# Mathematics of Computation 

A journal devoted to advances in numerical analysis, the application of computational methods, mathematical tables, high-speed calculators and other aids to computation


Published Quarterly for the
National Academy of Sciences-National Research Council By the
American Mathematical Society

# Editorial Committee <br> Division of Mathematics National Academy of Sciences-National Research Council <br> Washington, D. C. 

H. Polacher, Chairman, Applied Mathematics Laboratory, David Taylor Model

- Basin, Washington 7, D. C.

Philip J. Davis, Mathematics Department, Brown University, Providence, Rhode Island
Alan Fletcher, Department of Applied Mathematics, University of Liverpool, Liverpool 3, England
Alan J. Hoffman, IBM Research Center, Yorktown Heights, New York
A. S. Householder, Oak Ridge National Laboratory, Oak Ridge, Tennessee

Peter D. Lax, Courant Institute of Mathematical Sciences, New York University, New York 3, New York
Y. L. Luke, Midwest Research Institute, Kansas City 10, Missouri

Philip M. Morse, Massachusetts Institute of Technology, Cambridge 39, Massachusetts
Daniel Shanks, Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C.
R. S. Varga, Case Institute of Technology, Cleveland 6, Ohio
J. W. Wrench, Jr., Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C.

## Information to Subscribers

The journal is published quarterly in one volume per year with issues numbered serially since Volume I, Number 1. Volumes I-XVII (1943-1963), the price is $\$ 3.00$ per issue; $\$ 12.00$ per volume. Back issues are available for most issues. Write for specific information.

## Unpublished Mathematical Tables

The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section Reviews and Descriptions of Tables and Books. Readers may request copies of the tables from the editor, which are made available at a nominal cost.

## Microcard Edition

Volumes I-X (1943-1956), Nos. 1-56 are now available on Microcards and may be purchased from the Microcard Foundation, Box 2145, Madison 5, Wisconsin, at a cost of $\$ 20.00$ for the complete set. Succeeding volumes are available on request.

## Information to Contributors

All contributions intended for publication in Mathematics of Computation and all books for review should be addressed to H. Polachek, Technical Director, Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C. The author may suggest an appropriate editor for his paper. Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see Mathematical Reviews, v. 25, Index for January-June 1963. Authors should submit the original and one copy, and should retain one copy.
Subscriptions, address changes, business communications and payments should be sent to:

American Mathematical Society<br>190 Hope Street<br>Providence, Rhode Island 02906<br>Published Quarterly for the<br>NATIONAL ACADEMY OF SCIENCES-NATIONAL RESEARCH COUNCIL by the

## CLASSIFICATION OF REVIEWS

A. Arithmetical Tables, Mathematical Constants
B. Powers
C. Logarithms
D. Circular Functions
E. Hyperbolic and Exponential Functions
F. Theory of Numbers
G. Higher Algebra
H. Numerical Solution of Equations
I. Finite Differences, Interpolation
J. Summation of Series
K. Statistics
L. Higher Mathematical Functions
M. Integrals
N. Interest and Investment
O. Actuarial Science
P. Engineering
Q. Astronomy
R. Geodesy
S. Physics, Geophysics, Crystallography
T. Chemistry
U. Navigation
V. Aerodynamics, Hydrodynamics, Ballistics
W. Economics and Social Sciences
X. Numerical Analysis and Applied Mathematics
Z. Calculating Machines and Mechanical Computation

# Mathematics of Computation <br> TABLE OF CONTENTS 

July 1964

> New Monotone Type Approximations for Elliptic Problems
> James H. Bramble \& Bert E. Hubbard
> 349
> $\begin{aligned} & \text { Error-Bounds for the Evaluation of Integrals by the Euler-Maclaurin Formula } \\ & \text { and by Gauss-Type Formulae................................... } 368\end{aligned}$
> Chebyshev Approximations of a Function and Its Derivatives
> D. G. Moursund
> 382
> $\begin{aligned} & \text { On the Computation of } L_{1} \text { Approximations by Exponentials, Rationals, and } \\ & \text { Other Functions........................................................ RICE } 390\end{aligned}$
> Bounds for Pairs of Consecutive Seventh and High Power Residues
> John Brillhart, D. H. Lehmer \& Emma Lehmer 397
> Solutions of the Diophantine Equation $x^{3}+y^{3}=z^{3}-d$
> V. L. Gardiner, R. B. Lazarus \& P. R. Stein 408
> Vector Partitions and Combinatorial Identities...............M. S. Cheema 414
> The Evaluation of Some Definite Integrals Involving Bessel Functions which Occur in Hydrodynamics and Elasticity................ A. H. Van Tuyl
> 421
> Evaluation at Half Periods of Weierstrass' Elliptic Function with Rhombic Primitive Period-Parallelogram.... Chih-Bing Ling \& Chen-Peng Tsai 433
> Conditions on Minimization Criteria for Smoothing
> Boris Podolsky \& Harry H. Denman 4

A Comparison of Several Methods for Inverting Large Symmetric Positive Definite Matrices.
.M. H. Lietzke, R. W. Stoughton \& Marjorie P. Lietzke
A Comparison of Methods for Computing the Eigenvalues and Eigenvectors of a Real Symmetric Matrix...... Paul A. White \& Robert R. Brown ..... 457Laguerre's Method Applied to the Matrix Eigenvalue Problem

Beresford Parlett 464

## Technical Notes and Short Papers Tests of Parlett's Algol Eigenvalue Procedure Eig 3

George E. Forsythe 486 Differential Approximation Applied to the Solution of Convolution
Equations..Richard Bellman, Robert Kalaba \& Bella Kotkin 487 On the Numerical Solution of Equations of the Abel Type

Henry E. Fettis 491 A New Method of Numerical Integration of Differential Equations
W. H. Witty 497 On the Reduction of an Arbitrary Real Square Matrix to Tridiagonal
Form.........................H. H. WANG \& R. T. Gregory 501 A Note on La Budde's Algorithm.................. . Beresford Parlett 505 A Note on Projective Planes of Order Nine
E. T. Parker \& R. B. Killgrove 506 The First Power of 2 with 8 Consecutive Zeros....E. Karst \& U. Karst 508
Reviews and Descriptions of Tables and Books........................ 509
Peters 65, Gardiner, Lazarus \& Stein 66, Lehmer 67, Krull 68, Stewart 69, Khovanskit 70, Faddeev \& Faddeeva 71, Larsson 72, Gelfond 73, Finney, Latscha, Bennet \& Hsu 74, Weintraub 75, Bellman 76, Phllips 77, Spira 78, Zhurina \& Karmazina 79, Singh, Lumley \& Betchov 80, Concus 81, Krylov 82, Aiken \& Main 83, Gauss 84, Todhunter 85, Sobolev 86, Fox 87, Buchholz 88, Winograd \& Cowan 89
Table Errata................................................................... 532
Byrd \& Friedman 352, Erdélyi 353, Kraitchik 354, Owen 355, National Bureau of Standards 356, von Sterneck 357, Lebedev \& Federova, Burunova 358
Corrigenda
Newbery, Whittlesey

