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Approximations for the Psi (Digamma) Function

By William T. Moody

A series of approximations has been derived for the psi function. As used here, the psi function is defined as the derivative of the natural logarithm of the gamma function; that is

$$\psi(x) = \frac{d[\ln \Gamma(x)]}{dx} = \frac{\Gamma'(x)}{\Gamma(x)}.$$

The approximations are best in the Chebyshev sense, in that the magnitude of the maximum error in the prescribed interval is minimized. Each approximation is of the form

$$\psi(1 + x) = \frac{x}{1 + x} - \gamma + \frac{1}{2}x^{n+1} + \sum_{i=1}^{n} c_i(x^i - x^{n+1}) + \epsilon(x), \qquad 0 \leqq x \leqq 1,$$

wherein

$$\gamma = 0.5772 \cdots, \quad \text{(Euler's constant)}.$$

Values of the constants, c_i, and the limiting values of ϵ for $n = 4, 5, 6$ are given in Table 1 below. The error of the approximation vanishes at the end points.

TABLE 1
Values of Constants

n	4	5	6
$\epsilon <$	1.3×10^{-6}	1.3×10^{-7}	1.3×10^{-8}
i	c_i		
1	+0.644876	+0.6449266	+0.64493313
2	−0.201186	−0.2019040	−0.20203181
3	+0.077968	+0.0812656	+0.08209433
4	−0.026867	−0.0334532	−0.03591665
5	—	+0.0111653	+0.01485925
6	—	—	−0.00472050

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