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# Mathematics of Computation

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high-speed calculators and other aids to computation



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The editors of *Mathematics of Computation* and the *Siam Review* have agreed to exchange the printing of the tables of contents of these two journals. This permits the readers of each journal to keep informed concerning the papers appearing in the other journal. The Table of Contents of the current issue of the *Siam Review* follows:

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