(b) With a slightly different algorithm i.e.

$$x = P^2 - Q^2 - R^2,$$
$$y = 2PQ,$$
$$z = 2PR.$$

We find for $x = 495$, $y = 840$, $z = 448$,

$$x^2 + y^2 + z^2 = 1073^2, \qquad x^2 + y^2 = 975^2, \qquad z^2 + y^2 = 952^2,$$

$x^2 + z^2$ not a square.

(c) The sets (1008, 1100, 1155) and (1008, 1100, 12075) have two numbers in common.

(d) There are several sets of (x, y, z) which have one value in common e.g. (2964, 9152, 9405), (2964, 6160, 38475) and (5643, 43680, 76076), (5643, 14160, 21476).

Department of Mathematics
Memorial University of Newfoundland
St. John's, Newfoundland, Canada

1. W. Sierpiński, *A Selection of Problems in the Theory of Numbers* (English transl.), Pergamon Press, Oxford, 1964.
2. L. E. Dickson, *History of the Theory of Numbers*, Vol. II, Chelsea, New York, 1952.

Some Designs for Maximal $(+1, -1)$-Determinant of Order $n \equiv 2 \pmod 4$

By C. H. Yang

When $n \equiv 2 \pmod 4$, Ehlich [1] has shown that

(i) the maximal absolute value α_n of nth order determinant with entries ± 1 satisfies

$$\alpha_n^2 \leqq 4(n-2)^{n-2}(n-1)^2 = \mu_n,$$

(ii) matrices M_n of the maximal nth order $(+1, -1)$-determinant whose absolute value equals $\mu_n^{1/2}$ exist for $n \leqq 38$, provided that "$(n-1, -1)_p = 1$ (Hilbert's symbol) for any prime p," which is also equivalent to "any prime factor of squarefree part of $n - 1$ is not congruent to 3 (mod 4)."

It is found that M_{42}, M_{46} also exist by Ehlich's method and such maximal matrices M_n are likely to exist for all $n \equiv 2 \pmod 4$ if $(n-1, -1)_p = 1$ for any prime p. This means that for $n < 200$, all such matrices are likely to be found except for $n = 22, 34, 58, 70, 78, 94, 106, 130, 134, 142, 162, 166, 178$, and 190.

The maximal matrix M_n such that

$$M_n M_n^T = \begin{pmatrix} P & 0 \\ 0 & P \end{pmatrix}, \qquad \text{where } P = \begin{pmatrix} n & & 2 \\ & \ddots & \\ 2 & & n \end{pmatrix}$$

Received June 16, 1965.

and M_n^T = the transpose of M_n, can be constructed from the following (cf. Ehlich [1]):

$$M_n = \begin{pmatrix} A_1 & A_2 \\ -A_2^T & A_1^T \end{pmatrix},$$

where A_1, A_2 are circulant matrices of order $n/2$.

For $n = 42, 46$, the designs for the maximal matrices M_n are:

$$\begin{aligned} n = 42; \quad & A_1: ----+\ \ +-+-+\ \ --++-\ \ ++-++\ \ +, \\ & A_2: --+++\ \ -+-+-\ \ -++++\ \ +++++\ \ +; \\ n = 46; \quad & A_1: ----+\ \ +-+-+\ \ --++-\ \ +++-+\ \ +++; \\ & A_2: ---++\ \ -+-++\ \ +-++-\ \ +++++\ \ +++, \end{aligned}$$

where $-$ stands for -1, $+$ for $+1$.

Another design for $n = 38$ is found as follows:

$$\begin{aligned} & A_1: ---++\ \ -+-++\ \ +-++-\ \ ++++; \\ & A_2: ---++\ \ +-++-\ \ +++++\ \ ++-+. \end{aligned}$$

For $n = 50$, the maximal matrix M_n can be constructed by taking $A_1 = A_2$ = the matrix of Raghavarao [3], without circulancy.

As noted in the design of above maximal matrices, the numbers n_1 and n_2 of -1's respectively in each row of A_1 and A_2 can not be arbitrary. For example, when $n = 38$; n_1, n_2 must be either 6 or 7, provided $n_1, n_2 < n/4$. Similarly, when $n = 42$; n_1, n_2 must be either 6 or 10: when $n = 46$; either 7 or 10.

For $54 \leqq n < 200$, the following table of n_1 and n_2 is helpful to construct the maximal matrices. ($n_1, n_2 < n/4$)

n	54	62	66	74	82	86	90	98	102	110	114	118	122	126
n_1 or	9	10	12 (11)	13	16	16 (15)	16	18	20	21	21	22	25	25 (24)
n_2	11	15	13 (15)	16	16	18 (21)	21	22	21	24	28	28	25	27 (29)

n	138	146	150	154	158	170	174	182	186	194	198
n_1 or	27	30	29	31	31	34 (36)	36	36	38 (37)	39	42
n_2	31	31	36	34	37	39 (36)	38	45	42 (45)	46	43

4 Gunnison Park Boulevard
Oneonta, New York

1. H. Ehlich, "Determinantenabschätzungen für binäre Matrizen," *Math. Z.*, v. 83, 1964, pp. 123–132.

2. H. Ehlich & H. Zeller, "Binäre Matrizen," *Z. Angew. Math. Mech.*, v. 42, 1962, pp. 20–21.

3. D. Raghavarao, "Some optimum weighing designs," *Ann. Math. Statist.*, v. 30, 1959, pp. 295–303. MR **21** #3077.

4. D. Raghavarao, "Some aspects of weighing designs," *Ann. Math. Stastist.*, v. 31, 1960, pp. 975–986.