolic partial differential equations and applications; M. M. Schiffer, Boundary-value problems in elliptic partial differential equations; I. S. Sokolnikoff, The elastostatic boundary-value problem; N. Wiener, The theory of prediction; H. F. Bohnenblust, The theory of games; G. W. King, Applied mathematics in operations research; R. Bellman, The theory of dynamic programming; G. W. Brown, Monte Carlo methods; L. A. Pipes, Matrices in engineering; J. L. Barnes, Functional transformations for engineering design; E. F. Beckenbach, Conformal mapping methods; C. B. Morrey, Jr., Nonlinear methods; G. E. Forsythe, What are relaxation methods?; C. B. Tompkins, Methods of steep descent; D. H. Lehmer, High speed computing devices and their applications.

It is not possible to discuss all the contributions; we shall only comment briefly on a few which are specially relevant to the field of this journal.

In his essay in the first part, R. Courant indicates the importance of a double attack on problems, by modern computing techniques and by penetrating analysis. In the second part, the essays by H. F. Bohnenblust and G. W. Brown give an account of subjects which, although they have largely developed in the last few decades, are becoming more and more used in the programming and operational aspects of engineering.

Some of the articles in the third part are not very closely related with practical computational considerations. However, those of Morrey and Forsythe do get down to numerical cases. Morrey discusses the solution of functional equations, mainly arising in calculus of variation problems. Newton's method is discussed in the finite dimensional case, normed linear spaces are introduced and the Rayleigh-Ritz method is discussed. Examples of the solution of a pair of equations in two variables, a first order differential equation and of a calculus of variations problem are discussed in detail.

Forsythe gives a valuable introduction to the subject of relaxation methods, indicating the scope of the method and its extensions, giving illuminating examples, and discussing the basic questions of convergence which arise. There is an ample bibliography.

Tompkins discusses the method of steep descent in a general framework: the solution of many engineering and physical problems can be represented as minimizing some function of several variables, or some integral. There are accounts of the solution of a system of linear equations by the projection method of Kaczmarz and the conjugate gradient method of Hestenes and Stiefel. After appropriate generalizations, a representative problem of the calculus of variations is discussed and some account is given of recent work in this field which is relevant in connection with computational problems.

The concluding essay by D. H. Lehmer is a pleasant introduction to automatic computers and includes many significant remarks. After noting the inadequacy of current machines for handling differential equations involving three space variables and time he adds, "It is only fair to state that there are also inadequacies in the numerical analysis of such problems." He encourages engineers to do their own programming and coding—a list of the machine instructions is almost enough —and most of the tricks of coding will occur to an engineer "who uses his normal budget of native cunning."

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125[S].—A. RAHMAN, *Tables of Integrals* $A_n(\alpha) = \int_1^\infty \lambda^n e^{-\alpha\lambda} d\lambda$ *and* $F_n(\alpha) = \int_1^\infty Q_0(\lambda)\lambda^n e^{-\alpha\lambda} d\lambda$, Extrait des *Annales de la Societe Scientifique*, T. LXIX, Serie I, 1955, p. 123–128.

This paper gives the functions mentioned in the title $\left(\text{with } Q_0(\lambda) = \frac{1}{2}\ln\frac{\lambda+1}{\lambda-1}\right)$ to ten significant figures for $n = 0(1)10$, $\alpha = .1(.1)3$; 10S. The computations were carried out to 12 significant figures. Interpolation formulae are given. The functions here tabulated are used in calculating the wave functions for the hydrogen molecule.

A. H. T.

126[S].—K. M. CASE, F. DE HOFFMAN, & G. PLACZEK, *Introduction to the Theory of Neutron Diffusion*, v. 1, 1953, Los Alamos Scientific Lab., Los Alamos, New Mexico, viii + 174 p., 26 cm. U. S. Gov. Printing Office, Washington, D. C., Price $1.25.

This book discusses the determination of the distribution of neutrons in space and time in terms of the geometrical configuration and physical properties of the medium they are in under the restrictive assumption that the magnitude of the neutron velocity is unchanged on collision. A very detailed well written treatment of this restricted one-velocity theory is given. This theory is basic to the solution of more general and more realistic problems. Indeed, some of its results have immediate application in such problems as the slowing down of neutrons by elastic collisions and in other problems.

The book is organized into three parts: Part A, Introduction; Part B, Propagation in the absence of scattering collisions; and Part C, One-velocity theory of neutron differsion.

Part B discusses the solutions of the continuity equation for the angular demity function, $\psi(r, \Omega, t)$ the number of neutrons per unit volume and unit solid angle moving in the direction of the unit vector Ω; r and t denote the space coordinates and time respectively. The continuity equation is discussed for three types of streaming: with no sources in vacuum, with sources described by a function $q(r, \Omega)$, and with sources q and absorbers.

Included in this part are four place tables for the collision probability for a slab of half-thickness $\phi = a/2$, a sphere of radius a, and an infinite cylinder of radius a. These tables are listed in accordance with the values of b/l, a/l and a/l respectively where l is the mean free path. The variable a/l (b/l) ranges from 0.00 to 5.00 in steps of 0.01.

Part C starts with a discussion of the transport equation and some of its properties, discusses this equation for a uniform medium with isotropic scattering and finally shows how the results obtained for the uniform infinite medium may be applied to the solution of finite problems. In this part there are fairly complete tables and graphs of functions entering in the solution of the transport equation.

The Mathematical Tables Project is credited for the material in Tables 28a and 28b. The remaining computations reported in this work were carried out by the Los Alamos computing group under the direction of Bengt Carlson and Max Goldstein.

A. H. T.

127[S]—M. Ferentz & N. Rosenzweig, "Table of F Coefficients," U. S. Atomic Energy Commission Report ANL–5324, Argonne National Laboratory, Lemont, Illinois, 1955, 293 p. Price $6.30 (photostat), $3.00 (microfilm).

This report contains an extensive tabulation of a function $F_k(L, L', j', j)$ of five arguments known as the F coefficient, which occurs in the formulas for the angular correlation between successive nuclear radiations. The arguments of the function are restricted in accordance with various triangular and other conditions. The table is in two parts. Part I contains integral spins (7,189 entries on 151 pages) and Part II contains the half integral spins (6,413 entries on 133 pages). The values of F are given to eight decimal places. The computations were carried out on the UNIVAC at the AEC computing facility at New York University and the table was reproduced photographically from the original UNIVAC Output. The computations were checked by comparing the results with other computations and by computing $F_0(L, L', j', j)$ $(\equiv 1)$. On the basis of these checks it is asserted that the entries in the table are correctly given to the sixth decimal place.

A. H. T.

128[S].—A. Rahman, "Two centre integrals arising out of $2s$ and $2p$ atomic func tions," Acad. r. de Belgique, *Cl. d. Sciences, Memoires*, v. 14, fasc. 2, publ. No. 1660, 1955, 13 p., 29 cm. Price 10 Belgian francs.

In working with the wave functions for diatomic molecules from $2s$ and $2p$ atomic wave functions centered on the atomic nuclei certain linear combinations of integrals previously tabulated by Kopineck [1, 2, 3] turn out to be very useful. These combinations of Kopineck's integrals are tabulated herein to five places.

A. H. T.

1. Hermann-Josef Kopineck, "Austausch- und andere Zweizentrenintegrale mit $2s$- und $2p$-Funktionen," *Z. Naturforschung*, B. 5a, 1950, p. 420–431.
2. Hermann-Josef Kopineck, "Zweizentrenintegrale mit $2s$- und $2p$-Funktionen II," *Z. Naturforschung*, B. 6a, 1951, p. 177–183.
3. Hermann-Josef Kopineck, "Zweizentrenwechselwirkungsintegrale III," *Z. Naturforschung*, B. 7a, 1952, p. 785–801.

129[S, V].—SVERRE PETTERSSEN, *Weather Analysis and Forecasting*, v. 1, *Motion and Motion Systems*, Second Edition, McGraw-Hill Book Co., Inc., New York, 1956, xix + 428 p., 23 cm. Price $8.50.

As the author states explicitly, this book is designed for use as a general text on weather forecasting. Accordingly, its subject matter and style of presentation are strongly slanted toward the interests and needs of the practicing meterologist, and will be of little direct value to the student of mathematics and numerical analysis.

Two chapters, however, deal with computing methods that are interesting, if only for their novelty. In chapter 3, the author outlines a method for extrapolating the position and intensity of certain definite features of atmospheric pressure patterns (e.g., "high," "lows," "fronts," etc.) from one moment to the next. This method is based entirely on the differential geometry of the pressure field and its instantaneous time-derivatives (assumed known). Thus, since it contains essentially no element of physics, it is equally applicable to other time-dependent fields.

Chapter 19 describes a graphical method, originally due to Fjortoft, for solving an equation of the form

$$\nabla^2\psi_t + \psi_x\nabla^2\psi_y - \psi_x\nabla^2\psi_x = 0$$

with ψ constant along a fixed closed curve in the (x, y) plane at all times t, and with ψ given everywhere within that curve at $t = 0$. Interpreted geometrically, this equation states that the instantaneous local velocity of the $\nabla^2\psi$-field is a vector equal in magnitude, but normal to $\nabla\psi$. This provides the basis for a graphical method of extrapolating the $\nabla^2\psi$-field at a slightly later time t, by moving the initial $\nabla^2\psi$-field for a short time t with the initial local velocity. The remaining problem, that of recovering the ψ-field at time t from the extrapolated $\nabla^2\psi$-field, is solved by a graphical method of repeated averaging—a procedure that is roughly equivalent to Richardson's method of relaxation. Although such graphical methods are doubtless very ingenious, it is the reviewer's opinion that they are a step backward against the current trend toward the more easily automatized digital type of computation.

Chapter 18, on Numerical Forecasting, was specially prepared for this volume by Dr. Arnt Eliassen of the Oslo University. This section of the book is a clear, concise, and remarkably complete account of the application of numerical methods and high-speed automatic computing techniques to the problem of weather prediction, and is probably of more general interest than the sections discussed above.

On the whole, Professor Petterssen's new book is clearly written, well illustrated, and represents a truly heroic effort to bring the practicing weather forecaster up to date with new and improved tools of his profession.

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130[S, V].—E. L. HARRIS & G. N. PATTERSON, *The Boltzmann H-Function Applied to the Shock Transition*, Institute of Aerophysics, University of Toronto, UTIA REPORT No. 40, 1956, iv + 17 p. + 8 p. of figures + 6 p. of tables, 28 cm. Available from Institute of Aerophysics on an exchange basis only.

The authors evaluate an approximation to the Boltzmann H-function for a gas with a shockwave present, where

$$H(x) = n \int f(\xi, x) \log nfd\xi,$$

n is the number of molecules per unit volume, and $nf(x, \xi)d\vec{x}d\vec{\xi}$ is the number of molecules with velocity components in the range ξ_i to $\xi_i + d\xi_i$ in the volume containing the point between x_i and $x_i + dx_i$ $(i = 1, 2, 3)$. The function $f(\xi, x)$ is approximated by using the solutions of the hydrodynamical equations of conservation of energy, means and momentum. Numerical solutions of the latter equations are involved in the calculations. The results of these calculations are listed for Mach numbers 1.5, 2.00, 2.3238, 3.00, 3.3764 and 4.00.

A. H. T.

131[X].—CORNELIUS LANCZOS, *Applied Analysis*, Prentice Hall, Inc., Eaglewood Cliffs, N. J., 1956, xx + 539 p., 21 cm. Price $9.00.

This book is at first an abbreviated account of the author's work in different fields of numerical analysis. Therefore the reader may take it as a guide to the papers published by Lanczos in various periodicals. The specific contributions of Lanczos to numerical analysis can be listed as follows. First of all he discovered the usefulness of Chebyshev polynomials in many fields where they had not been used before. The reader will find much valuable material in this connection, in particular for solving large linear systems of equations by relaxation, for solving algebraic equations, for telescoping power-series and for the approximation of solutions of linear differential equations ("τ-method"). The author's method of computing eigenvalues of matrices by "minimized iterations" is outlined a little sketchily in chapter III and his more recent invention of "spectroscopic eigenvalue analysis" is described.

But the book gives moreover a general view of numerical analysis. It is very interesting for the specialist who will find even in its more elementary parts Lanczos' personal attitudes and his sometimes very original ideas. It may however be a little hard to read for the beginner. Chapter I deals with algebraic equations and much weight is given to Bernoulli's method called the "method of moments." Stability is discussed by mapping the left half-plane onto the unit circle. One should observe that by this procedure the nearest root to the imaginary axis does not correspond necessarily to the nearest root to the unit circle. Chapter II begins with a theoretical introduction in linear algebra and proceeds to the numerical methods of solving linear equations and eigenvalue problems. Iterative methods are given in Chapter III.

Chapter IV is devoted to harmonic analysis. One of the merits of the book is the extensive discussion of the numerical methods connected with network analysis and Laplace transforms. In particular the author develops four methods for numerical inversion of Laplace transforms. The chapter is a little heavy because some results are discussed twice in the languages of Fourier and Laplace transforms.

The more classical problems of data analysis (interpolation and smoothing) and quadrature are treated in Chapters V and VI. The reader will find some new and interesting points of view, for instance the improvement of Simpson's rule by end corrections and a very elegant treatment of Hermite's quadrature formulas based on Legendre polynomials.

Error estimates are sometimes a little vague. As an example the estimate 6–5.4 for Simpson's rule can be made exact by multiplying the right side by 5/4 and assuming that the fourth derivative of the integrand does not vanish inside the interval of integration.

At the end of chapter VI there is an interesting application of Hermite's quadrature to eigenvalue problems connected with linear differential equations.

The last chapter of the book gives various methods for the expansion into power series and analytical extension.

It seems to the reviewer that the modern literature on numerical analysis has been taken into account up to 1952 approximately. Throughout the whole book almost every computational method is illustrated by numerical examples.

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132[X].—HEINZ RUTISHAUSER, *Der Quotienten-Differenzen-Algorithmus*, Birkhauser Verlag, Basel, Switzerland, 1957, 74 p., 23 cm. Price DM 8.50.

The Quotient-Difference Algorithm in its simplest form is a numerically effective method for determining the poles of a rational (or meromorphic) function from one of its Taylor expansions. Based on ideas due to Hadamard, Aitken, Lanczos, and Stiefel, and on concepts of the theory of continued fractions, the algorithm was developed by Rutishauser in three articles in *Z. angew. Math. Physik* [1, 2, 3] and shown to be an efficient tool for the solution of a number of fundamental problems of numerical analysis.

Each of the mentioned articles forms the backbone of one of the chapters of the work under review, but there are many changes and additions. Chapter I furnishes the theoretical basis of the algorithm. Among the new material we mention the didactically valuable rhombus rules (§3) and an addition theorem for continued fractions (§10). The latter appears to be a contribution to the formal theory of continued fractions and has several applications in the two later chapters. Chapter II deals with applications of the Quotient-Difference Algorithm to the summation of slowly converging series, to the factorization of polynomials, and to the interpolation of functions by sums of exponentials. Much of the new material included here tends to bring the algorithm closer to the computer. One of the disadvantages of the original form of the algorithm is that it occasionally may require dividing by small numbers. It is now shown (§9) how this can be circumvented by a simple device. Also, the quadratically convergent modification of the algorithm given in the original article for determining the real roots of a real polynomial is now extended so as to yield also conjugate complex roots (§8). The original method for interpolation is now replaced by a numerically more stable process (§10). The section dealing with the summation of series is perhaps the least impressive of the chapter. No attempt is made to define a class of series for which application of the algorithm may speed up convergence. Chapter III describes the application of the algorithm to the determination of the eigenvalues and eigenvectors of a not necessarily symmetric or hermitian matrix. Again the author shows his concern with actual computation by including detailed flow diagrams and (as in the other chapters) numerical examples. In three appendices the author discusses several extensions of the algorithm. One of these, the so-called L-R-algorithm, is treated more fully by Rutishauser in an article to appear in volume 49 of the National Bureau of Standards Applied Mathematics Series.

On the whole the presentation follows the pattern of the underlying papers in *Z. angew. Math. Physik*. A few proofs that were originally missing (e.g., of Theorem 1 of chapter III) are now carried out in full. The algorithm is still based on the analytic theory of continued fractions, and the reader's task is not made easier by the fact that no references are made to Perron's standard works on the subject. [In an article which will appear also in the above-mentioned volume 49 of the NBS Applied Mathematics Series, the reviewer has given an introduction to the algorithm which is based solely on the elementary theory of complex variables.] The printing of the whole work is excellent, and the fraction bar has ample opportunity to demonstrate its much-neglected faculty to make complicated formulas easier to read.

The world of computation owes to the author already several pioneering contributions on automatic coding and on the numerical integration of ordinary differential equations. By his impressive work on the Quotient-Difference Algorithm and its ramifications, Rutishauser has firmly established his position as one of the most resourceful and inventive numerical analysts of the present.

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1. HEINZ RUTISHAUSER, "Der Quotienten-Differenzen-Algorithmus," *Z. angew. Math. Physik*, v. 5, 1954, p. 233–251.
2. HEINZ RUTISHAUSER, "Anwendungen des Quotienten-Differenzen-Algorithmus," *Z. angew. Math. Physik*, v. 5, 1954, p. 496–508.
3. HEINZ RUTISHAUSER, "Bestimmung der Eigenwerte und Eigenvektoren einer Matrix mit Hilfe des Quotienten-Differenzen-Algorithmus," *Z. angew. Math. Physik*, v. 6, 1955, p. 387–401.

133[X, P].—G. TEMPLE & W. G. BICKLEY, *Rayleigh's Principle and its Applications to Engineering. The Theory and Practice of the Energy Method for the Approximate Determination of Critical Loads and Speeds*, Dover Publications, Inc., New York, 1956, vi + 157 p., paper bound. Price $1.50.

The authors state: "The object of this book is to explain and justify a rapid method of determining the fundamental period of a vibrating system or the condition of stability of an elastic system, with the degree of accuracy usually demanded in engineering problems. . . . The fundamental principle is due to the third Lord Rayleigh, and it applies not only to vibrating systems with a finite number of degrees of freedom, but also to continuous systems such as a stretched string or metal reed."

The authors only mention that the Rayleigh Principle is intimately related to the principle of least action and state that whenever the differential equations of a problem, dynamical or otherwise, are equivalent to a variational principle, the problem is always soluble by methods analogous to those discussed in this book. However, this intimate relationship is nowhere discussed nor is the relation between variational principle for the system under discussion and the Induction Function $G(x, s)$ (the Green's function of the equations describing the system mentioned).

An outstanding feature of this book is the use of this function $G(x, s)$ to generate sequences of functions $f_n(x)$ which approach the first (and higher) proper function of the equations describing the system and the use of these approximate proper functions for the determination of approximations to the proper values of the system. The major portion of the discussion is devoted to a method for determining the first proper function and proper value. However lower and upper bounds are given for the latter quantity and the lower bound is shown to depend on the ratio of the first two proper values. A method for estimating this ratio is also given.

The introductory chapter gives a general account of Rayleigh's Principle, of the problems to which it can be applied, and of the results obtained in subsequent chapters.

Chapters I, II, V, and VI contain many illustrative examples in which the methods derived in some detail in Chapters III and IV are applied. Chapters I and II are mainly concerned with problems in elastic stability and Chapter V deals with vibrating systems.

Chapter VI deals briefly with theorems relating the fundamental frequencies of two systems which themselves are related by inequalities in their mass distributions, or potential energies.

Chapter VII entitled Numerical and Graphical methods discusses one problem: The determination of the fundamental frequency of a tapered Aeroplane strut.

Chapter VIII gives a few examples of the determination of equilibrium configurations of elastic systems.

The book is extremely well written and will be particularily useful to those who desire closed formulas for various approximations in engineering problems. Modern numerical analysts and other mathematicians will find this a thought-provoking book and very suggestive of methods for solving engineering problems. However it is an open question as to how suitable the methods discussed in this book are if numerical solutions are to be obtained by the use of automatic digital computers.

A. H. T.

134[X].—J. Greenstadt, "A method for finding roots of arbitrary matrices," *MTAC*, v. 9, 1955, p. 47–52.

The author proposes to triangularize an arbitrary matrix by a sequence of transformations of the form

$$A \rightarrow B = S^*AS$$

where S is a unitary matrix operating on two rows and columns and S^* denotes the transposed matrix. No proof is given that the sum of the squares of the absolute values of the sub-diagonal elements decreases to zero. It is known that this sum does not decrease monotonically. A variety of matrices whose elements were picked at random were treated by this method on an IBM 701 and it was shown that for this class of matrices the method converged.

A. H. T.

135[X].—E. Bodewig, *Matrix Calculus*, Interscience Publishers, Inc., New York, 1956, xi + 334 p., 23 cm. Price $7.50.

Experience shows that at least half the problems of a modern computing organization involve "linear algebra." Computers, in particular, will therefore welcome *Matrix Calculus*, which represents the first attempt to cover the whole field in a single volume.

The book is in four parts. Part I, in 85 pages, treats the theory needed in practical applications. It considers elementary properties of vectors and matrices; errors in A^{-1} caused by small uncertainties in A; measures of magnitude of A; decomposition theorems and orthogonalization; properties of latent roots and latent vectors; bounds for eigenvalues and determinants; and some theory of elementary divisors.

Part II, in 100 pages, considers practical methods for solving linear simultaneous algebraic equations. Direct methods include the elimination and condensation methods of Gauss and Jordan, both for exact and approximate solution; applications of matrix decomposition into triangles and its connections with elimination; the Gauss-Doolittle process, the methods of Banachiewicz and Cholesky, and Aitken's "triple-product" method. Details are given of desk-computing techniques and their variations of advantage for more automatic equipment. There is also a discussion of ill-conditioned equations and the "error" (accuracy) and "exactness" (precision) of solutions.

Iterative methods discussed include those of Gauss and Seidel, the acceleration devices of Aitken, and the steepest-descent methods of Hestenes, Stiefel, and others. The general theory of iteration, with theorems on convergence and its acceleration, is given at length. Methods for improving an approximate inverse, procedures which always converge and which are therefore considered suitable for automatic computing, and the practical advantages of various scaling operations, are also included.

The inversion of matrices, in about 50 pages, is the topic of Part III. In addition to the obvious applications and extensions of the methods of Part II there is a discussion of partitioning and bordering of matrices, an old but little known method for reducing a matrix to diagonal form, and a lengthy account of special treatment of the matrices arising in geodetic surveying.

The last 100 pages belong to Part IV, the determination of latent roots and vectors. Iterative methods are used to determine the dominant root and associated vector, and perhaps one or two sub-dominant solutions if convergence is slow, in this respect an advantage. For accelerating convergence, or causing convergence to a wanted solution, methods of "vector-deflation" and "matrix-deflation" are fully explained. The construction of the characteristic equation is also discussed, together with its ill-conditioned dangers. There is a mention of Aitken's sequences for "higher" eigenvalues, and a further discussion of elementary divisors. Then come the orthogonal transformation methods of Jacobi and Givens, perturbation methods for improving accuracy, the gradient methods of Hestenes and Karush and the use of Rayleigh's quotient.

Some direct methods are also mentioned, associated with the names of Leverrier, Krylov, Duncan, Hessenburg, Samuelson, and others. These methods effectively form the characteristic equation, and find favor only for matrices of low order.

There is a bibliography of 111 references and a somewhat terse index.

Numerous examples illustrate the text (even in Part I), the amount of computation in each method is carefully assessed, and there are many computing hints, close attention to checks, and a willingness, refreshing if not always believed, to give an opinion on the relative merits of the different methods, according to the nature of the matrix, the amount of information required about it, and the computing equipment available.

Few omissions come readily to mind. Perhaps the "escalator" method was worthy of passing mention, however, and the absence of the name of Lanczos, from the section on latent roots, is somewhat surprising.

This reviewer did not find the book easy to read. The emphasis on a "new matrix calculus," with stress on rows and columns, avoidance of the use of individual elements, and accompanying new ideas and notations, together with the vast amount of material contained in relatively few pages, cannot readily be assimilated from the depths of an arm chair. But there is nothing essentially difficult, and the student who really makes the effort with this book and succeeds, can claim, with its author, to be an expert in this field.

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136[X, I].—H. Fishman, "Numerical integration constants," *MTAC*, v. 11, 1957, p. 1–9.

Tables of the b_j and x_j in the approximate quadrature formula

$$\int_0^1 x^n g(x)\,dx \doteq \sum_{j=1}^{m} b_j g(x_j)$$

are given for $n = 0(1)5$, $m = 1(1)8$ to 12D. They were prepared on an IBM 650 and are an extension of tables of P. C. Hammer, O. J. Marlowe, and A. H. Stroud [1] which covered the range $n = 1, 2$, $m = 1(1)5$ and $n = 3$, $m = 1(1)4$, but give 18D. The x_j are the zeros of certain orthogonal polynomials $q_m(x)$ defined by

$$q_m(x) = \sqrt{(n + 2m + 1)} \sum_{k=0}^{m} (-1)^k \binom{m+n+k}{m} \binom{m}{k} x^k$$

which are given explicitly for the same range of m, n. The b_j are defined by

$$b_j = \left\{ \sum_{i=0}^{m-1} [q_i(x_j)]^2 \right\}^{-1}.$$

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1. PRESTON C. HAMMER, O. J. MARLOWE, & A. H. STROUD, "Numerical integration over simplexes and cones," *MTAC*, v. 10, 1956, p. 130–137.

137[X, Z].—F. J. WEYL, *Report on a Survey of Training and Research in Applied Mathematics in the United States*, Monograph No. 1, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, 1956, vi + 58 p. Price $2.00.

Durch die Erfindung der Rechenautomaten und die schnelle Entwicklung der numerischen Analysis ist der vorher ruhig dahinfliessende Strom der mathematischen Forschung in den letzten Jahren etwas turbulent geworden und wir stehen heute vor manchen Problemen grundsätzlicher Natur hinsichtlich des Verhältnisses der Mathematik zu Naturwissenschaft und Technik und hinsichtlich des mathematischen Unterrichts.

Das dringende Bedürfnis nach Neufestlegung der Standorte und abklärender Aussprache hat in den vereinigten Staaten zum vorliegenden Bericht geführt, der auf Grund von zwei Konferenzen und einer allgemeinen Umfrage bei den Universitäten, der Industrie und den militärischen Organisationen von der mathematischen Abteilung des national research council ausgearbeitet wurde.

Die Aufgabe war erstens, Charakter und Umfang der gegenwärtigen Forschung in angewandte Mathematik festzustellen und nach potentiellen Möglichkeiten zur Vertiefung zu suchen und zweitens, Anregungen für die Ausbildung angewandter Mathematiker zu geben.

Der Bericht umgrenzt zunächst den Begriff "angewandte Mathematik," der ja sehr schwankend gebraucht wird, und erklärt sehr zutreffend, dass angewandte Mathematik weniger ein Sachgebiet kennzeichnet als eine geistige Haltung, charakterisiert durch die bewusste Bereitschaft zur Zusammenarbeit mit Schwesterwissenschaften, mit dem grossen Ziel, unsere Umgebung und uns selbst besser zu verstehen. Mit Recht wird darüber geklagt, dass viele der führenden Mathematiker, welche die Atmosphäre des Hochschulunterrichts bestimmen, nicht imstande sind, Substanz und Ziel dieser Denkweise zu begreifen.

Er stellt weiter fest, dass die nächste Zukunft durch eine niedagewesene Mathematisierung gekennzeichnet sein wird, die sich nicht nur auf Physik und Technik beschränkt, sondern auch alle unsere Lebensgewohnheiten beeinflussen wird. Als Gründe dafür werden angegeben einmal die wachsende Kraft mathematischer Methoden, die Notwendigkeit feinerer Berechnungen in allen Zweigen der Technik und ferner das Erscheinen der Automaten.

Es werden die vielen verschiedenen Wege beschrieben, die von den einzelnen Universitäten und Amtsstellen beschritten worden sind, um der steigenden Nachfrage nach angewandter Mathematik gerecht zu werden. Vor allem ist hier die neue Erscheinung des mathematischen Instituts zu nennen, dessen Erfolge mehr auf der Aktivität einer Arbeitsgruppe beruhen, als auf spektakulären Leistungen Einzelner. Es wird auch durchaus hervorgehoben und als positiv gewertet, dass die Regierungsstellen und die militärischen Instanzen durch ihre Kontrakte oft diese Institute erst lebensfähig machen. Wiederum beklagt sich der Bericht darüber, dass viele Mathematiker diese nationalen Notwendigkeiten nicht einsehen und Kritik üben statt am Aufbau mitzuwirken. Es wird auch auf die drohende Gefahr hingewiesen, dass die mathematischen Wissenschaften in zwei Teile—den abstrakten und den angewandten—auseinanderfallen.

Das untersuchende Komitee findet aber, dass die Entwicklung zu langsam und zu unausgeglichen vor sich geht. Vor allem fehlt es an qualifizierten Studierenden, am Unterricht und an einer klaren Konzeption desselben. Die beunruhigenden Nachwuchsprobleme werden analysiert und auf verschiedene Quellen zurückgeführt.

Aehnliche Unzulänglichkeiten werden hinsichtlich der Rolle des Mathematikers in der Industrie festgestellt. Obwohl die Industrie in hohem Mass Mathematik konsumiert, stellt sie selten Mathematiker an, sondern übergibt mathematische Aufgaben ihren Ingenieuren.

Der Bericht gipfelt in folgenden Empfehlungen:

1) Es soll ein ständiges Komitee gebildet werden, das die Beziehungen zwischen der Mathematik und ihren Anwendungen fördert.
2) Die Universitäten sollen Forschungsprogramme in angewandter Mathematik für fortgeschrittene Studierende durchführen.
3) Alle Mathematikstudenten sollen gezwungen werden, sich auch mit anderen Wissensgebieten bekannt zu machen, in denen die Mathematik gebraucht wird.
4) Mathematiker aus der Industrie oder aus staatlichen Organisationen sollen zu Gastvorlesungen an Universitäten eingeladen werden.
5) Alle Dozenten der Mathematik sollen angehalten werden, Kurse in angewandter Richtung zu geben, sofern sie dies imstande sind.

Ein zweiter Teil des Berichtes befasst sich mit einigen typischen neueren mathematischen Entwicklungen. Es wird zunächst die axiomatische Methode geschildert, sodann die Rolle der Wahrscheinlichkeit und Statistik in den Theorien der mathematischen Physik auseinandergesetzt und die Bedeutung einer flexiblen und anpassungsfähigen Anwendung mathematischer Methoden in der Technik besprochen. Die ganz jungen Gebiete des "operational research," der mathematischen Theorien biologischer und soziologischer Phänomene werden gestreift und die numerische Analysis als Ganzes ist untersucht.

Als Anhang zum Bericht findet man eine Analyse der Antworten auf den vom Komitee versandten Fragebogen.

Es wurden hier absichtlich diejenigen Teile des Berichtes besonders gewürdigt, die unabhängig von den spezifisch amerikanischen Verhältnissen weltweite Gültigkeit haben. Alles Gesagte trifft wörtlich auch auf europäische Verhältnisse zu und zwar in verschärftem Mass, da in manchen europäischen Staaten auch noch das aktive Interesse der Behörden fehlt und somit die finanziellen Mittel oft knapp sind. Ferner veranlassen die besseren Arbeitsverhältnisse in Amerika sehr viele und oft die besten jungen Mathematiker aus Europa zur Auswanderung.

Der Wunsch nach einer gründlichen Neuorientierung des mathematischen Unterrichts hat einige europäische Länder veranlasst, internationale Besprechungen durchzuführen und die Fragen gemeinsam zu regeln. Eine erste Konferenz fand im Juni 1957 in München statt am mathematischen Institut der dortigen technischen Hochschule. Vertreter von Deutschland, Frankreich, Oesterreich und aus der Schweiz arbeiteten eine Resolution aus, die sich weitgehend mit den Forderungen des amerikanischen Berichtes deckt und in mehreren Fachzeitschriften (z.B. Zeitschrift für angewandte Mathematik und Physik, Heft VIII/5) veröffentlicht wird. Die an der Tagung gehaltenen Vorträge sollen ausserdem vom Münchner Institut herausgegeben werden.

Allen Fachleuten ist es wohl klar, dass angewandte Mathematik heute hauptsächlich ein Erziehungsproblem ist. Es handelt sich weniger darum, immer schnellere Automaten zu bauen, als das Personal heranzubilden, das imstande ist, mathematische Probleme schnell genug für den Automaten vorzubereiten.

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138[Z].—Milton H. Aronson, Editor, *The Computer Handbook*, First Edition, 55 p. *The Computer Handbook*, Second Edition, 1956, 71 p., The Instruments Publishing Co., Pittsburgh, Pennsylvania, 28 cm.

The first edition of this booklet contains six papers presented at the First Automation Exposition (New York, Nov. 29–Dec. 2, 1954) and published in the magazine *Instruments and Automation*. The second edition reprints three of these and adds eight others. In both cases about half of the papers are on analog computers and applications, the other half being on digital computers and applications.

The articles are written for the general reader, rather than being highly technical. They provide a limited amount of information about a number of specific computers and their business or industrial applications. The articles will not be useful to any person desiring detailed technical information about the computers discussed.

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139[Z].—*Proceedings of the Symposium* (Automatic Coding), 1957, Monograph No. 3, *J. of the Franklin Institute*, Philadelphia, Pa., 1957, vii + 118 p. Copies may be obtained from AUTO CODE, Franklin Institute, Philadelphia 3, Pa. Price $3.00.

The monograph contains the following papers:

AUTOMATIC CODING AT G. E. by Richard M. Petersen
SYSTEMS OF DEBUGGING AUTOMATIC CODING by Charles Katz
PRINT 1—AN AUTOMATIC CODING SYSTEM FOR THE IBM 705 by Robert W. Bemer

THE PROCEDURE TRANSLATOR—A SYSTEM OF AUTOMATIC PROGRAMMING by Henry Kinzler and Perry M. Moskowitz

OMNICODE—A COMMON LANGUAGE PROGRAMMING SYSTEM by Russell C. McGee

A MATRIX COMPILER FOR UNIVAC by Laurence C. McGinn

A MATHEMATICAL LANGUAGE COMPILER by Alan J. Perlis and Joseph W. Smith

A MECHANIZED APPROACH TO AUTOMATIC CODING by E. C. Yowell

There is a short introduction by John S. Burlew of the Franklin Institute, a transcription of the discussion of each paper, and a list of those attending the Symposium.

It seems to be a sad fact that workers in the field of automatic coding, who in general are scientific or research personnel, bring a most unscientific and unresearchlike attitude to these symposia. The papers presented at this symposium are, with two and possibly three exceptions, simply descriptions of individual and special-purpose coding systems which are of interest to users of the particular computer for which they were written, and to nobody else. Most of the information contained in these papers could more easily and efficiently have been obtained by interested parties through a users' organization; by this time such organizations have grown up about most large computers. Evidence of the inefficiency of communication is the turn generally taken in the discussions; questions are often requests for information already given in the paper, and comments generally consist of destructive or petty and obvious criticism. Constructive criticism—the contribution or generation of constructive ideas—is so rare as to be negligible.

Some of the papers deserve at least individual mention; one such is the Katz paper. This contains a short discussion of debugging problems that arise in automatically coded programs, which are difficult of solution because, by the very nature of automatic coding, the programmer is insulated from (and may be completely unfamiliar with) the computer. Mr. Katz describes an attempt to produce debugging routines for some compilers for UNIVAC. Automatic debugging is a most interesting and difficult logical problem, and this paper is one of the few in which more rather than less detail would have been welcome.

The Perlis-Smith paper is another refreshing exception. It is, in the final analysis, a description of an automatic coding scheme for the IBM 650, but the language and organization are a pleasure to see; further, the approach to the problem is at once scientific and comprehensible. There is a clear, concise description in logical terms of just what the automatic coding problem is, and a clear and convincing justification of the methods used in solution. It may or may not be significant that this is the only paper arising from work at a university.

The last paper, and the last deserving mention, is the Yowell paper. This is quite different from the others in that it describes an attempt (by The National Cash Register Company) to design a computer (the 304) with macro-commands; this is automatic programming by hardware rather than routines. The paper is at once wordy and short, but the novelty of the content makes it interesting.

The tenor of papers and comments makes it clear that the overwhelming majority of those attending the symposium have a distorted idea of just who and what automatic coding is for. Time and again the emphasis is on simplifying programming so that cheaper talent can be used to write programs. This is a case of being right for the wrong reasons. The saving in time and money in making do with low-priced programmers is as nothing when compared with the saving that can be effected if high-priced talent—scientists, engineers, executives—can have access to a computer without having to explain problems or learn programming. The only statement or comment made at the Symposium in support of this philosophy comes from Dr. Perlis and is, in part: "It is the man who proposes the problem who is important, and his ideas are the ones that count."

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140[Z].—D. D. McCRACKEN, *Digital Computer Programming*, John Wiley and Sons, Inc., New York, 1957, vii + 253 p., 24 cm. Price $6.50.

Digital Computer Programming offers for the first time an excellent introduction to the problems of coding for a high speed stored program computer. Such a work has long been needed in this field; McCracken's book will probably remain as the standard of comparison for some time. Since this book will undoubtedly be widely used, not only as a textbook in colleges but also as a training manual within industry, it is felt that a fairly detailed review of the work is indicated. In what follows, the plan of attack will be to cover the book more or less chapter by chapter, and, in general, point up some of its shortcomings. Most of these arise from viewing the book from the point of view of a neophyte in coding, and will possibly seem obvious to the old hands in the business, who can read *Digital Computer Programming* almost as if it were a novel, exclaiming "Yes, yes!" at every page. Following this survey will come a few remarks on the general philosophy of the book.

In chapter 1 the newcomer is given a brief introduction to stored program computers. It becomes obvious almost immediately that the work is intended as a textbook, as new terms are introduced with fair rapidity. Most of these terms are indicated in italics, and defined contextually, which makes for easy reading, but these terms must be mastered before going on, and will probably require several readings on the part of the student. In order to have a concrete machine with which to work, McCracken invents TYDAC, the TYpical Digital Automatic Computer, a decimalized version of a modern binary scientific machine. A brief history of computing follows, which is fairly complete with one exception. Most of the major users of large scale machines for scientific computing started out by applying standard punched-card machine techniques to engineering calculations, and many of the methods, traditions, and customs which the newcomer to scientific computing encounters have their roots in this early work. Some credit should be paid to these pioneers.

A fairly comprehensive outline of the steps in coding skips mention of one of the most important steps—that of making sure the proposed problem is really the one which should be solved. Often a great deal of labor may be saved by incorporating several small problems into one large systems analysis. On the other hand, it may be too early in the investigation of a problem to tackle more than a small piece at a time. The large amount of time spent recoding problems points up a need for more time spent in this phase of solution. Also, no mention is made of the large amount of feedback that ordinarily occurs between the scope of the problem under solution and the method employed.

There is a good survey here of typical applications of digital computers; this might also have included some examples of problems which are *not* suited to high-speed stored program machines.

A lucid, though conventional, explanation of coding fundamentals is presented in chapter 2. The going is still fairly fast here, and much reference will be made back to the terms previously defined in chapter 1, and to the appendix which contains a complete summary of TYDAC. (A worthy addition to the book would have been a small card, loosely inserted, containing a summary of the instruction codes, etc. Practically every coder for a real machine carries one of these supplied by the manufacturer; TYDAC should not be an exception.) Cogent points which must be remembered are often hidden in the middle of a paragraph; the book will probably be much underlined.

A good glossary of computing terms would be welcome here. Since this text will find such widespread usage, it is to be regretted that the author did not include what might have become a definitive dictionary of computing terms.

There is a good description of the execution of a three step program which adds two numbers, but the section on multiplication and division processes within the machine is probably overdone. A better use of space here would have been to present the arithmetic registers as they appear before and after the execution of the instructions, as is done in the section on scaling. It is pleasant to see an introductory work which mentions the necessity of clearing the MQ register before division takes place, although more emphasis might have been placed here—especially in connection with small-integer arithmetic.

In an example on p. 26 several opportunities have been missed to point up the possibility of subtle errors in even such a simple problem as $(10 \cdot A + B)C$. Here, as happens throughout the text, the examples seem fairly simple when the correct answer is given. Only three examples of coding in the entire text are pointed out as containing errors—certainly a far smaller proportion than maintains in real-life coding. A very worthwhile addition to the explanations would give examples which do not work, and the reasons why.

The exercises appended here, as throughout the work, are excellent. Perhaps the most frequently neglected part of teaching coding is in this realm, and it is refreshing to see examples more typical than $A + B = C$.

Chapter 3 leads us painlessly through the mysteries of binary and octal number systems. One addition to this chapter that could be made would explain the use of mixed-base number systems for compacting information within a word, as is often done to minimize input-output time or conserve storage space.

The Decimal Point Location Systems, as described in chapter 4, are exceedingly well done. McCracken is to be congratulated on his fearless approach to the so-called "mysteries" of fixed-point coding. There are many experienced coders who could profitably study this explanation. The first method, that of fixing the point in the middle of the word, seems rather awkward and artificial, and should probably have been cut a bit. Here would be a good place to explain the necessity of clearing the MQ before division, but this is not done.

The "graphic" method has long been the favorite of this reviewer, who finds here the first really good explanation of the method. The examples of multiplication miss the fact that in a large proportion of cases a left shift is necessary after the multiply, since both factors may not reach their maximum value at the same time. Also neglected is the problem of finding a full ten digit rounded quotient, which is never presented in the book. The scaling of the remainder after a division is never presented; this should be included. Some simplification could have been made by considering only the number of digits before the point, as the number after is redundant. To be complete, this method should include a set of rules similar to the 5 step rule given under the scale factor method, and would probably give the "graphic" method more widespread use.

The commonly used scale factor method is accompanied by a good set of rules for its employment, and should dispel much fear on the part of the coder approaching fixed point problems.

In the section on scaling, the author has missed an opportunity to present the coder with a check on the programming of the problem that is often valuable. In coding fixed point arithmetic, the values of the shifts required should normally be small; e.g., 3 or 4 places at the most. If a shift of eight or nine is arrived at in, say, an addition, an insignificant quantity is probably being added. The statement of the problem may be checked at this point, and many errors have been detected in such a fashion.

The coder should be warned, in conjunction with the problem of overflow, that in a large majority of cases the overflow indicator will be turned on by some part of the problem which does logical work, or by subroutines, and that it should be turned off prior to embarking on a sequence of fixed point arithmetical calculations and checked immediately following the sequence.

In this section, as elsewhere, a good deal more space could have been profitably devoted to pointing out possible pitfalls. The examples are generally good, although more requiring "tight" scaling to retain significance could have been included.

Chapters 5 and 6, which deal with address computation and loops, lead the coder quite easily into the real power of stored program machines. Particularly to be commended is the decision to separate the address computation from the testing computation; introductory examples here are often much too complicated. Two objections were noted here. In several examples some of the necessary information is buried in the text, and it would have been better to write out cells containing instructional constants, etc., following the examples. Where this is not done, the somewhat confusing notation "Loc 1400" is used rather than the more common "L(1400)" or "Loc. of 1400."

This section would be a good place to point up the advantages of standardization when writing loops. There are many ways to write a given loop; the important point is for the coder to pick a single method to use and stick with it—the temptation to be clever is seldom resisted by the neophyte, and usually leads to disaster.

The excellent coder usually develops his own style of coding, and this becomes a major factor in reducing his own mistakes.

There is an excellent paragraph at the bottom of page 77 which bears repeating here.

> "Depending on the nature of the loop and of the test, it is possible to make a truly remarkable variety of mistakes in testing. If the loop should be carried out n times, it is quite easy to make mistakes which will result in doing it: (1) not at all; (2) $n - 1$ times; (3) $n + 1$ times; (4) $2n$ times; (5) until the power fails or the machine breaks down. It is fairly safe to say that loops, although one of our most powerful tools, are also a *very* large source of errors. Whatever other prechecking systems may be used, it is always advisable to go back and check the loop-testing parts of the program. As indicated above, one simple way is to analyze what would happen if the loop were to be executed only once."

This succinctly conveys to the coder the main source of errors in coding. McCracken might have pointed out that once a coder learns to "count to one" (not only in writing loops, but in many other phases of a problem) he is well on the way towards eliminating mistakes in coding.

On page 80 there is a discussion of the time vs. space problem which always seems to bother the new coder. Perhaps this would be a good time to inject the point that very often the best loop is not the shortest, not the fastest, but the one which works the first time. The amount of time spent in debugging a slightly faster or shorter loop often far overshadows the time saved in running the problem. Many coders adopt the policy of writing all loops in a standard fashion, checking the program out, and then going back and "cleaning up the code," one loop at a time, when time considerations are really important.

There is also a gem of wisdom at the beginning of chapter 6 which might have been emphasized more—in coding a loop, start from the inside and work out. This philosophy can be carried to all parts of a problem, and bears repeating many times, although the student may quickly find this out for himself in working out the fine examples at the end of this section.

The chapter on flow charting is well introduced and presented; unfortunately no attempt has been made to give general rules for problem organization. Moreover, no symbolism has been adopted for indicating the execution of a closed subroutine, whose detailed flow chart may be given elsewhere. Some attention to this point might have helped to keep flow charts from growing almost beyond comprehension.

TYDAC contains, as do most modern scientific computers, a set of index registers. A rather arbitrary, though adequate, set of instructions is provided for operating on them, and an extremely good explanation is made of the distinction between indexable and non-indexable instructions. Another example of a program with an error occurs here, and again the opportunity to point up the advantages of standardized coding is missed.

In chapter 9 the student is introduced to one of the most powerful tools at his disposal—subroutines. The various methods for entering and leaving a subroutine are well presented, but two methods for communicating information to the subroutine that are commonly used are not emphasized. (These are transmittal via the machine's arithmetic registers and via standard cells in memory.) Several other points are missed that are vital to the discussion. A generally used subroutine must somehow take care of all possibilities of error in inputs, with appropriate error warnings, and must also conform to some prescribed set of conventions regarding the use of overflow indicators, index registers, etc. An example of a subroutine with an error exit is given in one of the examples in a later chapter, but the point is not emphasized. It is in the attention to details such as these that a good subroutine is distinguished from a poor one.

The statement is made, on p. 114, that it is not clear whether or not the use of index registers for linking to subroutines will grow. On the contrary, the reviewer feels that one of the most important advantages of the newer machines lies in their built-in features, such as the set index jump instruction and indirect addressing features, which permit subroutines to be constructed with great ease while coding. The closed nature of these subroutines greatly facilitates the check-out phase, and they will undoubtedly become a more and more important part of programming. It is at this point in the book that the coder might have been introduced to good coding practices, and it is unfortunate that the point was not made.

The introduction to floating point operation, as given in chapter 10, is generally good. His warning against the injudicious use of floating point is not too strongly made, in the face of the general trend toward hardware incorporating this feature. The new coder may be lulled into a false sense of security if he is not strongly confronted with the pitfalls that can arise. One of the most time-consuming problems for a programmer using built-in floating point operations in a large problem concerns the treatment of underflows and overflows; the routines necessary are not mentioned at all. It should also have been explained that built-in floating point operations are not always exactly analogous to the corresponding fixed point operations; e.g., TYDAC's MQ is cleared in floating point addition. This is often a big source of errors for the new coder.

Input-output methods can be treated only briefly in a book such as this, as McCracken points out, since these are strongly dependent on the actual machine used. This is unfortunate, as perhaps the best way for a coder to learn the differences between numbers, addresses, locations, operations, etc., and at the same time meet non-numeric and numeric data and conversion and scaling problems is to tackle an input or output routine. This is especially true on a binary machine. The statement that a coder need never know input-output will be contested by many, including this reviewer. The inclusion of a console on TYDAC leaves something to be desired in view of the general trend away from having the programmer touch the console, or indeed, operate the machine at all. There might have been a section included, more pertinently, on the problems of organizing the production phase of a problem so that it may easily be run by a professional operator who may have no knowledge at all of the coding or the problem itself.

A load program is presented which uses an initial address and a word count on each card to control loading and the advantages are detailed. The disadvantages should have been presented, too. Suppose the deck has had corrections added, and is somehow out of order? There is no indication that this has happened, and the program might run through to completion, giving wrong answers, at a large cost in machine time. Similarly, no indication is given if a card is missing; much running time can elapse before this is detected. Many problems arise in connection with input-output procedures, and the programmer should be made aware of these early in his career.

The presentation of magnetic tape procedures is again necessarily brief. The inclusion of check-summing as a machine checking feature is good, especially in view of the fact that TYDAC does have a redundancy check indication. The necessity of having programmed checks on the machine operation, even when checking circuits are built in, could probably have been emphasized elsewhere with equal advantage. Here again the coder should be warned that his problem will be run by professional operators unfamiliar with his routine and that he should try to simplify tape handling procedures as much as possible.

Chapter 13 presents a basic introduction to program checkout techniques. Since the average coder spends the largest amount of time in this phase of problem preparation, a much more extensive treatment of the subject would be indicated. Unfortunately, no general set of rules to guide the programmer has as yet been presented. McCracken fails to distinguish the two phases of program checkout: debugging and testing. By debugging is meant the process of making sure that the program does what the coder meant it to do, and by testing is meant the process of making sure that the program solves the problem it is intended to solve. The distinction is strong, and the testing phase is often woefully neglected, even by experienced programmers.

The debugging process is fairly well covered, including the standard techniques of tracing, memory dumping, and breakpoint printing. Also welcome is a description of the "decoding" process of checking a code before it is tried out on the machine, and various methods are presented. An excellent example points up the necessity for picking realistic and non-degenerate test case data, and should decrease the number of times the cry, "But the test case checked perfectly!" is heard.

No rule is given for the procedure to be followed when the newly coded program collapses; a simple one is to try to get the program to execute all of the instructions from beginning to end without regard to the correctness of answers, and then to tackle the arithmetic portion of the code. All such rules are, of course, rather dependent upon the programming system used, and will vary from machine to machine. Another fruitful area for making mistakes is ignored—the procedure for correcting mistakes once they are found. Nowhere is this problem covered, and as the neophyte programmer does not have symbolic techniques at his disposal at this point in the text, he may reach the conclusion that it is necessary to do a great deal of rewriting each time an error is discovered. A few words as to the necessity for leaving room for patching would be in order here. Also, were symbolic methods available to the coder at this point, the ease with which these permit the inclusion of personalized diagnostics could be presented.

It is difficult, in a short chapter, to present relative (or symbolic) programming techniques and give a good indication of their power. The advantages of these techniques are most strongly felt on large problems, and a student is not inclined to go through a large problem example in a new and unfamiliar notation. It is significant, however, that once a coder has mastered relative and symbolic coding systems he usually becomes much more skilled in programming actual machine language. The second exercise in this chapter requires the student to find a "small" error in a routine which has been presented in the text, and points up the fact that even advanced methods lead not to perfect programs, but to different types of errors, with which the coder must learn to cope.

The final sections on interpretive routines, double precision routines, and miscellaneous programming techniques are well written and the examples which accompany these chapters provide the opportunity for the student to apply the principles learned in the early part of the book.

In spite of the general level of excellence of this text, there are several areas which have been rather neglected if programming, rather than coding, is to be taught. The first of these is that of program checkout and testing, and the second is that of problem organization. In the majority of classes using this as a text, it is difficult to see how the student can be made aware of the many mistakes he will undoubtedly make in coding the examples given. Certainly few instructors will have time to discover, by the process of decoding, all of the subtle mistakes which an inexperienced coder can make in even the simplest examples. One solution which will undoubtedly be adopted at a few schools, is to code an interpretive routine for a high speed computer to simulate the features of TYDAC, and to provide laboratory sessions in which to try out the problems.

If this is not practical, or if a high speed machine is not available, the only alternative would seem to be to standardize coding techniques and penalize the student who uses fancy routines much as he would be penalized in actually trying to check them out on an operating machine. Certainly, as has been indicated in individual cases above, there are many opportunities in the text to point out the various mistakes that can be made, and it would be a great help if a greater portion of the text, and particularly the examples, was devoted to finding errors in routines. There is a trend, particularly among the users of the largest scientific machines, towards finding mistakes without the use of elaborate machine debugging programs. Examples might be given which would contain the original coding for a routine, and the results of running this coding on TYDAC. (This routine stopped at xxxx. Why?) If necessary, the relevant portions of a memory or console dump could be supplied.

Perhaps the greatest difference between the performance of a mediocre coder and a top-notch programmer lies in the way in which they organize a problem. It is a far cry from writing a set of twenty or fifty instructions to completing a problem containing many thousand orders, and many coders fail to bridge this gap. The good programmer thinks at all times about the possibility of errors, and so arranges his problem to eliminate as many mistakes before they occur as he can. One technique that is widely used is to break the problem into small, self-contained subroutines, trying at all times to isolate the various sections of coding as much as possible. At first glance this appears extremely wasteful of instructions—many of them are communicating information between routines that could well communicate directly with one another. It is in this direct communication, however, that many mistakes are made, and by isolating the various portions of the code as much as possible the problem is reduced to many much smaller ones. The truth of this seems very obvious to experienced coders, yet it is hard to put across to the newcomer. An extremely valuable addition to the McCracken text would be a rather large problem, perhaps requiring as many as several thousand instructions, broken down into small logical blocks, with descriptions of these blocks. Similar problems could be presented for analysis by the student. This sort of work also has the advantage of being quite easily checkable by the instructor.

The experienced programmer, on reading the text, will probably wonder, as did the reviewer, why symbolic coding was not introduced much earlier rather than being relegated to a few pages at the end of the book. Certainly everyone who is introduced to computer programming in industry is given symbolic coding shortly after he learns the basic operation of the machine. The aim of symbolic coding is to help alleviate some of the burdens of coding, help him in checking and testing his program, and it is probably most useful to start the would-be coder off with as much help as possible, leaving him to concentrate on the essential problems of programming. Many excellent programmers would be lost without such an aid, and the student is probably no exception.

One final criticism of the book will be raised by many. Why, it will be asked, bother to teach many students to program for a mythical machine which they will never actually use, and bears only a confusing relationship to the machine they will ultimately program, and, additionally, one with so many instructions? This reviewer feels, on the contrary, that it is probably worthwhile for the coder to be familiar with more than one machine, even if one of them is mythical. Machines are continually being manufactured which are larger, faster, and with a larger set of instructions, and the switch from TYDAC to an actual machine will adequately prepare the coder for future machine changes.

In conclusion, it should be stated once more that *Digital Computer Programming* will probably be the standard text on coding for some time to come, and rightfully so.

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141[Z].—W. SLUCKIN, *Mind and Machines*, Penguin Books, Inc., Baltimore, Maryland, 1954, 221 p., 18 × 11 cm. Price $0.50.

This book attempts a comparison of human brains with digital and analogue computing machines in the endeavor to answer some questions raised by the existence of these machines. These questions: of similarity of mind and machine, of how the human brain works, of what constitutes "thinking," of whether machines "think," are all open fields for controversy. The author presents a many-sided discussion, leaving it to the reader to make his own decisions, and without always disclosing where he himself stands on some of these problems.

Cybernetics, and the concept of negative feedback are discussed in an attempt to determine how the human brain functions. Machines, particularly electronic machines, use the negative feedback principle extensively. How valid are our attempts to introduce this concept into our discussions of the nervous system? The author shows clearly the value and the limitations of the principle of negative feedback in this application.

In one sense, however, he persistently ignores a fact which to this reviewer should be more often used in these arguments, particularly on the question of whether what a computing machine does constitutes real thinking. This fact is that man has built the machine and has put into it, by virtue of his designs, a predictable pattern of behavior. True, this may be a very complicated pattern, it may involve decisions simulating human thought processes, it may even simulate "learning." But in every case, in the final analysis, a machine has been built which can only do what the designer says it can do.

Thus it seems a little odd to take this simulator, this computing machine, and use it as a model of what we suppose the brain to be like. It would be very surprising if it did not resemble the brain in some sense, since we have built it with our own logical thought processes as a model. This seems to be a weakness of the arguments for a mechanistic interpretation of the human brain.

One minor criticism concerning usage should be made. The author states, on page 176, for example, that digital and analogue computing machines "think" in that they perform *mathematical* calculations. This is not correct in a strict sense. These machines perform *arithmetical* calculations.

This is an informative book, and while it avoids dogmatic answers to the problems it states, it does give a wide basis for consideration of validity of the many hypotheses which have been suggested.

W. Sluckin combines interests in two fields ideally situated to help in the discussion of these problems: he is both a psychologist and an electrical engineer.

Each chapter concludes with a pertinent bibliography for additional reading.

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142[Z].—*Elektronische Rechnenmaschinen und Informationsverabeitung* (*Electronic Computers and Information Processing*), Nachrichtentechnische Fachberichte (Technical Information Report), Friedr. Vieweg and Sohn, Braunschweig, Germany, v. 4, 1956, viii + 299 p., 29.5 cm. Price DM 26.

This volume reports the proceedings of an international meeting held in Darmstadt, Germany, from October 25th to 29th, 1955 under the sponsorship of the German Association for Applied Mathematics and Mechanics (GAMM), the German Society of Electrical Engineers (VDE), the German Mathematical Society (DMV), and the Association of German Physical Societies (VDPG).

Among the seven introductory papers, three were presented by U. S. representatives with H. H. Goldstine evaluating computer speeds required to solve significant differential and integral equations by existing methods; A. S. Householder discussing numerical mathematics; and H. H. Aiken projecting the future of automatic computing machinery. A. D. Booth of England discussed input-output systems. The problems of data processing, programming and switching and memory techniques were respectively considered by R. Piloty of the Technische Hochschule München, H. Rutishauser of the Technische Hochschule, Zürich, and H. Billing of the Max Planck Institute, Göttingen.

The largest group of papers concerned themselves with descriptions of European computer developments and will have value as source material not readily found elsewhere. The computers reported on are tabulated below:

COUNTRY	*ORGANIZATION*	*MACHINE*
Austria	Institute for Low Frequency Techniques, Technische Hochschule, Vienna	Logical Function Computer
Belgium	Institute for the Encouragement of Scientific Research in Industry and Agriculture and National Fund for Scientific Research, Antwerp	IRSIA-FNRS
Czechoslovakia	ARITMA, Prague	Calculating Punch
	Institute of Mathematical Machines, Academy of Sciences, Prague	SAPO
		Digital Plotter
		Linear Equation Solver
		Relay Network Synthesizer
Germany	Institute of Applied Math. Tech. Hochschule, Dresden	D 1
	Institute of Electrical Information Handling Tech. Hochschule, Munich	PERM
	Institute for Experimental Mathematics, Tech. Hochschule, Darmstadt	DERA
	Leitz Optical Works, Wetzlar	Z 5
	Max Planck Institute, Göttingen	G 1
		G 2
		G 3
Netherlands	PTT Laboratories, Hague	PTERA
	Mathematische Centrum Amsterdam	ARRA
		ARMAC
Sweden	Matematikmaskinnämnden Stockholm	BESK
Switzerland	Institute for Applied Math. Tech. Hochschule, Zurich	ERMETH
U. S. A.	IBM	705
USSR	Academy of Sciences, Moscow	BESM
		URAL

Eleven papers describe approaches to programming in Germany and the Netherlands, with consideration given both to changes in machine design to facilitate programming and to compiling and interpretive methods.

In the area of numerical analysis a dozen papers cover problems in linear algebra, iterative processes, interpolation, hyperbolic partial differential equations, quadratures, nonlinear differential equations, hydrodynamical equations, linear programming, aircraft design computation, weather forecasting, matrix inversion, and astronomy.

The concluding miscellaneous section includes a survey of logical function computers, a description of contact grids as an aid to relay network synthesis, a discussion of statistical programs in industry and description of an equivalence algebra for representation of digital machines.

Aside from the content of the papers, interested researchers will find a number of new references associated with several of the articles.

Moreover, the publisher has been quite considerate in providing a list of addresses of the authors as well as short translated summaries of all the papers.

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TABLE ERRATA

Reviews and papers in this issue mention errata in the following works:

A. J. C. CUNNINGHAM & H. J. WOODALL, *Factorization of* $y^n \pm 1, y = 2, 3, 5, 6, 7, 10, 11, 12$ *up to high powers* (n), Francis Hodgson, London, 1925. (See RAPHAEL M. ROBINSON note, "Some factorizations of numbers of the form $2^n \pm 1$," p. 265.)

R. B. DINGLE, D. ARNDT, & S. K. ROY, "The integrals

$$A_p(x) = (p!)^{-1} \int_0^\infty \epsilon^p(\epsilon + x)^{-1}e^{-\epsilon}d\epsilon \text{ and } B_p(x) = (p!)^{-1} \int_0^\infty \epsilon^p(\epsilon + x)^{-2}e^{-\epsilon}d\epsilon$$

and their tabulation," Review 119, p. 279.

D. N. LEHMER, *List of Prime Numbers from* 1 *to* 10006721, Review **107,** p. 272.

257. —C. A. COULSON & W. E. DUNCANSON, "Some new values for the exponential integral," *Phil. Mag.*, v. 33, 1942, p. 754–761.

A comparison of tables of $\text{Ei}(x)$ and $-\text{Ei}(-x)$ appearing in this paper with more elaborate tables by Harris [1] reveals a total of nine rounding errors and two more serious errors in the former.

Rounding errors appear in the 10-figure values given by Coulson and Duncanson for $\text{Ei}(x)$ when $x = 20, 31, 46$, and 47, and for $-\text{Ei}(-x)$ when $x = 16, 20, 23, 29$, and 39.

Their claim of accuracy to within 2 units in the last figure is refuted, however, by the following emendations. Corresponding to $x = 44$ and 45 their approximations to $\text{Ei}(x)$ should be increased by nearly 8 units in the last place; that is, for 2.990444711, read 2.990444719, and for 7.943916028, read 7.943916036, respectively.

The accuracy of Harris's table of the interpolation coefficients $R_n(1)$ was confirmed by me by an independent calculation, and these data were then used to