# Mathematics of Computation

A journal devoted to advances in numerical analysis, the application of computational methods, mathematical tables, high-speed calculators and other aids to computation



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All contributions intended for publication in Mathematics of Computation and all books for review should be addressed to H. Polachek, Technical Director, Applied Mathematics Laboratory, David Taylor Model Basin, Washington 7, D. C. The author may suggest an appropriate editor for his paper. Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see Mathematical Reviews, v. 25, Index for January-June 1963. Authors should submit the original and one copy, and should retain one copy.

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