105	1091265	2839935	6503595	8741145
16065	1615845	3243345	7187775	
19425	1954365	3400215	7641375	
43785	2822715	6005895	8062005	

Seven numbers out of 21 can be prime, eight out of 27 and nine out of 31. These all occur and are listed below. It is possible for ten numbers out of 33 to be prime and eleven out of 37, but these do not occur in the range; included in the list are such groups of nine out of 33 and ten out of up to 37 as occur in the range.

	1	3	5	7	9	1	3	5	7	9	1	3	5	7	9	1	3	5	7	9	7	8	9	10
																						out of		
1271	31	19	—	*	*	—	*	—	—	*	*	—	—	*	—	*	*	—	*	7	—	27	31	
5621	7	*	—	17	13	—	43	—	—	*	*	—	—	*	—	*	*	—	*	*	21			
88781	7	47	—	19	*	—	*	—	—	*	*	—	—	*	—	*	*	—	*	*	21	27	31	
113141	7	*	—	*	*	—	*	—	—	*	*	—	—	*	—	*	*	—	*	11	—	27	31	35
165701	*	*	—	*	*	—	*	—	—	*	*	—	—	103	—	53	17	—	11	7	21			
284711	7	11	—	23	101	—	*	—	—	*	*	—	—	*	—	*	*	—	*	*	21	27		
416381	7	11	—	*	*	—	*	—	—	*	*	—	—	*	—	29	*	—	*	*	—	—	33	
626591	7	11	—	*	*	—	17	—	—	*	*	—	—	*	—	*	*	—	*	*	21	—	33	
855701	7	19	—	149	*	—	*	—	—	*	*	—	—	*	—	*	*	—	*	*	21	27	31	
1068701	*	*	—	*	*	—	*	—	—	*	*	—	—	11	—	19	47	—	29	7	21			
1146761	7	*	—	43	13	—	*	—	—	*	*	—	—	*	—	*	*	—	*	*	21	27		
2580641	7	13	—	*	*	—	*	—	—	*	*	—	—	*	—	*	*	—	11	457	—	27		
6560981	7	11	—	*	29	—	*	—	—	*	*	—	—	*	—	*	*	—	*	*	21	27	33	
6937931	7	*	—	*	*	—	*	—	—	*	*	—	—	13	—	*	*	—	*	*	—	—	33	37
7540421	7	11	—	1879	13	—	41	—	—	*	*	—	—	*	—	*	*	—	*	*	21			
8573411	7	*	—	191	*	—	17	—	—	*	*	—	—	*	—	*	*	—	*	*	21			

In each line, the number in the left hand column is the first number for the line, an asterisk indicates a prime, a dash a multiple of 3 or 5 and a number the least factor of any other composite. There are thus seen to be eleven groups of seven primes in 21 numbers, eight of eight primes in 27, five of nine primes in 31 (two in the line beginning with 113141) and four more of nine in 33, and one group of ten in 35 and one of ten in 37. The range 1 to 50 is excluded as being altogether exceptional. A list of the groups of four and five has been deposited in the *UMT* file of *MTAC* (see Review **110**, *MTAC*, v. 11, 1957, p. 274).

King's College
Cambridge, England

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

60[A, B, C, D, H, I, K, L, M, P, R, V, X, Z].—George E. Forsythe, *Bibliography of Russian Mathematics Books*, Chelsea Publishing Co., New York, 1956, 106 p., 20 cm. Price $3.95.

Professor Forsythe precedes his very useful bibliography of Russian mathematical books with an informative introduction which contains, among others, a complete review of the book. To quote Prof. Forsythe:

"The subject matter of the books listed is mathematics, pure and applied, including tables beyond the most elementary, but excluding descriptive geometry. There are a few titles on quantum mechanics and other branches of mathematical physics, and more on mechanics, mathematical machines and nomography, but these topics are far from completely covered. For textbooks, the subject matter is more advanced than calculus.

The list is confined to books, excluding dissertations. Because of their mathematical interest, most of the serial monographs (Trudy) of the Steklov Mathematical Institute have also been included. The books in the list have been published (or reprinted) in Russian or Ukrainian since 1930. Translations into Russian are omitted."

The books listed cover a wide range of topics and are in very many cases written by some of the very best research men in Russia. Thus there is every reason for supporting Prof. Forsythe's thesis that these books "would be of the greatest value to mathematicians and students." If, as Prof. Forsythe seems to think, ignorance of the existence of these fine (and cheap) books and ignorance of how to obtain them are the major obstructions to their popularity, then the present bibliography ought to result in a run on Soviet mathematical books and periodicals.

Abraham Shenitzer

Rutgers University
New Brunswick, New Jersey

61[A, B, C, D, E, F, K, L, M, N].—*CRC Standard Mathematical Tables*, Tenth & Eleventh Editions (Formerly Mathematical Tables from Handbook of Chemistry and Physics), Chemical Rubber Publishing Company, Cleveland, Ohio, 1957, ix + 480 p., 19.5 cm. Price $3.00.

The section on Mathematical Tables from the Handbook of Chemistry and Physics considerably enlarged and greatly improved both in content and in the format of its tables made its first appearance in January 1954 as the Tenth Edition of the C. R. C. Standard Mathematical Tables.

Among the major improvements in the Tenth Edition are the following:

1. A new section on vector analysis
2. A table of Laplace transforms
3. A table of Bessel Functions of orders zero and one, Hyperbolic Bessel Functions and Spherical Bessel Functions
4. A table of sine, cosine and exponential integrals
5. A table of Elliptic Integrals of the First and Second Kinds
6. Extension of the tables of the hyperbolic functions $\sinh x$, $\cosh x$, and $\tanh x$ to $x = 10$
7. Extension of the table of the exponential function to $x = 10$
8. The revision and enlargement of the table of integrals to include an additional ninety-four formulas
9. A table of χ^2
10. A table of F and t for 1% and 5% distributions
11. Commissioners 1941 Standard Ordinary Mortality Table with its auxiliary table of the commutation symbols at $2\frac{1}{2}\%$.

In addition to the above, the following are included in the Eleventh Edition:

1. An enlarged section on Fourier series with special power series, Bernoulli numbers and finite sine and cosine transforms
2. A new section on partial fractions with examples
3. A table of multiples of $\pi/2$
4. A table of sums of the powers of natural numbers
5. A method for the tenfold extension of the tables of Factors and Primes
6. A table of particular solutions to ordinary and partial differential equations with constant coefficients.

These and other minor changes tend to make this volume more legible and more serviceable to all.

I. A. STEGUN

National Bureau of Standards
Washington, D. C.

62[B].—TOKYO NUMERICAL COMPUTATION BUREAU, *Report No. 10. Table of Square Roots of Complex Numbers*, 1956, i + 23 p., 25 cm. Not for sale.

This table, with explanation by T. Sasaki, lists the real and imaginary parts of the square roots of $1 + ix$ and $x + i$ to $11D$ for $x = 0(.002)1$. Second differences are given. Legibility is only moderate, but the table may be useful, because existing tables for the range $0 \leq x \leq 1$ give fewer arguments and far fewer decimals; see Fletcher, Miller, and Rosenhead *Index* [1].

A. F.

1. A. FLETCHER, J. C. P. MILLER, & L. ROSENHEAD, *An Index of Mathematical Tables*, Scientific Computing Service Limited, London, 1946, p. 33 (*MTAC*, Review **233**, v. 2, 1946–47, p. 13–18).

63[D, L].—G. E. REYNOLDS, *Table of* $(\sin x)/x$, Antenna Laboratory, Electronics Research Directorate, Air Force Cambridge Research Center, Bedford, Mass., AFCRC-TR-57-103, ASTIA Document No. AD117063, March 1957, iii + 5 + 200 p., 26 cm.

This work was prompted by the frequent appearance of the function $(\sin x)/x$ in the mathematics of antenna design. Values are given to 8D for $x = 0(.001)49.999$. The table is machine printed in block form, the digit in the third decimal place of x being given as the top argument, and the earlier digits as left (and right) argument. There are no differences. The table was printed from punched cards produced by Datamatic Corporation of Newton Highlands, Mass. and the AFCRC Statistical Services Division.

As far as the reviewer's information goes, there are only two other tables which, although slighter on the whole, complement this new work to a significant extent. A manuscript table by C. Blanc [1] gives $(\sin x)/x$, as well as its even derivatives to order 16, to 10D for $x = 0(.01)4$, and is worth mention in the present context because of its ten decimal places. A well-known volume by K. Hayashi [2] includes a table of $(\sin x)/x$ to 8D for $x = 0(.01)10(.1)20(1)100$, and is worth mention because of its values for $x = 50(1)100$ and its general relevance. Hayashi's table, which contains errors, is very suitable for comparison with the new table, since both are strictly eight-place. The reviewer compared some scores of corrected Hayashi values without finding any error in those of Reynolds.

A. F.

1. C. Blanc, see *MTAC*, v. 7, 1953, p. 188 (*UMT* 160).
2. K. Hayashi, *Tafelu der Besselschen, Theta-, Kugel- und anderer Funktionen*, Berlin, 1930, p. 30. See *MTAC*, v. 1, 1943–45, Review 12, p. 4.

64[D].—Clovis Faucher, *Tables trigonometriques contenant les valeurs naturelles des sinus et des cosinus de centigrade en centigrade du quadrant avec dix decimales*, Gauthier-Villars, Paris, 1957, 51 p., 27 cm. Price 500 francs.

These tables, of $\sin x$ and $\cos x$ to 10D for $x = 0(0^g.01)50^g$ with first differences, have been photo-offset from manuscript. The author states (in effect) that the values were derived by chopping the last two figures of 12D values calculated by quadratic interpolation in the fundamental tables of Andoyer. A dot appended after the 10th digit indicates that the value should be rounded up. The only checking specifically mentioned is a comparison with an existing 8D table. These tables, so far as is known to the reviewer, are the only published ones to 10D with argument interval $0^g.01$. (Peters has such a table in manuscript form.) Linear interpolation gives 8D, but full accuracy is attainable only with quadratic interpolation.

T. H. Southard

University of California
Los Angeles, California

65[H, X].—Jean Peltier, *Resolution Numerique des Equations Algebriques*, Gauthier-Villars, Paris, 1957, iv + 244 p., 21 cm. Price $7.29.

The equations considered are polynomial equations of high degree in a single variable, and the treatment is from the standpoint of the desk-machine user. This means that to some extent the book is behind the times because it is usually quicker and cheaper to mail such problems to an organization equipped with a high-speed automatic computer and set of subroutines for iterative root-finding.

Only the most powerful methods are described and the reviewer is in agreement with the choice made. The opening chapters deal with elementary algebraic properties of the roots and the arithmetic operations required in the evaluation, multiplication and division of polynomials. Chapters 3 and 4 are devoted to the calculation of the moduli of the roots by Graeffe's root-squaring process and the subsequent determination of the phases by the "highest common factor" method. An omission here is the use of the latter method merely to resolve the finite number of ambiguities in the determination of the accurate phases from the root-squaring computations. Chapter 5 deals with iterative methods, and later chapters cover error propagaton and equations with complex coefficients. The recommended general computational procedure is summarized in the final chapter.

Many numerical examples are included, but little indication is given of checking procedures. In particular, the use of current checks on the root-squaring computations, which is vital in hand computing, does not appear to be mentioned.

F. W. J. Olver

National Bureau of Standards
Washington 25, D. C.

66[I].—L. N. KARMAZINA & L. V. KUROČHKINA, *Tablit͡sy interpolt͡at͡sionnykh koėffit͡sientov* (*Tables of interpolation coefficients*), Press of the Academy of Sciences of the USSR, Moscow, 1956, 376 p., 26.5 cm. Price 37.65 rubles.

These tables have been prepared at the Computing Centre of the Academy of Sciences. The coefficients are given throughout to 10 decimals. The fractional part of the interval being p, the following tables of Lagrangean coefficients are given: Table I, 3-point, $p = -1(0.001)1$; II, 4-point, $p = -1(0.001)2$; III, 5-point, $p = -2(0.001)2$; IV, 6-point, $p = -2(0.01)0(0.001)1(0.01)3$; V, 7-point, $p = -3(0.01) -1(0.001)1(0.01)3$; VI, 8-point, $p = -3(0.01)0(0.001)1(0.01)4$. Table VII gives 3 to 11-point Lagrangean coefficients for sub-tabulation to tenths.

All the coefficients for interpolation by differences are at interval 0.001, except Bessel's which are at 0.0001; coefficients up to those of the seventh difference are given throughout and the error is said not to exceed 6×10^{-11}. The even Bessel coefficients are those of mean, not double mean, differences (i.e. they are $2B_2$, $2B_4$, $2B_6$ in the notation of *Interpolation and Allied Tables*, London, 1956). Table VIII gives the Newton-Gregory coefficients, IX gives Bessel's, X gives Stirling's ($-0.5 \leq p \leq 0.5$), XI gives Everett's (to E_6). None of the tables of coefficients is provided with differences.

All the coefficients have been newly computed except the Lagrangean ones which, apart from the portion of the eight-point table at interval 0.01, have been taken from the National Bureau of Standards Tables (1944). One may hazard a guess that the Russian equivalent of the National accounting machines has eight registers; it is hard to see what else should make one stop at B_7; the predilection for Bessel's formula, which agrees with this reviewer's personal preference, is noteworthy; more striking is the absence from the Introduction of any mention of throw-back. In view of Chebyshev's connection with it, this is strange indeed.

J. C. E. JENNINGS

Birbeck College
University of London
London, W.C. 1., England

67[I, X].—H. E. SALZER, "Formulas for calculating Fourier coefficients," *Jn. Math. and Physics*, v. 36, 1957, pp. 96–98.

If a function is defined in the range $(0, 2\pi)$ as the polynomial of degree not exceeding eight which takes nine given values f_n at the nine equally spaced arguments $x = n\pi/4$, where $n = 0(1)8$, then all the coefficients in the Fourier expansion of the function in the range $(0, 2\pi)$ are defined, and may be calculated from the given values by using equations of the form

$$\frac{1}{2\pi}\int_0^{2\pi} f(x) \cos nx dx = a_1(f_0 + f_8) + a_2(f_1 + f_7) + a_3(f_2 + f_6) + a_4(f_3 + f_5) + a_5 f_4,$$

$$\frac{1}{2\pi}\int_0^{2\pi} f(x) \sin nx dx = b_1(f_0 - f_8) + b_2(f_1 - f_7) + b_3(f_2 - f_6) + b_4(f_3 - f_5).$$

The table on page 97 lists values of the nine coefficients a_i and b_i to 5D for $n = 0(1)24$, so that means are provided for calculating all Fourier coefficients up to those of cos $24x$ and sin $24x$ inclusive. The values of a_i and b_i tabulated are said to be "most probably correct to within around $1\frac{1}{2}$ units in the 5th decimal, even though they cannot be absolutely guaranteed to less than $3\frac{1}{2}$ units in the 5th decimal."

A. F.

68(I).—H. E. Salzer & Peggy T. Roberson, *Table of Coefficients for Obtaining the Second Derivative without Differences*, Convari-Astronautics, San Diego 12, California, 1957, ix + 25 p., 26 cm., deposited in the *UMT* files.

If n function values are given at n equidistant arguments at interval h, this table lists coefficients for determining the value of the second derivative at any point in the range of width $(n - 1)h$. The tables relate to the cases $n = 5(1)9$, for which the Lagrange interpolation polynomials are of degrees 4(1)8 respectively, and their second derivatives consequently of degrees 2(1)6 respectively. For $n = 5(1)7$, the coefficients are given for hundredths of h; for $n = 8, 9$, they are given for tenths of h. The coefficients are in each case multiplied by an integer (6, 12, 360, 360, 10080 for $n = 5(1)9$ respectively) such that the tabulated values, instead of involving recurring decimals, are exact in a convenient number of decimals (4, 6, 8, 5, 6 respectively.) This useful table was calculated on the IBM 650 Magnetic Drum Data-Processing Machine.

A. F.

69[I].—H. O. Rosay, *Interpolation Coefficients* $\binom{S}{K}$ *for Newton's Binomial Interpolation Formula*, 20 tabulated sheets, 28 × 38 cm., deposited in the UMT File.

Binomial coefficients $\binom{S}{K}$, $K = 1(1)5$, $S = 0(.01)5$, 11D; however in many cases the last three digits are not reliable and accuracy is probably 8D.

The tables were computed on SWAC for use with the Gregory-Newton interpolation formula ([1], p. 96, equation 4.3.11). Similar tables for $K = 1(1)6$, $S = 0(.01)100$, 7D are in [2] according to [3].

T. H. Southard

University of California
Los Angeles, California

1. F. B. Hildebrand, *Introduction to Numerical Analysis*, McGraw-Hill, New York, 1956.
2. G. Vega & J. A. Hülsse, *Sammlung Mathematischer Tafeln*, Weidmann, Leipzig, 1933.
3. A. V. Lebedev & R. M. Fedorova, *Spravochnik po Matematicheskim Tablitsam*, Moscow, 1956 [*MTAC*, Rev 49, v. 11, 1957, p. 104–106.]

70[L].—A. A. Abramov, *Tablitsy ln* $\Gamma(z)$ *v Kompleksnoĭ oblastĭ* (*Tables of ln* $\Gamma(z)$ *in a complex region*), Akad. Nauk SSSR, Moscow, 1953, 333 p., 26 cm. Price $3.50.

This volume gives 6D tables of the real and imaginary parts of $\ln \Gamma(x + iy)$ with second differences for $x = 1(.01)2$ and $y = 0(.01)4$. Real and imaginary parts are tabulated on opposite pages, each page having six columns corresponding to six values of x (the columns $x = 1.05(.05)1.95$ are repeated occurring as last columns on one page and first columns on the following page) and 51 lines corresponding to fixed y (with repetitions for $y = .5(.5)3.5$). The introduction contains formulas for computation, error estimates, references both to books and tables (not including, however, Sibagaki's tables mentioned below), and a nomogram for $\frac{1}{2}(\xi - \eta)(\xi + \eta - 1)\Delta_2$. The volume is well printed.

For other tables of the gamma function in the complex domain see RMT **234** (*MTAC*, v. 2, p. 19), RMT **855** (*MTAC*, v. 5, p. 25–26), and RMT **1143** (*MTAC*, v. 7, p. 246). A small number of obvious misprints in the present work are corrected in pencil in the reviewer's copy. Comparison of 1300 values given both in the present tables and those by Sibagaki (RMT **1143**) showed about 100 discrepancies, mostly of one unit of the last decimal.

A. Erdélyi

California Institute of Technology
Pasadena, California

71[L].—Carl-Erik Fröberg, *Complete Elliptic Integrals*, Lund University Department of Numerical Analysis, Table No. 2, G. W. K. Gleerup, Lund, Sweden, 1957, 82 p., 25 cm. Price 12 Kr.

Ten decimal tables of the complete elliptic integrals of the first and second kinds as functions of the modulus. The functions tabulated are $K(k)$, $E(k)$, $K'(k) = K(k')$, $E'(k) = E(k')$ and two auxiliary functions

$$S(k) = K'(k) - \ln\frac{4}{k}, \quad S'(k) = K(k) - \ln\frac{4}{k'}.$$

The intervals are $k = 0.000(0.001)\ 0.900,\ 0.9000(0.0001)\ 1.0000$. Second differences, modified where necessary, are given for the ranges in which Everett's second order interpolation formula will give full accuracy. Elsewhere differences are omitted and auxiliary formulae are given, for use at the extreme ends of the table. In point of fact these auxiliary formulae could have been dispensed with had the author observed that while his function $S(k)$ is not interpolable at the given interval for $k < 0.005$, the function

$$K'(k) - \frac{2K(k)}{\pi}\ln\frac{4}{k}$$

is interpolable without restriction. Since the function $S(k)$ fails just where it is most needed, it is difficult to see the point of introducing it throughout the range of k, or indeed at all.

The tables were calculated in two ways namely by Landen's transformation and by power series but no information is given of the comparative labor of the two methods nor of the agreement of the results.

The argument which presents itself naturally in calculations with elliptic functions and integrals is k^2 rather than k. Furthermore the use of k^2 enables the size of the table to be halved without loss.

L. M. Milne-Thomson

Brown University
Providence, Rhode Island

72[L].—CARL-ERIK FRÖBERG & HANS WILHELMSSON, "Table of the function $F(a, b) = \int_0^a J_1(x)(x^2 + b^2)^{-\frac{1}{2}}dx$," *Fysiografiska Sallskapets I Lund Förhandlingar*, Bd. 27, 1957, p. 201–215.

The function $F(a, b)$ defined in the title is tabulated to 6D (which for most of the given values means 6S) for $a = .1(.1)2(.2)10$ and $b = 0(.1)2(.2)10$.

A short introduction mentions some physical applications of the function $F(a, b)$ and lists some limiting and special cases. Five-point Lagrangian interpolation is said to give full accuracy except for Max $(a, b) \leq 1$; to facilitate interpolation in this region, an auxiliary function $f(a, b)$ defined by

$$F(a, b) = \tfrac{1}{2}[(a^2 + b^2)^{\frac{1}{2}} - b] - [f(a, b)]^3$$

is tabulated for $a = .1(.1)1$, $b = 0(.1)1$, 6D.

The tables were calculated on SMIL, the electronic computer of Lund University, by numerical quadrature. For $b \leq 2$ a power series expansion was used as a check. The error is said to be in general less than 1 unit in the last place.

PETER HENRICI

University of California
Los Angeles, California

73[L].—J. F. NICHOLAS, *A Table of $\int_0^x exp(-1/u)du$ for Small Values of x.* 28 Mimeographed pages, 22 × 16.5 cm., deposited in the UMT File.

Let $I(x) = \int_0^x e^{-1/u}du$ and $f(x) = x^{-2}e^{1/x}I(x)$. The table lists $f(x)$ and $I(x)$, $x = .01(.0005).05075$, 6S. The values given for $f(x)$ are said to be accurate except for small errors in the sixth place, the values given for $I(x)$ may be accurate to no more than four places for some values of the argument.

The tables are intended for use in connection with chemical and other reactions which are thermally activated and governed by an equation of the form $p = g(p)e^{-Q/RT}$.

They are related to other tables listed in the bibliography as well as a table whose existence is mentioned in a comment by Hastings [1]. They are also related to Hasting's rational approximations for the function $-\text{Ei}(-x)$ listed in [1]. This relation follows from the identity listed by the author, $I(x) = xe^{-1/x} + \text{Ei}^{-1/x}$.

The tables are reproduced photographically from machine tabulations, with a 7 page introduction reproduced photographically from typescript. A few copies may be available directly from the author at The Division of Tribophysics, Commonwealth Scientific and Industrial Research Organization, University of Melborne, Australia.

There is a minor misprint on page 3 of the introduction, where the author should have written $I(x) = xe^{-1/x} - \int_0^x u^{-1}e^{-1/u}du$; he has a + sign between the two main terms.

C. B. T.

1. CECIL HASTINGS, JR., JEANNE T. HAYWARD, & JAMES P. WONG, JR., *Approximations for Digital Computers*, Princeton, 1955, p. 187–190. See *MTAC*, v. 9, 1955, Review No. 56, p. 121–113.

74[L, P].—ROBERT L. STERNBERG, J. S. SHIPMAN, & S. R. ZOHN, *Table of the Bennett Functions $A_m(h)$ and $A_{mn}(h, k)$*, Laboratory for Electronics, Inc., Boston 14, Massachusetts, 18 p., 28 cm., deposited in *UMT* files.

The announced purpose of these undated and unpublished tables is to provide for the simple or multiple Fourier series expansion of the output from a cut-off power law rectifier when responding to a simple or multiple frequency oscillatory input.

The definitions of the functions are:

$$A_m(h) = \frac{2}{\pi}\int_{R_1} (\cos u - h) \cos mu \, du$$

where R_1: $\cos u \geq h, \quad 0 \leq u \leq \pi$

$$A_{mn}(h, k) = \frac{2}{\pi^2}\int\int_{R_2} (\cos u + k \cos v - h) \cos mu \cos nv \, du \, dv,$$

where R_2: $\cos u + k \cos v \geq h, \quad 0 \leq u, \quad v \leq \pi.$

Reference is made to two papers by W. R .Bennett [1].

$A_m(h)$ is given to 6D for $m = 0(1)9$ and $h = 0(.05)1$. $A_{mn}(h, k)$ is given to 6D for m, n non-negative integers on the range $0 = m + n = 4$ and $h = 0(0.1)2$, $k = 0.1(0.1)1$.

Computation was done on the IBM C.P.C. by from two to five different methods. All results are stated to have an absolute error of about 10^{-6} at most in absolute value.

THOMAS H. SOUTHARD

University of California
Los Angeles, California

1. W. R. BENNETT, "New results in the calculation of modulation products," *Bell System Technical Journal*, v. 12, 1933, pp. 228–243; also "The biased ideal rectifier," *Bell System Technical Journal*, v. 26, 1947, pp. 139–169.

75[L, P].—CHIAO-MIN CHU, GEORGE C. CLARK, & STUART W. CHURCHILL, *Tables of Angular Distribution Coefficients for Light-Scattering by Spheres*, University of Michigan Press, 1957, xv + 58 p., 22 × 28 cm. Price $3.00.

Expressions for the differential scattering of light by a spherical particle are well known. Quite recently the present authors, following a suggestion of W. Hartel, derived forms which facilitate both numerical evaluation and application. The normalized differential scattering cross section is expressed by an expansion in Legendre polynomials. The coefficients are the quantities of interest here. They depend on two parameters, one the index of refraction of the medium surrounding the particle, the other a quantity, α, expressing the ratio of particle diameter to wave length.

The coefficients are evaluated for a series of fifteen refractive indices, ranging from 0.90 to 2.00 and also for the limit at infinity. The values of α are selected integral ones from the range 1–30. The accuracy is usually ± 1 in the fifth decimal digit. The output of the computer is directly printed without transcription. It is not explicitly stated to what extent the computations have been checked; such checks should be straightforward here. Perhaps Part II should have been labelled "Total Scattering Coefficients" rather than "Total Scattering Cross Sections." It would also have been interesting to include some of the computing times.

Together with the table of Legendre polynomials prepared by two of the authors (Clark and Churchill), calculations of differential scattering of light should now be simple and straightforward.

N. METROPOLIS

University of Chicago
Chicago, Ill.

76[M, P].—GUSTAV DOETSCH, *Anleitung zum praktischen Gebrauch der Laplace-Transformation*, R. Oldenbourg Verlag, Munich 8, Germany, 1956, 198 p., 23 cm. Price DM 22.00.

The author of this work has written several well-known books on the Laplace transformation and its applications. The present book was undertaken at the request of the publishers and aims at providing a handbook of the Laplace transformation for the use of engineers, special consideration being given to the needs of the theory of servomechanisms. The presentation is lucid, and the style is free of mathematical sophistication; at the same time the statements are precise and mathematically sound. Proofs are not given, except where they contribute to a full understanding of the results. Conditions of validity are not only stated but also fully discussed; and the reader is often warned against neglecting to observe proper caution in using these methods.

After introducing the Laplace transformation in chapter 1, the so-called *rules* are discussed in chapter 2, applications to ordinary linear differential equations with constant coefficients, and to systems of such equations, are given in chapter 3, difference equations and recurrence relations are presented in chapter 4, partial differential equations in chapter 5, and integral equations and integral relations in chapter 6. The problem of inversion is discussed in chapter 7, and asymptotic expansions and stability in chapter 8.

An Appendix of some sixty pages, almost one third of the book, was compiled by Rudolf Herschel and gives transform pairs with emphasis on those needed in servoengineering and other engineering problems. There are 41 general formulas followed by nearly 300 transform pairs arranged according to image functions (116 rational functions, 109 algebraic and elementary transcendental functions, and 56 functions arising in the solution of differential equations).

Since the theoretical part covers a wide field in something like 130 pages, it is inevitable that not all topics are covered thoroughly. The presentation of ordinary differential equations with constant coefficients is very satisfactory as are, for a book of this kind, the unusually thorough discussions of the inversion problem and of asymptotic expansions. By comparison, the chapter on partial differential equations is somewhat meagre, and that on integral equations, skimpy. Differential and integral equations with time-lag are not discussed at all.

In the field which it does cover thoroughly, the book will prove a reliable guide to the engineer whose mathematical training enables him to understand fully simple and precise mathematical statements, and who needs precise and unambiguous instructions for using Laplace transforms.

A. ERDÉLYI

California Institute of Technology
Pasadena, California

77[S].—MANFRED VON ARDENNE, *Tabellen der Elektronenphysik Ionenphysik und Ubermikroscopie*, I. Band. Hauptgebiete, Deutscher Verlag der Wissenschaften, Berlin, 1956, xvi + 614 p., 27 cm. Price DM 74.

These two volumes contain a very complete collection of formulas, tables, references, and selections from the very extensive physical literature on electron physics, ion physics and the application of these subjects to microscopy. The material contained in these books relating to mathematics and mathematical methods is very meager. The latter topic is confined to a brief discussion (2 pages) of the Laplace equation and a reference to the Liebmann method for integrating this equation numerically.

Volume II contains six pages devoted to mathematical formulas and references. Of these, two pages are devoted to references, one page to a four place tables of exponential functions, two pages to formulas pertaining to statistics, one page to a figure illustrating the logarithmic scale used in the book, and one page giving the first few terms of the MacLaurin expansion of various simple functions.

A. H. T.

78[V, X].—B. ETKIN, *Numerical Integration Methods or Supersonic Wings in Steady and Oscillatory Motion*, UTIA Report No. 36, Institute of Aerophysics, University of Toronto, 1955, v + 37 p. + 22 figures + 4 tables, 28 cm. Available only on an exchange basis.

The paper deals with the problem of the determination of a perturbation velocity potential on supersonic wings with such a geometry that analytic methods are not applicable and, therefore, a numerical integration is unavoidable. It is based on the linearized theory of supersonic flow, and the cases of a supersonic as well as of a subsonic leading edge are considered.

The first part contains the results of the author's two previous papers on the problem of steady flow. The essential novelty consists of an extension of the method for oscillating wings. Insofar as the reduced frequency is concerned, three different stages of complexity appear. For a very slow oscillation the computation formulae are nearly identical with those for steady motion. If the frequency is sufficiently small to permit the linearization of the equations with respect to the frequency, then the formulae generated are still relatively simple. Finally, if this simplification is not possible, then the method still gives a system of equations, but the process for their numerical solution seems to be rather complicated.

Emphasis is placed on the presentation of the method in a form suitable for automatic digital computation. In some simple cases, such as those involving a flat or a cambered delta wing, the numerical results are compared with the exact (linear) solutions and the agreement is found to be very good.

PENTTI LAASONEN

Kasarmikatu 2B11
Helsinki, Finland

79[W, Z].—RICHARD G. CANNING, *Electronic Data Processing for Business and Industry*, John Wiley and Sons Inc., New York, 1956, xi + 332 p., 23 cm. Price $7.00

The objective of this book "is the outline of a program of study and planning leading to the preparation of a proposal to top management." As the germ of his method, the author advocates (sic) systems engineering which he interprets for management personnel. The book is readable, many examples are given and the individual parts fit together quite well. Furthermore, the author recognized that hardware aspects could be expected to change (as they have) and accordingly qualified his material. This reviewer believes that the author has met his objective; this book probably will continue to prove useful to management personnel, particularly if it passes through revisions designed to keep it abreast of both technology and experience.

This reviewer does not wish to criticize this book on the basis of what it was not designed to provide. He does believe that it represents a reasonable set of case studies for clerical systems problems. The book is thus recommended for examination with the hope that such an observation process may stimulate or assist work on underlying numerical techniques or ingenious gadgetry, or both.

W. H. MARLOW

The George Washington University
Logistics Research Project

80[W, Z].—RICHARD G. CANNING, *Installing Electronic Data Processing Systems*, John Wiley and Sons Inc., New York, 1957, vii + 193 p., 22 cm. Price $6.00.

Most managers who are considering Electronic Data Processing Systems, EDPS, should find this book of considerable value and interest. It should be of particular interest to those managers who have decided to include an EDPS in their organizations. The book is written in nontechnical language and assumes that the reader (the MANAGER) is relatively unacquainted with electronic computers. However, familiarity with such basic concepts as files, unit records, fields and some knowledge of programming is presumed.

The author contends throughout the book . . . "that top management should not remain so aloof from the *details* of electronic data processing. . . . It is time for management to recognize that EDP is a *management tool* and it is the responsibility of management to learn the advantages and limitations of this new tool. Perhaps the most effective way to learn about these new machines is to learn how to program a machine. Or to put it slightly differently, it is questionable whether a person really understands electronic data processing until he knows how to program." This reviewer concurs with the above statements.

This book consists essentially of a narrative description of the typical problems encountered (and how they are solved) by an organization which is planning the installation of an EDPS. The organization is hypothetical (yet realistic); the problems encountered are not hypothetical, they are real problems. The discussion begins at a point in time where management has concluded a feasibility and application study and is beginning to plan for the installation of an EDPS. The discussion extends through a point in time (some three years later) where the installed system is in operation and the conversion period is almost complete. (The conversion means the changing over from the old operational system to the new electronic system.)

The EDPS organization (be it a section or branch or department) is first fitted in perspective, i.e., relative to the entire organizational structure. Management then begins plans for acquiring and training the personnel needed to staff the new EDPS section. Included are detailed job descriptions, qualifications and basis for selection for each of the various types of personnel required to staff the EDPS section. Approximately one half of the book discusses the personnel problems, i.e., acquisition, training, duties and performance. The case history reveals that the programming effort required should not be underestimated, viz., one might expect programming efforts to expend one man hour per one single address instruction coded and checked. The last half of the book discusses the problems encountered in the physical installation of the EDPS. This includes both hardware and personnel facilities and terminates with a thorough discussion of the problems encountered in the conversion period.

In summary, this book should be of considerable value to anyone involved in the planning, installation, or operation of an Electronic Data Processing System.

M. J. Romanelli

Computing Laboratory
Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

81[W, Z].—G. Kozmetsky & P. Kircher, *Electronic Computers and Management Control*, McGraw-Hill, New York, 1956, 296 p., 23 cm. Price $5.00.

While it is difficult for this reviewer to judge, it would seem that this book, written primarily for the business executive and presupposing no technical training, succeeds in its purpose "to explain certain new developments which may have a greater influence on the management of enterprise than any other single group of events have had since the first industrial revolution."

A principal shortcoming is a complete lack of documentation. There are four footnotes and at least a dozen quotations from various pronouncements and articles; however, this reviewer did not see a single precise bibliographical reference within the text. In his opinion this greatly detracts from what seems to be a large collection of accurate and suggestive accounts of management experiences with computing equipments. So far as this reviewer could tell, the authors are reasonably meticulous in their limiting use of the present tense to computers and processes in being.

The authors' exposition on management control is capable and generally stimulating. Among the aspects described and related to computer applications are the following: model-making, programming (planning), scheduling and feed-back, and simulation.

In conclusion, this reviewer believes that the book would be of interest to those readers of this journal who wish to obtain some of the flavor of "business" data processing problems. He differs with the authors (p. 12) by believing that mathematicians must lead the way toward eventual resolution of difficulties in this particular area of numerical analysis.

W. H. MARLOW

The George Washington University
Logistics Research Project

82[W, X, Z].—PURDUE UNIVERSITY COMPUTER RESEARCH PROGRAM, *Proceedings of Symposium I*, held at Purdue University, November 8 and 9, 1956, Purdue Research Foundation, iv + 76 p., 28 cm.

The book is a transcript of seven papers presented at Purdue University Computer Research Program, a symposium held November 8 and 9, 1956; the table of contents follows:

C. B. T.

83[X].—WILLIAM E. MILNE, *Numerical Solution of Differential Equations*, John Wiley and Sons, Inc., New York, 1953, xi + 275 p., 23 cm. Price $7.25.

The work under review has in the years since its appearance been widely used both as a text and as a reference book. The book owes this success to several very attractive features. Eminently teachable and lavishly illustrated with worked examples, it leads the student along a gentle path. While providing a great deal of useful information, it seldom exposes the reader to the stern discipline of analytical rigor. Moreover, Professor Milne does not jump to the sterile conclusion that, because there is some good and some bad in every method, there is little to choose between them; on the contrary, he comes out with very definite opinions and thus offers firm guidance to the inexperienced worker in the field.

If in the following we shall comment critically on some aspects of Milne's book, we do not mean to belittle the author's achievement, which was quite substantial at the time the book was written. Rather we believe that our remarks will illustrate the enormous development which has taken place in the field of numerical computation during the last decade and which Professor Milne was one of the first to help to bring about.

Although high-speed machinery is mentioned at a few scattered places, it is probably fair to say that the whole outlook of Milne's book is dominated by the idea that all computations are carried out on a desk machine. The faithful disciple of the book is imagined to work doggedly with pencil and paper, erasing predicted values when replacing them by corrected ones. This basic philosophy of the book is, in this reviewer's opinion, a serious detriment to its value as a guide for use in connection with automatic computers. The following remarks may serve to substantiate this statement.

The vexing problems that arise in the numerical integration of ordinary differential equations over a long range are not given adequate attention. The discussion of round-off is relegated to a rather superficial one-half page appendix. Although much attention is paid to the local error, no attempt is made to obtain realistic appraisals for the inherited error. Few numerical examples are carried further than ten steps.

In the sections on ordinary differential equations, which together comprise about 100 pages, the emphasis is on difference methods. Methods of Runge-Kutta type are discussed on two pages (without proofs), and everything is done to discourage their use. The simple second order Runge-Kutta methods are not even mentioned. While it is true that difference methods are somewhat less involved from the point of view of pencil and paper work, this advantage is hardly relevant from the point of view of computing machinery. Here all genuine one-step methods (such as the various Runge-Kutta methods or their modern modifications) enjoy the tremendous advantage of being self-starting (and thus cutting the length of any code in half) and of rendering trivial the operation of changing the basic interval.

Those who, in view of earlier publications of the author, expect the book to contain a wealth of relations involving finite differences will not be disappointed. Since for a given number of ordinates central difference formulas are the most accurate ones, these are particularly emphasized. Unfortunately these formulas are frequently unstable (in the sense of H. Rutishauser) if used for the integration of ordinary differential equations. A simple example may illustrate this. Milne discusses the classical Euler method ($y_{n+1} = y_n + hy'_n$) in two lines. He then proceeds immediately to a discussion of the midpoint rule ($y_{n+1} = y_{n-1} + 2hy'_n$) to which he devotes five pages. It so happens that the midpoint rule, quite apart from requiring a starting procedure, is unstable; it will not produce useful results for such a simple differential equation as $y' = -y$ over any reasonably long interval. Euler's method, on the other hand, will produce good, if not very accurate results for any differential equation. The celebrated Milne method (known in European countries as method of central differences, or simply as Simpson's rule) is also unstable, as was shown by Rutishauser [1]. While it is true that unstable methods usually produce good results over short ranges, their blind use in problems involving many steps can lead to disastrous effects.

The chapters on the solution of partial differential equations are handicapped by the fact that it was apparently not possible to give proper attention to several fundamental papers which became available shortly before the book was published. There is a long section on linear equations and matrices, intended to provide the student with the tools for solving implicit partial difference equations in particular those arising from elliptic problems. But the text does not mention the merits of the successive overrelaxation method, discussed independently by S. P. Frankel [2] and D. M. Young [3, 4], which has proved to be so eminently successful in problems of this type. Again a great variety of partial difference operators is listed, but the discussion of numerical stability adds nothing to the 1928 Courant-Friedrichs-Lewy paper.

It has been pointed out that the general mathematical level of the book is elementary. To be specific, the level is well below the comparable book by L. Collatz [5]. While this is a desirable feature from the point of view of making the field accessible to students with a limited interest in mathematics, this reviewer has experienced a less desirable consequence of the lack of intellectually challenging material. To the bright mathematics student, if this is his first acquaintance with numerical analysis, the subject is made to appear a dull collection of recipes rather than a logical piece of mathematics. He has no opportunity to see the many interesting problems which merit his interest, and he will not feel attracted to a science which today more than ever needs his talents.

Peter Henrici

University of California
Los Angeles, California

1. Von Heinz Rutishauser, "Uber die instabilitat von mathoden zur integration gemohnlicher differentialgleichungen," *Zeit. f. ang. Math. u. Phys.*, v. 3, 1952, p. 65–74.
2. Stanley P. Frankel, "Convergence rates of iterative treatments of partial differential equations," *MTAC*, v. 4, 1950, p. 65–75.
3. D. M. Young, *Iterative Methods for Solving Partial Difference Equations of Elliptic Type*, Doctoral thesis, Harvard University, 1950.
4. D. M. Young, "Iterative methods for solving partial difference equations of elliptic type," *Amer. Math. Soc., Trans.*, v. 76, 1954, p. 92–111.
5. L. Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd Ed., Springer-Verlag, Berlin, 1955.

84[Z].—John Robert Stock, *Die mathematischen Grundlagen für die Organisation der elektronischen Rechenmaschine der Eidgenössischen Technischen Hochschule*, Inst. f. angew. Math., *Mitt.*, Zurich, No. 6, 1956, 73 p., 23 cm. Price Sw. Fr. 7.30.

This seventy-three page book is the sixth in the current series on computers and applied mathematics prepared by the Institute for Applied Mathematics at the Eidgenossishe Technische Hochschule in Zurich, Switzerland.

The publication consists of a complete description of the ERMETH (Elektronische Rechenmaschine der Eidgenossishen Technischen Hochschule). In four Chapters, the mathematical and engineering characteristics, and the internal organization of the machine, are described. The first chapter presents a description of the mathematical characteristics, indicating number representation, instruction codes and machine operation. The second chapter gives the fundamentals of floating point, normalization and rounding, fixed point and index registers. The third chapter discusses the engineering fundamentals, including basic decision elements and arithmetic principles. The fourth chapter includes a discussion of internal organization and the manner in which the arithmetic unit performs addition, subtraction, multiplication and division. The control unit and error control methods are also described.

A large volume of information is given in the five appendices. Appendix I contains a complete description of the external features:

TECHNICAL DATA:

Drum storage capacity	10,000 words
Word length	16 decimal digits
Drum speed	6,000 RPM
Pulse repetition rate in storage	352 Kc
Pulse repetition rate in control and arithmetic unit	32 Kc
Cycle time	0.5 milliseconds
Average drum access time	5.0 milliseconds
Average access time in the index—instruction counter—register	2.5 milliseconds

OPERATION TIMES:

Add, excluding storage access, average	4 milliseconds
Multiply, excluding storage access, average	18 milliseconds
Divide, excluding storage access, average	28 milliseconds
Logical instruction	1 millisecond

COMPONENTS:

Tubes	1,900
Germanium diodes	7,000

Of these, 40% are in the arithmetic unit
25% are in the control unit
20% are in the storage switching unit
15% are in the input-output

WORD STRUCTURE:

Floating point number words

Check digit (left end)	1
Exponent	3
Absolute value of mantissa (<10)	11
Sign (right end)	1

Fixed point number words

Check digit (left end)	1
Absolute value of mantissa (<1)	14
Sign (right end)	

Instruction word

Check digit (left end)		1
Left or first instruction		7
Operation	2	
Index	1	
Address	4	

Right or second instruction		7
Operation	2	
Index	1	
Address	4	
Sigh (right end)		1

Appendix II contains block diagrams, electrical schematics and logical diagrams, of the arithmetic unit, including a description of the micro-steps performed by the arithmetic unit during the execution of various types of instructions.

Appendix III gives similar information concerning the control unit. Included in Appendix IV is a detailed description of the networks contained in the adder.

Appendix V contains four different typical programs that have been prepared for the computer. A bibliography is included, however all of the references are dated 1953 or earlier.

The publication is written in clear, concise, easily understandable language. Although the ERMETH is a relatively slow computer (no high-speed storage other than the three arithmetic registers), the discussion and analysis of general computer organization and principles of operation are very interestingly presented. The reader will be quite pleased with the manner in which such subjects as number systems, arithmetic schemes, instruction codes, and programming techniques are handled. Sufficient information is given to permit the reader to code problems for the ERMETH. The book is worthwhile reading for anyone who wishes to familiarize himself with the general techniques and principles of computer construction, organization, and operation, even though the ERMETH itself may not be the reader's particular concern.

Martin H. Weik

Computing Laboratories
Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland

85[Z].—Takashi Kojima, *The Japanese Abacus—Its Use and Theory*, Charles E. Tuttle Co., Tokyo, Japan, 1957, 102 p., 18 cm. Price $1.25.

There has been some interest in the abacus as the so-to-speak most elementary digital calculating instrument in use, but there has been no book in English which describes the abacus adequately.

This book appears to be the first of that kind.

Beginning with the story of the contest held in Japan in 1946 between operators of the Japanese abacus and the electro-mechanical desk calculator, it deals briefly with the history of the abacus in general, then goes on to explain step-by-step manipulations of beads of the Japanese abacus to do four arithmetic operations.

The book shows that by following various rules, the arithmetic operations (especially addition and subtraction) can be reduced to quite mechanical operations of the abacus requiring very small amounts of mental effort (it is only necessary to remember 10's complements of $1 - 9$ and 5's complements of $1 - 4$).

The book also contains some exercises for those who wish to acquire skill in manipulating the Japanese abacus.

The abacus is used quite extensively in Japanese business establishments. In the hands of experts, it is faster than the usual desk calculator for addition or subtraction and about as fast for multiplication or division of 10 to 12 digit numbers. The fact that the price of the Japanese abacus is approximately $3.00 may be one of the big reasons for its popularity among the small business establishments in Japan. However it must be borne in mind that to attain reasonable proficiency in the use of the abacus requires a number of months of practice.

The monograph provides interesting reading for those who are curious about the Japanese abacus.

MASANAO AOKI

University of California
Los Angeles, California

TABLE ERRATA

261.—LE CENTRE NATIONAL D'ÉTUDES DES TÉLÉCOMMUNICATIONS, *Tables des fonctions de Legendre associées*, Paris, 1952. [*MTAC*, v. 7, RMT **1110**, p. **178**; *MTAC*, v. 8, Table Erratum **233**, p. 28.]

The authors report the following errors:

p. 38, $P^0{}_{9,6}(\cos 17^0) = -0.144118$ *and not* -0.144072
p. 42, $P^0{}_{9,6}(\cos 17^0) = -0.261274$ *and not* -0.261234
p. 82, $P^1{}_{8,6}(\cos 17^0) = -4.053574$ *and not* -4.053695
p. 86, $P^1{}_{9,6}(\cos 17^0) = -3.47195$ *and not* -3.47219
p. 136, $P^2{}_{9,6}(\cos 17^0) = 49.29970$ *and not* $-49{,}30035$.

They further report that there are a number of instances of poor printing, which might lead to confusion. The list follows:

p. 17, $P_3{}^0(\cos 80^0) = -0.2473819$—the 8 is illegible
p. 18, $P^0{}_{3,9}(\cos 14^0) = 0.7338195$—the 9 is poorly printed and could be mistaken for an 8
p. 60, $P^1{}_{3,1}(\cos 13^0) = -1.3328662$—the 8 is poorly printed
p. 78, $P^1{}_{7,9}(\cos 31^0) = 2.068962$—the 6 is poorly printed and could be mistaken for an 8.

LOUIS ROBIN

Recherches Mathématiques
L'Ingénieur en Chef des Télécommunications
3 Avenue de la Republique
Paris, France

262.—CARL-ERIK FRÖBERG, *Hexadecimal Conversion Tables*, C. W. K. Gleerup, Lund, Sweden, 1957. [*MTAC*, Review **82**, v. 11, 1957, p. 208; *MTAC*, Table Erratum, v. 11, 1957, p. 309.]

On p. 10, *for* 0.38 30A3D 70A3D *4*0A3D *read* 0.38 30A3D 70A3D *7*0A3D.

B. ASKER