A Formula for the Approximation of Definite Integrals of the Normal Distribution Function

A simple expression has been found which may be used to yield approximate numerical values for definite integrals of the normal distribution function. The expression is as follows.

$$\int_X^\infty e^{-t^2/2}dt = \frac{e^{-X^2/2}}{X + .8e^{-.4X}} ; X \geq 0.$$

This approximation is satisfactory for certain applications over a broad range of values for the finite integration limit, as may be judged from the table below.

X	$\int_X^\infty \exp(-t^2/2)dt$	$\frac{\exp(-X^2/2)}{X + .8\exp(-.4X)}$	Difference	% Difference
0	1.253	1.250	−.003	− .2
.2	1.055	1.044	−.011	−1.0
.5	.773	.764	−.009	−1.2
1.0	.398	.395	−.003	− .8
1.5	.1675	.1674	−.0001	0
2.0	.0570	.0573	+.0003	+ .5
2.5	.0156	.0157	.0001	.6
3.0	.00338	.00343	.00005	1.5
3.5	.00058 3	.00059 2	.00000 9	1.5
4.0	.00007 95	.00008 06	.00000 11	1.4
5.0	.00000 0718	.00000 0730	.00000 0012	1.7

Some of the properties of Mill's ratio, the function here approximated by $X + .8\, e^{-.4X}$, have been recently described by Sampford [1].

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1. M. R. Sampford, "Some inequalities on Mill's ratio and related functions," *Annals Math. Stat.*, 1953, v. 24, p. 130.

Some Factorizations of Numbers of the Form $2^n \pm 1$

The author has prepared a factorization routine for use on an IBM 701 computer. In this note, we describe the routine briefly, and report on some results obtained during the period February–April 1957 on the computer at the University of California, Berkeley.

In the basic routine, an arithmetic progression is given in which divisors of a number N are to be sought. Only single word divisors, that is, divisors less than 2^{35}, are considered, but the number N may be many words. After deleting those terms of the progression which are multiples of 2, 3, 5, 7, or 11, the remaining terms up to $N^{\frac{1}{2}}$, or up to a prescribed bound, are tried as divisors of N. In order to delete the multiples of 2, 3, 5, 7, and 11 efficiently, use is made of the fact that the differences of the remaining terms of the progression repeat with a period $\phi(2\cdot3\cdot5\cdot7\cdot11) = 480$. The 480 required differences are computed in advance, and used repeatedly.