

# Mathematics of Computation

Volume 20, Number 94

April, 1966

*Published by the American Mathematical Society*

PROVIDENCE, RHODE ISLAND



## Editorial Committee

- EUGENE ISAACSON, Chairman, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012  
AVRON DOUGLIS, Department of Mathematics, University of Maryland, College Park, Maryland 20740  
ALAN FLETCHER, Department of Mathematics, University of Liverpool, Liverpool 3, England  
ALAN J. HOFFMAN, IBM Research Center, Yorktown Heights, New York 10598  
A. S. HOUSEHOLDER, Oak Ridge National Laboratory, Oak Ridge, Tennessee  
PETER D. LAX, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012  
Y. L. LUKE, Midwest Research Institute, Kansas City, Missouri 64110  
PHILIP M. MORSE, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139  
HARRY POLACHEK, Research Division, U. S. Atomic Energy Commission, Washington, D. C. 20545  
DANIEL SHANKS, Applied Mathematics Laboratory, David Taylor Model Basin, Washington, D. C. 20007  
R. S. VARGA, Case Institute of Technology, Cleveland, Ohio 44106  
J. W. WRENCH, JR., Applied Mathematics Laboratory, David Taylor Model Basin, Washington, D. C. 20007

## Information for Subscribers

The journal is published quarterly in one volume per year, with issues numbered serially since Volume 1, Number 1. All back volumes are available. For Volumes 1-19 (1943-1965), prices are \$20.00 per volume and \$6.00 per issue.

## Unpublished Mathematical Tables

The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section *Reviews and Descriptions of Tables and Books*. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost.

## Microcard Edition

Volumes 1-10 (1943-1956), Nos. 1-56 are now available on Microcards and may be purchased from the Microcard Foundation, Box 2145, Madison, Wisconsin 53705.

## Information for Contributors

Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see *Mathematical Reviews*, v. 28, Index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "Information for Contributors to Mathematics of Computation" and "Manual for Authors", both of which are available upon request from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to Eugene Isaacson, Chairman, Editorial Committee, Mathematics of Computation, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. Beginning with the January, 1965 issue, institutions sponsoring research reported in the journal are assessed page charges.

Subscriptions, address changes, business communications and payments should be sent to:

AMERICAN MATHEMATICAL SOCIETY  
P. O. Box 6248  
Providence, Rhode Island 02904

Copyright © 1966, American Mathematical Society  
Printed in the United States of America

Second-class postage paid at Providence, Rhode Island and at additional mailing offices.

# New from Saunders

**New! 3100 Transform Pairs in a Single Source**

## **TABLE of LAPLACE TRANSFORMS—Roberts & Kaufman**

**By George Roberts, RCA Victor Company, Ltd., and Hyman Kaufman, McGill University.**

This superb reference offers the mathematician, the engineer, and the physicist a comprehensive collection of 3100 Laplace transforms—probably the most complete single source in print today.

The utility of the work is immeasurably enhanced by inclusion of separate tables for direct transforms and inverse transforms. Drs. Roberts and Kaufman have devised a unique indexing system for this volume. Through the use of a simple coding system, they enable the user to locate individual entries with a minimum of labor and time. The scientist will find this volume especially valuable in such applications as: *solution of initial and boundary value problems, control and feedback systems, transients.*

*368 pages \* \$6.75 \* New! Published January, 1966.*

**New! A Geometric Approach to Advanced Programming**

## **LINEAR & CONVEX PROGRAMMING—Zukhovitskiy & Avdeyeva**

**By S. I. Zukhovitskiy and L. I. Avdeyeva, both at Kiev Institute**

A text and reference for mathematicians interested in advanced programming, this volume is a translation from the original Russian. The authors' approach in their book is geometric, in that they attempt to give a thorough understanding of the geometry involved in the problems and methods. The computational methods are based on Jordan eliminations. The basic simplex method, as applied to linear programming, is explained in the first four chapters. The fifth chapter is devoted to Chebyshev approximation, while the last chapter treats the subject of convex programming.

An extensive international bibliography is included in the book. A substantial number of examples in each chapter illustrate the points made by the authors.

*About 350 pages \* Illustrated \* About \$10.00 \* New! Ready May.*

---

**W. B. Saunders Company West Washington Square, Phila., Pa. 19105**

Please send and bill me:

*(Discount accorded to full-time teachers listing affiliation)*

☐ Roberts & Kaufman—TABLE OF LAPLACE TRANSFORMS.....\$6.75

☐ Zukhovitskiy & Avdeyeva—LINEAR & CONVEX PROGRAMMING.....about \$10.00

Name\_\_\_\_\_Affiliation\_\_\_\_\_

Address\_\_\_\_\_Zip\_\_\_\_\_

# FOR MATHEMATICIANS...

## **ELEMENTARY NUMERICAL ANALYSIS: An Algorithmic Approach**

By S. D. CONTE, Purdue University. McGraw-Hill Series in Information Processing and Computers. 278 pages, \$7.95 . . . This computer-oriented text derives and analyzes algorithms of numerical analysis proved most valuable for the solution of industrial problems on modern high speed computers.

## **HOW TO GAMBLE IF YOU MUST: Inequalities for Stochastic Processes**

By LESTER E. DUBINS, University of California, Berkeley; and LEONARD J. SAVAGE, Yale University. McGraw-Hill Series in Probability and Statistics. 250 pages, \$12.75 . . . Based on mathematical research begun by the authors in 1956, this report adduces inequalities for stochastic processes from the graphic idea of making the best of a bad situation at the gambling table.

## **NUMERICAL METHODS FOR SCIENTISTS AND ENGINEERS**

By RICHARD W. HAMMING, Bell Telephone Laboratories. International Series in Pure and Applied Mathematics. 432 pages, \$11.00 . . . Emphasizes insight into the meaning of the computation rather than machine efficiency. Begins with discrete finite difference calculus and progresses to classical numerical analysis through ordinary differential equations, least squares, and Chebyshev polynomials.

## **A FIRST COURSE IN NUMERICAL ANALYSIS**

By ANTHONY RALSTON, State University of New York, Buffalo. 600 pages, \$12.50 . . . A modern, introductory text, assuming a background of advanced calculus and matrix algebra. Aimed directly at applications on digital computers. Includes 80 worked examples and 500 problems.

## **THEORY OF RECURSIVE FUNCTIONS AND EFFECTIVE COMPUTABILITY**

By HARTLEY ROGERS, Jr., Massachusetts Institute of Technology. International Series in Pure and Applied Mathematics. Off press . . . An outstanding mathematical logician offers this semi-formal, intuitive approach to recursive functions. More than three-quarters of the material appears here for the first time in book form.

## **A SURVEY OF NUMERICAL ANALYSIS**

Edited by JOHN TODD, California Institute of Technology. 608 pages, \$13.00 . . . The 14 nationally known contributors are experts in the use of automatic computers. They examine classical and modern numerical analysis and supporting areas of mathematics and statistics not adequately covered in current literature.

## **INTRODUCTION TO APPROXIMATION THEORY**

By E. W. CHENEY, University of Texas. International Series in Pure and Applied Mathematics. Available in late Spring . . . Although the approach is theoretical, emphasis is placed on topics useful in computer applications. The subject of uniform approximation is stressed throughout, with techniques of convexity employed to prove the main features.



**McGRAW-HILL BOOK COMPANY**

330 West 42nd Street / New York, N.Y. 10036



*Special Prepublication Offer!*

# **SÉMINAIRE BOURBAKI**

**W. A. Benjamin, Inc., is pleased to announce a reprinting of the entire series of the Séminaire Bourbaki covering the years 1948 through 1965.**

WE HAVE BEEN AUTHORIZED by N. Bourbaki to issue this special facsimile reprinting of all the past Séminaire Bourbaki; photo offset reproduction has been used and the combined volumes (almost entirely in French) are copied directly from the revised edition of the original seminar volumes. A special name index has been compiled for the complete set and appears in each volume.

DUE TO THE FACT that the Séminaire Bourbaki have been previously available in paperback in limited quantities, we plan a single printing which is intended to supply libraries and faculties with a complete set in permanent bindings at a reasonable price. Several of the early Séminaire have been put together into a single volume with the result that the original seventeen paperback volumes have been combined into twelve volumes, all bound in hard cover.

TO TAKE ADVANTAGE of the prepublication offer, all orders—whether for the complete twelve volume set (1948 through 1965) or for individual volumes—must be received by W. A. Benjamin, Inc., by April 30, 1966.

	Special Price	Regular Price	You Save
Each Volume	\$ 9.75	\$ 12.25	\$ 2.50
Complete Twelve Volume Set	\$102.00	\$129.00	\$27.00

Volume 1948/49–1949/50, 243 Pages  
Volume 1950/51–1951/52, 308 Pages  
Volume 1952/53–1953/54, 256 Pages  
Volume 1954/55–1955/56, 350 Pages  
Volume 1956/57–1957/58, 377 Pages  
Volume 1958/59, 239 Pages

Volume 1959/60, 214 Pages  
Volume 1960/61, 274 Pages  
Volume 1961/62, 290 Pages  
Volume 1962/63, 207 Pages  
Volume 1963/64, 205 Pages  
Volume 1964/65, 215 Pages

**PLEASE NOTE: TO TAKE ADVANTAGE OF THE PREPUBLICATION OFFER, ORDERS MUST BE RECEIVED BY APRIL 30, 1966; PREPAID ORDERS RECEIVE AN ADDITIONAL 10% DISCOUNT FROM THE PREPUBLICATION PRICE.**

**W. A. Benjamin, Inc.**  
**One Park Avenue New York 10016**



## Blaisdell Publishing Company

### **The Calculus of Variations**

N. I. Akhiezer, *Kharkov University*; translated by Aline H. Frink, *The Pennsylvania State University*

Suitable for a one-semester course at the advanced undergraduate and graduate level, this book acquaints the reader with some of the problems and main methods, both classical and direct, of the calculus of variations. It includes necessary conditions for an extremum, various sufficient conditions, and direct methods for establishing the existence of solutions of extremum problems and approximating these solutions practically.

1962. 247 pp. \$9.50

### **Interpolation and Approximation**

Philip J. Davis, *Brown University*

Appropriate for use in graduate study, this tested text can be used in courses in interpolation and approximation theory or as a supplementary text for courses in real variables, complex variables, functional analysis, and numerical analysis. It is assumed the reader has some knowledge of real and complex variable theory, linear algebra, and integration theory. Students, pure mathematicians, applied mathematicians, and numerical analysts will find the book of great value, as should computation laboratories, libraries, and research institutes.

1963. 393 pp. \$12.50

"This book fulfills its aim very well and will be indispensable for every student of applied mathematics." *Mathematical Reviews*

### **Theory of Matrices in Numerical Analysis**

Alston S. Householder, *Oak Ridge National Laboratory*

This book develops the theory of matrices with special reference to those aspects of the theory that are most relevant to computational processes. Basic knowledge of matrices and determinants is presupposed. The author examines how the theory of norms provides error bounds, convergence criteria for infinite processes, nonsingularity criteria, localization of characteristic roots, and includes other important information. Numerous exercises and a long bibliography are included.

1964. 257 pp. \$9.00

### **Introduction to Basic FORTRAN Programming and Numerical Methods**

William Prager, *University of California, San Diego*

The organization of this second-year course and the choice of its contents are motivated by the desire to introduce students in engineering and the sciences to automatic computation as early as possible. Graduated exercises at the end of each chapter enable the reader to practice what he has learned and to check his progress.

1965. 203 pp. Paper, \$6.50

### **Theoretical and Mathematical Biology**

Edited by Talbot H. Waterman and Harold J. Morowitz, both of *Yale University*

The basic plan of this advanced text in biology and biophysics is to present a series of case histories demonstrating how special attention to adequate conceptual and quantitative methods has promoted substantial advances in many parts of the life sciences. The volume is organized in four parts following the introduction: Physical and Chemical Analysis, Statistical Analysis, Computer Applications, and Systems Analysis.

1965. 426 pp. \$12.50

BLAISDELL PUBLISHING COMPANY / A Division of Ginn and Company

275 WYMAN STREET, WALTHAM, MASS. 02154



# ***Computer oriented texts from Prentice-Hall . . .***

## **INTRODUCTION TO COMPUTING**

*by Thomas E. Hull, University of Toronto.* Provides an introductory, non-calculus approach to computer procedures and problem solving. Emphasis is on a clear-cut understanding and effective use of algorithms with special stress on FORTRAN IV used throughout. Enables students to write programs quickly after covering only the first four chapters. Includes numerous problems and projects for more effective implementing of algorithms. *June 1966, 240 pp., \$6.95*

## **BRIEF NUMERICAL METHODS**

*by Wendell E. Grove, General Motors Institute.* This new text acquaints the reader with some of the elementary numerical methods found useful in the world of computing. It covers iterative solution of equations, interpolation versus curve fitting, numerical integration and solution of ordinary differential equations. There are numerical problems (with answers) at the end of each chapter. *May 1966, approx. 128 pp., \$6.75*

## **NUMERICAL SOLUTION OF INITIAL VALUE PROBLEMS**

*by F. Ceschino and J. Kuntzman, Translated by D. Boyanovitch, Grumman Aircraft Engineering Co.* Deals with the exposition of problems relative to numerical integration of initial value problems. Theory is based on approximate representation of the derivative and integral by discontinuous expressions. The principal tool used in the development of techniques presented here is Taylor's series. (In the Automatic Computation Series.) *June 1966, approx. 352 pp., \$10.50*

## **ALGEBRAIC STRUCTURE THEORY OF SEQUENTIAL MACHINES**

*by J. Hartmanis, Cornell University, and R. E. Stearns, General Research Institute.* Presents concepts of machine theory and machine design so that a mathematician may consult the text without and experience with computers or switching theory. Similarly, a preliminary chapter on basic algebraic concepts supplies adequate mathematics to make the book self-contained for a non-mathematician. *March 1966, 224 pp., \$9.00*

## **ITERATIVE SOLUTION OF ELLIPTIC SYSTEMS AND APPLICATIONS TO THE NEUTRON DIFFUSION EQUATIONS OF REACTOR PHYSICS**

*by Eugene L. Wachpress, General Electric Co.* Gives a comprehensive description of iterative methods for solving elliptic systems with digital computers. It includes mathematical theory and useful computational techniques for anyone who must solve such systems. (In the P-H Series in Applied Mathematics.) *January 1966, 299 pp., \$12.95*

(PRICES SHOWN ARE FOR STUDENT USE.)

*for approval copies, write: Box 903*

**PRENTICE-HALL, Englewood Cliffs, New Jersey 07632**

# SCIENTIFIC PROGRAMMERS

Responsible for analysis and preparation of computer programs for solution of engineering and scientific problems utilizing IBM 360. Application includes design automation, missile trajectory simulation and missile flight test data reduction. Opportunities available in Milwaukee, Los Angeles, Boston, Houston, New York and Cape Kennedy. Degree plus 2 years or more experience in scientific programming for digital computers.

For further information call or write Mr. R. W. Schroeder, Director of Scientific Employment.

Area Code and Phone Number is 414-762-7000 (Ext 412).

## Wanted

**ASSISTANT PROFESSOR OF MATHEMATICS**, for active undergraduate mathematics department with sophisticated program of studies. Highly selective student body. Candidate must have Ph.D. degree with specialization in numerical analysis or allied field. Computer facilities available on campus. College has good salary scale, TIAA insurance and annuity plans, and master medical plan. For details write to Chairman, Department of Mathematics, College of the Holy Cross, Worcester, Massachusetts, 01610.

## **FIVE-YEAR MATHEMATICAL REVIEWS INDEX**

*Index, in two volumes, covers the years 1960-1964, volumes 21-28 of MATHEMATICAL REVIEWS. It follows the same format, includes over 6,000 reviews, and is nearly as large as the Twenty-Year Index.*

*In addition, the new index presents a list of common Chinese characters, a table including all current systems of transliterations of Chinese names, and an errata applicable to the Twenty-Year Index.*

*List Price \$39.30*

*Member Price \$29.48*

PLEASE ORDER FROM  
AMERICAN MATHEMATICAL SOCIETY  
P. O. Box 6248,  
Providence, R. I. 02904



## CLASSIFICATION OF REVIEWS

- |  |  |
|--|--|
| A. Arithmetical Tables, Mathematical Constants | N. Interest and Investment                         |
| B. Powers                                      | O. Actuarial Science                               |
| C. Logarithms                                  | P. Engineering                                     |
| D. Circular Functions                          | Q. Astronomy                                       |
| E. Hyperbolic and Exponential Functions        | R. Geodesy   |
| F. Theory of Numbers                           | S. Physics, Geophysics, Crystallography            |
| G. Higher Algebra                              | T. Chemistry                                       |
| H. Numerical Solution of Equations             | U. Navigation                                      |
| I. Finite Differences, Interpolation           | V. Aerodynamics, Hydrodynamics, Ballistics         |
| J. Summation of Series                         | W. Economics and Social Sciences                   |
| K. Statistics                                  | X. Numerical Analysis and Applied Mathematics      |
| L. Higher Mathematical Functions               | Z. Calculating Machines and Mechanical Computation |
| M. Integrals                                   |  |



# Mathematics of Computation

## TABLE OF CONTENTS

APRIL 1966

Computation of Successive Derivatives of $f(z)/z$ .....	WALTER GAUTSCHI	209
Complex Zeros of Cylinder Functions.....	BORO DÖRING	215
Reduction and Evaluation of Elliptic Integral W. J. NELLIS & B. C. CARLSON		223
Numerical Calculation of Integrals with Strongly Oscillating Integrand A. I. VAN DE VOOREN & H. J. VAN LINDE		232
Exponential Differences.....	L. M. DELVES	246
Minimax Polynomial Approximation.....	HARRY H. DENMAN	257
The Chebyshev Polynomial of Best Approximation to a Given Function on an Interval.....	O. SHISHA	266
Convective Difference Schemes.....	K. V. ROBERTS & N. O. WEISS	272
The Number of Lattice Points in a $k$ -dimensional Hypersphere W. C. MITCHELL		300
A Division Algebra for Sequences Defined on <i>all</i> the Integers D. H. MOORE		311
TECHNICAL NOTES AND SHORT PAPERS		
A Note on Equal Sums of Like Powers.....	IAN BARRODALE	318
Numerical Solutions of the Diophantine Equation $y^2 - x^2 = k$ M. LAL, M. F. JONES & W. J. BLUNDON		322
Experiments on Gram-Schmidt Orthogonalization.....	JOHN R. RICE	325
A Matrix Reduction Problem.....	J. W. MOON & L. MOSER	328
Evaluation of $I_n(b) = 2\pi^{-1} \int_0^\infty \left(\frac{\sin x}{x}\right)^n \cos(bx) dx$ and of Similar Integrals.....	RORY THOMPSON	330
Evaluation of Some Integrals Involving the $\psi$ -Function..	M. L. GLASSER	332
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS.....		334
NIKIFOROV, UVAROV & LEVITAN 47, MARTIN 48, MARTIN 49, SWITZER 50, COURTILOTT, BASSEUR & COHEN, LETTELIER & WEBER 51, MARCUS & MINC 52, SOURIAU 53, KRISHNAN 54, FRISCH-FAY 55, MICHAUD 56, BENEŠ 57, KHABAZA 58, 4-ÈME CONGRÈS DE CALCUL ET DE TRAITEMENT DE L'INFORMATION 59, BROOKS & IVERSON 60, WEISS 61, BURROWS 62, KEMENY 63.		
TABLE ERRATA		
ABRAMOWITZ & STEGUN 388, BYRD & FRIEDMAN 389, SPIRA 390.		
CORRIGENDUM		
ALEXANDROFF.....		344
NOTE		
NEW JOURNAL.....		345