18 [F].—ROGER OSBORN, *Tables of All Primitive Roots of Odd Primes Less than 1000*, University of Texas Press, Austin, 1961, 70 p., 30 cm. Price $3.00.

This slim volume lists all 28,597 primitive roots of the 167 odd primes less than 1000. These tables were computed on an IBM 650. The program and running times are not indicated. The most extensive earlier table, as noted by the author, is due to Chebyshev and extends to $p = 353$.

There also is a small table of statistical information. Perhaps the most interesting column here lists the number of (positive) primitive roots less than $p/2$ for each prime p. Of the 87 primes $\equiv -1 \pmod 4$, eight have exactly one-half of their primitive roots less than $p/2$. The seven primes 223, 379, 463, 631, 691, 883, and 907 have more than one-half less than $p/2$. The remaining 72 primes have less than one-half there. The author associates this preponderance with the well-known fact that more than one-half of the quadratic residues of such primes lie in this interval.

For the primes $\equiv +1 \pmod 4$) this column is clearly redundant, since it is easily seen that if g is a primitive root for such a prime then so is $p - g$. For these primes the real interval of interest is $p/4 < g < 3p/4$. Since the quadratic non-residues are in excess here, one would expect the primitive roots to also be preponderantly in excess, since approximately three-fourths of all non-residues are primitive roots.

D. S.

19 [I, X].—D. S. MITRINOVIĆ & R. S. MITRINOVIĆ, *Sur les nombres de Stirling et les nombres de Bernoulli de l'ordre supérieur*, *Publ. Fac Élect. Univ. Belgrade* (Série: *Math. et Phys.*), No. 43, 1960, 64 p. (French with Serbian summary.)

The tables in this paper extend those given in previous papers, especially the three reviewed in *Mathematics of Computation*, v. 15, 1961, p. 107. The notation used is explained in that review.

Table I (p. 15–44) gives $(-)^m C_m^k$ for $k = 0(1)32$, $m = 33(1)50$, and for $k = 33(1)49$, $m = k + 1(1)50$,

Table II (p. 45–50) gives S_n^{n-m} for $m = 33(1)49$, $n = m + 1(1)50$, and also for $m = 50$, $n = 51$.

Table III (p. 51–63) gives S_n^{n-m} for $m = 1(1)3$, $n = 201(1)1000$.

The tables were computed on desk machines. Checks made by the authors were supplemented by comparison with Miksa's unpublished tables and by many-figure computations made in laboratories at Liverpool, Rome, and Munich. A bibliography of 26 items is given.

A. F.

20 [K].—B. M. BENNETT & P. HSU, *Significance Tests in a 2×2 Contingency Table: Further Extension of Finney-Latscha Tables*, February 1961. Deposited in UMT File.

These manuscript tables constitute an extension for $A = 21(1)30$ of tables prepared by Latscha for $A = 16(1)20$, and supersede the previous tables by the present authors for $A = 21(1)25$. (See Review 9, *Math. Comp.*, v. 15, 1961, p. 88–89.) The format and precision of those tables (four decimal places) is retained in this addendum.

J. W. W.

21 [K].—Colin R. Blyth & David W. Hutchinson, *Tables of Neyman Shortest Unbiased Confidence Intervals* (*a*) *for the Binomial Parameter* (*b*) *for the Poisson Parameter*, (reproduced from *Biometrika*, v. 47, p. 381–391, v. 48, p. 191–194, respectively) University Press, London, 1960, 16 p., 28 cm. Price 2s. 6d.

Anscombe [1] observed that exact confidence intervals for a parameter in the distribution function of a discrete random variable could be obtained by adding to the sample value, X, of the discrete variable a randomly drawn value, Y, from the rectangular distribution on $(0, 1)$. Eudey [2] has applied this idea in the case of the binomial parameter, p, to find the Neyman shortest unbiased confidence set. The present authors use Eudey's equations for a uniformly most powerful level $1\text{-}\alpha$ test of $p = p^*$ vs $p \neq p^*$ based on an X in a sample of n, which give the acceptance interval $a(p^*)$ determined by a value of Y in the form $n_0 + \gamma_0 \leqq X + Y \leqq n_1 + \gamma_1$ in which n_0 and n_1 are integers and $0 \leqq \gamma_0 \leqq 1$, $0 \leqq \gamma_1 \leqq 1$. These are solved for γ_0 and γ_1 in terms of n_0 and n_1 and the given X, n, and α. Then trial values of n_0 and n_1 are used until the resulting γ_0 and γ_1 are both on $(0, 1)$. The computation was carried out on the University of Illinois Digital Computer Laboratory's ILLIAC. The program used for arbitrary n, α prints out $n_0 + \gamma_0$, $n_1 + \gamma_1$ for any equally spaced set of p^* values. From these the Neyman shortest unbiased α-confidence set for p, $X + Y \in \alpha(p^*)$ can be read off to 2D. The tables give such 95% and 99% confidence intervals for p to 2D for $n = 2(1)24(2)50$ and $X + Y = 0(.1)5.5$ for $n \leqq 10$, $0(.1)1(.2)10$ for $11 \leqq n \leqq 19$, $0(.1)1(.2)6(.5)15(1)17$ for $20 \leqq n \leqq 32$, and $0(.2)2(.5)23(1)26$ for $34 \leqq n \leqq 50$. For n, $X + Y$ not tabled, one enters the table at n, $n + 1 - (X + Y)$ and takes the reflection about $p = \frac{1}{2}$ of the interval given.

Similar confidence intervals for the Poisson parameter, λ, were found by the same method. The table gives Neyman shortest unbiased 95% confidence intervals for λ to 1D for $X + Y = .01(.01).1(.02).2(.05)1(.1)10(.2)40(.5)55(1)59$ and to the nearest integer for $X + Y = 60(1)250$. For the same values of $X + Y$, 99% confidence intervals are given to 1D for $X + Y \leqq 54$ and to the nearest integer for $X + Y > 54$.

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1. F. J. Anscombe, "The validity of comparative experiments," *J. Roy Statist. Soc. Ser. A*. v. 111, 1948, p. 181–211.
2. M. W. Eudey, *On the Treatment of a Discontinuous Random Variable*, Technical Report No. 13 (1949), Statistical Laboratory, University of California, Berkeley.

22 [L].—M. I. Zhurina & L. N. Karamazina, *Tablit͡sy funkt͡siĭ Lezhandra* $P_{-1/2+i\tau}(x)$, Tom I (Tables of the Legendre functions $P_{-1/2+i\tau}(x)$, Vol. I), Izdatel'stov Akad. Nauk SSSR, Moscow, 1960, 320 p., 27 cm., 2700 copies. Price 34.50 (now 37.95) rubles.

This important volume belongs to the well-known series of Mathematical Tables of the Academy of Sciences of the USSR, and the tables were computed on the high-speed electronic calculator STRELA at the Computational Center of the Academy.

The Russian work has been concerned with the functions $P_{-1/2+i\tau}(x)$, where τ is real and $x > -1$. The functions are real, and satisfy the differential equation

$$(1 - x^2)u'' - 2xu' - (\tfrac{1}{4} + \tau^2)u = 0.$$

The functions occur in potential problems relating, for example, to cones and hyperboloids of revolution; they also occur in the Mehler-Fock inversion formulas [1]. The tables for $-1 < x < 1$ and $x > 1$ are given in Volumes I and II, respectively. The formulas given in the Introduction to Vol. I are limited to those which have some application in the range $-1 < x < 1$. The values were computed from

$$P_{-1/2+i\tau}(x) = F(\tfrac{1}{2} - i\tau, \tfrac{1}{2} + i\tau; 1; \tfrac{1}{2} - \tfrac{1}{2}x),$$

where $F(a,b;c;z)$ denotes the hypergeometric function, and were checked by differencing. The main table (pages 13–312) gives values of $P_{-1/2+i\tau}(x)$ to 7S for $\tau = 0(0.01)50$, $x = +0.9(-0.1)-0.9$, without differences. (It is stated that Vol. II, which the reviewer has not seen, gives values for $x = 1.1(0.1)2(0.2)5(0.5)10(10)60$.) The interval in τ has been made narrow because applications in mathematical physics frequently require integration with respect to τ. It is stated that interpolation in τ may be performed by the three-point Lagrange formula with an error not exceeding 1.6 final units; it may be added that such an error can occur in only a small part of the table. Interpolation in x is naturally more troublesome, even well away from a logarithmic singularity at $x = -1$.

An auxiliary table on pages 315–318 facilitates use of an asymptotic series for large τ; arc cos x and four coefficients which are functions of x are tabulated to 7D for $x = 0.99(-0.01)-0.90$, without differences. Values of the Bessel functions I_0 and I_1 are required to be available for use with the auxiliary table.

A useful bibliography of 16 items averages about one misprint per item in the five non-Russian titles, the most entertaining being MacRobert's well-known book on "Spherical Harmonies" and a paper by Barnes on "Veneralized Legendre Functions."

The reviewer differenced about a hundred values without finding any error. Assuming its accuracy, this must be reckoned a valuable table.

A. F.

1. A. Erdélyi et al, *Higher Transcendental Functions*, Vol. 1, McGraw-Hill, New York, 1953, p. 175.

23 [X].—A. Charnes & W. W. Cooper, *Management Models & Industrial Applications of Linear Programming*, v. 1, John Wiley & Sons, Inc., New York, 1961, xxiii + 471 p., 26 cm. Price $11.95.

This book is addressed to persons interested in the application of linear programming techniques to various aspects of management planning. Much of the material has been published previously by the authors in scattered journals and texts; however, this volume offers the advantage of a unified mathematical treatment of sundry topics in mathematical programming and managerial economics within the framework of adjacent-extreme-point techniques.

The earlier parts of this volume do not require mathematics beyond college algebra. The rudiments of linear programming theory and techniques are illustrated by means of simple numerical examples. An elementary machine loading problem is introduced to elucidate such concepts as linear model formulation, approximation of model types by scaling, the dual linear programming problem, and data accuracy and program sensitivity. The stepping-stone method for the classical Hitchcock transportation problem and transshipment problem are described at length. The procedure for dealing with degeneracy is also discussed. To explicate the concept of input-output analysis, a three-industry input-output model as an example of a "static, open Leontief model" is given. Feasible solutions are obtained by the Gauss elimination method.

With the exception of the transportation algorithm, a rigorous mathematical treatment of the foregoing topics are presented in the succeeding parts of this volume. Background material from the fields of matrix algebra, convex sets, and linear systems are developed and interpreted to provide an essentially self-contained account of the mathematics relevant to the managerial applications covered in the rest of the volume.

Considerable attention is devoted to Dantzig's simplex method for solving the general linear programming problem. The basic simplex algorithm is carefully explained and illustrated with the aid of numerical examples and geometrical interpretations. Additional by-products and interpretations are obtained, such as the extension of the simplex calculations for analyzing the effects of altering (a) the stipulations vector, (b) the coefficients of the objective function, and (c) the structural vectors. Also, the role of the simplex procedure as a tool for securing proofs of several important duality theorems in the field of linear inequalities is deftly portrayed.

The application of delegation models to managerial economics is first examined along the lines of T. C. Koopman's "activity analysis models." A major purpose of such models is the determination of rules which might be applied to guide the activities of a decentralized management organization. Koopman's formulation is reduced to a series of special linear programming problems and their duals. "Efficient" solutions are obtained by the "spiral" method. Koopman's concept of "efficiency" is then generalized to provide under certain circumstances more suitable criteria for managerial applications.

Linear programming approaches to statistical problems involving inequality relationships are delineated and applied to a problem of determining an executive-compensation formula for an industrial concern. Moreover, the techniques employed to solve this problem provide an introduction to the use of adjacent-extreme-point methods to a variety of nonlinear problems encountered in management planning. Modifications of simplex criteria and procedures are developed for the case where a functional subject to linear constraints may be decomposed by linear transformations into a sum of functionals involving only a single variable. The basic shortcoming of this approach is that, in general, only a local optimum is guaranteed.

A dynamic model for production scheduling at minimum cost when the costs are unknown is solved by means of "surrogate" techniques and "subhorizon" methods. Optimizing rules are enumerated and expounded for solving an actual example for which these methods were first devised. This is followed by a proof of the optimizing properties of the rules. The effects of introducing costs, such as inventory charges, and additional constraints, such as storage limitations, are touched upon from the standpoint of possible variations in the length of sub-horizons. A generalized approach to this class of problem is explored via the Kuhn-Tucker theorem for nonlinear programming.

The "classical" models of linear programming are presented with commendable clarity. Moreover, the adaptation of linear programming methods for solving non-linear types of management problems is aptly demonstrated. However, this reviewer's enthusiasm was tempered by the fact that the present edition abounds with errors resulting from an apparent cursory attempt at editing and proofreading. This reviewer recommends that the publishers prepare an errata sheet; otherwise, the intolerable number of typographical errors will vitiate the intrinsic merits of this book as a textbook and reference.

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24 [X].—Roman Jakobson, Editor, *Proceedings of Symposia in Applied Mathematics*, Vol. XII, "Structure of Language and its Mathematical Aspects," American Mathematical Society, Providence, 1961, vi + 279 p., 26 cm. Price $7.80.

Sponsored by the American Mathematical Society, the Association for Symbolic Logic, and the Linguistic Society of America, and cosponsored by the Institute for Defense Analyses under an Office of Naval Research contract, the symposium, held in April, 1960, included the following papers:

W. V. Quine	Logic as a Source of Syntactical Insights
Noam Chomsky	On the Notion "Rule of Grammar"
Hilary Putnam	Some Issues in the Theory of Grammar
Henry Hiż	Congrammaticality, Batteries of Transformations and Grammatical Categories
Nelson Goodman	Graphs for Linguistics
Haskell B. Curry	Some Logical Aspects of Grammatical Structure
Yuen Ren Chao	Graphic and Phonetic Aspects of Linguistic and Mathematical Symbols
Murray Eden	On the Formalization of Handwriting
Morris Halle	On the Role of Simplicity in Linguistic Descriptions
Robert Abernathy	The Problem of Linguistic Equivalence
Hans G. Herzberger	The Joints of English
Anthony G. Oettinger	Automatic Syntactic Analysis and the Pushdown Store
Victor H. Yngve	The Depth Hypothesis
Gordon E. Peterson and Frank Harary	Foundations in Phonemic Theory
Joachim Lambek	On the Calculus of Syntactic Types
H. A. Gleason, Jr.	Genetic Relationship Among Languages
Benoit Mandelbrot	On the Theory of Word Frequencies and on Related Markovian Models of Discourse
Charles F. Hockett	Grammar for the Hearer
Rulon Wells	A Measure of Subjective Information
Roman Jakobson	Linguistics and Communication Theory

Some of the authors are concerned with preformal questions, i.e., with a discursive characterization of the substance of language; Quine, Putnam, Chao, Herzberger, and Jakobson seem to have such interests. Others are fully engaged with the construction of formal systems: Chomsky, Hiż, Curry, Halle, Abernathy, Peterson and Harary, Lambek, Mandelbrot, and Wells. Oettinger, Yngve, and Hockett aim at description of linguistic processors—natural or artificial—rather than at characterizations of language, although all three have formalisms to display. Eden, working on handwriting, might be placed with one of the latter two groups. Goodman's contribution is the exposition of a branch of mathematics in its potential application to linguistic theory. Gleason shows the application of classification theory to a major branch of linguistics, the tracing of historical connections among languages.

A cursory inspection of this volume would suggest that the "structure of language" is just its grammatical—or, more narrowly, syntactic—structure. Mandelbrot objects to the identification of "linguistics" and "grammar" (pp. 211–214), but mathematical formalization of linguistic theory is going forward more rapidly in syntax than in any other area, and it is, as Jakobson remarks (p. vi), mathematical logic and the theory of recursive functions in particular that is being applied. Mandelbrot seems to agree with his opponents that "statistical" and "grammatical" models are "contradictory." He supposes that they must remain so; a different possibility is that grammatical models will furnish a structure on which statistical models can be developed. Grammar in any case is not the whole of linguistics, and problems like Gleason's will probably be brought to computing centers more often in the future.

Computational linguistics has been hampered by lack of sufficient and sufficiently sound publications in mathematical linguistics; this volume should be studied by any linguist or mathematician who proposes to program syntactic operations, whether for research purposes or in connection with such applications as machine translation.

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25 [Z].—Donald P. Eckman, Editor, *Systems: Research & Design*, John Wiley & Sons, Inc., New York, 1961, xiii + 310 p., 23 cm. Price $8.50.

This book is the Proceedings of the First Systems Symposium at Case Institute of Technology. It contains a Foreword, a Preface, and fourteen papers concerning systems research and systems design. The fourteen papers vary in style, most noticeably with regard to bibliographic reference. Some are simply advice from the author without reference to other work, others have extensive bibliographies. Only one pertains directly to the mathematics of computation, "A problem in the design of large-scale digital computer systems" by R. J. Nelson. This paper is devoted almost entirely to the problem of designing a machine which would be efficient in selecting the largest number of a set and (by implication) in other sorting problems. No specific design is arrived at, but a facility for scanning a region of the memory is suggested; the ideas may mislead some readers if they are unfamiliar with threshold search commands such as that of the Control Data Corporation 1604 computer and with the engineering details of comparison circuits.

Other papers have implications connected with the mathematics of computation, as would be expected in any current book on large systems. Thus in the Foreword, Simon Ramo remarks that "it could be said that systems engineering in today's sense became possible only with the introduction of the large digital computer." However, the papers in this volume contribute few direct suggestions concerning this use, and concern themselves largely with other general and specific aspects of systems engineering.

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26 [Z].—Daniel D. McCracken, *A Guide to FORTRAN Programming*, John Wiley & Sons, Inc., New York, 1961, viii + 88 p., 28 cm. Price $2.95.

The usefulness of Fortran as an automatic programming system available on many different computers has prompted Dr. McCracken to publish this guide. It is addressed to people who have no programming experience but have a requirement to accomplish scientific computation or wish to get some appreciation of how this can be done.

The guide is developed pedagogically, with numerous examples, and includes a set of detailed case studies which provide examples from several fields of effort. These case studies illustrate the essential features of Fortran and suggest the range of its applicability.

An appendix summarizes the characteristics of a number of Fortran systems that have been established for different computers.

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27 [Z].—Francis J. Murray, *Mathematical Machines*, Vol. 1 and 2, Columbia University Press, New York, 1961. V. 1, vii + 300 p., 26 cm. Price $12.50. V. 2, vii + 365 p., 26 cm. Price $17.50.

Volume one of Professor Murray's two-volume work on mathematical machines, is concerned with digital computers. There are two parts in Volume 1: part I on desk calculators and punched card machines, and part II on automatic sequence digital calculators. These digital devices are presented in the order of increasing competence and complexity.

In part I, there are eight chapters. The first four chapters describe desk calculators, from the basic idea of register and counter to the description of many commercial automatic calculators. Chapter 5 covers electrical counters and accumulators. Punched card machines are presented in Chapters 6 and 7, and sequence calculators such as calculating punch and electronic calculator in Chapter 8.

Part II consists of ten chapters. The first four chapters describe the logic aspect of the computer as well as digital arithmetic. Chapter 5 is a general discussion on the use of Boolean analysis. Chapter 6 is concerned with circuit elements. The programming aspects are covered in Chapters 7, 8, and 9. Chapter 10 is a very brief survey of digital computers.

In this volume, the author succeeded in many cases in bringing out the principles and fundamental ideas. An example is the exposition on desk calculators. Although the material is mostly descriptive, it will serve a useful purpose as a general reference.

Volume two of Professor Murray's work on mathematical machines presents the subject of analog devices. There are three parts: part III on continuous computers, part IV on true analogs, and part V on mathematical instruments.

Part III consists of fifteen chapters. After a brief introduction in Chapter 1, Professor Murray describes mechanical adders, multipliers, dividers, and other mechanical components in Chapter 2. Cams, gears, and their computing applications constitute Chapter 3. This is followed by an excellent presentation on mechanical integrators, differentiators, and amplifiers in Chapter 4. Chapter 5 is a review of circuit theory. Computation by using potentiometers and condensers are described in Chapter 6, vacuum tube amplifiers in Chapter 7, electromechanical components of D'Arsonval movement, watt-hour meters, and synchros in Chapter 8, electrical multipliers including time division multipliers, strain gauge multipliers, step multipliers, cathode ray multipliers in Chapter 9, and function generation by using mechanical, electromechanical and electronic means in Chapter 10. Chapters 11 through 13 describe equation solution: linear equations in Chapter 11, harmonic analysis and polynomial equations in Chapter 12, differential equations in Chapter 13, and error analysis in Chapter 14. Chapter 15, the last chapter of this part, discusses the use of digital check solutions obtained by using numerical methods when the analog solution has narrowed down the range of parameters.

Part IV, consisting of nine chapters, presents the idea of true analogs. True analogs are direct analogies on which measurements can be taken more conveniently or more economically than the analog devices described in part III. The author examines the theory of true analogs and includes descriptions of dimension theory, models, and principles of spatial relationships. True analogs that are described include the use of electrolytic tanks, electrically conductive sheets, stretched membranes, photoelastic models, and electromechanical analogies.

Part V consists of five chapters. It deals with mathematical instruments that operate on data in a specified form and perform a few mathematical operations. These devices include slide rule, plotting devices, planimeters, integrometers, integraphs, and other geometrical and trigonometrical devices. This part is rather unique.

This volume again emphasizes principles. A significant portion describes mechanical analog devices. The treatment of analog devices in volume two is more extensive than that of digital computers.

As mentioned in the book, this work was sponsored by the Office of Naval Research. These two volumes are a contribution to the study of mathematical machines, and Columbia University Press deserves credit for an excellent printing job.

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28 [Z].—E. L. WILLEY, MARION TRIBE, A. D'AGAPEYEFF, B. J. GIBBENS & MICHELLE CLARK, *Some Commercial Autocodes*, Academic Press, Inc., New York, 1961, vii + 53 p., 23 cm. Price $2.50.

Some Commerical Autocodes is a study of nine programming languages applicable to commercial data processing problems, compiled in a tabular form by language elements. The study is based upon information available in December 1960 and does not represent the final specifications for some languages which have been, or are being, implemented for the various computers.

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