Mathematics of Computation

Volume 20, Number 96
October, 1966

Published by the American Mathematical Society

PROVIDENCE, RHODE ISLAND

Editorial Committee

EUGENE ISAACSON, Chairman, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012

AVRON DOUGLIS, Department of Mathematics, University of Maryland, College Park, Maryland 20740

ALAN FLETCHER, Department of Mathematics, University of Liverpool, Liverpool 3, England

WALTER GAUTSCHI, Computer Sciences Department, Purdue University, Lafayette, Indiana 47907

ALAN J. HOFFMAN, IBM Research Center, Yorktown Heights, New York 10598

A. S. HOUSEHOLDER, Oak Ridge National Laboratory, Oak Ridge, Tennessee

PETER D. LAX, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012

Y. L. LUKE, Midwest Research Institute, Kansas City, Missouri 64110

PHILIP M. MORSE, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

HARRY POLACHEK, Research Division, U. S. Atomic Energy Commission, Washington, D. C. 20545

DANIEL SHANKS, Applied Mathematics Laboratory, David Taylor Model Basin, Washington, D. C. 20007

R. S. VARGA, Case Institute of Technology, Cleveland, Ohio 44106

J. W. WRENCH, JR., Applied Mathematics Laboratory, David Taylor Model Basin, Washington, D. C. 20007

Information for Subscribers

The journal is published quarterly in one volume per year, with issues numbered serially since Volume 1, Number 1. The subscription price is $16.00. All back volumes are available. For Volumes 1–19 (1943–1965), prices are $20.00 per volume and $6.00 per issue.

Unpublished Mathematical Tables

The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section *Reviews and Descriptions of Tables and Books*. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost.

Microcard Edition

Volumes 1–10 (1943–1956), Nos. 1–56 are now available on Microcards and may be purchased from the Microcard Foundation, Box 2145, Madison, Wisconsin 53705.

Subscriptions, address changes, business communications and payments should be sent to:

AMERICAN MATHEMATICAL SOCIETY
P. O. Box 6248
Providence, Rhode Island 02904

Copyright © 1966, American Mathematical Society
Printed in the United States of America

Second-class postage paid at Providence, Rhode Island and at additional mailing offices.

Mathematics of Computation

A Quarterly Journal
Edited by

EUGENE ISAACSON, Chairman

AVRON DOUGLIS
ALAN FLETCHER
WALTER GAUTSCHI
ALAN J. HOFFMAN
A. S. HOUSEHOLDER
PETER D. LAX
Y. L. LUKE
PHILIP M. MORSE
HARRY POLACHEK
DANIEL SHANKS
R. S. VARGA
J. W. WRENCH, JR.

XX

NOS. 93–96

1966

TABLE OF CONTENTS TO VOLUME 20

Pages

BOROSH, I. AND FRAENKEL, A. S. Exact Solutions of Linear Equations with Rational Coefficients by Congruence Techniques 107–112

BRANDT, ACHI. Estimates for Difference Quotients of Solutions of Poisson Type Difference Equations 473–499

BUTCHER, J. C. On the Convergence of Numerical Solutions to Ordinary Differential Equations 1–10

CARLSON, B. C. AND NELLIS, W. J. Reduction and Evaluation of Elliptic Integrals .. 223–231

DE BALBINE, GUY AND FRANKLIN, J. N. The Calculation of Fourier Integrals 570–589

DELVES, L. M. Exponential Differences 246–256

DENMAN, H. H. Minimax polynomial approximation 257–265

DÖRING, BORO. Complex zeros of Cylinder Functions 215–222

DURIS, C. S. A Simplex Sufficiency Condition for Quadrature Formulas 68–78

FRAENKEL, A. S. AND BOROSH, I. Exact Solutions of Linear Equations with Rational Coefficients by Congruence Techniques 107–112

FRANKLIN, J. N. AND DE BALBINE, GUY. The Calculation of Fourier Integrals 570–589

FYFE, D. J. Economical Evaluation of Runge-Kutta Formulae 392–398

GAUTSCHI, WALTER. Computation of Successive Derivatives of $f(z)/z$ 209–214

GODART, M. An Iterative Method for the Solution of Eigenvalue Problems 399–406

HABER, SEYMOUR. A Modified Monte-Carlo Quadrature 361–368

HUBBARD, BERT. Some locally One-Dimensional Difference Schemes for Parabolic Equations in an Arbitrary Region 53–59

JAGERMAN, D. Investigation of a Modified Mid-Point Quadrature Formula 79–89

KANIEL, SHMUEL. Estimates for Some Computational Techniques in Linear Algebra . 369–378

KING, RICHARD. Runge-Kutta Methods with Constrained Minimum Error Bounds .. 386–391

LAMBERT, J. D. AND SHAW, B. A Method for the Numerical Solution of $y' = f(x, y)$ Based on a Self-Adjusting Non-Polynomial Interpolant 11–20

LEAVITT, J. A. Methods and Applications of Power Series 46–52

LEHMAN, R. S. Separation of Zeros of the Riemann Zeta-Function 523–541

LESS, MILTON. A Linear Three-Level Difference Scheme for Quasilinear Parabolic Equations 516–522

LETHER, F. G. The Use of Richardson Extrapolation in One-Step Methods with Variable Step Size 379–385

MANGAD, MOSHE. Bounds for the Two-Dimensional Discrete Harmonic Green's Function 60–67

MECHEL, FR. Calculation of the Modified Bessel Functions of the Second Kind with Complex Argument 407–412

MITCHELL, W. C. The Number of Lattice Points in a k-dimensional Hypersphere .. 300–310

MOORE, D. H. A Division Algebra for Sequences Defined on *All* the Integers 311–317

NELLIS, W. J. AND CARLSON, B. C. Reduction and Evaluation of Elliptic Integrals . 223–231

ROBERTS, K. V. AND WEISS, N. O. Convective Difference Schemes 272–299

SATO, CHIKARA. Correction of Stability Curves in Hill-Meissner's Equation 98–106

SCHMID, L. P. AND SHANKS, DANIEL. Variations on a Theorem of Landau. Part I 551–569

SHANKS, DANIEL AND SCHMID, L. P. Variations on a Theorem of Landau. Part I ... 551–569

SHANKS, E. B. Solutions of Differential Equations by Evaluations of Functions ... 21–38

SHAW, B. AND LAMBERT, J. D. A Method for the Numerical Solutions of $y' = (x, y)$ Based on a Self-Adjusting Non-Polynomial Interpolant 11–20

SHISHA, O. The Chebyshev Polynomial of Best Approximation to a Given Function on an Interval 266–271

SOLOMON, ALAN. Some Remarks on the Stefan Problem 347–360

SPIRA, ROBERT. Zeros of Sections of the Zeta Function. I 542–550

STROUD, A. H. Some Fifth Degree Integration Formulas for Symmetric Regions ... 90–97

Pages

TRAUB, J. F. A Class of Globally Convergent Iteration Functions for the Solution of Polynomial Equations 113–138
TREANOR, C. E. A Method for the Numerical Integration of Coupled First-Order Differential Equations with Greatly Different Time Constants 39–45
VAN DE VOOREN, A. I. AND VAN LINDE, H. J. Numerical Calculation of Integrals with Strongly Oscillating Integrand 232–245
VAN LINDE AND VAN DE VOOREN, A. I. Numerical Calculation of Integrals with Strongly Oscillating Integrand 232–245
VERMA, ARUN. A Note on an Expansion of Hypergeometric Functions of Two Variables 413–417
———, Expansions Involving Hypergeometric Functions of Two Variables 590–596
WEISS, N. O. AND ROBERTS, K. V. Convective Difference Schemes 272–299
WIDLUND, O. B. On the Rate of Convergence of an Alternating Direction Implicit Method in a Noncommutative Case 500–515

TECHNICAL NOTES AND SHORT PAPERS

BARRODALE, IAN. A Note on Equal Sums of Like Powers 318–322
BEN-ISRAEL, ADI. A Note on an Iterative Method for Generalized Inversion of Matrices 439–440
BERGER, B. S. Inversion of the N-Dimensional Laplace Transform 418–421
BLUNDON, W. J. AND LAL, M. Solutions of the Diophantine Equation $x^2 + y^2 = l^2$, $y^2 + z^2 = m^2$, $z^2 + x^2 = n^2$ 144–147
BLUNDON, W. J., LAL, M. AND JONES, M. F. Numerical Solutions of the Diophantine Equation $y^3 - x^2 = k$ 322–325
BURROWS, J. W. Maximization of a Second-Degree Polynomial on the Unit Sphere 441–444
DAY, J. T. A Note on Best Approximation in E^n 599–600
DECELL, H. P., JR., GUSEMAN, L. F. AND LEA, R. N. Concerning the Numerical Solution of Differential Equations 431–434
EDELBLUTE, D. J. Matrix Inversion by Rank Annihilation 149–151
FAIR, WYMAN AND LUKE, Y. L. Rational Approximations to the Solution of the Second Order Riccati Equation 602–606
GLASSER, M. L. Evaluation of Some Integrals Involving the ψ-Function 332–333
GLASSER, M. L., WOOD, V. E. AND KENAN, R. P. Doppler Broadening Integrals 610–611
GUSEMAN, L. F., DECELL, H. P., JR. AND LEA, R. N. Concerning the Numerical Solution of Differential Equations 431–434
HART, R. G. A Close Approximation Related to the Error Function 600–602
JARRATT, P. Some Fourth Order Multipoint Iterative Methods for Solving Equations 434–437
JONES, M. F., LAL, M. AND BLUNDON, W. J. Numerical Solutions of the Diophantine Equation $y^3 - x^2 = k$ 322–325
KENAN, R. P., WOOD, V. E. AND GLASSER, M. L. Doppler Broadening Integrals 610–611
KRAVITZ, SIDNEY. Distributions of Mersenne Divisors 448–449
LAL, M. AND BLUNDON, W. J. Solutions of the Diophantine Equations $x^2 + y^2 = l^2$, $y^2 + z^2 = m^2$, $z^2 + x^2 = n^2$ 144–147
LAL, M., JONES, M. F. AND BLUNDON, W. J. Numerical Solutions of the Diophantine Equation $y^3 - x^2 = k$ 322–325
LANDER, L. J. AND PARKIN, T. R. Equal Sums of Biquadrates 450–451
LEA, R. N., DECELL, H. P., JR. AND GUSEMAN, L. F. Concerning the Numerical Solution of Differential Equations 431–434
LOW, R. D. On the First Positive Zero of $P_{\nu-1/2}^{(-m)}(\cos\theta)$ Considered as a Function of ν 421–424
LUKE, Y. L. AND FAIR, WYMAN. Rational Approximations to the Solution of the Second Order Riccati Equation 602–606
MOON, J. W. AND MOSER, L. A Matrix Reduction Problem 328–330
MOSER, L. AND MOON, J. W. A Matrix Reduction Problem 328–330
MUSKAT, J. B. On Divisors of Odd Perfect Numbers 141–144

Pages

NEWBERY, A. C. R. Interpolation by Algebraic and Trigonometric Polynomials.... 597–599
PARKIN, T. R. AND LANDER, L. J. Equal Sums of Biquadrates..... 450–451
PARLETT, BERESFORD. Singular and Invariant Matrices Under the *QR* Transformation..... 611–615
RICE, J. R. Experiments on Gram-Schmidt Orthogonalization..... 325–328
RUDNICK, PHILIP. Note on the Calculation of Fourier Series..... 429–430
SHANKS, DANIEL AND WRENCH, J. W., JR. Questions Concerning Khintchine's Constant and the Efficient Computation of Regular Continued Fractions..... 444–448
TAKENAGA, ROY. On the Evaluation of the Incomplete Gamma Function..... 606–610
THOMPSON, RORY. Evaluation of $I_n(b) = 2\pi^{-1} \int_0^\infty ((\sin x)/x)^n \cos(bx)\, dx$ and of Similar Integrals..... 330–332
TUNG, SHIH-HSIUNG. Boundedness of Difference Kernels of Bessel and Fourier Series..... 157–163
UCHIYAMA, SABURÔ. A Note on a Theorem of J. N. Franklin..... 139–140
VARAH, J. M. Certifications of Parlett's ALGOL Eigenvalue Procedure Eig 3..... 437–438
VERMA, ARUN. Certain Expansions of the Basic Hypergeometric Functions..... 151–157
VOIGT, R. G. On a Numerical Solution of an Integral Equation with Singularities... 163–166
WOOD, V. E. Some Integrals of Ramanujan and Related Contour Integrals..... 424–429
WOOD, V. E., KENAN, R. P. AND GLASSER, M. L. Doppler Broadening Integrals..... 610–611
WRENCH, J. W., JR. AND SHANKS, DANIEL. Questions Concerning Khintchine's Constant and the Efficient Computation of Regular Continued Fractions..... 444–448
YANG, C. H. Some Designs for Maximal $(+1, -1)$-Determinant of Order $n \equiv 2 \pmod 4$..... 147–148

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

Abramowitz & Stegun, 1. Alder, Fernbach & Rotenberg, 37. Arsac, Lentin, Nivat & Nolin, 114. Attwood, 4, 5, 6, 7, 8. Barnett, 19. Barrière, 2. Bary, 38. Bishop, Gladwell & Michaelson, 32. Becker, 39. Beeger, 68, 69. Beneš, 57. Bennett & Horst, 92. Billingsley, 73. Blanch & Clemm, 23. Blundon, Lal & Jones, 89. Brooks & Iverson, 60. Bruyevich, 44. Burrows, 62. Caslin & Fettis, 99, 100, 101. Chervonenkis, Yanpol'skii & Lyusternik, 64. Chisholm & Morris, 35. Christenson, 75. Clemm & Blanch, 23. 4-ème Congrès de Calcul et de Traitement de l'Information, 59. Copson, 27. Courant & John, 109. Davis, Martin, 45. Davis, P. J., 15, 18. Deleeuw & Southworth, 74. Ditkin, 94. Eckhaus, 108. Emersleben, 102. Fernbach, Rotenberg & Alder, 37. Fettis & Caslin, 99, 100, 101. Freeman, 20. Frisch-Fay, 55. Fry, 21. Gardiner & Wright, 66. Gauss, 86. Germain, 82. Gladwell, Bishop & Michaelson, 32. Golden, 83. Gradshteyn & Ryshik, 85. Hamashita, Kilpatrick, Katsura & Inoue, 98. Hartman, 28. Henrici, 29. Hildebrand, 40. Hollingdale & Tootill, 111. Horst & Bennett, 92. Inoue, Hamashita, Kilpatrick & Katsura, 98. Instituto Gulbenkian de Ciencia, 84. Iverson & Brooks, 60. Jennings, 79. John & Courant, 109. Jones, Blundon & Lal, 89. Karpov, 91. Katsura, Inoue, Hamashita & Kilpatrick, 98. Kaufman & Roberts, 104. Kemeny, 63. Khabaza, 58. Khrenov, 67. Khintchine, 11, 12. Kilpatrick, Katsura, Inoue & Hamashita, 98. Kleinrock, 22. Krishnan, 54. Kronrod, 30. Lal, Jones & Blundon, 89. Lebedev, 26. Lecht, 113. Lefschetz, 105. Leondes, 33, 106. Lether, 78. Levitan, Nikiforov & Uvarov, 47. Lind, Morris & Shapiro, 88. Ling, 16. Lyusternik, 65. Lyusternik, Chervonenkis & Yanpol'skii, 64. McCoy, 9. Marcus & Minc, 52. Margulis, 17. Marshall, 72. Martin, D. G., 48, 49. Martin, James, 76. Mathews & Walker, 36. Michaelson, Bishop & Gladwell, 32. Michaud, 56. Miller & Powell, 3. Minc & Marcus, 52. Morris & Chisholm, 35. Morris, Shapiro & Lind, 88. Murnaghan & Wrench, 71. Muskat, 10. Nadler, 34. Narasimha, 95. Nikiforov, Uvarov & Levitan, 47. Nixon, 31. Norkin, 80. Osipova & Tumarkin, 70. Pennington, 43. Plumb, 46. Pollack, 112. Powell & Miller, 3. Rall, 41. Revue Française de Traitement de l'Information, 51. Richardson, 107. Roberts & Kaufman, 104. Rotenberg, Alder & Fernbach, 37. Ryshik & Gradshteyn, 85. Saaty, 13, 14. Saleh, Zabransky & Sibuya, 93. Saul'yev, 77. Shapiro, Lind & Morris, 88. Sibuya, Saleh & Zabransky, 93. Slater, 103. Souriau, 53. Southworth & Deleeuw, 74. Spira, 24. Stamper, 81. Stegun & Abramowitz, 1. Switzer, 50. Syret & Wilson, 25.

Tootill & Hollingdale, **111.** Thompson, **97.** Tumarkin & Osipova, **70.** Universal Encyclopedia of Mathematics, **42.** Uvarov, Levitan & Nikiforov, **47.** Wai-Kwok Ng, **96.** Walker & Mathews, **36.** Weiss, **61.** Wilkinson, **90.** Wilson & Syret, **25.** Wrench & Murnaghan, **71.** Wright & Gardiner, **66.** Wunderlich, **87.** Yanpol'skii, Lyusternik & Chervonenkis, **64.** Zabransky, Sibuya & Saleh, **93.** Zonneveld, **110.**

ERRATA

Abramowitz & Stegun, **379, 388, 393, 399.** Adams & Hippisley, **391.** Barakat, Houston & Levin, **380.** Byrd & Friedman, **389, 397.** Caslin & Fettis, **398.** Clark & Hansen, **381.** Epperson, Spenceley & Spenceley, **396.** Erdelyi, Magnus, Oberhettinger & Tricomi, **400, 401.** Fettis & Caslin, **398.** Friedman & Byrd, **389, 397.** Friedman, Sheldon & Zondek, **394.** Glover, **395.** Gradstein & Ryshik, **392.** Hansen & Clark, **381.** Hippisley & Adams, **391.** Houston, Levin & Barakat, **380.** Humbert, Poli & McLachlan, **383.** Kaufman & Roberts, **402.** Lance, **382.** Lehmer, **403, 404.** Levin, Barakat & Houston, **380.** McLachlan, Humbert & Poli, **383.** Magnus, Oberhettinger, Tricomi & Erdelyi, **400, 401.** Natrella, **384.** Oberhettinger, Tricomi, Erdelyi & Magnus, **400, 401.** Poli, McLachlan & Humbert, **383.** Ritchie & Sakakura, **385.** Roberts & Kaufman, **402.** Ryshik & Gradstein, **392.** Sakakura & Ritchie, **385.** Sheldon, Zondek & Friedman, **394.** Spenceley, Epperson & Spenceley, **396.** Spira, **390.** Stegun & Abramowitz, **379, 388, 393, 399.** Tallqvist, **386.** Tricomi, Erdelyi, Magnus & Oberhettinger, **400, 401.** Walther, **387.** Zondek, Friedman & Sheldon, **394.**

CORRIGENDA

Alexandroff, 344. Armerding & Gruenberger, 207. Burgoyne, 643. Cody, 207. Editor's Note, 643. Gruenberger & Armerding, 207. Wrench, 643.

NOTES

Announcement	Delay Line Sieve, 645
New Journals	Cybernetics, 644
	Computing: Archives for Electronic Computing, 644
	Journal of Combinatorial Theory, 645
	Journal of Computational Physics, 345
	Journal of Computer and Systems Sciences, 644
Newsletter	Special Interest Committee on Numerical Mathematics (SICNUM), 646

Current and forthcoming titles in the Prentice-Hall

AUTOMATIC COMPUTATION SERIES

Edited by Dr. George E. Forsythe, Professor of Mathematics and Computer Science at Stanford University and Executive Head of the Computer Science Dept.

COMPUTATION: FINITE AND INFINITE MACHINES by Marvin Minsky, Massachusetts Institute of Technology

An introduction to the theories of finite-state machines, programmed computers, Turing machines and formal languages (in the form of Post Systems). Topics range from basic principles to current research problems with extensive discussions of the meaning and motivation of the theory, its practical value and limitations. This book includes guidance for self-study and further reading and study. Solutions to problems are in the text. February 1967, approx. 320 pp., $10.50

NUMERICAL SOLUTION OF INITIAL VALUE PROBLEMS by F. Ceschino, Civil Engineer, Armament, France; J. Kuntzmann, Grenoble, France (Translated by D. Boyanovitch, Grumman Aircraft Engineering Co.)

Deals with the exposition of problems relative to numerical integration of initial value problems. Theory is based on approximate representation of the derivative and integral by discontinuous expressions. The principal tool used in the development of techniques presented here as Taylor's series. June 1966, approx. 352 pp., $10.50

INTERVAL ANALYSIS by Ramon E. Moore, University of Wisconsin

Presents a new set of techniques by which a computer can be programmed to provide solutions of guaranteed accuracy to a variety of types of mathematical problems. All the necessary analysis is carried out by the computer itself for each specific application of the resulting programs. For graduate level courses and as a reference book for professional computer programmer's, mathematicians, and numerical analysts. October 1966, approx. 192 pp., $9.00

CHEBYSHEV METHODS IN NUMERICAL APPROXIMATION by Martin Avery Snyder, Courant Institute of Mathematical Sciences, New York University

Designed for numerical analysts and mathematical programmers—to show how the Chebyshev polynomials can be used in approximation to obtain nearly optimal or minimax approximations. This is the first time it has been brought together as a unified point of view. September 1966, approx. 144 pp., $7.50

(Prices shown are for student use.)

For approval copies, write: Box 903
PRENTICE-HALL
ENGLEWOOD CLIFFS,
NEW JERSEY 07632

MATHEMATICS NEWS FROM POLAND

Mieczyslaw Warmus

TABLES OF LAGRANGE COEFFICIENTS FOR CUBIC INTERPOLATIONS

(In English)

These tables permit interpolation of functions when used with the tables for arguments with k significant figures to find the values of the function with k + 5 significant figures.

Published by
Polish Scientific Publishers

In two volumes, each *501 pp. Cloth.*
$25.00 both volumes

ORDER FROM:

ARS POLONA
FOREIGN TRADE ENTERPRISE
7 Krakowskie Przedmiescie,
WARSAW, Poland

Oxford University Press

Numerical Solution of Partial Differential Equations

G. D. SMITH, Senior Lecturer in Mathematics, Brunel College of Advanced Technology, London

Designed as a textbook for students with no previous knowledge of numerical methods, this volume is also intended as a bridge to the increasing number of advanced treatises on the numerical solution of partial differential equations. The work assumes no prior training in finite-difference calculus and develops the subject clearly and in detail. Exercises and worked solutions included. *Contents:* Introduction and Finite-Difference Formulae; Parabolic Equations; Convergence, Stability, and Systematic Iterative Methods; Hyperbolic Equations and Characteristics; Elliptic Equations. References, index.

1965 190 pp. $5.00

An Introduction to Numerical Linear Algebra *With Exercises*

L. Fox, Director, University Computing Laboratory, and Professor of Numerical Analysis, Oxford University

This introduction covers the practice of matrix algebra and manipulation, and the theory and practice of direct and iterative methods for solving simultaneous algebraic equations, inverting matrices, and determining the latent roots and vectors of matrices. "It seems to be an excellent book, particularly for teaching matrix algebra to students in the computer sciences."—Thomas P. Bogyo, Washington State University

1965 344 pp. $8.50

Oxford University Press
200 Madison Ave.
New York, N.Y. 10016

From Pergamon

ANNUAL REVIEW IN AUTOMATIC PROGRAMMING

Edited by **Richard Goodman,** Automatic Programming Information Center, Brighton College of Technology, England.

Here is a continuing review of the present state of knowledge in the field. Each volume contains papers written by experts on recent advances and problems yet to be solved. Vol. 1 *1960* $12.00 Vol. 3 *1963* $12.00 Vol. 2 *1961* $12.00 Vol. 4 *1964* $12.00

U.S.S.R. COMPUTATIONAL MATHEMATICS AND MATHEMATICAL PHYSICS

Translated from *Zhurnal vychislitel'noi matematiki i matematicheskoi fiziki. Translation Editor:* **R. A. Buckingham,** London.

This translation from the original Russian contains the collated works of leading Russians concerned with computational mathematics and other branches of applied mathematics. *Bi-monthly* $140.00

ADVANCES IN PROGRAMMING NON-NUMERICAL APPLICATIONS TO COMPUTING MACHINES

Edited by **L. Fox,** University of Oxford Computing Library, England.

An effective collection of recent work with many illustrated examples.
1966 230 pp. $10.00

NUMERICAL ANALYSIS

By **I. M. Khabaza,** Queen Mary College, University of London.

"An elementary approach to desk and computer calculation, yet it contains much advanced material. The basic approach, after a pair of chapters on digital computers and desk machines, is classical via many interpolation formulas derived by symbolic operator methods." — *Mathematics of Computation*
1965 264 pp. *flexi-cover* $5.00

DIGITAL COMPUTERS IN ACTION

By **A. D. Booth,** University of Saskatchewan.

". . . one of the best introductions to digital computers and their uses that has been written. . . ." *Engineering*

The book provides a full technical exposition of recent developments.
1965 160 pp. *flexi-cover* $1.95

AN INTRODUCTION TO DIGITAL COMPUTING

By **F. H. George,** Teaching Programmes Ltd., England.

The construction and operation of digital computers is described. Familiarity with elementary arithmetic is assumed.
1966 248 pp. *flexi-cover* $4.50

THE DIGITAL COMPUTER

By **K. C. Parton,** The General Electric Company Ltd., Birmingham, England.

A survey of the use and scope of a modern computer, describing how computers deal with a wide range of technical and commercial problems. The structure of computers, automatic coding and related organizational problems are discussed. *1964* 132 pp. *flexi-cover* $2.95

Order from your bookstore or direct from:

PERGAMON PRESS

44-01 21st Street / Long Island City, New York 11101
New York • Toronto • Oxford • London • Sydney
Edinburgh • Paris • Braunschweig • Tokyo

STATISTICAL ECONOMISTS

...at ORI

New responsibilities at Operations Research Incorporated involving the evaluation and analysis of governmental economic assistance programs have created unusually attractive career opportunities for economists with training and experience in mathematics and statistics.

The work is long-range and has three objectives: (1) development of effectiveness criteria for program evaluation, (2) analysis of program goals and purposes, and (3) development and comparison of alternative programs. You will find that ORI, a small, profit-making company, offers an environment in which your efforts are quickly recognized and rewarded. Salaries are excellent, and ORI staff policies and benefits programs are geared to the needs of creative professionals. Location is at ORI's modern headquarters facility in suburban Washington, D. C.

For further information, please send your resume to: Mr. Carlton Robinson, Professional Staffing, ORI, 1400 Spring Street, Silver Spring, Maryland 20910 (suburb of Washington, D. C.)

OPERATIONS RESEARCH INCORPORATED

Washington ● Los Angeles ● Boston

An equal opportunity employer

LITTON SCIENTIFIC SUPPORT LABORATORY SEEKS SENIOR STATISTICIANS AND MATHEMATICIANS FOR POSITIONS IN MONTEREY, CALIFORNIA

The Litton Scientific Support Laboratory of the Data Systems Division, Litton Industries, has been awarded a long-term contract by the United States Army to provide scientific support to its Combat Developments Command Experimentation Command.

The CDCEC is established to serve as a field laboratory in conjunction with the Combat Developments Command. The mission of CDC is to formulate and document current Army doctrine, and to determine the kinds and numbers of forces and materiel needed . . . how such forces and materiel should be employed in anticipation of the nature of land warfare in the future. CDC is responsible for answering such questions as: how should the Army be equipped? how should it be organized? and how should the Army fight?

The Combat Developments Command Experimentation Command conducts field experiments at Fort Ord and the Hunter-Ligget Military Reservation to provide inputs to CDC. CDCEC engages a contractor to help plan, design, instrument and conduct its field experimentation program. The Litton Scientific Support Laboratory, headquartered at Fort Ord in the Monterey, California area, is the current contractor.

As a project member, or in a support capacity, you will assist in the design of field experiments; participate in or monitor the experiments to assure collection of data in a meaningful format; analyze field generated data and prepare inputs into the project reports. Positions require a master's degree and several years' in one or more of the following fields: error analysis, simulation and modeling, estimation theory, hypothesis testing, analysis of variances and regression analysis. A bachelor's degree plus additional experience will be acceptable in lieu of the master's degree.

Contact Mr. M. C. Harrold, P. O. Box 7601, Van Nuys, Calif.

DATA SYSTEMS DIVISION
LITTON INDUSTRIES

— an equal opportunity employer M/F —

CLASSIFICATION OF REVIEWS

A. Arithmetical Tables, Mathematical Constants
B. Powers
C. Logarithms
D. Circular Functions
E. Hyperbolic and Exponential Functions
F. Theory of Numbers
G. Higher Algebra
H. Numerical Solution of Equations
I. Finite Differences, Interpolation
J. Summation of Series
K. Statistics
L. Higher Mathematical Functions
M. Integrals
N. Interest and Investment
O. Actuarial Science
P. Engineering
Q. Astronomy
R. Geodesy
S. Physics, Geophysics, Crystallography
T. Chemistry
U. Navigation
V. Aerodynamics, Hydrodynamics, Ballistics
W. Economics and Social Sciences
X. Numerical Analysis and Applied Mathematics
Z. Calculating Machines and Mechanical Computation

Information for Contributors

Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see *Mathematical Reviews*, v. 28, Index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "Information for Contributors to Mathematics of Computation" and "Manual for Authors", both of which are available upon request from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to Eugene Isaacson, Chairman, Editorial Committee, Mathematics of Computation, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. Beginning with the January, 1965 issue, institutions sponsoring research reported in the journal are assessed page charges.

Mathematics of Computation

TABLE OF CONTENTS

OCTOBER 1966

Estimates for Difference Quotients of Solutions of Poisson Type Difference Equations ... ACHI BRANDT 473

On the Rate of Convergence of an Alternating Direction Implicit Method in a Noncommutative Case ... OLOF B. WIDLUND 500

A Linear Three-Level Difference Scheme for Quasilinear Parabolic Equations ... MILTON LEES 516

Separation of Zeros of the Riemann Zeta-Function ... R. SHERMAN LEHMAN 523

Zeros of Sections of the Zeta Function. I ... ROBERT SPIRA 542

Variations on a Theorem of Landau. Part I. ... DANIEL SHANKS & LARRY P. SCHMID 551

The Calculation of Fourier Integrals ... GUY DE BALBINE & JOEL N. FRANKLIN 570

Expansions Involving Hypergeometric Functions of Two Variables ... ARUN VERMA 590

TECHNICAL NOTES AND SHORT PAPERS

Interpolation by Algebraic and Trigonometric Polynomials ... A. C. R. NEWBERY 597

A Note on Best Approximation in E^n ... J. T. DAY 599

A Close Approximation Related to the Error Function ... ROGER G. HART 600

Rational Approximations to the Solution of the Second Order Riccati Equation ... WYMAN FAIR & YUDELL L. LUKE 602

On the Evaluation of the Incomplete Gamma Function ... ROY TAKENAGA 606

Doppler Broadening Integrals ... VAN E. WOOD, R. P. KENAN & M. L. GLASSER 610

Singular and Invariant Matrices Under the *QR* Transformation ... BERESFORD PARLETT 611

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS ... 616

GRADSHTEYN & RYSHIK **85**, GAUSS **86**, WUNDERLICH **87**, LIND, MORRIS & SHAPIRO **88**, LAL, JONES & BLUNDON **89**, WILKINSON **90**, KARPOV **91**, BENNETT & HORST **92**, ZABRANSKY, SIBUYA & SALEH **93**, DITKIN **94**, NARASIMHA **95**, WAI-KWOK NG **96**, THOMPSON **97**, KATSURA, INOUE, HAMASHITA & KILPATRICK **98**, FETTIS & CASLIN **99**, FETTIS & CASLIN **100**, FETTIS & CASLIN **101**, EMERSLEBEN **102**, SLATER **103**, ROBERTS & KAUFMAN **104**, LEFSCHETZ **105**, LEONDES **106**, RICHARDSON **107**, ECKHAUS **108**, COURANT & JOHN **109**, ZONNEVELD **110**, HOLLINGDALE & TOOTILL **111**, POLLACK **112**, LECHT **113**, ARSAC, LENTIN, NIVAT & NOLIN **114**

TABLE ERRATA ... 639

BYRD & FRIEDMAN **397**, FETTIS & CASLIN **398**, ABRAMOWITZ & STEGUN **399**, ERDÉLYI, MAGNUS, OBERHETTINGER & TRICOMI **400**, ERDÉLYI, MAGNUS, OBERHETTINGER & TRICOMI **401**, ROBERTS & KAUFMAN **402**, LEHMER **403**, LEHMER **404**

CORRIGENDA ...

BURGOYNE, WRENCH, EDITOR'S NOTE ... 643

NOTES ...

NEW JOURNALS ... 644

ANNOUNCEMENT ... 645

NEWSLETTER ... 646

INDICES TO VOLUME XX ...

Index of Papers and Technical Notes by Authors ... 647

Index of Reviews by Author of Work Reviewed ... 649

Index of Reviews by Subject of Work Reviewed ... 652

Index of Table Errata ... 660

Index of Corrigenda ... 661

Index of Notes ... 661