REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[F].—Margaret Ashworth & A. O. L. Atkin, *Tables of $p_k(n)$*, copy deposited in UMT file.

Define $p_k(n)$ by

$$\prod_{m=1}^{\infty} (1 - x^m)^k = \sum_{n=0}^{\infty} p_k(n)x^n.$$

Then $p_{-1}(n)$ is the well-known partition function, generally written $p(n)$, and $p_{24}(n - 1)$ is the famous Ramanujan function $\tau(n)$. The 688 computer sheets here give two tables, the first being

$$p_k(n), \qquad k = 10(1)15, \qquad n = 1(1)16200,$$

225 entries listed per page, and the second gives

$$p_k(n), \qquad k = 16(1)31, \qquad n = 1(1)1920,$$

120 entries listed per page.

There are very few published tables of these functions. In [1], Newman gives $k = 1(1)13$ to $n = 800$, and $k = 14, 15, 16$ to $n = 750, 500$, and 400, respectively. There are a number of tables of $\tau(n)$, the best published being that of Watson to $n = 1000$ [2]. The present table ($k = 24$) goes to $n = 1921$. There is a table by Lehmer [3] to 2500, said to be in the UMT file, but in fact not there at this time.

The present table was computed at the Atlas Computer Laboratory by recursive multiplication with the gap series for $k = 1$, using the Chinese remainder theorem for the needed multi-length precision. No indication of checking was included, but the reviewer has made some spot comparisons for $k = 24$ with the table in [2], and verified that Kolberg's relation for $k = 23$:

$$p(n + 1) \equiv p_{23}(n) \quad \mod 5 \quad (n \equiv 0, 1 \mod 5)$$

is valid for $n < 200$ and $800 < n < 1000$.

At some future date the second-named author plans to discuss the zeros:

$$p_k(n) = 0.$$

The copy that was deposited here was the personal copy of the reviewer. This may be superseded by a more formal copy from the authors, including the tables for $4 < k < 10$, and details of the computation.

D. S.

1. M. Newman, "A table of the coefficients of the powers of $\eta(\tau)$," *Nederl. Akad. Wetensch. Proc. Ser.* A, v. 59, 1946, pp. 204–216.
2. G. N. Watson, "A table of Ramanujan's function $\tau(n)$," *Proc. London Math. Soc.*, (2), v. 51, 1950 (paper is dated 1942), pp. 1–13.
3. D. H. Lehmer, *Tables of Ramanujan's* $\tau(n)$, UMT **101**, *MTAC*, v. 4, 1950, p. 162. Copy said to be deposited in UMT file.

2[F, G].—DANIEL B. LLOYD, "The use of finite polynomial rings in the factorization of the general polynomial," *J. Res. Nat. Bur. Standards Sect.* B, v. 69, 1965, pp. 189–212.

In the Foreword to this paper the author describes the evolution of his manuscript factor tables, which extend through degree 11 for moduli 2 and 3, through degree 8 for modulus 5, and through degree 6 for modulus 7. His first set of tables, extending through degree 6 for these four moduli, were calculated at the University of Oklahoma; the present tables, designed with a more concise format, were computed on an IBM 7090 system at the University of Maryland.

Because of the size of these tables, only excerpts for moduli 2, 3, and 5 are included in this paper, and the modulus 7 table is entirely omitted. Condensation of the tabular entries was achieved by using the octal form of the detached coefficients in the modulus 2 table, and by writing successive pairs of detached coefficients in the nonary system for the modulus 3 table. On the other hand, a condensed notation is not used in the excerpt given of the modulus 5 table.

A further abridgment of the tables was accomplished by listing only monic polynomials and by omitting reciprocal polynomials (that is, those formed by writing tabulated coefficients in reverse order), since the factors of such polynomials are the reciprocals of the factors of the original polynomials.

The author describes his procedure for finding the factors of a general polynomial over the ring of the rational integers through a study of a limited set of congruent polynomials over finite rings such as those modulo 2, 3, or 5. He restates, with proofs, three theorems [1] that underlie this procedure. Use of the tables is illustrated by the factorization of three reducible polynomials of degree 5, 8, and 10, respectively.

It is interesting to note, as the author does, that previously published tables have been restricted to the listing of *irreducible* polynomials for small prime moduli.

These tables should be useful in studies of the structure of Galois fields and to the application of congruential polynomials to error-correcting codes such as those discussed by Peterson [2].

Pertinent references, in addition to the list of 20 appended to this paper, include papers by Swift [3], Watson [4], and Tausworthe [5].

J. W. W.

1. D. B. LLOYD, "Factorization of the general polynomial by means of its homomorphic congruential functions," *Amer. Math. Monthly*, v. 71, 1964, pp. 863–870.
2. W. W. PETERSON, *Error-Correction Codes*, Wiley, New York, 1961.
3. J. D. SWIFT, "Construction of Galois fields of characteristic two and irreducible polynomials," *Math. Comp.*, v. 14, 1960, pp. 99–103.
4. E. J. WATSON, "Primitive polynomials (mod 2)," *Math. Comp.*, v. 16, 1962, pp. 368–371.
5. R. C. TAUSWORTHE, "Random numbers generated by linear recurrence modulo two," *Math. Comp.*, v. 19, 1965, pp. 201–209.

3[I].—ALEX M. ANDREW, *Table of the Stirling Numbers of the Second Kind*, with an Introduction by H. VON FOERSTER, Tech. Rep. No. 6, Electrical Engineering Research Laboratory, Engineering Experiment Station, University of Illinois, Urbana, Illinois, December 1965, 22 + 154 pp., 28 cm. Price $2.00.

This report contains the most extensive tabulation to date of the Stirling numbers of the second kind, herein designated $S(n, k)$. The table is complete for $k \leqq n = 1(1)95$; however, for $k \leqq n = 96(1)100$, a total of 98 tabular omissions occur because of the arbitrary restriction that all entries shown be less than 10^{109}.

These numbers occur naturally in the study of distributions, as is noted in the Introduction. Also of importance in combinatorial analysis is the sum

$$\sum_{k=1}^{n} S(n, k),$$

which is included in the present table, for $n = 1(1)95$.

The most extensive previous table of Stirling numbers of the second kind appears to have been in a manuscript of Miksa [1], for the range $k \leqq n = 1(1)50$, part of which has been reproduced in the NBS *Handbook* [2]. The sums of $S(n, k)$ over k were also given by Miksa, and a more extensive tabulation, for $n = 1(1)74$, has been given by Levine and Dalton [3]. None of these references is cited in this report.

The introduction to the present table includes the definition of the Stirling numbers of both the first and second kinds and the derivation of several of their properties. For more details the table-user is referred to the well-known book of Riordan [4].

The arrangement of the tabular data and their use is also described in the Introduction.

Immediately preceding the table is a description of the computer program used in performing the underlying calculations on the ILLIAC II system. The printed output consists of juxtaposed computer words in which high-order zeros were not printed; consequently, all such spaces are to be read as zeros, as noted on p. 12 of the Introduction.

Despite this imperfection in the editing of the computer output, the rather poor reproduction of the tabular material, and the omission of a bibliography, this table is a valuable addition to the literature dealing with Stirling numbers and their applications.

J. W. W.

1. F. L. Miksa, *Table of Stirling Numbers of the Second Kind*, deposited in the UMT file. (See *MTAC*, v. 9, 1955, p. 198, RMT **85**.)
2. M. Abramowitz & I. A. Stegun, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964, p. 835.
3. J. Levine & R. E. Dalton, "Minimum periods, modulo p, of first-order Bell exponential integers," *Math. Comp.*, v. 16, 1962, pp. 416–423.
4. J. Riordan, *An Introduction to Combinatorial Analysis*, Wiley, New York, 1958.

4[J, L, P, X].—V. Mangulis, *Handbook of Series for Scientists and Engineers*, Academic Press, New York, 1965, viii + 134 pp., 24 cm. Price $6.95.

Suppose one is given a function and one asks for its power series' representation or, for example, its representation in series of Bessel functions. On the other hand, suppose one is given a power series or, for example, a series involving Legendre polynomials, and one desires to identify the sum of the given series. This handbook should prove a convenient tool to answer the posed problems. As in the case of tables of pairs of Laplace transforms, no table can be complete, since such tables are by their nature infinite in character. Nonetheless, pure and applied workers should find this compendium useful.

The volume is divided into three parts. Part I states some tests for the convergence of a series and conditions for series rearrangement, multiplication, etc. Some expansion methods are briefly outlined. These include Taylor's theorem, Fourier series and Euler's summation formula. This section is by no means complete since, for example, general expansion formulas in series of orthogonal functions and Bessel functions are not mentioned, though many samples of such expansions are listed in Parts II and III. Part II is a list of series corresponding to a given function. This is divided into 12 subsections, for example, rational algebraic functions, trigonometric functions and Bessel functions of the first kind. Part III gives sums of series and is in a way the inverse of Part II. Here we are given a series, and we seek the function it represents. This portion is divided into 6 subsections, for example, series involving only natural numbers, series of algebraic functions and series of Bessel functions.

In Parts II and III, some data are given beside each entry to identify the source from which the series was taken or was deduced. This is useful to check entries and to aid in the evaluation of similar series not given in the table. In one instance, see p. 107, the author incorrectly deduces the "formula" $\int_x^\infty Y_\nu(t)dt = 2\sum_{n=0}^\infty Y_{2n+1+\nu}(x)$, $R(\nu) > -1$, a divergent expansion, from the source's correct formula $\int Y_\nu(z)dz = 2\sum_{n=0}^{m-1} Y_{2n+1+\nu}(z) + \int Y_{2m+\nu}(z)dz$, $m = 1, 2, \cdots$. Our casual reading has revealed some typographical errors. On p. 5, top of page, line 2, for $(2m + 2)$ read $(2m + 1)$. On p. 27, the letter c has been omitted in the spelling of the inverse hyperbolic functions. Aside from these and other possible discrepancies, we believe this to be a worthwhile volume.

Y. L. L.

5[K, S, X].—E. B. Dynkin, *Markov Processes*, Vols. I and II, Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, Vol. 121, Springer-Verlag, Berlin and Academic Press, New York, 1965, xii + 365 pp., 24 cm. Price $12.00.

This is a translation from the Russian book which appeared in 1963. In the last decade the theory of Markov processes in continuous time has become a serious subject. Thus it was found necessary to re-lay the foundation as the author did in his book *Die Grundlagen der Theorie der Markoffschen Prozesse* (Springer-Verlag, Berlin, 1961). The present book developed the general concepts and tools (characteristic operators, additive and multiplicative functionals, transformations, stochastic integrals), related them to known theories of harmonic functions and partial differential equations, and examined certain particular cases such as processes with continuous paths and diffusion and Wiener processes. These topics are still undergoing intensive research for which this book will be a valuable guide. It is fortunate that it is written in the author's usual careful, explicit and expansive style. Nonetheless, a casual perusal would not be easy owing to rather heavy cross-references within the book itself and to the *Grundlagen* cited above. The book would become even more useful if another more specialized and more concrete summary of the basic results in the *Grundlagen*, but restricted to the cases actually applied here, could be appended to a latter edition.

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6[L].—M. I. ZHURINA & L. N. OSIPOVA, *Tablitsy vyrozhdennoy gipergeometricheskoy funktsii* (*Tables of the confluent hypergeometric function*), Computing Center of the Academy of Sciences of the USSR, Moscow, 1964, xviii + 244 pp., 27 cm. Price 2.50 rubles.

This volume is one of a well-known series edited by V. A. Ditkin, and lists values of solutions of the confluent hypergeometric equation

$$xu'' + (\gamma - x)u' - \alpha u = 0$$

in the case $\gamma = 2$. The solution called $F(\alpha, \gamma, x)$ is regular at the origin, and is identical with the usual one, often denoted by ${}_1F_1(\alpha; \gamma; x)$, $M(\alpha, \gamma, x)$ or $\Phi(\alpha, \gamma; x)$. The solution called $G(\alpha, \gamma, x)$ is the $\Psi(\alpha, \gamma; x)$ of [1] and the $U(\alpha; \gamma; x)$ of [2]; in general, for $\gamma = 2$, it has a singularity at the origin, but $G(0, 2, x)$ is unity.

Table I (pp. 2–121) and Table II (pp. 124–243) give $F(\alpha, \gamma, x)$ and $G(\alpha, \gamma, x)$ respectively; since F and G are not mentioned in page headings, and the two tables are similarly arranged, the user has to keep his wits about him to avoid dipping into the wrong table. Both F and G are given to 7S or 7D for $\alpha = -0.98(0.02) + 1.10$, $x = 0(0.01)4.00$. No differences are given; Lagrange interpolation is used when necessary in the illustrative examples, which never involve interpolation in both α and x.

For the two integral values of α included in the range ot tabulation, namely 0 and 1, the solutions tabulated reduce to:

$$F(0, 2, x) = 1, \qquad G(0, 2, x) = 1,$$

$$F(1, 2, x) = (e^x - 1)/x, \qquad G(1, 2, x) = 1/x.$$

Thus for $\alpha = 0$ the tabulated G solution fails to be independent of the F solution; but for this value of α an independent second solution, $Ei(x) - x^{-1}e^x$, may easily be calculated from tables of the exponential integral and function.

The early part ot the Introduction contains a number of formulas relating to confluent hypergeometric functions; they include the usual formulas connecting "contiguous" functions, which allow F and G to be calculated for values of α outside the range of tabulation, and integral values of γ different from 2.

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1. A. ERDÉLYI, W. MAGNUS, F. OBERHETTINGER & F. G. TRICOMI, *Higher Transcendental Functions*, Vol. 1, McGraw-Hill, New York, 1953; Ch. 6.
2. L. J. SLATER, *Confluent Hypergeometric Functions*, Cambridge Univ. Press, New York, 1960.

7[P, Q, S, X, Z].—R. E. Bellman & R. E. Kalaba, *Quasilinearization & Nonlinear Boundary-Value Problems*, Vol. 3, American Elsevier Publishing Co., New York, ix + 206 pp., 24 cm. Price $8.50.

In this book a number of nonlinear boundary value problems are discussed, using analytic and numerical methods. Various applications—like a control problem, an inverse problem in radiation transfer theory and even in cardiology—are considered, but clearly the methods dominate the applications to these problems. The methods which the authors apply mainly consist of approximating the solution of the nonlinear problems by solving successively a sequence of linear problems as in the case of the well-known Newton-Raphson method. This approach is explained in very simple terms in the initial chapter and it is observed that for a number of problems the monotonicity of the approximating sequence can be insured and used for convergence proofs. The linear equations which one is led to are then treated by various numerical methods. The authors emphasize that this approach—which they call "quasilinearization"—is not the same as the Newton-Raphson method, but in all cases treated, the methods seem to be identical and the subtle distinction remained unclear to the reviewer. The main part of the book is restricted to boundary value problems for ordinary differential equations and a brief chapter is devoted to partial differential equations.

The book does not require a strong background in mathematics or numerical analysis. It is written in a fluent and informal style. However, while simple concepts and ideas are explained very clearly, the uninformed reader will be stopped by terms like dynamic programming, invariant imbedding techniques, which are not at all explained but frequently used in a casual manner. For example, on p. 52, one finds a derivation of the Hamilton-Jacobi equations for a simple example, however, reference is made to dynamic programming, but not to Hamilton-Jacobi. This is certainly misleading.

To sum up: This books contains an informal—not so informative—approach to some nonlinear boundary value problems with a variety of applications which are discussed briefly and supplied with computer programs.

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8[P, S, X].—Louis A. Pipes, *Operational Methods in Nonlinear Mechanics*, Dover Publications, New York, 1965, viii + 99 pp. 22 cm. Price $1.50.

Perturbation and iteration procedures are frequently employed to obtain approximate solutions of nonlinear vibration problems. Thus the nonlinear problem is replaced by a sequence of linear problems. In this book, the Laplace transform method is applied to solve the resulting linear problems. The author claims that this method reduces the algebraic labor involved in obtaining solutions. It is applied to a variety of problems most of which arise in circuit theory and are of nonautonomous type.

The purposes of the book are best described by quoting from the Introduction: ". . . the object of . . . this book (is) to present some useful methods for the solution of certain classes of important nonlinear technical problems in a manner available and understandable to the engineer and physicist interested primarily in applications. To do this, it appeared particularly desirable to present the mathematical techniques by applying them to definite examples of physical interest even at the expense of mathematical generality and elegance."

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9[P, X].—WENDELL E. GROVE, *Brief Numerical Methods*, Prentice-Hall, Englewood Cliffs, N. J., 1966, ix + 117 pp., 24 cm. Price $6.75.

This short book of 114 pages is an introduction to some elementary numerical methods. The emphasis is on presenting the numerical method and how to apply it. Although there is essentially no mathematical analysis, the methods discussed are sufficiently motivated.

The level of the book is sophomore-junior. It is probably more appropriate for engineering students than mathematics majors. It is computer oriented. The following list of chapter titles indicates the scope of the book: Iterative solution of algebraic and transcendental equations; Complex roots; Simultaneous equations; Interpolation techniques; Curve fitting; Numerical integration; Solution of differential equations; A simple boundary value problem.

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10[P, X, Z].—N. E. KOBRINSKII & B. A. TRAKHTENBROT, *Introduction to the Theory of Finite Automata*, translated from the Russian, North-Holland Publishing Co., Amsterdam, 1965, x + 337 pp., 23 cm. Price $8.40.

This book presents automata theory as applied mathematics and so is quite distinct in its treatment from much work on "abstract" automata in the United States.

The book begins with a self-contained lucid account of elementary logic. Real-time devices for processing digital data are introduced and shown to be associated with a definite class of mathematical operators. The physical characteristics of vacuum tubes (valves!), diodes, transistors, and ferromagnetic elements, are briefly discussed and their use in constructing flip-flops and in realizing basic logical operators is indicated. The problems of analyzing (going from a physical circuit to the operator it realizes) and synthesizing (going from a mathematical operator to a circuit realizing it) are discussed in detail. A final chapter describes the work of Shannon and Lupanov on asymptotic estimates for nets realizing a given operator.

The book is very well written and the English translation reads quite smoothly. It is an important contribution to the developing literature of automata theory.

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11[X].—ROBERT G. BUSACKER & THOMAS L. SAATY, *Finite Graphs and Networks, An Introduction with Applications*, McGraw-Hill Book Co., New York, 1965, xiv + 294 pp., 24 cm. Price $11.50.

In contrast to most other authors of recent books in graph theory, these writers have done little original research in the field, and this may have affected their generally excellent choice of material for an introductory book. The work is in two parts: basic theory and applications. The first deals with most of the standard topics and ideas (an exception being the four pages devoted to hypo-hamiltonian graphs). The second part has two chapters, one of which is on network flows. The other, entitled "A Variety of Interesting Applications," is over one hundred pages in length and includes sections on applications to economics and operations research, puzzles and games, engineering, and the physical and human sciences. This collection is unique, and many of the sections were written with the assistance of appropriate specialists.

The material is generally well referenced, but there are exceptions. The proof of Kuratowski's theorem characterizing nonplanar graphs is called "a refinement due to Berge." This error is partly the fault of Berge, who uses an uncorrected proof by Dirac and Shuster, and that proof is also given here. Another example of poor referencing is that for the result on the nonbiplanar character of the complete 9-point graph. There are at least two proofs in English and in more accessible journals than the article in French cited.

Graph theory is notorious for its proliferation of terminology, and this book has a selection which could (excepting such terms as *inarticulate* graphs) be adopted for general usage. There is a good selection of exercises of varying difficulty, and answers and hints are provided. Summarizing, the book is a very good one for anyone interested in learning some basic graph theory.

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12[X].—G. N. POLOZHII, *The Method of Summary Representation for Numerical Solution of Problems of Mathematical Physics*, translated from the Russian by G. J. Tee, Pergamon Press, New York, 1965, xx + 283 pp., 23 cm. Price $10.00.

In this book a method is given for the numerical solution of a class of boundary and initial value problems for second and fourth order linear partial differential equations. The method rests on the transformation of the finite difference approximation to the differential equation into a vector difference equation in one variable, and the explicit solution of the latter. This explicit solution contains open constants determined by the initial or boundary conditions.

Let the finite difference approximation be

$$R\bar{u}(x) + T\bar{u}(x) = f(x), \tag{1}$$

where (a) $\bar{u}(x) = (u_1(x), \cdots, u_n(x))$, $u_k(x) = u(x, y + kh_1)$, $k = 0, 1, \cdots$, and $h_1 > 0$; (b) $R\bar{u}(x) = \sum_{i=1}^{m} a_i[\bar{u}(x + ih) + \bar{u}(x - ih)]$, with m some (small) positive integer, a_i particular constants, and $h > 0$; (c) T is a tridiagonal matrix of the form $T = PLP$, with L diagonal and $P^2 = I$, for I the identity matrix; (d)

f is some vector function. Defining $U = P\bar{u}$, and $F = Pf$, Eq. 1 becomes

$$RU(x) + LU(x) = F, \tag{2}$$

which is a system of difference equations in U as a function of x; Eq. 2 is then solved explicitly.

The method is most effective when applied to the basic differential equations of mathematical physics for rectangular regions with many mesh points, for it does not require excessive computation and so prevents the accumulation of computational errors. For more general equations and regions it becomes difficult to use this method, for solutions to Eq. 2 cannot then be easily obtained.

The author stresses the fact that the development of new and more efficient mathematical methods is fully as important as the development of faster computing machines; his method represents a useful and interesting step in this direction.

The book is divided into two chapters and one appendix. In Chapter 1, explicit solutions are obtained for various difference equations in one variable, corresponding to Eq. 2; in Chapter 2 problems associated with Laplace's equation, the wave and heat equations and others are examined, using the author's method. In the Appendix, additional examples and some extensions of the theory are given.

The presentation of the material is at times hard to follow, and no clear explanation of the author's method is given at any point. There are some slight misprints, and two references (Nos. 16, 71) are missing. Nevertheless, the book is of definite value and interest.

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13[X].—James F. Price, *Numerical Analysis and Related Literature for Scientific Computer Users*, 2nd ed., Math. Note No. 456, Mathematics Research Laboratory, Boeing Scientific Research Laboratories, Seattle, Washington, March 1966, viii + 191 pp., 28 cm.

The explosive growth of publications in the field of numerical analysis is reflected in the relative size of the second edition of this annotated bibliography. Thus, we now find listed the contents of 151 books in English, whereas the first edition, dated May 1961, listed 69 book titles. Furthermore, the author states in the Preface that the present list is not quite as complete as the original one, and several books previously reviewed have now been dropped.

It is stated in the Introduction that this document was prepared to assist the large number of computer programmers who are not specialists in numerical analysis.

The body of this document consists of three main subdivisions. The first, entitled Numerical Procedures in Books, lists alphabetically by author the great majority of the texts in English on numerical analysis, as well as a very limited selection of related books. Those references considered by the present author to be especially helpful for computer users are designated by a double asterisk. For each book listed a brief summary is provided, indicating the level of difficulty, and the table of contents is reproduced.

Part II, entitled How to Find What You Want, describes a procedure for looking up given subjects in the literature. General guides to mathematical literature, abstracting journals, and specialized bibliographies are cited and briefly described. Special attention is given the problem of obtaining information about the large number of existing mathematical tables. Also, a list of recommended periodicals is included to assist the reader in keeping up with the current literature in numerical analysis. The four periodicals especially recommended are *Mathematics of Computation*, *Numerische Mathematik*, *Siam Journal on Numerical Analysis*, and *Computing Reviews*.

The third and concluding part of this document consists of a 27-page subject index, which is particularly useful. When more than five references are listed for a given subject the author has underlined a smaller number, which might be consulted first by the reader.

As he states in the Introduction, the author has not attempted to make this bibliography complete in any sense, nor has he included any foreign language references. He does point out, however, that many good Russian books have been translated into English and these are accordingly listed herein.

This attractively printed, conveniently arranged bibliography should provide considerable assistance to anyone searching through the English-language literature in numerical analysis.

J. W. W.

14[X].—A. H. Stroud & Don Secrest, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966, 24 cm. Price $14.95.

This is a valuable reference book for the use and application of Gaussian quadrature formulae. Not too many years ago, quadrature almost always was done with equally spaced data and the familiar Newton-Cotes quadrature formulae. The advantages of this approach are that most tables of special and other functions are given in equally spaced form so that no interpolation is required and further, much of the same data can be used for more than one formula of the Newton-Cotes variety. A disadvantage is the fact that the Newton-Cotes formulae are usually asymptotically divergent with respect to the number of points used. On the other hand, a disadvantage of Gaussian quadrature formulae is that the data are not equally spaced and that data for an n-point formula cannot be used for an m-point formula, $m > n$. Advantages of Gaussian quadrature formulae are that they converge under conditions which are most always realizable in practice and the order of precision with reference to the degree of the polynomial for which a specific formula is exact is much greater than that for the corresponding Newton-Cotes formula. The Gaussian quadrature formula nearly always uses data at points with numerical values which are irrational. But in current usage, as so much of computing is done on automatic computers, this presents no problem provided the computer can be given an efficient algorithm to compute the intergrand. Thus, Newton-Cotes formulae are by no means a relic of the past, especially if only a few calculations are needed and a desk calculator is handy, but the advent of the automatic computer enables one to exploit the advantages of Gaussian quadrature formulae over Newton-Cotes formulae while minimizing the disadvantages noted above.

The text portion of this volume is divided into five parts. Chapter 1 delineates properties of Gaussian quadrature formulae, culminating in the proof of convergence. The subject of error estimates is deferred to Chapter 4. Computation of the formulae is taken up in Chapter 2. There, beginning on p. 28, Fortran programs to compute the abscissae and weights for quadrature formulae based on the classical Jacobi, Laguerre, and Hermite polynomials are presented. Applications of the tabulated formulae to the evaluation of multiple integrals and the solution of integral equations are discussed in Chapter 3. Chapter 5 summarizes tables of quadrature formulae found in the literature.

Chapter 6 gives tables of coefficients for Legendre polynomials, $n = 1(1)16$; Chebyshev polynomials of the first and second kinds, $n = 1(1)12$; Hermite polynomials, $n = 1(1)12$; and Laguerre polynomials $n = 1(1)10$. Also given are exact coefficients for the orthogonal polynomials, with respect to the weights $|x|^\alpha$ on $[-1, 1]$, $n = 1(1)8$; $|x|^\alpha e^{-x^2}$ on $[-\infty, \infty]$, $n = 1(1)8$; $\ln(1/x)$ on $[0, 1]$, exact for $n = 1(1)4$ and to 30S for $n = 1(1)16$. The coefficients in the recursion formula for orthogonal polynomials with weight $|x|^\alpha e^{-|x|}$ on $[-\infty, \infty]$ are given to 30S for $\alpha = 1, 2$, and 3. Exact orthogonal polynomials for the evaluation of the inverse Laplace transforms are given for $n = 1(1)12$.

Let us write

$$\int_a^b w(x)f(x)\,dx \sim \sum_{i=1}^{N} A_i f(x_i).$$

The tables below summarize the $w(x)$ and $[a, b]$ for which the x_i and A_i are tabulated to 30S in Chapter 6. For each table there is a corresponding table of error coefficients to 4S. There are ten such tables like the one generally described above. Four other tables for the above integral but with slightly different right hand sides are also given. These are described below.

Table	$w(x)$	a, b	N
1	1	$[-1, 1]$	2(1)64(4)96(8)168, 256, 384, 512
2	$(1 - x^2)^\alpha$, $\alpha = -\frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2}$	$[-1, 1]$	2(1)20
3	$(1 + x)^\beta$, $\beta = 1(1)4$	$[-1, 1]$	2(1)20. For $\beta = 1$, 2(1)30
4	x^α, $\alpha = 1(1)4$	$[-1, 1]$	2(1)20
5	e^{-x^2}	$[-\infty, \infty]$	2(1)64(4)96(8)136
6	e^{-x}	$[0, \infty]$	2(1)32(4)68
7	$\vert x\vert^\alpha e^{-x^2}$, $\alpha = 1, 2, 3$	$[-\infty, \infty]$	2(1)20
8	$\vert x\vert^\alpha e^{-x}$, $\alpha = 1, 2, 3$	$[-\infty, \infty]$	2(1)20
9	$\ln(1/x)$	$[0, 1]$	2(1)16
10	$(2\pi i x)^{-1} e^x$	$[c - i\infty, c + i\infty]$	2(1)24

$$\int_a^b w(x)f(x)\,dx \sim Af(-1) + \sum_{i=1}^{N} A_i f(x_i) + Bf(1)$$

11	1	$[-1, 1]$	2(1)32(4)96, $A = B$
12	1	$[-1, 1]$	2(1)19(4)47, $B = 0$

$$\int_a^b w(x)f(x)\,dx \sim \sum_{i=1}^{N} A_i f(x_i) + \sum_{k=0}^{M} B_{2k} f^{(2k)}(0)$$

13	1	$[-1, 1]$	$N = 2(2)16$, $M = 1, 2, 3$
14	e^{-x^2}	$[-\infty, \infty]$	$N = 2(2)16$, $M = 1, 2, 3$

Y. L. L.

15[X, Z].—H. Hermes, *Enumerability, Decidability, Computability*, translated by G. T. Herman & O. Plassmann, Academic Press, New York, 1965, x + 245 pp., 24 cm. Price $9.75.

This book is a translation from the German. It is an introduction to the theory of computability (variously known as the theory of recursive functions, the theory of algorithms, etc.), in many ways comparable with the reviewer's *Computability and Unsolvability*, and requiring little of the reader by way of prerequisite.

After an introductory intuitive account of the notion of algorithm, Turing's analysis of computation is presented and used to introduce the formal theory of Turing machines. Turing computability is then defined and shown to be equivalent to μ-recursiveness (a function is μ-recursive if it is obtainable from suitable initial functions by composition, primitive recursion and minimalization) and to Herbrand-Gödel-Kleene recursiveness. Unsolvability (undecidability) results are obtained for Turing machines, Three systems (word problem for semigroups), the first and second order predicate calculi, and formal arithmetic. There are brief accounts of the arithmetic hierarchy, universal Turing machines, Church's notion of λ-definability, and recursive analysis. There is no discussion of partial recursive functions, reducibility (relative computability), or the pathology of recursively enumerable sets (e.g., simple sets are not discussed).

The exposition is careful and clear and the book is to be recommended to those seeking an introduction to computability theory.

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