REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

65[D].—Jean Peters, *Eight-Place Tables of Trigonometric Functions for Every Second of Arc*, Chelsea Publishing Company, Bronx, New York, 1963, xi + 954 p., 29 cm. Price $18.50.

The title listed above is that on the title page; the back of the binding merely has "Trigonometric Tables—Peters".

The main table here is a 900-page table giving the sine, cosine, tangent, and cotangent for every second of arc from 0° to 45°. The first three functions are given to 8D, while the cotangent has the same number of significant figures as the corresponding value of the tangent. Bounds for first differences are listed for each minute of arc. Peters' original (1939) table [1] was followed by two war-time photographic reprints, and this volume consists of a third photographic reprint. The previous editions have been reviewed at length in [2]. Peters' table is considered to be the standard (i.e., the best) 8-place trigonometric table [3].

The two (rather obvious) errors in [1] on pages 54 and 585, noted in [4], have not been corrected. Nor has the poorly printed digit 6 on page 783, that already appeared in the previous American reprint, and which is also noted in [4]. The printing here has the expected variation in digit blackness but is generally very good. The paper is of a fine quality.

The "appendix" contains reproductions of Peters' 1911 twenty-one place tables [5]. Specifically, Table II lists sin α and cos α to 21D for $\alpha = 0°0'(10')45°0'$, and Table III gives the same quantities for $\alpha = 0'0''(1'')10'0''$ together with first, second, and third differences. With these there is included an explanation, in English, for computing sin α and cos α to 20D from these two tables by the use of interpolation and addition formulas. These two tables were *not* included in the three earlier editions reviewed [2].

There also are three "supplementary" tables: nM to 21D for $n = 1(1)100$ where M is the Modulus $0.43429 \cdots$; likewise there is given n/M; and finally the values of n seconds of arc, expressed in radians, to 21D for $n = 1(1)100$.

For biographical remarks concerning Johann Theodor Peters, "the greatest table-maker of all time," see [4, p. 889] and [2, p. 168–169].

D. S.

1. J. Peters, *Achtstellige Tafel der trigonometrischen Funktionen für jede Sexagesimalsekunde des Quadraten*, Reichsamt für Landesaufnahme, Berlin, 1939.
2. R. C. Archibald, RMT 78, **Notes 5** and **6**, RMT **128**, *Math. Comp.*, v. 1, 1943–1945, p. 11–12, 64–65, 147–148.
3. Fletcher, Miller, Rosenhead & Comrie, *An Index of Mathematical Tables*, second edition, Addison-Wesley, Reading, Massachusetts, 1962, Vol. 1, p. 178–179.
4. *Ibid*, Vol. 2, p. 890.
5. J. Peters, *Einundzwanzigstellige Werte der Funktionen Sinus und Cosinus*, Reimer, Berlin, 1911.

66[F].—V. L. Gardiner, R. B. Lazarus & P. R. Stein, *Tables of Solutions of the Diophantine Equation* $x^3 + y^3 = z^3 - d$, Los Alamos Scientific Laboratory, Los Alamos, New Mexico, 1963, 69 p., 28 cm. Copy deposited in the UMT file.

These three tables are described in this issue of *Mathematics of Computation* [1].

The brief introduction repeats the specifications concerning these tables. They were computed on an IBM STRETCH and the MANIAC II. The output was placed on cards and paper tape prior to printing in the present form.

D. S.

1. V. L. GARDINER, R. B. LAZARUS & P. R. STEIN, "Solutions of the diophantine equation $x^3 + y^3 = z^3 - d$," *Math. Comp.*, v. 18, 1964, p. 408–413.

67[F].—D. H. LEHMER, "On a problem of Störmer," *Illinois J. Math.*, v. 8, 1964, p. 69–79, Tables I, II, III.

The problem of the title consists of finding all pairs of integers N, $N-1$ such that both numbers have as their prime divisors only primes contained in a preassigned set. For instance, if the set is that of the six smallest primes, an example is

$$N = 123201 = 3^6 \cdot 13^2, \qquad N - 1 = 123200 = 2^6 \cdot 5^2 \cdot 7 \cdot 11.$$

In Table I Lehmer gives *all* 869 pairs where the set consists of the 13 smallest primes, 2 through 41. Embodied in this are also all solutions where the set is that of the t smallest primes, with $t = 1(1)13$. Factorizations of N and $N - 1$ are also given if $N > 10^5$; for smaller N the author suggests the use of existing factor tables.

In Tables II and III he gives the analogous pairs of *odd* numbers N, $N - 2$ and N, $N - 4$, respectively, for the set of the first 11 primes.

The text of the article gives the underlying theory and mentions several number-theoretic applications.

D. S.

68[G].—WOLFGANG KRULL, *Elementare und klassische Algebra, vom modernen Standpunkt*, Band I, Walter De Gruyter & Co., Berlin, 1963, 148 + 31 p., 16 cm. Price 3.60 DM.

This is the third edition of a Göschen book which first appeared under the title *Elementare Algebra vom höheren Standpunkt* in 1939. The second edition appeared under the present title in 1952. Since the book carries the volume number I, it seems that a sequel is planned. The book deals with polynomial equations and does include some chapters on their solutions, as is customary in books of this type; it also includes an account of the Sturm theory. However, the first edition included a whole chapter on numerical calculation of the roots. The book is written by an expert who had helped shape the modern treatment of this subject. "Modern" means here, of course, "abstract"; hence, the book is not of immediate concern for the readers of this journal. The day may come when "modern" may mean "numerical".

O. TAUSSKY

California Institute of Technology
Pasadena, California

69[G].—FRANK M. STEWART, *Introduction to Linear Algebra*, D. Van Nostrand Co., Inc., Princeton, New Jersey, 1963, xv + 281 p. Price $7.50

This book is written in the belief that "linear algebra provides an ideal introduction to the conceptual, axiomatic methods characteristic of mathematics today". Accordingly, the symbolism, and to some extent the phraseology, is borrowed from functional analysis. Nevertheless, the book is intended to be introductory, so that only the real and the complex fields come into consideration.

The author endeavors to lead the reader by the hand and assist him over every possible rough spot. Thus, proofs and explanations are made at length and, in addition, several appendices are added, explaining what is meant by equations and identities, by a function, by a set, by a proof, etc. Unfortunately, the first chapter, which could be omitted, may very possibly frighten away a number of prospective readers. The chapter is intended to illustrate the power of analytical methods by proving that a triangle is isosceles if the bisectors of two of its angles are equal. After stating the theorem and commenting on its difficulty, the author promises to "use new concepts to abbreviate the dreadful computations a little".

The chapter headings indicate the extent of the coverage: Introduction; The Plane; Linear Dependence, Span, Dimension, Bases, Subspaces; Linear Transformations; The Dual Space, Multilinear Forms, Determinants; Determinants: A Traditional Treatment; Inner Product Spaces. In the final chapter there appear three versions of the Spectral Theorem for symmetric transformations, and the first version is stated for unitary transformations in one of the exercises.

For self-instruction the book should do very well; for class use, the instructor can be free to provide supplementary material, and to give attention to the rather long list of problems.

A. S. H.

70[G, H, J, L, X].—A. N. KHOVANSKII, *The Application of Continued Fractions and their Generalizations to Problems in Approximation Theory*, Noordhoff, Groningen, 1963, xii + 212 p., 22 cm. Translated by PETER WYNN. Price $7.85.

This book confines itself to analytic continued fractions and, as implied in the title, the orientation is toward practical computation.

The long Chapter I is concerned first with transformations from one continued fraction to another (including such operations as contraction), and between continued fractions and series. Next are presented several sections on convergence theory and tests.

The equally long second chapter develops many known analytic continued fractions (binomials, logarithm, tangent, hypergeometric, exponential integral, etc.) primarily by the use of Lagrange's method as applied to Riccati equations.

Chapter III presents some miscellaneous methods including a use of Obreschkoff's Formula to obtain certain rational approximations directly in closed form, and the application of the elegant Viskovatoff Algorithm to the difficult cases sin x, cosh x, Stirling's series, etc.

Finally the last chapter evaluates the roots of algebraic equations by matrix methods. Since these linear transformations are analogous to the recursion formulas for continued fraction convergents, these sequences are called *generalized* continued fractions.

The book is a useful compendium of these techniques and is especially valuable, since relatively little is available in English on the subject. A nice feature is the frequent inclusion of historical references, from which one learns the origin of names, notation, and formulas.

There are a fair number of typographical errors, and the reader must therefore proceed with some caution, especially where misplaced minus signs change the meaning entirely. Also to be guarded against is a terminological discrepancy. For

$$b_0 + \frac{1}{b_1 +} \frac{1}{b_2 +} \frac{1}{b_3 +} \cdots$$

most books refer to the b_i as *partial quotients*, while here (page 2) this is the name for the quantities $1/b_i$.

The forthcoming translation of Khintchine's *Continued Fractions* is also by Wynn and should complement the present volume for the arithmetical and theoretical aspects of continued fractions.

D. S.

71[G, X].—D. K. Faddeev & V. N. Faddeeva, *Computational Methods of Linear Algebra*, W. H. Freeman & Company, San Francisco, California, 1963, xi + 621 p., 24 cm. Price $11.50.

This volume should not be confused with an earlier one having the same title, and written by the second of the present authors. The earlier volume appeared in the USSR in 1950, and a translation, published by Dover, appeared in 1959 and was reviewed briefly in this periodical [v. 15, 1961, p. 201, RMT **36**].

The Russian edition of the volume here translated appeared in 1960, and bears little resemblance to the earlier one. It is nearly three times as large; it contains an extensive bibliography (40 pages as compared with two in the earlier one); and the original printing of 10,150 copies was evidently soon exhausted, since a second edition, somewhat larger (734 pages as compared with 656) appeared in 1963 in a printing of 12,000 copies. Each edition was, at the time of its appearance, by far the most complete and up-to-date treatment of the subject in print. Perhaps the most serious criticism that could be made of either is, curiously, the scant use of norms, in spite of the fact that the 1950 volume had already called attention to their usefulness. Since that time this reviewer, A. M. Ostrowski, F. L. Bauer, J. H. Wilkinson, and others, have developed the theory extensively and made numerous applications, but little or no account of this work is taken in the present volume or its successor. Otherwise, however, the first chapter gives a fairly complete and self-contained development of the theory of matrices so far as it is relevant to computational problems. Thereafter, there is discussion with numerical illustrations of virtually every known method of solving the standard problems of finding inverses, solutions, and characteristic roots and vectors.

Regretfully, though, it must be said that the translation by no means does justice to the original. A first glance at the bibliography arouses apprehensions that are, alas, fulfilled by an examination of the text proper. In the original, names are listed alphabetically according to the Russian spellings. In the translation, Aitken is properly transported from the end of the alphabet to the beginning, but the first four names on the first page of references in the translation are Abramov, Azbelev, Albert, and Aitken, in that order. Curiously, starting with Rushton, who follows Růžička, the ordering is nearly correct (there is one inversion, and the misspelled Scherman is placed properly for that spelling). For several pages at the start diacritical marks are completely omitted from French and German titles, then suddenly they appear. In the original, the normal spelling of non-Russian names is given, sometimes incorrectly, in parentheses following the Russian spelling. The errors are retained in the translation, and at least one new one added: Joung for Young, Scherman for Sherman, an incorrect initial for Wilkinson, two distinct misspellings of the name of this reviewer. A "supplementary" list in the original, presumably added after the main one had been compiled, remains separate in the translation.

In the table of contents, there are nine cases of poor or even misleading phraseology: "gradients of functionals" becomes "functional gradients"; "condition of matrices" becomes "conditioned matrices"; "resolution" into factors becomes "expansion" into factors; a method of "supplementation" (*popolnenija*, what the reviewer calls a method of modification) becomes the "reinforcement" method; "some methods of conjugate directions" is translated clumsily as "some conjugate directions methods." These are perhaps the worst.

Since the translator is not (presumably) a mathematician or a numerical analyst, it is not surprising that, for example, "deflation" becomes "exhaustion," and in a footnote it is remarked that occasional nonstandard terminology should not cause trouble for readers who are "literate enough." This is true, but it can cause trouble for beginners, who could otherwise find in this an excellent introduction to the subject. And mere nonstandard technical terminology is not the only fault. "Suitably generalized" becomes "throughly [*sic*] reviewed" (p. 32); "terms" becomes "elements" (p. 61); plurals become singular and prepositions are omitted; reference to an example in "paragraph 7" is said to be on p. 80 (actually it is on p. 55); symbols are omitted; subscripts are raised; definite articles improperly supplied (in Theorem 7.5). These slips were noted in scarcely more than a casual scanning of a small portion of the whole; in this last list they represent purely mechanical faults in translating and in proofreading, and there were no corresponding slips in the original to have led to them.

To those who are "literate enough," in the subject but not in Russian, the translation can be highly recommended, because the faults in translation should cause only occasional annoyance. The lost symbols can be supplied after a little thought; the definite articles in Theorem 7.5 can be recognized as not belonging. The material is abundant, and, with proper corrections, the presentation very good. Those who are less literate should be warned to read with care.

A. S. H.

72[G, X, Z].—ROBERT D. LARSSON, *Equalities and Approximations with "FORTRAN" Programming*, John Wiley & Sons, New York, 1963, x + 158 p., 24 cm. Price $5.50.

This is a book written in response to the needs of the high school teacher who wishes to enrich the curriculum for high-ability students. It also might find use in the programs of many junior colleges. Basically this book contains topics from linear algebra together with some topics in elementary approximation theory. The latter serves as a vehicle for introducing the basic ideas of integral calculus.

The prerequisites for this book are elementary algebra and trigonometry. Chapter I introduces the idea of a group and its properties. In Chapter II matrices are introduced and their properties discussed. Matrix inversion and the solution of linear equations are combined through the use of the Gauss-Jordan elimination method. In Chapter IV rings, integral domains, and fields are introduced and illustrated. Chapter V considers inequalities and the ideas of this chapter are applied to find upper and lower bounds to simple problems of area estimation in Chapter VI. Finally some iteration techniques and polynomial interpolations are considered in Chapter VII.

An unusual feature of the book is the introduction of FORTRAN programming in Chapter III. In the following chapters many problems are solved by means of computer programs. The sections on programming appear adequate, but could well be supplemented by additional material. Any teacher who is unfamiliar with FORTRAN programming cannot depend on this text for all of the answers.

The reviewer feels that this book is carefully done and the mathematics is rigorously presented. Occasional theorems of a more difficult nature are stated without proof. This book may well find considerable use in assisting teachers confronted with rapidly changing curricula in our nation's high schools and junior colleges.

Richard C. Roberts

U. S. Naval Ordnance Laboratory
White Oak, Silver Spring, Maryland

73[I].—A. O. Gelfond, *Calcul des Différences Finies*, Dunod, Paris, 1963, x + 378 p., 25 cm. Price 58 F.

This is a translation into French of A. O. Gelfond's *Ischislenie Konechnikh Ratznostei*, Moscow, 1952, and I, for one, am glad to have the work of this great analyst in a more accessible tongue. The French edition embodies corrections and some emendations. For example, Chapter I now includes a section entitled "Interpolation and the Moment Problem in the Complex Plane".

As one looks at the chapter titles, one is apt to get the impression that this book contains only "standard material". This is not so; there are new and interesting results throughout, many of them due to the author. Bernard Berenson once said that a book is worth its place on the shelf if it contains *one* thing that no other book has. Gelfond's book passes Berenson's test by a wide, wide margin, and those of us who like interpolatory function theory will enjoy browsing and studying his work in this new edition.

Table of contents is as follows: Chapter 1, Interpolation; Chapter 2, Newton Series; Chapter 3, Construction of an Entire Function given certain Interpolatory Information; Chapter 4, The Summing of Functions; Bernoulli Numbers and Polynomials; and Chapter 5, Finite Difference Equations.

P. J. D.

74[K].—D. J. Finney, R. Latscha, B. M. Bennett & P. Hsu, *Tables for Testing Significance in a 2 × 2 Contingency Table,* Cambridge University Press, American Branch, New York, 1963, vi + 102 p., 27 cm. Price \$3.25.

These tables are used for testing the significance of deviations from proportionality in a 2 × 2 contingency table

	With Attribute	Without Attribute	Total
Series I	a	$A - a$	A
Series II	b	$B - b$	B
	r	$N - r$	N

with fixed marginal totals $A \geqq B$, and $a/A \geqq b/B$.

The present volume is the result of computations carried out by several authors over a number of years. Finney [1] gave a table of the one-tail significance levels of b, that is, of the largest integers b_p such that $\text{Prob}\{b \leqq b_p\} \leqq p$, for $p = .05, .025, .01$, and $.005$ and all permissible combinations of a, A, and B with $A = 3(1)15$, $B \leqq A$, together with the corresponding values of actual tail probability $\text{Prob}\{b \leqq b_p\}$ to 3D. Latscha [2] extended the Finney table for $A = 16(1)20$. B. M. Bennett and P. Hsu have extended the full computations in the Finney format for $21 \leqq A \leqq 45$, $B \leqq A$, adding a fourth decimal place in the exact probabilities. (See *Math. Comp.*, v. 16, 1962, p. 252–253, RMT **20**; ibid., p. 503, RMT **58**.)

Table 1 in the present volume includes the Finney and Latscha tables, with known errors corrected, and the full Bennett-Hsu tables up to $B \leqq A \leqq 30$. As in the original Finney format, the listed significant value of b is such that $\text{Prob}\{b \leqq b_p\}$ does not exceed the nominal significance level p, and is sometimes much less than the nominal level, because of the discrete nature of the probability distribution. The inclusion of the exact probability is therefore useful for the practical man who may wish to use that value of b for which $\text{Pr}\{b \leqq b_p\}$ is closest to p. For example, for $A = 14$, $B = 11$, and $a = 14$, Table 1 gives $b_{.025} = 6$ with $\text{Prob}\{b \leqq b_{.025}\} = 0.009$, whereas $b_{.05} = 7$ with $\text{Prob}\{b \leqq b_{.05}\} = 0.026$. Consequently, by taking 7 as the critical value of b, the test will be conducted more nearly at the 0.025 level of significance.

Table 2 gives Bennett and Hsu's values for $31 \leqq A \leqq 40$, $B \leqq A$, in abridged form, i.e. gives only the two significant values $b_{.05}$ and $b_{.01}$, and not the exact probabilities. The whole of the Bennett and Hsu table for $A = 21(1)45$, $B \leqq A$, has been deposited in the UMT file.

Mary G. Natrella

National Bureau of Standards
Washington, D. C.

1. D. J. Finney, "The Fisher-Yates test of Significance in 2 × 2 contingency tables," *Biometrika*, v. 35, Parts 1 and 2, May 1948, pp. 145–156. [*MTAC*, v. 3, 1948, p. 359]

2. R. Latscha, "Tests of significance in a 2 × 2 contingency table: Extension of Finney's table," *Biometrika*, v. 40, Parts 1 and 2, June 1953, p. 74-86. [*MTAC*, v. 8, 1954, p. 157]

75[K].—Sol Weintraub, *Tables of the Cumulative Binomial Probability Distribution for Small Values of p*, The Free Press of Glencoe, New York, 1963, xxix + 818 p., 28 cm. Price $19.95.

This book of tables gives to 10 decimals the cumulative binomial sums

$$E(n, r, p) = \sum_{i=r}^{n} \binom{n}{i} p^i (1 - p)^{n-i}$$

for $p = .00001, .0001(.0001).001(.001).100$, $n = 1(1)100$, $r = 1(1)n$. Through the relation $E(n, r, p) = 1 - E(n, n - r + 1, 1 - p)$ one may also readily obtain the cumulative sums for $p \geqq .9$ from this volume. The tables, which are arranged in order of increasing n, and within each n in order of increasing p, were reproduced directly from computer print-outs. Previously published binomial tables generally have a p-mesh of .01, and give either 5 or 7 decimals. Anyone, therefore, who frequently has need of binomial tables for $p \leqq .1$ or $p \geqq .9$ will find this volume a valuable addition to the existing tables. The necessity for interpolation can now often be eliminated.

This reviewer spot checked certain entries for $n = 15$ and 25. Of 64 values checked, all were found to be correct to 10 decimals. A comparison of all possible entries in the present tables (471 in number) for $n = 45, 49$ with the 7D tables of the National Bureau of Standards [1] and for $n = 97$ with the 7D tables of the Army Ordnance Corps [2] showed that the author's tables agree with the earlier tables except in two instances. Here, a further check proved certain NBS entries to be incorrect.

The introductory material, which contains examples of applications, notes on the computer program, and an error analysis, suffers, unfortunately, from a number of defects. Nowhere, for example, does the author state explicitly and concisely what he has tabulated and what range of arguments is covered by his tables. One would expect such a statement to appear either on the "contents" page or early in the introduction.

The error analysis attempts to prove that the upper bound of the error is sufficient to insure 10-decimal accuracy. But (as pointed out to me by H. Oser) in the proof of case B on page xxi, it is assumed that $e(n, r, p) = \binom{n}{r} p^r(1 - p)^{n-r}$ as a function of r always attains a maximum M at a point $r = m$ such that there exist $r_1 > m$, $r_2 < m$ with $e(n, r_1, p) < M$ and $e(n, r_2, p) < M$. It is also assumed that for $r + 1 > m$ there exists an integer $k < m$ such that $e(n, k, p) = e(n, r + 1, p)$. These assumptions are incorrect and render the proof invalid. So the conclusion that the error in each tabulated value is less than 7×10^{-11} may not hold. If it does, this is still not a small enough bound to guarantee the 10-decimal accuracy claimed.

The example on pages xiii–xiv ends with a disappointing error. It is stated that "we . . . find that the chances for success are now approximately a little more than 1 in 100," whereas the correct chances for success in this example are *less* than .01.

The statement on page xxiii that formula (1) on that page (the formula for $E(n, r, p)$ given above) is equal to $(p + q)^n$, where $q = 1 - p$, is true only if one takes $r = 0$. This the author neglected to include.

In short, for material on the cumulative binomial distribution and its applications, one will do much better to consult the introductions of previously published tables than that of the present work. Particularly noteworthy for a full discussion of applications is the Harvard University Computation Laboratory's publication of binomial tables [3].

Roy H. Wampler

National Bureau of Standards
Washington, D. C.

1. National Bureau of Standards, *Tables of the Binomial Probability Distribution*, Applied Mathematics Series, No. 6, U. S. Government Printing Office, Washington, D.C., 1950.
2. L. E. Simon & F. E. Grubbs, *Tables of the Cumulative Binomial Probabilities*, Ballistic Research Laboratories, Ordnance Corps Pamphlet ORDP 20-1, Aberdeen Proving Ground, Md., 1952.
3. Harvard University, Computation Laboratory, *Annals*, v. 35, *Tables of the Cumulative Binomial Probability Distribution*, Harvard University Press, Cambridge, Mass., 1955.

76[K, X].—RICHARD BELLMAN (Editor), *Mathematical Optimization Techniques*, University of California Press, Berkeley and Los Angeles, California, 1963, xii + 346 p., 23 cm. Price $8.50.

The papers appearing in this volume were presented at the Symposium on Mathematical Optimization Techniques held in Santa Monica, California, October 18–20, 1960. The first four chapters by Miele, Dergabedian and Ten Dyke, Breakwell, and Dreyfus concern themselves with optimizing performance of aircraft, rockets, and problems of guidance. In Chapter 5 Parzen describes a new approach to the synthesis of optimal smoothing and prediction systems. Adaptive matched filters are discussed by Kailath in Chapter 6 and statistical communication theory by Middleton in Chapter 7. Hall in Chapter 8 presents a theory of minimum-bias estimators analogous to minimum-risk theory and points to the need for some compromise in design between bias and risk. Optimal replacement rules based on periodic inspection are considered by Derman in Chapter 9. The simplex method of linear programming and its relation to theory is examined by Tucker in Chapter 10. Wolfe, in Chapter 11, surveys computational procedures of nonlinear programming. Kruskal presents in Chapter 12 a theorem on the number of simplices in a complex. Chapter 13 consists of a discussion by Prager of optimal structural design based on plastic analysis. Recent developments in the theory of experimental design are outlined by Elfving in Chapter 14. Generalities on automation and control in the Soviet Union are given by the reviewer in Chapter 15. In Chapter 16 Kalman discusses the problem of optimal control from the Hamiltonian point of view. In the last chapter Bellman formulates the making of mathematical models as an adaptive control process.

This impressive volume is an excellent source of information on and a guide to recent developments in optimization. It covers with authority a wide range of topics.

JOSEPH P. LASALLE

RIAS
7212 Bellona Avenue
Baltimore, Maryland

77[K, Z].—ROBERT D. PHILLIPS, *Tables Useful in Statistics and Information Theory*, Federal Systems Division, International Business Machines Corporation, Rockville, Maryland, 5 November 1963, iii + 174 p. (spiral bound).

The four tables comprising the body of this report are intended for use in statistics and information theory.

Table I gives to 5D the channel capacity in bits and the associated maximizing input probabilities for a general binary channel. If the binary channel is characterized by the noise matrix (conditional probabilities)

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

then the capacity C and input probability p_1^* are tabulated for $p_{11} = 0.00000$, 0.00001, 0.0001, 0.0005, 0.001, 0.005, 0.01(0.01)0.50, and $p_{21} = 0.00001$, 0.00005, 0.0001, 0.0005, 0.001, 0.005, 0.01(0.01)0.99, 0.995, 0.999, 0.9995, 0.9999, 0.99995, provided $p_{11} \neq p_{21}$. This table represents an extension of one by Sakaguchi [1].

Table II consists of 5D values of $-\ln p$, $-p\ln p$, $-p\ln p - (1-p)\ln(1-p)$, $-(1-p)\ln(1-p)$, and $-\ln(1-p)$ for $p = 0.001(0.001)0.500$. This represents an extension of certain portions of a table by Bartlett [2].

Table III gives 5D values of

$$p_1 \ln (p_1/p_2), \quad (1-p_1)\ln[(1-p_1)/(1-p_2)],$$

$$p_1 \ln (p_1/p_2) + (1-p_1)\ln[(1-p_1)/(1-p_2)],$$

$$p_1 \ln [p_1/(1-p_2)], \ (1-p_1)\ln[(1-p_1)/p_2],$$

and

$$p_1 \ln [p_1(1-p_2)] + (1-p_1)\ln[(1-p_1)/p_2]$$

for p_1, $p_2 = 0.00001, 0.00005, 0.0001, 0.0005, 0.001, 0.005, 0.01(0.01)0.50$. This represents an extension of a table by the reviewer [3].

Table IV gives 5D values of

$$(p_1 - p_2)\ln(p_1/p_2),$$

$$(p_2 - p_1)\ln[(1-p_1)/(1-p_2)],$$

$$(p_1 - p_2)\ln(p_1/p_2) + (p_2 - p_1)\ln[(1-p_1)/(1-p_2)],$$

$$[p_1 - (1-p_2)]\ln[p_1/(1-p_2)], \ [(1-p_1) - p_2]\ln[(1-p_1)/p_2],$$

and

$$[p_1 - (1-p_2)\ \ln[p_1/(1-p_2)] + [(1-p_1) - p_2]\ln[(1-p_1)/p_2]$$

for the same values of p_1 and p_2 as in Table III.

All these tables were computed on an IBM 709 system, using floating-point double-precision arithmetic. According to the author, a comparison with the corresponding logarithms published by the National Bureau of Standards [4] revealed that the error in the logarithm routine used in the construction of the present tables did not exceed $3 \cdot 10^{-8}$ for arguments in the range (0.02, 0.99999) and was at most $9 \cdot 10^{-8}$ for arguments in the range (0.00001, 0.02). This instills considerable confidence in the reliability of these 5D tables.

Examples illustrating the use of the tables appear in the accompanying explanatory text, to which is appended a bibliography consisting of eight titles.

S. Kullback

Statistics Department
The George Washington University
Washington, D. C.

1. Minoru Sakaguchi, "Table for the capacity of binary communication channels," *J. Operations Res. Soc. Japan*, v. 4, 1962, p. 55–66.

2. M. S. Bartlett, "The statistical significance of odd bits of information," *Biometrika*, v. 39, 1952, p. 228–237.

3. S. Kullback, *Information Theory and Statistics*, John Wiley and Sons, Inc., New York, 1959.

4. National Bureau of Standards, *Table of Natural Logarithms for Arguments between Zero and Five to Sixteen Decimal Places*, Applied Mathematics Series 31, U. S. Government Printing Office, Washington 25, D. C.

78[L].—ROBERT SPIRA, *Table of the Riemann Zeta Function*, ms. volume of 35 typewritten sheets bound with 904 unnumbered computer sheets, with two supplementary volumes of unrounded computer output (3p. + 600 sheets and 3p. + 720 sheets, respectively) deposited in UMT File.

This calculation of the Riemann zeta function $\zeta(\sigma + it)$ is for the values $\sigma = -1.6(.1)2.6$, $t = 0(.1)100.0$, 13D. The limit of 2.6 exceeds the line where $\zeta'(s) \neq 0$. The calculations were carried out with a minimum of 18 significant digits. For uniformity, all numbers are given to 13 decimals. If there are 15 figures or less, one can reasonably expect to have only rare rounding errors. For 16 figures, the accuracy appears to be ± 1 in the last place. There is also a manuscript table giving the unrounded results to 18 significant figures.

The values of the Haselgrove-Miller table were useful as check values, and they appear to be as accurate as claimed. On the $\sigma = \frac{1}{2}$ line, 63 values were found to be off by a unit in the last digit, while on $\sigma = 1$, a total of 79 such values were found. The actual errors were around .5 or .6 units except for $\text{Im}\, \zeta(1 + 90.8i) = -0.075218\,6942183$ versus their -0.075218. The number and size of the errors increased with t. The values of Shafer on the real axis and the value of $\zeta(\frac{1}{2})$ by Gram also served as check values [1, Sec. 22].

A bisection method was used to calculate the first 30 zeros. An independent calculation of the first 50 zeros to 50 places by M. D. Bigg showed small discrepancies in the 16th decimal, the largest being ± 4 in zeros 11, 12, and 23.

The computing was done using the Euler-Maclaurin sum formula method used by Hutchinson [2]. The general method used for checking was the functional equation:

$$\zeta(1 - s) = \left[2 \cos \frac{\pi}{2} s\right] [(2\pi)^{-s} \Gamma(s)] \zeta(s).$$

First $\zeta(s)$ was calculated, and then the reflected value. Then $\zeta(1 - s)$ was calculated and compared. The results of the comparison gave a running account of how the calculation was proceeding. Along the line $\sigma = \frac{1}{2}$, the calculations were performed in two different ways by varying parameters. The coefficients $c_n = -B_{2n+2}/(2n + 2)(2n + 1)B_{2n}$ were calculated using

$$B_{2n} = (-1)^{n+1} (2\pi)^{-2n} 2(2n)!\, \zeta(2n),$$

so

$$c_n = \frac{1}{4\pi^2} \left[\frac{\zeta(2n + 2)}{\zeta(2n)}\right],$$

and the coefficients of the Dirichlet series for the quantity in brackets are easily found.

The complete list of tables is as follows:

I	$\arcsin x$,	$x: .1(.1)1.0$	18S
IIa	c_n	$n: 1(1)7$	exact
		$n: 8$	80D
		$n: 9$	63D
		$n: 10(1)47$	30D

IIb	Dirichlet series coefficients $\sum_{d\mid k} d^2\mu(d)$		
		$k: 1(1)50$	exact
III	$J_n\left(\frac{\pi}{2}\right)$,	$n: 0(1)28$	30S
		$n: 29(1)38$	15S
IV	Coefficients for $\cos \frac{\pi}{2} x = \sum a_n x^{2n}$		20D
V	$\sin x$, $\cos x$		
		$x: 10^2(10^2)10^3(10^3)10^4(10^4)10^5$	18D
VI	$I_n(.05)$,	$n: 0(1)10$	31S
VII	Coefficients for $e^{.05x} = 1 + \sum a_n x^n$		20D
VIII	$t = 3 + 2\sqrt{2} - 2\sqrt{4 + 3\sqrt{2}}$		32S
	$4t^n/n$,	$n: 1(1)22$	30S
IX	$k = 3 + 2\sqrt{2}$		36S
	Coefficients for $\log\left(\frac{1+kz}{1-kz}\right) = \sum a_n z^n$		20D
X	Chebyshev coefficients for $\arcsin\left(\frac{x}{\sqrt{2}}\right) = \sum a_n T_{2n+1}(x)$		
		$n: 0(1)30$	30S
XI	Coefficients for $\arcsin\left(\frac{x}{\sqrt{2}}\right) = \sum a_n x^{2n+1}$		
		$n: 0(1)22$	20D
XII	Coefficients for asymptotic expansion of Γ-function		
		$n: 1, 2, 3$	exact
		$n: 4(1)18$	31S
	A_n/A_{n+1}	$n: 1, 2$	exact
		$n: 3(1)17$	31S
XIII	Chebyshev polynomial coefficients		
	$T_n(x)$,	$n: 1(1)97$	exact
XIV	$\zeta(\sigma + it)$,	$\sigma: -1.6(.1)2.6;\ t: 0(.1)100$	13D

There is also a copy of Bigg's values of the first 50 zeros to 50 decimal places.

A more complete discussion of the calculation and the formulas used is attached to the original tables. There is also a discussion of the theory of the arcsin Chebyshev series, which is rather complicated theoretically. It is planned to have the tables available on microcards.

The planning and programming took about one and a half years. The running time was several hundred hours. Procedures and documentation standards were set up on the basis of advice from Dr. Francis J. Murray of Special Research in Numerical Analysis. That such an involved program could be prepared and running in such a short period of time is certainly a tribute to the powerful and effective tools he supplied. Especially noteworthy was the systematic application of a 2000-command tracer program. The program itself had about 6000 commands, and there were at least another 6000 commands in the auxiliary programs for calculating the constants, testing, etc. The 7072 machine was extremely reliable.

There have been several effects on the theory of the Riemann zeta function as a result of this computation. First of all, zeros of $\zeta'(s)$ were discovered in the critical strip. The evidence of this calculation and further calculations results in the following conjecture: There are zeros of $\zeta'(s)$ on dense set of vertical lines for $\frac{1}{2} < \sigma \leqq 1$, and no zeros of $\zeta'(s)$ for $\sigma \leqq \frac{1}{2}$ except on the negative real axis. A paper is being prepared to show that $\zeta'(s) \neq 0$ in a very large portion of the left half plane $(\sigma \leqq \sigma_0, t \geqq t_0)$.

A second result is that $|\zeta(1 - s)| > |\zeta(s)|$ for $\frac{1}{2} < \sigma \leqq 1$, $t \geqq 10$, if $\zeta(s) \neq 0$. Thus, always for this region $|\zeta(1 - s)| \geqq |\zeta(s)|$, and the strictness of this inequality is equivalent to the Riemann hypothesis.

AUTHOR'S SUMMARY

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, Addison-Wesley, Reading, Massachusetts, 1962.
2. J. I. HUTCHINSON, "On the roots of the Riemann Zeta Function," *Trans. Amer. Math. Soc.*, v. 27, 1925, p. 49–60.

79[L].—(a) MARIYA I. ZHURINA & LENA N. KARMAZINA, *Tablitsy funktsiĭ Lezhandra* $P_{-1/2+i\tau}(x)$, Tom II (*Tables of the Legendre functions* $P_{-1/2+i\tau}(x)$, Vol. II), Akad. Nauk SSSR, Moscow, 1962, iv + 414 p., 27 cm. Price 4.42 rubles.

(b) M. I. ZHURINA & L. N. KARMAZINA, *Tablitsy i formuly dlya sfericheskikh funktsiĭ* $P^m_{-1/2+i\tau}(z)$ (*Tables and formulas for the spherical functions* $P^m_{-1/2+i\tau}(z)$), Akad. Nauk SSSR, Moscow, 1962, xivii + 58 p., 27 cm. Price 0.58 rubles.

Both volumes are in the series of *Mathematical Tables* of the Computational Centre of the Academy of Sciences of the USSR.

(a) This second volume, promised in the first volume and mentioned in the review of that volume (*Math. Comp.*, v. 16, p. 253–254, April 1962, where *for* Karamazina *read* Karmazina and *for* Izdatel'stov *read* Izdatel'stvo), has now been published. It will be remembered that Vol. I was for $x^2 < 1$ and that Vol. II was to be for $x > 1$. Replacing $-\frac{1}{2} + i\tau$ by s for convenience in the whole of the present reviews, the second volume does indeed give values of $P_s(x)$ to 7D without differences for $\tau = 0(0.01)50$ and $x = 1.1(0.1)2(0.2)5(0.5)10(10)60$. The main table occupies pages 11–270, i–iv, 271–407, a total of 401 pages. In principle there are four pages for each of the hundred ranges of width 0.50 in τ, but the table for $\tau = 32.50(0.01)33.00$ occupies five pages, the material having been skillfully and hardly noticeably spaced out (presumably to retrieve an error in pagination).

On pages 408–413 is an auxiliary table which for $x = 1.01(0.01)3(0.05)5(0.1)10$ gives, to 7D without differences, values of $\theta = \cosh^{-1}x$ and of the first four coefficients in the expansion of $P_s(\cosh \theta)$ in multiples of $\tau^{-n}J_n(\tau\theta)$. The values of θ have been read against the Harvard 9D tables [1], and appear to be correct on the convention that rounding is always downward, except that upward rounding occurs at $x = 1.61, 1.68, 1.72, 1.83, 2.00, 4.45, 7.60$. Nine decimals are not enough to decide at $x = 2.04$, but special calculation shows that upward rounding occurs here also.

(b) In this slim volume, which relates to both $x^2 < 1$ and $x > 1$, the same authors give first, on pages v–xxxviii, a collection of formulas relating to $P_s^m(z)$. Then follow a description of the tables and a bibliography of 43 items.

The eight tables on pages 1–56 fall into two groups.

Tables 1 and 2 list for $x = -0.99(0.01) + 0.99$, to 7D, $\theta = \cos^{-1}x$ and coefficients for the calculation of $P_s(\cos\theta)$ and $P_s^1(\cos\theta)$ when $I_0(\tau\theta)$ and $I_1(\tau\theta)$ are known, while Tables 3 and 4 list for $x = 1.01(0.01)3(0.05)5(0.1)10(1)60$, to 7D, $\eta = \cosh^{-1}x$ and coefficients for the calculation of $P_s(\cosh\eta)$ and $P_s^1(\cosh\eta)$ when $J_0(\tau\eta)$ and $J_1(\tau\eta)$ are known.

Tables 5 to 8 do not require values of Bessel functions to be available. Tables 5 and 6 list for $x = -0.90(0.01) + 0.99$, to 7D, values of $\theta = \cos^{-1}x$ and the first eight coefficients in the expansions of $P_s(\cos\theta)$ and $(1 + 4\tau^2)^{-1}P_s^1(\cos\theta)$ in powers of τ^2. Tables 7 and 8 list for $x = 1.01(0.01)3(0.05)5(0.1)10(1)60$, to 7D, $\eta = \cosh^{-1}x$ and the first eight coefficients in the expansions of $P_s(\cosh\eta)$ and $(1 + 4\tau^2)^{-1}P_s^1(\cosh\eta)$ in powers of τ^2.

There are no differences. Roundings in $\cosh^{-1}x$ for $x \leqq 10$ are as in (a) above, while for $10 < x \leqq 60$ there are upward roundings at $x = 35$ and 59, and unfortunately a major error at $x = 11$, where final 689 should be 699.

Taking the three volumes as a whole, the authors have achieved a gratifying fullness of coverage.

A. F.

1. Harvard University, *Annals of the Computation Laboratory*, v. 20, *Tables of Inverse Hyperbolic Functions*, Harvard University Press, Cambridge, Massachusetts, 1949.

80[L, M].—K. Singh, J. F. Lumley & R. Betchov, *Modified Hankel Functions and their Integrals to Argument 10*, Engineering Research Bulletin B-87, The Pennsylvania State University, University Park, Pennsylvania, October 1963, v + 29 p., 28 cm. Price $1.00.

Let

$$h_1(z) = (12)^{1/6}e^{-i\pi/6}[Ai(-z) - iBi(-z)] = (\tfrac{2}{3}z^{3/2})^{1/3}H_{1/3}^{(1)}(\tfrac{2}{3}z^{3/2}),$$

$$h_2(z) = (12)^{1/6}e^{i\pi/6}[Ai(-z) + iBi(-z)] = (\tfrac{2}{3}z^{3/2})^{1/3}H_{1/3}^{(2)}(\tfrac{2}{3}z^{3/2})$$

where the usual notation for Airy functions and Hankel functions is used. Tables are presented for the real and imaginary parts of

$$h(z), \int_0^s h(iu)\,du, \int_0^s \int_0^v h(iu)\,du\,dv, z = is,$$

for $s = -10(0.1)10$, where h stands for h_1 or h_2. The number of significant figures varies from 8 to 4. Most of the tables are new, though there is some overlap with the tables of M. V. Cerrillo and W. H. Kautz (see *Math. Comp.*, v. 16, 1962, p. 390). The functions were computed using ascending series and asymptotic series representations. The latter are not given in the text. For these and other representations, see Y. L. Luke, *Integrals of Bessel Functions*, McGraw-Hill Book Co., 1963. I find it most irritating that this report containing work sponsored by the U. S. government should carry a price tag. This petty practice should be discontinued.

Y. L. L.

81[M, X].—PAUL CONCUS, *Additional Tables for the Evaluation of* $\int_0^\infty x^\beta e^{-x} f(x)\,dx$ *by Gauss-Laguerre Quadrature*, ms. of 8 typewritten pages, deposited in UMT File.

Tables of abscissae and weight coefficients to fifteen places are presented for the Gauss-Laguerre quadrature formula

$$\int_0^\infty x^\beta e^{-x} f(x)\,dx \sim \sum_{k=1}^{N} H_k f(a_k), \quad \text{for} \quad \beta = -\tfrac{2}{3} \text{ and } -\tfrac{1}{3}, \quad \text{and} \quad N = 1(1)15.$$

These tables supplement those presented previously [1] for $\beta = -\frac{1}{4}, -\frac{1}{2}$, and $-\frac{3}{4}$. The same computer program was used, and the accuracy of the tables is the same. The values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ used in the calculations were taken from papers of Zondek [2] and of Sherry and Fulda [3].

AUTHOR'S SUMMARY

1. P. CONCUS, D. CASSATT, G. JAEHNIG & E. MELBY, "Tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)\,dx$ by Gauss-Laguerre quadrature," *Math. Comp.*, v. 17, 1963, p. 245–256.

2. B. ZONDEK, "The values of $\Gamma(\frac{1}{3})$ and $\Gamma(\frac{2}{3})$ and their logarithms accurate to 28 decimals," *MTAC*, v. 9, 1955, p. 24–25.

3. M. E. SHERRY & S. FULDA, "Calculation of gamma functions to high accuracy," *MTAC*, v. 13, 1959, p. 314–315.

82[M, X].—V. I. KRYLOV, *Approximate Calculation of Integrals*, The Macmillan Company, New York, 1962, x + 357 p., 23 cm. Price $12.50.

This is a translation of Krylov's *Priblizhennoe Vychislenie Integralov*, which appeared in 1959. The translation is in the American idiom and is clear and readable.

As the translator remarks, the book might more accurately have been named "Approximate Integration of Functions of One Variable", specifically one *real* variable. Multiple integrals are not treated, except for a short aside in Chapter 7. As a treatise on numerical evaluation of single integrals, this is an excellent book: it is clearly written and comprehensive, developing almost all known practical methods of integration and providing thorough treatments of error estimation and of the question of convergence. A great deal of the material appears for the first time in book form; much had been available before this only in the Russian journals. The text proceeds at a fairly, but not excessively, rapid pace, with frequent pauses to linger over the qualities of a particular formula, or subfamily of formulas, from a wide family under consideration.

The contents are divided into three parts. The first, of about 60 pages, is preparatory, developing certain mathematical topics that will be needed subsequently. These are: The Bernoulli Polynomials, Orthogonal Polynomials, Interpolation, and Banach Spaces. In each case the author moves by a short route from the basic definitions and theorems to the material he will need in his treatment of numerical quadrature; for example, he starts with the definition of Banach space, introduces the most important spaces and linear operators, and proves the uniform boundedness principle, in a very readable chapter only 12 pages long.

The second part, of over 200 pages length, is the heart of the book. It begins, in Chapter 5, with a general discussion of linear quadrature methods and their errors, and derives a general formula for the remainder in approximate quadrature. Chapter 6 takes up interpolatory quadrature methods, and derives the Newton-Cotes formulas and their remainders. The trapezoidal rule, Simpson's rule, and the three-eighths rule are discussed in greater detail. Chapter 7 begins the use of an approach that recurs throughout the book—the construction of formulas of highest degree of precision, sometimes referred to as "Gaussian" formulas. The general theory is set down on the basis of the theory of orthogonal polynomials, and the Gauss-Legendre, Gauss-Jacobi, Gauss-Hermite, and related families, are found, together with their remainder terms. In Chapters 9 and 10 formulas subject to special conditions—the use of some preassigned abscissas, or the restriction to equal coefficients—are treated along similar lines. Chapter 8 is devoted to the so called "best integration formulas"—those defined by regarding the remainder as a linear functional on an appropriate function space, and making its norm minimal. Not much is actually known about these formulas, though they have excited considerable interest. Chapter 11 is devoted mainly to a development of Krylov's: an expansion of the remainder of any quadrature formula in a form similar to the Euler-Maclaurin formula (which is Krylov's formula for the trapezoid rule), and the use of terms of this expansion to improve the accuracy of estimates of integrals. Chapter 12 takes up, in considerable detail, the question of convergence of sequences of quadrature formulas. For analytic integrands a variety of interesting results is proven by function-theoretic methods. Theorems dealing with more general integrands are also given.

The last section of the book, about 55 pages long, is devoted to a topic not usually treated in American texts—numerical indefinite integration; that is, the evaluation of an integral for a considerable number of (equally spaced) values of the upper limit. After an introductory chapter devoted to a very careful discussion of the propagation of errors and stability conditions, two interesting methods due to the author are treated. Each is designed to maximize the degree of precision of the formula while minimizing the number of new values of the integrand to be computed at each step. Some tables of abscissas and coefficients for these methods are given.

The book concludes with appendices listing abscissas and coefficients for the most important Gaussian quadrature formulas. The translator has expanded these tables beyond those given in the original edition.

As this is the only book in English on its subject, it is hardly necessary to recommend it. We are fortunate in the high quality of its writing. The translator has made a real contribution to the English mathematical literature.

Seymour Haber

National Bureau of Standards
Washington 25, D. C.

83[P, X, Z].—Howard Aiken & William F. Main (editors), *Switching Theory in Space Technology*, Stanford University Press, Stanford, California, 1963, x + 357 p., 26 cm. Price $11.50.

This volume consists of twenty-five papers on switching theory, switching circuits, and related topics presented at the Symposium on the Application of Switching Theory in Space Technology, held at Sunnyvale, California, February

27–28 and March 1, 1962. Despite the title of the symposium, only a minority of the papers are concerned with switching theory or space. Roughly, one-third of the papers are concerned with switching theory; another third are concerned with new switching elements, their physical attributes, fabrication and application (primarily as storage elements); and the remaining papers are concerned with systems and related subjects. The rather broad spectrum of topics should, as the editors intended, make the volume serve as something of a survey of the whole area and make it of interest to a quite broad class of reader. Unfortunately, a number of the papers suffer from inadequate introductions, heavy reliance on prior papers and prerequisite knowledge of rather specialized subjects, thus severely limiting (or, at least, taxing) their audience. As the well-rounded, self-contained presentation of some of the other papers in the volume indicates, such specialized presentation is unnecessary, and there is no doubt that it detracts from the value of the volume.

To supply a somewhat more detailed idea of the contents of the volume, we present the following list of authors and titles:

G. Polya, "Intuitive Outline of the Solution of a Basic Combinatorial Problem"; Günter Hotz, "On the Mathematical Theory of Linear Sequential Networks"; William H. Kautz, "Totally Sequential Switching Circuits"; Eiichi Goto, "Threshold, Majority, and Bilateral Switching Devices"; Franz E. Hohn, "Tyron's Delay Operator and the Design of Synchronous Switching Circuits"; David G. Willis, "Minimum Weights for Threshold Switches"; Heinz Zemanek, "Switching and Information"; V. Belevitch, "On the Realizability of Graphs with Prescribed Circuit Matrices"; J. Paul Roth, "The Theory of Algorithms"; G. W. Patterson, "Analysis and Design Confirmation of Controlled-Flow Nets"; Warren Semon, "General *E*-Algebras"; Frank F. Stuki, "New Techniques for the Machining and Shaping of Ferrites"; Robert C. Minnick, "Magnetic Comparators and Code Converters"; Jan A. Rajchman, "Computer Memories: Remarks on Possible Future Developments"; Jose Garcia Santesmases, "Nonlinear Resonance Switching Devices"; A. E. Slade, "Superconductive Switches and Storage Devices"; A P. Speiser, "Hydraulic Switching Devices"; J. I. Raffel, "Magnetic Films: New Possibilities, New Problems"; M. E. Browne, J. A. Cowen, and D. E. Kaplan, "Electron Spin-Echo Storage"; A. Van Wijngaarden, "Switching and Programming"; W. R. Abbot, "The Application of System Theory to Space Missions"; David E. Muller, "Asynchronous Logics and Application to Information Processing"; L. B. Heilprin, "Information Storage and Retrieval as a Switching System"; Daniel Hochman, "Space-Borne Digital System for Data Bandwidth Compression"; and Nicholas Szabo, "Recent Advances in Modular Arithmetic".

Eric G. Wagner

IBM Corporation
Yorktown Heights, New York

84[Q, X].—Karl Friedrich Gauss, *Theory of the Motion of the Heavenly Bodies Moving about the Sun in Conic Sections*, Dover Publications Inc., New York, 1963, xvii + 376 p., 24 cm. Price $2.95 (paperbound).

This unaltered reprint of the 1857 English translation of Gauss's *Theoria Motus* is timely, and especially welcome since so little of Gauss is available in English. In it he gives the development of those computational techniques that he devised for the relocation of the planetoid Ceres. Because of the few existing observations, and the unusual elements of its orbit, existing methods did not suffice. But from Gauss's calculations, as he proudly announced, Ceres was rediscovered "the first clear night".

The appendix contains an account of other computational techniques by Encke and Peirce, and 39 pages of tables by Le Verrier, Bessel, and others to facilitate certain astronomical calculations.

D. S.

85[R].—I. Todhunter, *A History of the Mathematical Theories of Attraction and the Figure of the Earth*, Dover Publications Inc., New York, 1962, xxxvi + 984 p., 22 cm. Price $7.50.

This is a timely and most welcome reprint of Todhunter's history, which was originally published in 1873. In it he gives a detailed and critical account of all the work in this field from the time of Newton to that of Laplace. This includes that of Newton, Huygens, Maupertius, Clairaut, Maclaurin, D'Alembert, Boscovich, Laplace, Legendre, Poisson, Ivory, and others.

The volume is not only of current physical interest but also contains valuable historical accounts of the origins of potential theory and of many investigations in partial differential equations. The style is simple and pleasant, and is enlivened by classical descriptions and original observations of his own. Thus: "Maupertius . . . who flattened the poles and the Cassinis"; "Madame la Marquise du Chastellet . . . from the fluctuation of her opinions, it seems as if she had not yet entirely exchanged the caprice of fashion for the austerity of science"; and "Gauss's writings are distinguished for the combination of mathematical ability with power of expression: in his hands Latin and German rival French itself for clearness and precision."

For the hurried reader the long preface and table of contents give a good idea of the volume's scope.

D. S.

86[S, X].—S. L. Sobolev, *Applications of Functional Analysis in Mathematical Physics*, Volume Seven, *Translations of Mathematical Monographs*, American Mathematical Society, Providence, Rhode Island, 1963, viii + 239 p., 24 cm. Price $6.70.

The development of the theory of distributions and generalized functions has its roots in the works of many famous mathematicians, such as J. Hadamard, M. Riesz, S. Bochner, and J. Leray, to mention a few. In 1936, S. L. Sobolev introduced a concept of generalized functions and derivatives which is essentially equivalent to the one now used. However, it was only with the appearance of L. Schwartz's comprehensive books on distributions in 1950 and 1951 that the field began to receive the systematic and extensive treatment we now know.

In the same year, namely 1950, S. L. Sobolev published the original Russian edition of the present monograph. It was the outgrowth of courses given at Leningrad State University and presented a unifying treatment of a number of problems in partial differential equations, using Sobolev's own approach to the concepts of generalized functions and their applications. The book appears not to have been noticed much by the non-Russian speaking mathematical world. In turn, the presentation in the book does not take the least bit of notice of any related work in the Western world.

In the 13 years that have elapsed between the publication of the original and this translation, a large body of knowledge has developed concerning the application of the theory of generalized functions to differential equations. Sobolev's earlier approaches to these problems and his results now appear in one way or another as part of a systematic and comprehensive theory. Nevertheless, this monograph is a veritable classic of mathematics; it is the work of a great mathematician who presents his own unified treatment of an important class of problems, with great clarity and without any attempt at being as general as possible. As such, it certainly remains an eminently worthwhile work to read.

Chapter I provides the necessary functional-analytic background and begins with a brief discussion of L_p spaces. Next, generalized derivatives are introduced; the definition is essentially identical to that for distributions, except that the generalized derivative of a function is itself a summable function. The linear space of all summable functions having on a finite domain Ω all generalized derivatives of order l summable to power $p > 1$ is denoted by $W_p^{(l)}$. $W_p^{(l)}$ with the norm

$$\| u \|_{W_p^{(l)}}^p = \| u \|_{L_p}^p + \int_\Omega \Big[\sum_{|q|=l} | D^q u |^2 \Big]^{p/2} d\omega$$

are the spaces upon which the discussions of all following parts of the book are based. The remainder of Chapter I is devoted to an investigation of the properties of $W_p^{(l)}$. Two theorems play an essential role here, assuring that under certain conditions $W_p^{(l)}$ can be imbedded in C or in some L_{q^*}, and that in both cases the imbedding operators are completely continuous. These theorems also give a natural setting for some inequalities between different norms on $W_p^{(l)}$.

Chapter II, entitled Variational Methods in Mathematical Physics, begins with an application of the theory of Chapter I using variational methods to prove the existence and uniqueness of the solution of Dirichlet's problem for the Laplace equation in the space $W_2^{(1)}$. This is then continued with a similar approach to the existence and uniqueness of the solutions of the Neumann problem and of the basic boundary value problem for the polyharmonic equation. The chapter ends with a detailed discussion of the eigenvalue problem

$$\Delta u + \lambda u = 0, \qquad \left.\frac{\partial u}{\partial n}\right|_S - hu \,|_S = 0$$

Chapter III concerns the solution of the Cauchy problem in $W_p^{(l)}$ spaces for the n-dimensional wave equation and for linear hyperbolic equations with variable coefficients. This discussion is based on the classical integration theory for these equations under the assumption of sufficiently smooth data. All necessary results are fully developed using the theory of characteristics for the $(2k + 1)$-dimensional case and Hadamard's method of descent for the space of $(2k)$-dimensions. The results for the generalized Cauchy problem then follow naturally from the classical results and the properties of the $W_p^{(l)}$ functions.

There is no question that the book plays an even more unique and distinct role today than at the time of its first appearance. It should certainly not be compared with any of the newer systematic and comprehensive treatises of the field, such as L. Hörmander's excellent work (Springer-Academic Press, 1963). As stated before, the value of this monograph is rather the clear and detailed presentation of one unified and original approach to the solution of some of the basic problems in the theory of partial differential equations, even though this approach has now become part of a larger theory. It appears to this reviewer that the value of the translation might have been enhanced even more if an up-to-date, annotated bibliography had been provided to supply the student with the necessary bridge to the present state of the field.

WERNER C. RHEINBOLDT

University of Maryland
College Park, Maryland

87[X].—L. Fox, *Numerical Solution of Ordinary and Partial Differential Equations*, Addison-Wesley Publishing Company, Inc., Reading, Mass., 1962, ix + 509 p., 23.5 cm. Price $10.00.

This volume is another useful addition to the expanding library in the field of numerical analysis devoted to the solution of differential equations by numerical methods. The material is based on a series of lectures presented at the Oxford University Computing Laboratory during the summer of 1961. The areas covered include the following: (1) ordinary differential equations; (2) integral equations; (3) introduction to partial differential equations; and (4) practical problems in partial differential equations.

Although the lectures were delivered by a number of workers in the field, the book achieves a remarkable degree of coherence. The editor, L. Fox, and the contributors (D. F. Moyers, *et al.*) also deserve much credit for the promptness of the publication and the lucidity of the presentation.

The first section treats the solution of ordinary differential equations by the method of finite differences. It covers such topics as the Runge-Kutta method, eigenvalue problems, and Chebyshev approximation. Of special interest is the discussion of stability as it relates to the solution of ordinary differential equations. The author finds it useful to classify several types of instability, such as inherent instability, partial instability, and strong instability.

Section 2 discusses the numerical solution of integral equations, including such topics as Fredholm equations of the first, second, and third kinds, equations of Volterra type, integro-differential equations in nuclear collision problems, and the Hartree-Fack equation.

Section 3 contains a readable exposition of the methods in common use for the numerical solution of partial differential equations of hyperbolic, elliptic, and parabolic types.

Section 4 contains a discussion of illustrative problems involving partial differential equations solved by the methods of finite differences, selected from a representative cross-section of modern physics and engineering.

Although generally well done, the book does show the signs of haste in many spots, and should be improved in later editions.

H. P.

88[Z].—WERNER BUCHHOLZ, editor, *Planning a Computer System*, McGraw-Hill Book Company, Inc., New York, 1962, xviii + 322 p., 23 cm. Price $9.75.

The conception, design, simulation, operational details, and finally the limitations that must be accepted in the creation of a very large and powerful logical machine, STRETCH, are documented in a satisfyingly complete and thorough manner in this volume of more than 300 pages. Much of the material has appeared in the form of journal articles elsewhere, but this bringing together of the many considerations that entered into this mammoth undertaking provides the serious student of systems design with a compact, concise, and convenient handbook of valuable and workable ideas.

It is strongly recommended that this volume be studied and kept as a shelf reference by any who would be conversant with the latest techniques of computer organization—and this includes programmers as well as logicians. This first group would surely find writing routines for such a magnificently endowed machine a tremendous challenge and stimulating experience. Surely only the truly first-rate members of the profession can accept this challenge and learn to make use of the facilities so carefully provided by the designers.

Project STRETCH, as the book is sub-titled, is a welcome library addition for all of these reasons, and still more so as a volume to record, for easy distribution, the key ideas and developments in a remarkable achievement of a dedicated team.

The book never glosses over difficulty, and with remarkable candor acknowledges wherever design goals could not be met.

Chapter 1 traces the history and gives the design objectives for the project, and gives some indications of how well these were, or were not, met. Chapter 2, entitled Architectural Philosophy, presents a most intelligent approach to satisfying the conflicting needs of a variety of potential users. Chapter 3, entitled System Summary, presents enough of the characteristics of the STRETCH to make it sound like an exciting machine to have at one's disposal. The FORTRAN influence is quite evident.

Chapter 4 begins the detailed technical presentation with a discussion of why "words" are made of variable numbers of "bytes" rather than a fixed length, as is more common. Chapter 5 is a perhaps unnecessary rationalization of the old "bits versus digits" or "binary versus decimal" dichotomy. For the initiated, who are sufficiently computer wise to appreciate Project STRETCH, it is needless to belabor the point. Chapter 6 presents an interesting discussion of the character-set choice, including a fine awareness of the multiple-faceted implications which are not as obvious as those in Chapters 4 and 5. One comes to the conclusion that it is indeed unfortunate that the computer-user, i.e., programmer-expert, will rarely read and appreciate such design-viewpoint computer "biographies" as this, missing the insights into the design procedures, the often difficult choices and elegant solutions which render to them many extra conveniences. Many of these are often taken lightly, ignored, or rarely appreciated and used, in optimizing machine usage. The search for the ultimate, the effort to attain logical perfection, is again illustrated in the discussion in Chapter 7 of the variable-field-length operational feature. Aesthetics and utility meet when it is shown how 16 Boolean connectives were furnished at little more than the price of a "basic set" of four. We have here a beautiful example of the futility of eliminating an innovation on the basis of its probable cost or utility. How much better to let it be found both valuable and easy by actual trial and usage. What use has been, or will be, made of an evidently powerful Boolean data-handling facility may determine whether whole new areas are to be opened to mechanization or whether it is an essentially superfluous "frill", put in before its time!

The discussions of floating-point arithmetic, and particularly the Noisy Mode in Chapter 8 are of above-average interest to the analyst. Chapter 9 details the machine-language instruction formats, including the relationships to earlier machines and to automatic programming requirements. Chapters 10 and 11 show the complexities and intricacies that arise in attempting to implement the flexibility and speed of STRETCH, with particular reference to program-interrupts, multiprogramming, and indexing.

The general philosophy and some discussion of details of Input-Output Control occupy Chapter 12. Flexibility, speed, simplicity-in-complexity, and refraining from "freezing" the system to today's hardware again pervade this chapter. In Chapter 13 a closer look is taken at Multiprogramming, its needs and advantages. The balancing of features in the supervisory executive routine vis-à-vis hardware is well presented. Chapter 14, the Central Processing Unit, emphasizes the organization of hardware to achieve a performance gain of several orders of magnitude over that achievable by component technology alone; i.e., the effective 0.5 μsec interval of memory access with a 2.1 μsec memory cycle; the simultaneity approach in the Exchange, disk synchronizers, C. P. U. overlapping of up to 11 instructions by use of its sub-computers (Instruction Unit, Look-Ahead, Main Arithmetic Unit with a serial and a parallel unit); strictly binary multiply and divide with subroutines for decimal; multiplexing of every imaginable kind to achieve extreme speeds; retaining accuracy by parity checks with one-bit error correction on transfers and duplication, parity and "casting out three's" on arithmetic. The total hardware to realize this Central Processing Unit includes 170,000 transistors, consumes 21 KW of power, and attains average speeds of 1.5 μsec for Add, 2.7 μsec for Multiply, and 9.9 μsec for Divide.

Chapter 15 focuses in greater detail on the Look-Ahead portion of the Central Processing Unit, describing its behavior in providing maximum computer utilization despite work-load peaks, which cause conflicts between the Arithmetic Unit and Input-Output memory access demands. Despite its very complex overlapping and non-sequential completion of parts of several instructions, the logic must function so as to appear perfectly sequential, to the user who writes the instruction program! The actual configuration and relationships between Look-Ahead complexity, the Arithmetic Unit speeds, and memory unit access restrictions were arrived at with the help of simulations on the IBM 704 and timing comparisons of five typical problem types with the various C. P. U. configurations. Obvious savings of time by giving maximum control of input-output to the Exchange, guessing "no-branch" on conditional transfers, and "forwarding" data from common memory-address references are described. The implications of "yes" on a branch instruction, or any "interrupt", are discussed under the "housecleaning" mode.

A more detailed description of input-output control follows in Chapter 16, devoted to the Exchange, which is a highly specialized, fixed-program computer with its own 1 μsec memory. Finally, the book concludes with Chapter 17, a discussion of non-arithmetic data processing, i.e., byte streaming through the table look-up, statistical aid, and adjustment units of the 7951 auxiliary computer to the 7030 STRETCH. With tape units furnishing up to 140,000 64-bit words per second, a rate of approximately 3.3 million bytes per second is achieved for merging, sorting, searching, or file maintenance.

In summary, a wealth of information, a candid view of practical solutions to grand concepts, and an insight into the philosophy of large computing machine design are all to be found herein. It is a readable and highly worth-while book for those whose interest in computers extends beyond the "where" and "when" to the "how" and "why" they are what they are!

Herbert M. Ernst

Applied Mathematics Laboratory
David Taylor Model Basin
Washington, D. C. 20007

89[Z].—S. Winograd & J. D. Cowan, *Reliable Computation in the Presence of Noise*, M. I. T. Press, Cambridge, Massachusetts, 1963, xiv + 112 p., 24 cm. Price \$5.00.

Reliable computation in the presence of noise, meaning errors in the machine, is a problem of increasing interest as we come to depend more and more upon computers which we are not in a position to repair immediately.

The authors consider not only the construction of reliable machines from unreliable components, but also the effects of errors in the basic wiring. The treatment is mathematical and reasonably precise as opposed to past speculations by philosophers on these matters.

Behind all the search for a theory to enable us to construct reliable machines is the fascinating fact that we ourselves seem to be constructed with an unreliable nervous system whose individual components seem to die at a surprisingly high rate. We are each of us apparently a living proof that reliable large scale operation can be achieved from unreliable components, and it naturally is of considerable interest to us to learn about possible models of how we might be constructed. The authors wisely refrain from too much premature speculation in this area, but almost every reader will do his own anyway.

R. W. Hamming

Bell Telephone Laboratories
Murray Hill, New Jersey