REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

56[A-F, H, I, K-N, P, X, Z].—KARL SCHÜTTE, *Index Mathematischer Tafelwerke und Tabellen* (*Index of Mathematical Tables*), second edition, R. Oldenbourg, München, 1966, 239 pp., 24 cm. Price DM 49.00.

This new edition of Professor Schütte's index of mathematical tables constitutes a considerable enlargement of the first edition, which appeared in 1955. The format of that edition has been retained, including the use of both English and German on the title page and in the preface, the table of contents, and the headings and subheadings.

As the author notes in the preface, the number of references cited has been increased to more than 2800 from approximately 1200 in the earlier edition. The same 16 general classifications of publications are used; namely: I. Numerical and practical calculating, II. Logarithms of natural numbers, III. Logarithms of circular functions, IV. Natural values of circular functions, V. Simple functions derived from elementary functions, VI. Primes, prime factors, compound interest and rent; theory of numbers and algebra, VII. Factorials, gamma functions, exponential and hyperbolic functions; elementary transcendental functions, VIII. Elliptic functions and integrals, spherical, Bessel and other higher functions, IX. Integral tables, statistics, numerical solution of equations and differential equations, other higher functions, X. Tables applicable to physics, chemistry and other sciences, XI. Astronomy and astrophysics, XII. Geodesy, geophysics and geography, XIII. Nautical and aeronautical determination of position, XIV. Meteorology, XV. Astronautics, XVI. Tables without detailed table of contents, collections of formulas; tables of measures, weights, monetary units; miscellaneous tables.

Each of these classifications is further subdivided into sections and subsections, totalling more than 210, in place of 130 in the first edition. Within each of these subdivisions of the index, the arrangement of reference material is chronological. Mathematical tables are further grouped according to their precision; thus, for example, on p. 62 one finds 4D tables of circular functions listed chronologically, followed by 5D tables of such functions also arranged chronologically, and similarly for more precise tables.

When examining this index one is naturally led to compare it with the FMRC *Index* [1], which appeared in a second edition in 1962, following extensive revision and updating. A defect in the Schütte index that is immediately apparent upon such a comparison is the lack of any indication of the interval or range of arguments or of provision for interpolation. These deficiencies have been pointed out in a review [2] of the first edition of this index. Thus, in contrast to the FMRC *Index*, the present index is essentially merely a bibliographic listing of mathematical tables and related works, with occasionally some additional information such as the precision of a table. Certainly, this cannot rival the comparative wealth of detail available in the FMRC *Index*.

No attempt is made to present a comprehensive list of statistical tables; instead, the author refers the interested reader to the elaborate index of Greenwood & Hartley [3].

As in the first edition of this present index, there is included a list of abbreviations used in the book, an index of authors of works cited, and an index of institutes referred to in the body of the book.

Although it is disappointing to note the perpetuation of errors and deficiencies noted previously in the first edition, it should be pointed out that this new edition does serve as a valuable supplement to the FMRC *Index*, particularly with respect to the listing of publications that have appeared since about 1961.

J. W. W.

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, second edition (in two volumes), Addison-Wesley, Reading, Mass., 1962. (See *Math. Comp.*, v. 17, 1963, pp. 302–303, RMT **33**.)
2. *MTAC*, v. 10, 1956, pp. 100–102, RMT **34**.
3. J. A. GREENWOOD & H. O. HARTLEY, *Guide to Tables in Mathematical Statistics*, Princeton Univ. Press, Princeton, N. J., 1962. (See *Math. Comp.*, v. 18, 1964, pp. 157–158, RMT **13**.)

57[A, K].—RUDOLPH ONDREJKA, *The First* 100 *Exact Subfactorials*, ms. of 9 pp. (handwritten) deposited in the UMT file.

The subfactorial of n, designated here by the symbol $n¡$ following the notation of Chrystal [1], is most commonly associated with the number of derangements of n objects so that none is in its original place. This interpretation yields the well-known formula

$$n¡ = n! \sum_{k=0}^{n} (-1)^k/k!\,,$$

which implies the useful recurrence relation $n¡ = n(n-1)¡ + (-1)^n$.

The author has thereby calculated the present carefully checked table of the exact values of the first one hundred subfactorials, which appears to be by far the most extensive tabulation of its kind.

Examples of previous compilations are to be found in books by Whitworth [2] and by Riordan [3]. These extend to only $n = 12$ and $n = 10$, respectively.

J. W. W.

1. G. CHRYSTAL, *Textbook of Algebra*, 6th ed., Chelsea, New York, 1952, Vol. II, p. 25.
2. W. A. WHITWORTH, *Choice and Chance*, 5th ed., Bell, Cambridge and London, 1901, p. 107.
3. J. RIORDAN, *An Introduction to Combinatorial Analysis*, Wiley, New York, 1958, p. 65.

58[G, H, X].—FRANK S. CATER, *Lectures on Real and Complex Vector Spaces*, W. B. Saunders Co., Philadelphia, Pa., 1966, x + 167 pp., 24 cm. Price $5.00.

This is an abstract development, some of which is considered suitable for undergraduates, and all of it for first-year graduates. The presentation is quite condensed and an amazing amount of material is covered.

There are five "Parts," the first, on "Fundamental Concepts," consists of three "Lectures." The Maximum Principle and the Axiom of Choice are stated and their equivalence asserted. Other topics include the factorization of polynomials and the definition of vector spaces and linear combinations. The remaining Parts are made up of six or seven Lectures each, and each Lecture is followed by a page or more of problems. The Cayley-Hamilton Theorem and the Jordan normal form occur in Part 3. Part 4 deals with infinite-dimensional spaces and operator algebras; Part 5 with finite-dimensional unitary spaces.

As a text for presenting an abstract development the book should do very well. As a reference book for the numerical analyst who needs to look up something about matrix theory there are more accessible sources.

Not many misprints were noted, but on p. 123 "the contraction of M" appears as "the contradiction of M."

A. S. H.

59[G, H, X].—Paul A. White, *Linear Algebra*, Dickenson Publishing Co., Inc., Belmont, Calif., 1966, x + 323 pp., 24 cm. Price $8.50 (Text List), $11.35 (Trade List).

This is a carefully written, introductory text. It contains all of the material essential to such a text. The subject is introduced concretely, using ordered n-tuples, after which geometry is done within this context. Abstract, finite-dimensional, vector spaces are then developed, followed by matrices and linear transformations. Attention is paid to congruence and similarity invariants (Jordan forms, minimal polynomials, etc.). The geometric content of the subject is emphasized throughout. The logical structure is clear, since the definition-theorem-proof approach is used. Finally there are many worked-out examples, as well as a varied selection of exercises.

One apparent bonus at this level, is the introduction of the exterior product $\mathbf{u}_1 \wedge \cdots \wedge \mathbf{u}_k$, for $\mathbf{u}_i \in V$, an n-dimensional space. Unfortunately, in this reviewer's opinion, this noble attempt fails. First, the definition is very much dependent on coordinates, hence highly computational and unmotivated. Next, the definition is not standard, nor even unique, since if $\mathbf{e}_1, \cdots, \mathbf{e}_n$ is the usual basis in coordinate space, $\mathbf{e}_{i_1} \wedge \cdots \wedge \mathbf{e}_{i_k} \wedge (i_1 < \cdots < i_k)$ is defined only up to a multiplicative constant $c_{i_1 \ldots i_k}$, which leads to complications when the author speaks of "the" exterior product. Furthermore, the author (uncharacteristically) neglects to state $c_{i_1 \ldots i_k} \neq 0$ —clearly required if the usual results on linear dependence are to hold.

According to the author, the book follows the CUPM recommendations for a linear algebra course. The material has been used in NSF Institutes and in regular undergraduate classes, and despite the above objection, it is easy to believe that it proved highly successful.

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60[K].—M. J. Alexander & C. A. Vok, *Tables of the Cumulative Distribution of Sample Multiple Coherence*, Research Report RR 63–37, Rocketdyne Division of North American Aviation, Inc., Canoga Park, Calif., November 1963, nine volumes totalling 5440 pp., 32 cm. Price $50.00 (not postpaid).

The multiple coherence parameter plays a role in spectral analysis of multidimensional time series analogous to that of the squared multiple correlation coefficient in multivariate analysis. In fact, these tables can be used for the latter under the conditions described below.

If n, p, R^2, and x represent, respectively, the number of degrees of freedom, the number of records, the true coherence, and the square of the sample coherence, then under appropriate conditions [1], the sample multiple coherence is approximately distributed with probability density function

$$C(x|n, p, R^2) = \frac{\Gamma(n)}{\Gamma(p-1)\Gamma[n-(p-1)]} (1-R^2)^n x^{p-2} \times (1-x)^{n-p} F(n, n, p-1; R^2 x)$$

where $F(n, n, p-1; R^2x)$ is the hypergeometric series

$$\sum_{k=0}^{\infty} \frac{\Gamma^2(n+k)\Gamma(p-1)}{\Gamma^2(n)\Gamma(p-1+k)} \frac{(R^2x)^k}{k!}.$$

The tables presented in these nine volumes give 5D values of the cumulative distribution function $\int_0^x C(u|n, p, R^2)du$ for $p = 2(1)10$, $n = p(1)25$; $R^2 = 0(0.01)0.69$, $x = 0(0.01)1$, and $R^2 = 0.70(0.01)1$, $x = 0(0.01)0.66(0.005)1$. Each volume contains the tabular entries for a specific value of p, with the values of n arranged in ascending order.

The tables are in agreement with Pearson's tables of the incomplete beta function [2], which correspond to $R^2 = 0$, and with the Amos-Koopmans tables [3], which give the cumulative distribution of sample multiple coherence for $p = 2$. The tables were also checked internally. It is believed that the tabular errors do not exceed a unit in the final decimal place.

The tables can also be used for the distribution of the square of the multiple correlation coefficient [4]. Thus, if p', n', R^2, and x represent, respectively, the number of variables, the number of degrees of freedom, the true square of the multiple correlation coefficient, and the square of the sample multiple correlation coefficient, then the tables include entries for $p' = 3(2)19$, $n' = p + 1(2)50$, with the same ranges for R^2 and x as before.

AUTHORS' SUMMARY

1. N. R. GOODMAN, "Statistical analysis based on a certain multivariate complex Gaussian distribution (an introduction)," *Ann. Math. Statist.*, v. 34, 1963, pp. 152–177.
2. KARL PEARSON, *Tables of the Incomplete Beta-Function*, Cambridge Univ. Press, Cambridge, 1956.
3. D. E. AMOS & L. H. KOOPMANS, *Tables of the Distribution of the Coefficient Coherence for Stationary Bivariate Gaussian Processes*, Sandia Corporation Monograph SCR-483, March 1963.
4. R. A. FISHER, *Contributions to Mathematical Statistics*, Wiley, New York, 1950.

61[K, P, W, X].—W. GRANT IRESON, Editor, *Reliability Handbook*, McGraw-Hill Book Co., New York, 1966, 720 pp., 24 cm. Price $22.50.

This closely packed 720-page volume contains such a wealth of practical and useful information that it is difficult for a reviewer to write an analytical description. No other work in the reliability area comes to mind with the broad scope, the depth of detail, and the clarity of exposition of this *Handbook*. The editor and authors can justifiably take pride in the fruit of their labors.

To say that the first five sections, for example, are concerned with background mathematical and statistical concepts and tools, does not give the flavor of the content. The section on system effectiveness provides a basis for the quantitative evaluation of a system. This section, as is true of most of the others, has an aura of authority based on the author's evident intimacy with his subject. The section on characteristic life patterns contains a large portion on the construction and uses of probability paper which is too frequently completely overlooked, or arbitrarily dismissed, in statistical texts.

Test plan selection is the topic of the third section. Consideration is given to both practical and theoretical considerations. However, it might have been better to introduce this problem after the following section which is a self-contained exposition of the "Application of Mathematics and Statistics to Reliability and Life Studies." This section, running 75 pages, commences with set theory as an introduction to probability and goes through sequential analysis and the analysis of variance. Frequent graphs, tables, and examples illustrate the text.

The section on reliability estimation is at its strongest in its discussion of practical problems. Looseness exists in the statistics, such as writing the equation at the top of page 5.6 and saying "This technique is based on assumptions that are usually obtained," without specifying the assumptions. One wonders, too, what value is served by specifying, on page 5.26, that the circuit analysis program is written for the IBM 709. There is nothing in the text or the extensive flow charts which is machine-related.

Section 6 provides a comprehensive discussion of the acquisition, organization, storage, manipulation and retrieval of reliability data. The 59 pages can stand alone as a detailed introduction to this aspect of the information sciences.

The balance of the book tends to be less mathematical and machine-oriented, but no less important for a balanced exposition of the total problem area. Sections are devoted to such engineering aspects as test programs, failure analysis, engineering design and development, maintainability, human factors, and production. Practical experience pervades all these sections as it does the concluding four managerial sections on specification and procurement, acceptance testing, organization, and cost aspects.

As indicated above, the coverage is encyclopedic. The vast amount of content led the publisher to use a small type which makes extended reading or browsing more difficult. The text is supplemented by three appendices of useful tables, charts, and references. A 14-page index completes the volume. The index shows some carelessness in referring to N. Sobel and C. Moer instead of M. Sobel and C. Mooers, respectively. The term "Jacobian" has one reference in the index which neither is the first nor the only place Jacobians are mentioned. It is possibly due to the individual judgment of different authors as to importance and a consequence of a joint endeavor such as this *Handbook*. A puzzling observation, which was jarring when first noticed, was the use of the symbol "$\overline{<}$" in the table of contents of the appendix, even though the actual table or chart used the more customary "$\leq$".

Minor points aside, the enthusiasm shown in the initial sentences of this review remains. It is a handbook; it is a reference work; and in many cases it is a complete introductory treatment of many individual facets of reliability which may profitably be used in its own right.

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62[K, P, X].—L. Radanović, Editor, *Sensitivity Methods in Control Theory*, Pergamon Press, New York, 1966, xiii + 442 pp., 24 cm. Price $13.50.

This volume contains 30 papers that were presented at the International Symposium on Sensitivity Analysis, Dubrovnik, Yugoslavia in 1964 under the auspices of the Theory Committee of IFAC and is divided into five sections: I. Basic approaches, II. Sensitivity functions, III. Compensation of parameter variations, IV. Synthesis of insensitive structures, and V. Sensitivity and optimality. Among the basic approaches proposed are stability theory (I. Gumowski and Ya. Z. Tsyphin), invariant imbedding (R. Bellman, R. Kalaba, R. Sridhar), optimality and game theory (P. Dorato, R. F. Drenick), sensitivity operators for linear problems (W. R. Perkins, J. B. Cruz, Jr.), and computer methods and simulation (R. Tomović). Section II is concerned with quantitative measures of sensitivity and their use in the design of systems. Here one sees special methods for special problems. One paper (J. Vidal, W. J. Karplus, and G. Kaludjian) discusses the correction of quantization errors in hybrid computer systems. The systems in Section III are either "self-adjusting" (also called "adaptive") or "insensitive" to parameter variations and various design schemes are proposed. One of these schemes is the use of what the Russians call "the theory of invariance." "Invariance" equals complete insensitivity to a variation of certain parameters. Section IV deals exclusively with the design of insensitive systems. The sensitivity of optimal control systems is discussed in Section V.

As admitted by the editor in the Preface and substantiated by the papers, there is "no unified opinion" as to the meaning of "sensitivity" even though, like the word "stability," everyone has a feeling for what it means. The view of Bellman is that sensitivity is a concept which cannot be defined except relative to a system and what is expected of it. And this suggests mentioning that an important problem is not when is optimality insensitive (Section V) but when does optimality imply insensitivity in this pragmatic sense.

The Proceedings may disappoint someone looking for applicable results but it should be remembered that sensitivity theory is neither well defined nor well developed. The volume contributes to an understanding of the state of development of the theory, its objectives, and proposed methods of attack.

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63[K, X].—J. Keilson, *Green's Function Methods in Probability Theory*, Hafner Publishing Co., New York, 1965, viii + 220 pp., 22 cm. Price $6.50.

The central subject of the book is the theory of one-dimensional spatially and temporally homogeneous Markov processes, both unrestricted and in the presence of absorbing barriers. The theory is comparable to that in J. H. B. Kemperman's *The Passage Problem for a Stationary Markov Chain*, University of Chicago Press, Chicago, Ill., 1961. Just as Kemperman, the author relies heavily on the analysis of characteristic and moment generating functions in the complex domain. This leads to unified complex variable proofs of central limit theorems, renewal theorems, and other asymptotic results required in the applications to queues, dams, risk, and inventory problems. The emphasis on Green's functions is partly a matter of terminology, as the author so labels all transition measures (which indeed are Green's functions of the space-time process). The book is more highly recommended to the reader engaged in sophisticated applications than to the serious beginner in stochastic processes.

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64[L].—HENRY E. FETTIS & JAMES C. CASLIN, *An Extended Table of Zeros of Cross Products of Bessel Functions*, Report ARL 66-0023, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1966, v + 126 pp., 28 cm. [Copies obtainable from the Defense Documentation Center, Cameron Station, Alexandria, Virginia.]

This useful report presents 10D tables of the first five roots of the equations: (a) $J_0(\alpha)Y_0(k\alpha) - Y_0(\alpha)J_0(k\alpha) = 0$, (b) $J_1(\alpha)Y_1(k\alpha) - Y_1(\alpha)J_1(k\alpha) = 0$, (c) $J_0(\alpha)Y_1(k\alpha) - Y_0(\alpha)J_1(k\alpha) = 0$.

In particular, Table 1a gives such roots of Eq. (a) for $k = 0.01(0.01)0.99$, while Table 1b gives the corresponding normalized roots $\gamma_n = (1 - k)\alpha_n/(n\pi)$, which are better adapted to interpolation, as originally observed by Bogert [1].

The same information for Eq. (b) is given in Tables 2a and 2b. In Tables 3a and 4a we find the corresponding roots of Eq. (c) for the respective ranges $k = 0.01(0.01)0.99$ and $k = 1.01(0.01)20$; the corresponding normalized roots $\gamma_n = |1 - k|\alpha_n/[(n - \frac{1}{2})\pi]$ appear in Tables 3b and 4b. The last two tables (5a and 5b) give the roots of Eq. (c) and their normalized equivalents for $k^{-1} = 0.001(0.001)0.050$.

As the authors note, because of symmetry it suffices for Eq. (a) and Eq. (b) to tabulate the roots corresponding to $0 < k < 1$.

The values of the roots γ_n were calculated by the method of false position on an IBM 7094 system, subject to the requirement that the corresponding values of the left member of the appropriate equation not exceed 10^{-16} numerically. These values of γ_n were then converted to the corresponding values of α_n, and both sets of data were then rounded to 10D.

Previously published tables of this kind have been very limited in scope and precision; one of the most extensive of these appears in a compilation (to 5D and 8D) on page 415 of the NBS *Handbook* [2]. The present authors have announced [3] a number of errors therein as a result of their more extensive calculations.

This reviewer has compared entries in Table 2a with the corresponding 5D approximations appearing in the table of roots of $\Delta_0(\xi) = 0$ in a recent paper by Bauer [4]. The accuracy of at least 4D claimed by Bauer is now confirmed.

In a brief introduction the authors show how such equations involving Bessel functions arise in certain boundary-value problems. This is elaborated upon in Appendix 1, which shows the relation of the tables to the solution of a problem in heat conduction involving three sets of boundary conditions.

An asymptotic series for the higher roots of the equation $J_p(\alpha)Y_q(k\alpha) -$

$Y_p(\alpha)J_q(k\alpha) = 0$ is derived in Appendix 2. Two supplementary tables are included therein. The first table consists of floating-point 14S approximations to the first 20 coefficients in the asymptotic expansion of the phase angle of the Hankel function $H_p^{(1)}(x) = J_p(x) + iY_p(x)$ when $p = 0$ and 1. The second table gives floating-point 15S values of the coefficients of the first 15 partial quotients in the continued-fraction expansion of $H_0^{(1)}(x)$ and $H_1^{(1)}(x)$. This expansion was used by the authors in their evaluation of the Bessel functions $J_p(x)$, $Y_p(x) (p = 0, 1)$ for x exceeding 5; otherwise the standard power series were used.

An insert sheet clarifies a number of illegibly printed tabular entries and corrects one erroneous table title (on p. 79).

These extensive tables constitute a significant contribution to the relatively limited tabular literature relating to this class of transcendental equations.

J. W. W.

1. B. P. BOGERT, "Some roots of an equation involving Bessel functions," *J. Math. and Phys.*, v. 30, 1951, pp. 102–105.
2. M. ABRAMOWITZ & I. A. STEGUN, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series No. 55, Washington, D. C., 1964.
3. *Math. Comp.*, v. 20, 1966, pp. 469–470, MTE **393**.
4. H. F. BAUER, "Tables of zeros of cross product Bessel functions $J_p'(\xi)Y_p'(k\xi) - J_p'(k\xi)Y_p'(\xi) = 0$," *Math. Comp.*, v. 18, 1964, pp. 128–135.

65[L].—HENRY E. FETTIS & JAMES C. CASLIN, *Jacobian Elliptic Functions for Complex Arguments*, ms. of 75 computer sheets deposited in the UMT file.

These tables of the Jacobian elliptic functions $\text{sn}(u + iv)$, $\text{cn}(u + iv)$, and $\text{dn}(u + iv)$ consist of 5D values of these functions for the ranges $u/K = 0(0.1)1$, $v/K' = 0(0.1)1$, and $\sin^{-1}k = 5°(5°)80°(1°)89°$, where K and K' represent the complete elliptic integral of the first kind for modulus k and complementary modulus k', respectively.

These tabular data resulted from a test run of an IBM 1620 subroutine prepared by the authors.

Entries corresponding to a given function and a prescribed value of $\sin^{-1}k$ are arranged on a single page of computer output. No provision has been made for interpolation in the tables. Beneath the heading of each page appears a 7D approximation to the Jacobi nome, $q = \exp(-\pi K'/K)$, for the corresponding value of k.

These new tables supplement both in precision and in range the published tables of Henderson [1].

J. W. W.

1. F. M. HENDERSON, *Elliptic Functions with Complex Arguments*, The University of Michigan Press, Ann Arbor, 1960. [See *Math. Comp.*, v. 15, 1961, pp. 95–96, RMT **18**.]

66[L].—M. I. ZHURINA & L. N. KARMAZINA, *Tables and Formulae for the Spherical Functions $P^m_{-1/2+i\tau}(z)$*, Pergamon Press, New York, 1966, vii + 107 pp., 26 cm. Price $3.50.

This is an English translation of the Russian edition previously reviewed in these annals (*Math. Comp.*, v. 18, pp. 521–522, 1964, item b). The former reviewer noted a major error in the table for arc cosh x at $x = 11$ where final 689 should read 699. This error is retained in the English translation. The previous reviewer also noted that the bibliography had 43 items. The number in the present edition is 44. We should like to add that the bibliography is quite extensive though not complete. In the applications one often needs integrals involving $P^{m}_{-1/2+i\tau}(z)$ where the integration may be with respect to τ or z. In this connection and for additional references to applications, one should consult F. Oberhettinger and T. P. Higgins, *Tables of Lebedev, Mehler and Generalised Mehler Transforms,* Math. Note No. 246, October, 1961, Boeing Scientific Research Laboratories, Seattle, Washington, (*Math. Comp.*, v. 17, 1963, p. 95) the references given there, and J. Wimp, "A class of integral transforms," *Proc. Edinburgh Math. Soc.*, (2), v. 14, 1964, pp. 33–40.

Y. L. L.

67[L].—C. W. Clenshaw & Susan M. Picken, *Chebyshev Series for Bessel Functions of Fractional Order,* Mathematical Tables, Vol. 8, National Physical Laboratory, London. Her Majesty's Stationary Office, 1966, iii + 54 pp., 28 cm. Price 17s. 6d.

These short tables are a noteworthy addition to the NPL Mathematical Tables Series started in 1957.

The main body of this volume (Tables 1–28) lists the Chebyshev coefficients for the Bessel functions of real and imaginary argument for the following arguments and orders:

For $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$:

$$x \leqq 8, \nu = 0, \tfrac{1}{4}, \pm \tfrac{1}{3}, \pm \tfrac{1}{2}, \pm \tfrac{2}{3}, \pm \tfrac{3}{4}, 1 ,$$
$$x \geqq 8, \nu = 0, \tfrac{1}{4}, \tfrac{1}{3}, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{3}{4}, 1 .$$

For $K_\nu(x)$:

$$x \leqq 8, \nu = 0, 1 ,$$
$$x \geqq 8, \nu = 0, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{3}{4}, 1 .$$

The next 14 tables give for the same range of ν, in the range $x \leqq 8$, Chebyshev coefficients such that $J_\nu(x)$ and $I_\nu(x)$ can be computed from a single auxiliary function and in the range $x \geqq 8$, the Chebyshev series expansion for two auxiliary functions which permit the computation of $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$, and $K_\nu(x)$.

The last table is a double-series expansion to permit the calculation of $J_\nu(x)$ and $I_\nu(x)$ for any value of ν in the range $-1 \leqq \nu \leqq 1$ when $x \leqq 8$. For all tables the coefficients are given to a high degree of accuracy, usually 20 decimal places.

In order to use the coefficients tabulated in this report one should be familiar with the discussion of the properties of Chebyshev series and with the methods for their computation and manipulation found in Volume 5 of this series, *Chebyshev Series for Mathematical Functions* (1962) by Clenshaw. It would have been extremely useful if the pertinent formulas on summation by recurrence and on the transformation of argument necessary for even series, from Section 5 of Volume 5, were included in the present volume.

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68[L, M, T].—Y. A. KRUGLIAK & D. R. WHITMAN, *Tablitsy integralov kvantovoĭ khimii* (*Tables of quantum chemistry integrals*), Computing Center, Acad. Sci. USSR, Moscow, 1965, xlix + 440 pp., 27 cm. Price 4.76 rubles.

Approximations to solutions of the Schrödinger molecular equation are generally constructed with the help of *atomic orbitals*, that is, atomic one-electron functions, centered on the various nuclei in a molecule. In order to carry through this approach, it is necessary to evaluate a great number of difficult integrals between atomic orbitals on various centers. The various known methods for effecting these quadratures all require the evaluation of certain auxiliary functions as intermediate quantities.

The simplest of these auxiliary functions are represented by the integrals

$$A_n(\alpha) = \int_1^\infty x^n e^{-\alpha x} dx, \; B_n(\alpha) = \int_{-1}^1 x^n e^{-\alpha x} dx$$

which are related to the incomplete gamma function. Methods for evaluating these functions are well known, computer programs are available in many institutes, and a number of tables [1] have been published in the past. The numerous and considerable difficulties inherent in a study of molecular integrals are quite unrelated to these simple functions $A_n(\alpha)$ and $B_n(\alpha)$.

The present volume deals exclusively with these two functions. An excellent introduction in Russian, probably the most detailed one on this subject, sets forth the properties of the tabulated functions, the methods used in calculating the tables, and details of interpolation in the tables. Included is a complete, detailed, and accurate review of all tables of $A_n(\alpha)$ and $B_n(\alpha)$ hitherto published.

Table I (pp. 1–401) gives values of $A_n(\alpha)$ and $B_n(\alpha)$ to 6S (in floating-point form) for $n = 0(1)15$, $\alpha = 0(0.01)50$. Table II (pp. 403–439) continues this tabulation to 10, 12, or 14S for $n = 0(1)17$, $\alpha = 0(0.125)25$.

This book appears to have been published under the sponsorship of the Computing Center of the Academy of Sciences of the USSR, the Gorky State University in Kharkov, and the Case Institute of Technology in Cleveland. Table I appears to have been calculated essentially at Case Institute and was published in 1961 as a technical report of that institution. Table II is, except for two values, a copy of a previous table by Miller, Gerhauser & Matsen [2].

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1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, Vol. I, 2nd ed., Addison-Wesley, Reading, Massachusetts, 1962, pp. 305–306.
2. J. MILLER, J. M. GERHAUSER & F. A. MATSEN, *Quantum Chemistry Integrals and Tables*, Univ. of Texas Press, Austin, 1959. (For a review see *Math. Comp.*, v. 14, 1960, pp. 211–212, RMT **36**.)

69[M, X].—R. E. BARNHILL & J. A. WIXOM, *Tables Related to Quadratures with Remainders of Minimum Norm*. I, ms. of 22 typewritten pages deposited in the UMT file and reproduced on the Microfiche page attached to this issue.

These tables contain the weights w_k for a family of quadrature formulas of the following type:

$$\int_{-1}^{+1} f(x)dx = \sum_{k=1}^{n} w_k f(x_k) + R_n ,$$

where R_n denotes the error associated with using the sum in place of the integral. Different groups of weights are tabulated, one for each of ten sets of abscissas $x_1, x_2, \cdots, x_n$. These sets of abscissas are identical to those used in the following rules: trapezoïdal, Simpson, Weddle, and Gauss 2, 3, 4, 5, 7, 10, 16 point rules. A bound for the quadrature error of the form

$$|R_n| \leq ||R_n|| \; ||f||$$

exists. The norm $||R_n||$ (cf. [1]) is also tabulated. The norm $||f||$ is defined by

$$||f|| = \iint_{\epsilon(a)} |f(z)|^2 dxdy$$

or by the same relation with $f(z)$ replaced by $f'(z)$, the first derivative of $f(z)$, depending on the choice of tabulated weights; the double integral is taken over an ellipse in the complex plane with semimajor axis a and semiminor axis $b = (a^2 - 1)^{1/2}$. Weights are tabulated for different a ranging from 1.0001 to 5.0. These weights have been determined for each a and each set of abscissas by the condition that the norm $||R_n||$ be minimized. It is therefore possible for these weights to yield a smaller quadrature error than that associated with the corresponding "ordinary" weights and same abscissas; comparison of the quadrature errors for some special cases is given in reference 1.

Eleven-digit numbers are tabulated; the calculations were carried out in double precision (16 digits). The results of $||R_n||$, using the standard weights, agreed with the results obtained by Lo, Lee and Sun [2], which gives an external check on the computations. An explanation of the headings—No Precision—and—Precision for Constants—can be found in [1].

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1. R. E. Barnhill & J. A. Wixom, "Quadratures with remainders of minimum norm. I," *Math. Comp.*, v. 21, 1967, pp. 66–75.
2. Y. T. Lo, S. W. Lee & B. Sun, *Math. Comp.*, v. 19, 1965, p. 133.

70[P, S, X, Z].—Berni Alder, Sidney Fernbach & Manuel Rotenberg, Editors, *Methods in Computational Physics: Advances in Research and Applications, Vol. 5: Nuclear Particle Kinematics*, Academic Press, New York, 1966, xi + 264 pp., 23 cm. Price $11.50.

The fifth volume of this admirable series describes some applications of computers and computing technology to high-energy physics experiments and to the reduction of data from such experiments.

The last three of the five chapters are devoted to methods (hardware and software) for reclaiming experimental information from photographic records. The first chapter describes an alternative recording method, using the discharges of a spark chamber to set ferrite cores which can then be read conventionally. The second chapter tells of some uses of small computers to acquire and analyze experimental data.

The contributions are as follows.

"Automatic Retrieval Spark Chambers," by J. Bounin, R. H. Miller, and M. J. Neumann.

"Computer-Based Data Analysis Systems," by Robert Clark and W. F. Miller.

"Programming for the PEPR System," by P. L. Bastien, T. L. Watts, R. K. Yamamoto, M. Alston, A. H. Rosenfeld, F. T. Solmitz, and H. D. Taft.

"A System for the Analysis of Bubble Chamber Film Based upon the Scanning and Measuring Projector (SMP)," Robert I. Hulsizer, John H. Munson, and James M. Snyder.

"A Software Approach to the Automatic Scanning of Digitized Bubble Chamber Photographs," Robert B. Marr and George Rabinowitz.

This volume is a valuable documentation of the efforts of the authors. Even those who do not know what PEPR means may find that it and other techniques described here may have other applications (Precision Encoder and Pattern Recognition).

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71[P, W, X].—William R. Smythe, Jr. & Lynwood A. Johnson, *Introduction to Linear Programming, with Applications*, Prentice-Hall, Englewood Cliffs, N. J., 1966, xiii + 219 pp. 24 cm. Price $7.50.

This is an extremely well written introduction to linear programming and its business applications. Of the many textbooks dealing with this subject which are now available, this is certainly one of the clearest expositions which this reviewer has read. Although it requires very little mathematical background on the part of the reader, it is remarkably thorough in its coverage and includes discussions of degeneracy, finding initial solutions and other similar areas sometimes omitted in a first course. It is highly recommended as a text for a one-semester course.

Since computers have played such a large part in the development of the applications of linear programming, it is a little disappointing to find the use of computers completely ignored in this text. Instead, the authors dwell on tableaus and detailed calculations with them. It would have been preferable to change to computer programs about halfway through the text and relieve the reader from the tedium of numerical calculations. This would also have opened the possibility of discussing much more realistic applications.

Chapter 1 contains an excellent geometrical introduction to linear programming in two dimensions. All of the possibilities such as an unbounded constraint set with a finite solution, infinitely many solutions, and the like are covered in a logical, coherent way.

Chapter 2 is an introduction to linear algebra including matrices, vectors, linear dependence, rank, etc. Indeed, the reader will have an algorithm for determining the rank of a matrix after he has completed this chapter. This is in distinct contrast to most introductions to matrices where rank is defined and used in subsequent developments without giving the reader any method by which he can find the rank. With all of this elegance, however, it is rather surprising to find the transpose of a matrix conspicuous by its absence. The authors use row and column vectors interchangeably, and the use of the transpose would have made for less ambiguity in places.

Chapter 3 develops the simplex method first from a purely algebraic point of view. The geometrical ideas are discussed later in that same chapter. Problems of convergence are treated quite carefully and the appropriate theorems are proved rigorously.

Chapter 4 discusses network problems and includes the important max-flow, min-cut theorem. This chapter closes with a discussion of the transshipment and transportation problems.

Chapter 5 covers applications including the caterer problem, the cutting stock or trim problem (noninteger version), blending problems, production planning and distribution problems. It is somewhat of a disappointment not to find any engineering applications.

There are a few other minor criticisms which this reviewer had. (1) Duality is buried in Section 4-4 on the transportation problem and is discussed in a rather cursory way. Such an important idea deserves better treatment. (2) The authors never point out that a minimization problem can be changed to a maximization problem by changing the sign of the objective function. Omitting this observation keeps them concerned about whether the problem is a maximization one or a minimization one and forces them to consider two separate cases at several points. (3) The authors define a standard linear programming problem, but never say why that particular form is "standard." The observant reader will draw his own conclusions, but he could have been saved some worry with a sentence or two. (4) Early in the text (p. 5) the set of points satisfying $x_2 \leqq mx_1 + b$ is said to be those points lying on or below the line $x_2 = mx_1 + b$. The choice of the word below is unfortunate. If m is very large, it may be difficult to decide which half-plane is below the line. (5) When treating unrestricted variables, the authors make the standard substitution of $x_j = y_{2j-1} - y_{2j}$ where the $y_i \geqq 0$. However, they do not point out that this produces two columns in the tableau which are identical except for sign and the final entry. This observation can be used to reduce the number of columns which need to be carried, and hence computed, at every stage.

The work is remarkably free of typographical errors. The only one which this reviewer noticed occurred on p. 101, where the fourth line of the proof should read "Lemma 2" rather than "Theorem 2."

All in all, the criticisms raised here are of a rather carping nature and are really rather minor compared with all of the excellent features of this book. It will be difficult to improve upon this treatment of the subject. It should please both mathematicians and engineers.

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72[X].—ASKO VISAPÄÄ, *A Procedure for Finding the Coefficients of the Best Fitting Second and Third Degree Polynomials by Application of the Method of Least Squares*, The State Institute for Technical Research, Helsinki, Finland, 1966, 32 pp.

This booklet gives explicit formulas for the coefficients of second and third degree polynomials fitted to data by the method of least squares. It also gives a procedure for carrying out the computations on a desk calculator (including, in an appendix, an illustrative "program" for use on a particular electronic calculator). The given procedure, however, is defective, and this reviewer cannot recommend it. One defect of the scheme is that it provides no checks to detect arithmetic errors. The author's fifth example, where a transposition error goes undetected with the result that the calculated coefficients are wrong, clearly illustrates this pitfall. A more serious defect is the failure to consider the effect of rounding errors—errors which can easily be severe in solving linear equations. That it is usually necessary to code the data in order to reduce the rounding error, as well as to reduce the volume of computations, is ignored in the procedure given here.

A much better scheme of computations oriented to desk calculators is given in P. G. Guest's *Numerical Methods of Curve Fitting*, Cambridge University Press, 1961, pp. 147–160. The Doolittle method as presented by Guest includes a check column, and the question of coding is discussed. Moreover, Guest furnishes formulas and methods for calculating standard deviations and an analysis of variance table. Such features were omitted by Visapää.

Alternative methods of solving linear equations by desk calculators are described in considerable detail and illustrated by numerical examples in L. Fox's "Practical Solution of Linear Equations and Inversion of Matrices," a paper in *Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues*, edited by O. Taussky, National Bureau of Standards, Applied Mathematics Series, No. 39, U. S. Government Printing Office, Washington, D. C., 1954.

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73[X].—S. D. CONTE, *Elementary Numerical Analysis: An Algorithmic Approach*, McGraw-Hill Book Co., New York, 1965, x + 278 pp., 24 cm. Price $7.95.

This book offers an excellent introduction to the subject of Numerical Analysis. As the subtitle implies, computational procedures are summarized in the form of algorithms.

Many of the illustrative examples throughout the text are accompanied by flowcharts, Fortran IV programs, and computer results (obtained on an IBM 7090 system). Accordingly, the author presupposes knowledge of, or concurrent instruction in, a procedure-oriented computer language. However, he explicitly states that his emphasis is on an analysis of the accuracy and efficiency of algorithms.

The scope of the text may be inferred from the following enumeration of the main topics treated: number systems and errors, solution of nonlinear equations, interpolation and approximations, numerical differentiation and integration, matrices and systems of linear equations, and the solution of initial-value and boundary-value problems in ordinary differential equations. Unusual features include discussions of Muller's method for solving nonlinear equations, Romberg integration, and minimax polynomial approximation.

Of course, within the space limitations of any text, especially one designed for a one-semester course, a number of omissions are unavoidable. For example, in the present case we have to refer elsewhere for discussions of such computational tools as asymptotic series, continued fractions, the Monte Carlo method, and curve fitting, to name just a few.

This reviewer also noted a number of errors, most of them typographical. For example, on p. 51, 1.11 the reference should be to Eq. (2.28) instead of Eq. (2.27). On pp. 73 and 74 the value of $K(1)$ should read 1.5709 instead of 1.5708. In Chapter 4, beginning on p. 130 the numbers in the headings of the tables should be increased by a unit in the decimal digit, for example, Table 4.3 in place of Table 4.2. This correction of course entails corresponding changes in the references to these tables. On p. 134, lines 2 and 3 from the bottom, for $O(h^3)$ read $O(h^5)$. On p. 137, in formula (4.62 d) the error term involves $f^{\text{vi}}(\xi)$, not $f^{\text{iv}}(\xi)$. On p. 138, line 6 from the bottom, for $i + 0$ read $i = 0$. On p. 141, 1.11, the second and third letters in "those" have been transposed. On p. 246, in Eq. (6.71) the expression for β_2 should read

$$-\left(1 - \frac{Ah}{3}\right) + O(h^2) \quad \text{instead of} \quad -(1 - Ah) + O(h^2)\,,$$

and in Eq. (6.72) the last term should read

$$C_2(-1)^n e^{-Ax_n/3} \quad \text{instead of} \quad C_2(-1)^n e^{-Ax_n}\,.$$

Despite these minor flaws, the over-all impression is that of an attractively written, teachable textbook, supplied with a good selection of exercises for the student and an appended list of carefully selected references for further study.

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74[X].—Henry L. Garabedian, Editor, *Approximation of Functions*, Proceedings of the Symposium on Approximation of Functions, General Motors Research Laboratories, Warren, Michigan, 1964, Elsevier Publishing Company, New York, 1965, viii + 220 pp., 25 cm. Price $13.00.

This book contains the following thirteen articles:

(1) J. L. Walsh, The Convergence of Sequences of Rational Functions of Best Approximation with Some Free Poles.

(2) Arthur Sard, Uses of Hilbert Space in Approximation.

(3) R. C. Buck, Applications of Duality in Approximation Theory.

(4) Lothar Collatz, Inclusion Theorems for the Minimal Distance in Rational Tschebyscheff Approximation with Several Variables.

(5) P. Fox, A. A. Goldstein, and G. Lastman, Rational Approximation on Finite Point Sets.

(6) E. L. Stiefel, Phase Methods for Polynomial Approximation.

(7) Michael Golomb, Optimal and Nearly-Optimal Linear Approximation.

(8) E. W. Cheney, Approximation by Generalized Rational Functions.

(9) J. R. Rice, Nonlinear Approximation.

(10) F. L. Bauer, Nonlinear Sequence Transformations.

(11) P. J. Davis, Approximation Theory in the First Two Decades of Electronic Computers.

(12) Garrett Birkhoff and C. R. DeBoor, Piecewise Polynomial Interpolation and Approximation.

(13) G. G. Lorentz, Russian Literature on Approximation in 1958–1964.

The theory of approximation of functions by means of simpler functions has been on the mathematical scene for several centuries now. As the editor of this volume remarks in his preface, in recent years there has been a great resurgence of interest in this topic stimulated, without doubt, by the needs of the electronic digital computer. The collection of articles under review is a particularly good one; (2), (3), (7), (11), (12), and (13) are extremely useful surveys of the areas with which they are concerned. In (2) Sard makes the point that it is worth some effort to recast a problem in Hilbert space in view of the ease with which approximations in Hilbert space can be constructed. In (3) Buck surveys applications of a duality principle which replaces the problem of finding a closest element in a subspace to a given element of a Banach space by one of maximizing the value of a linear functional on the given element among all linear functionals of norm one which annihilate the subspace. In (7) Golomb examines the possibility of approximating best approximations by means of linear operators on the approximees. (11) is a delightful overview of the approximation scene by Davis, a genial blending of Gilbert and Hilbert. In (12) Birkhoff and De Boor give us an excellent survey of interpolation and approximation by spline functions, an area of much current research, particularly in this country. Lorentz' brief discussion in (13) of the recent Russian literature, encompassing as it does more than 160 bibliographic items, is a tour de force of incalculable value to those unable to read Russian (but able to read English).

It is clear from these papers that the single area of greatest current interest in this field is approximation by nonlinear families (i.e., functions which depend nonlinearly on their parameters. Thus polynomials are linear, since they depend linearly on their coefficients) and, in particular, rational function approximation. In (1) Walsh continues his important work in the complex domain. Walsh's book, *Interpolation and Approximation by Rational Functions in the Complex Domain*, which appeared in 1935, was one of the foundation stones on which recent developments in this area have been built, and its author remains one of the leading builders. Collatz in (4) discusses some configurations of what he calls H-sets, relatives of the alternating sets of the linear theory, in the little understood case of functions of more than one variable. In (5) the authors present algorithms for approximating functions of several variables on a discrete point set, while in (8) Cheney studies approximation by a ratio of functions each of which is a linear combination of linearly independent functions. In (9), Rice places the general nonlinear approximation problem in a geometric framework, the beginning of an ambitious and important program.

The remaining papers are (6), Stiefel's elegant new approach to polynomial approximation and (10), Bauer's survey of sequence transformations and their convergence accelerating potentialities.

All in all, this volume gives a panoramic view of a rich and flourishing field of mathematics, where new theoretical developments go hand in hand with important practical applications.

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75[X].—N. L. JOHNSON, *Tables to Facilitate Fitting S_U Frequency Curves*, New Statistical Tables Series No. 32, Biometrika Office, University College, London, University Printing House, Cambridge, England, 1965, 12 pp. Price 5s.

Let

$$z = \gamma + \delta \sinh^{-1} y$$

where y is a normal random variable with mean 0 and variance 1. The moments of z are involved functions of γ and δ. Tables with four significant figures for γ and δ are given in terms of the moment ratios $\sqrt{\beta_1}$ and β_2. The domain is $\sqrt{\beta_1} = 0.05(.05)2.00$ and β_2 from 3.2 to 15.0, first in steps of 0.1 and then in steps of 0.2.

Methods of interpolation, related tables, examples, and the method by which this table was constructed are presented.

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EDITORIAL NOTE: These tables appeared originally in *Biometrika,* v. 52, 1965, pp. 547–558.

76[X].—EUGENE ISAACSON & HERBERT BISHOP KELLER, *Analysis of Numerical Methods*, John Wiley and Sons, Inc., New York, 1966, xv + 541 pp., 24 cm. Price $11.95.

This book on numerical analysis has certain special features which should make it a welcome addition to the array of texts on this subject. Its position is somewhere in between a text for a stiff undergraduate course and a text for a moderate first graduate course. It contains a great deal of material, which is somewhat surprising since it is written in a style which avoids conciseness in presentation. This almost breezy approach to a mathematics text is, from my point of view, good because it gives a feeling of familiarity or of being comfortable with the ideas and techniques of the subject.

The book suffers from a complete absence of numerical examples, which must be supplied independently.

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77[X].—C. BALLESTER & V. PEREYRA, *Supplement to Bickley's Table for Numerical Differentiation*, ms. of 19 typewritten pages deposited in the UMT file and reproduced on the Microfiche page attached to this issue.

This unpublished table consists of the exact values of the integer coefficients ${}_{mn}A_{pr}$ and the coefficients to 5S (in floating-point form) of the error terms ${}_{mn}E_p$ for the discrete approximations

$$\frac{h^m}{m!} y^{(m)}(x_p) = \frac{1}{(n-1)!} \sum_{r=0}^{n-1} {}_{mn}A_{pr} y(x_r) + {}_{mn}E_p ,$$

where $x_r = x_0 + rh$, $p = 0(1)(n-1)$ for the ranges $m = 1(1)6, n = 7, 9; m = 5, 6, n = 8, 10$. The underlying calculations were performed on a CDC 3600 system.

An abridgement of Bickley's table [1] is given in the NBS *Handbook* [2]. The present authors have generated his entire table by the method of Gautschi [3] and thereby confirmed its accuracy.

The error term ${}_{mn}E_p$ can be expressed as ${}_{mn}e_p h^n y^{(n)}(\xi)$, where

$${}_{mn}e_p = -\frac{1}{n!(n-1)!} \sum_{j=0}^{n-1} (j-p)^n {}_{mn}A_{pj} .$$

For derivatives of even order the quantity ${}_{mn}e_{1/2(n-1)}$ vanishes, and the resulting symmetric formula is then accurate to an extra order of magnitude in h. Such error coefficients are identified in this supplementary table by an asterisk.

The authors include references to publications by Gregory [4] and Collatz [5]; however, they have not cited the relatively inaccessible tables of Kuntzmann [6], [7], which contain similar information for the first 10 derivatives.

J. W. W.

1. W. G. Bickley, "Formulae for numerical differentiation," *Math. Gaz.*, v. 25, 1941, pp. 19–27.

2. M. Abramowitz & I. A. Stegun, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover, New York, 1965, p. 914. (See *Math. Comp.*, v. 20, 1966, p. 167, RMT **1**.)

3. W. Gautschi, "On inverses of Vandermonde and confluent Vandermonde matrices," *Numer. Math.*, v. 4, 1962, pp. 117–123.

4. R. T. Gregory, "A method for deriving numerical differentiation formulas," *Amer. Math. Monthly*, v. 64, 1957, pp. 79–82.

5. L. Collatz, *The Numerical Treatment of Differential Equations*, 3rd ed., Springer, Berlin, 1959.

6. J. Kuntzmann, *Formules de Dérivation Approchée au Moyen de Points Équidistants*, Report No. 1.373/1, Société d'Électronique et d'Automatisme, Courbevoie (Seine), France, 1954. (See *MTAC*, v. 10, 1956, pp. 171–172, RMT **51**.)

7. J. Kuntzmann, *Evaluations d'Erreur dans les Représentations Approchées de Derivées*, Société d'Électronique et d'Automatisme, Courbevoie (Seine), France, 1955. (See *MTAC*, v. 12, 1958, pp. 104–105, RMT **56**.)

78[X, Z].—Edmund C. Berkeley & Daniel G. Bobrow, Editors, *The Programming Language LISP: Its Operation and Applications*, The M.I.T. Press, Cambridge, Mass., 1966 (second printing), ix + 382 pp., 20 cm. Price $5.00.

This is a reprint of a collection of articles prepared several years ago by Information International for the Department of Defense. It was formerly available from the Department of Defense Documentation Center.

The LISP programming language was devised by John McCarthy in 1960 for symbol manipulation. It is based on function composition and evaluation according to Church's notion of lambda-conversion. Its data structures are in the form of lists of elements, with each element, which itself can be either an atom or a list, containing a pointer linking it to its successor. An ingenious scheme for automatic collection of unused words, together with convenient handling of recursive functions, relieves the programmer of most of the book-keeping.

Perhaps more important than the precise handling of the data structures is the fact that the program is stored in the form of such a structure, with the result that it can be manipulated by the program itself. Interpreters and compilers within LISP are simply functions which do such manipulation. In particular, the interpreter can be written in about a page of LISP. The simplicity of the interpretive process reflects the clarity of syntax and semantics in the language. Unfortunately, such clarity is not appreciated by the casual Fortran programmer, who soon tires of prefix notation for arithmetic and assignment operations and the innumerable parentheses. The latest version of LISP, LISP 2, will allow Algol-like notation, as well as more types of data structure.

LISP provides unquestionably the best existing introduction to nonnumeric programming. Although this book is far from ideal for teaching purposes, it is the only one available other than the LISP 1.5 manual, also by M.I.T. Press. The first five articles are tutorial in nature, including exercises and comments on debugging and programming styles. The second section contains descriptions of implementations for the Q-32 and M460 computers, articles describing applications of LISP to problems in logic and inference, and descriptions of extensions of LISP. Lengthy appendices contain code for a number of the papers. Three articles, the two by Saunders and the one by Hart and Evans, provide a reasonable introduction to LISP and its implementation.

As machines become faster, and it becomes apparent that mere speed does not solve the more significant nonnumeric problems, programming languages with the power and flexibility of LISP will become increasingly important.

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79[X, Z].—TORGIL EKMAN & CARL-ERIK FRÖBERG, *Introduction to ALGOL Programming*, Studentlitteratur, Lund, Sweden, 1965, 123 pp., 25 cm.

This informal exposition of all of Algol 60 is very carefully done and stresses elegance of expression in programming. The text contains many examples and there are about fifty exercises with solutions. Diagrams are used to explain conditional expressions and block structure, and as a unique feature, the book contains two photographs of some of the personalities behind Algol.

The first of the twenty chapters gives a brief history of computers and programming, Chapters 2–14 explain Algol programming, and the last six chapters deal with the following topics: The Algol report, peculiar and controversial features of Algol, the IFIP input-output primitives, the IFIP Algol subset, stack compilation of an arithmetic expression, and "the future of Algol."

A bibliography and a two-page index complete the book.

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80[X, Z].—RICHARD V. ANDREE, *Computer Programming and Related Mathematics*, John Wiley & Sons, Inc., New York, 1967, 284 pp., 24 cm. Price $6.50.

Computer Programming and Related Mathematics by Professor Richard V. Andree introduces the reader to the basics of computer programming through a fundamental symbolic language called GOTRAN and its more sophisticated relative FORTRAN. Both these languages are suitable for processing on the IBM 1620, a computer which is quite common in educational establishments. Towards the end of the text the reader is introduced to both SPS and machine-language coding, although it should be stated that these are not emphasized and serve merely to whet the appetite of the ambitious reader.

It is refreshing to note that Professor Andree successfully resists the temptation to couch his ideas in overly technical language; even the examples he draws upon are explained simply and accurately. The mathematics he introduces is minimal and thus he is able to lead even an apprehensive student through the intricacies of programming in a somewhat painless fashion.

The text is replete with flow-charts, problems of a wide variety, and specimen programs to enable the student to follow the concepts with maximum ease. It is to the credit of the author that at no time does he "talk down" to the reader.

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81[Z].—MARTIN GREENBERGER, MALCOLM JONES, JAMES H. MORRIS, JR. & DAVID N. NESS, *On-Line Computation and Simulation: The OPS-3 System*, The M.I.T. Press, Cambridge, Mass., 1965, xi + 126 pp., 26 cm. Price: $4.95.

This book describes a conversational programming system which is being used on M.I.T.'s CTSS time sharing system. Although evidently written as a user's manual, it should also be of interest to those who will have no opportunity to use the system. It is written in a simple and readable style, introducing the various features gradually, so that a single reading produces a reasonable working knowledge.

OPS-3 provides facilities for input, editing, and execution of programs from a teletype-like terminal. The basic unit is the "operator," a pre-compiled subroutine written in FORTRAN, MAD, or FAP. The additional flexibility necessary for convenient on-line use is provided by a permanently-available symbol table, and flexible interpretation of operator parameters. Standard operators in the system include the usual numerical operators, statistical operations, operators for simulation and polynomial manipulation. In addition, there are facilities for creating compound operators, which are then available for interpretive execution or, if necessary, for compilation.

The language made available to the user by means of these operators is somewhat clumsy, in general using prefix notation without delimiters. Thus, while

$$\text{FIT } Y \text{ TO } X1\ X2\ X3$$

looks perfectly reasonable

PRINT A SUM I 2 DIFF J 3

does not. Prefix notation is not used for certain special operators, so that

$$\text{SET } A = B + \text{LOG}\cdot(C + D)$$

is a valid statement, and can be used to process either scalars, vectors, or arrays. This flexible treatment of arguments is the core of OPS-3, and details of its implementation are given. One interesting trick used by the system to determine whether an operator wants its arguments evaluated is to examine its transfer vector for certain routines, and to evaluate them if not found.

The over-all impression given is that OPS-3 evolved by addition of new facilities to earlier versions. The resulting system appears something of a patchwork, with very little structure. However, it does contain a number of features which will be useful in future systems.

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82[Z].—THE STAFF OF COMPUTER USAGE COMPANY, ASCHER OPLER, Editor, *Programming the IBM System*/360, John Wiley & Sons, Inc., New York, 1966, xii + 316 pp., 24 cm. Price $7.50.

This book is intended chiefly for experienced programmers who wish to program in assembly language. For the programmer who is thoroughly familiar with the System/360 assembly language instruction set, the book provides a concise, tightly written account of the applications of the instructions and provides techniques for the accomplishment of a large number of tasks. The book is thus an excellent reference.

The book contains very little explanation of the specific functions and effects of the instructions, and for this reason it is essential to have a publication such as *The* IBM *System*/360: *Principles of Operation* available, as the preface suggests.

For the programmer who is not familiar with the instruction set, much time is spent referring to *Principles of Operation*, and therefore the book is somewhat inefficient as a text.

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83[Z].—MARIO V. FARINA, *Fortran* IV *Self-Taught*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xi, 426 pp. 28 cm. Price $5.95.

Of the many Fortran IV instructional books which have recently appeared this self-educational introduction by Mario V. Farina is one of the best I have seen to date. It assumes no previous knowledge whatever on the part of the reader. He is exposed to the many facets of the Fortran IV language and repertoire in a gradual and well planned manner. There are over 400 pages to this work but the examples and explanations are carefully placed and spaced so that the reader does not have to fight his way through a mire of complicated diagrams and descriptions. All of the fundamentals of Fortran IV are present but little of the programming techniques. Perhaps one would seek such techniques elsewhere. There is a slight bias in the presentation towards the GE 625 computer, but this is to be expected since the author works for the General Electric Company's Telecommunications and Information department.

Included in the text are 30 so-called "think questions" which are to be found scattered in the various exercises. In view of their irrelevance to programming I would have thought twice about including them.

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