**84[A, F].**—M. Lal, *Expansion of* $\sqrt{3}$ *to* 19600 *Decimals*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, ms. of 2 typewritten pp. + computer printed table, deposited in UMT file.

The result here is very similar to Lal's previous work on $\sqrt{2}$. (See UMT **17,** this volume of *Math. Comp.*, for a detailed review of that computation.) The method, computer, and computation time here are the same as in the previous computation. Each of 19 blocks of 1000 digits has a decimal-digit count and an evaluated $\chi^2$ to 2D. The distribution appears to be random.

Lal's decimal-digit count for $\sqrt{3}$ at 14000D agrees with that of Takahashi & Sibuya (UMT **18,** this volume). His digits 13901–14000 also were checked against theirs and complete agreement was found.

D. S.

**85[F, J].**—D. E. Knuth & T. J. Buckholtz, *Tables of Tangent Numbers, Euler Numbers, and Bernoulli Numbers*, California Institute of Technology, Pasadena, California, January 1967, ms. of 311 computer sheets (unnumbered), 28 cm., deposited in the UMT file.

The first part of this manuscript consists of a 95-page table of the exact values of the first 404 Euler numbers, designated $E_{2n}$ and taken here as all positive. The most extensive table of these numbers previously calculated appears to be that of Joffe [1], consisting of 50 entries, reproduced by Davis [2].

The remaining table in this manuscript is a 216-page compilation of the first 418 tangent numbers and corresponding numbers $C_{2n}$, for $n = 1(1)418$, from which the nonvanishing Bernoulli numbers can be obtained by the relation $B_{2n} = C_{2n} - \sum 1/p$, where the sum is taken over all primes $p$ such that $(p - 1)|2n$, by virtue of the von Staudt-Clausen theorem. These primes are listed with each $C_{2n}$.

The tangent numbers, designated $T_{2n-1}$ by the present authors, are integers defined by the Maclaurin expansion

$$\tan x = \sum_{n=1}^{\infty} T_{2n-1}x^{2n-1}/(2n-1)!\,, \qquad |x| < \frac{\pi}{2},$$

and are, accordingly, related to the Bernoulli numbers by the formula

$$(-1)^{n+1}T_{2n-1} = 2^{2n}(2^{2n} - 1)B_{2n}/(2n)\,.$$

Previous tabulations of the exact values of $T_{2n-1}$ and $B_{2n}$ extend to at most $n = 30$ and $n = 110$, respectively [3], [4].

The calculations underlying the present tables constitute an extension of similar calculations of Bernoulli numbers carried out by Dr. Knuth in the course of his evaluation [5] of Euler's constant on a Burroughs 220 system. The present calculations, on the other hand, were performed on an IBM 7094 system, in a total running time of approximately 25 minutes. Further details of the method of calculation of these tables are set forth in a paper [6] appearing elsewhere in this journal.

J. W. W.

1. S. A. Joffe, "Calculation of eighteen more, fifty in all, Eulerian numbers from central differences of zero," *Quart. J. Math.*, v. 48, 1920, pp. 193–271.

2. H. T. Davis, *Tables of the Mathematical Functions*, vol. II, revised edition, Principia Press of Trinity University, San Antonio, Texas, 1963.

3. J. Peters, *Ten-Place Logarithm Tables*, new, revised edition, Ungar, New York, 1957, v. 1, *Appendix*, pp. 83–86, 88.

4. D. H. Lehmer, "An extension of the tables of Bernoulli numbers," *Duke Math. J.*, v. 2, 1936, pp. 460–464.

5. D. E. Knuth, "Euler's constant to 1271 places," *Math. Comp.*, v. 16, 1962, pp. 275–281.

6. D. E. Knuth & T. J. Buckholtz, "Computation of tangent, Euler, and Bernoulli numbers," *Math. Comp.*, v. 21, 1967, pp. 663–688.

**86[G, H, I, M, X].**—A. M. Ostrowski, *Solution of Equations and Systems of Equations*, Second Edition, Academic Press, New York, 1967, xiv + 338 pp., 24 cm. Price $11.95.

In his inimitable style, the author produces a major revision of the first edition which is best described in the words of his Preface to the second edition:

"For this second edition, the entire text was thoroughly revised and much new material added so that the size of the book is almost doubled.

"The new material deals with those methods which can be used with the automatic computer without any special preparation. The Laguerre iteration and its modifications are extensively analyzed, since this iteration can be used, at least for polynomials with only real zeros, starting with an arbitrary real value. In two chapters and one appendix we treat the approximation of a zero by zeros of interpolating polynomials, as the extensive experimentations by D. I. Muller make it appear probable that in many cases this method is not sensitive with respect to the choice of the starting value. In several chapters we deal with the method of steepest descent. Although, in the case of one variable, the complete working through of this method to its practical use with a computer was achieved too late to be included in the book, the method of steepest descent gives a nonsensitive although rather slow approach for large classes of systems of equations, a subject that was somewhat neglected in the first edition. In this respect the four chapters preceding the generalization of the Newton-Raphson method to the case of several variables may be welcome to 'pure' as well as to 'applied' numerical analysts. Finally, the discussion of the theory of divided differences, added in this edition, may help to make this theory more widely known."

The review of the first edition by H. Schwerdtfeger [1] states in its final paragraph,

"This is certainly a remarkable book which will be welcome to everybody who wants suggestions for original work or for the preparation of lectures in numerical analysis with stress on mathematical rigour. The absence of exercises finds its counterbalance in the style, which calls for constant attention and collaboration of the reader. Finally, the excellent printing and generally pleasing appearance of the book deserve to be mentioned."

The effort required of the reader of the second edition is somewhat greater than was required for reading the first edition. The reward is correspondingly larger.

E. I.

1. Review B571, *Math. Reviews*, v. 23, 1962, p. 97.

**87[G, H, X].**—F. A. Ficken, *Linear Transformations and Matrices*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1967, xiii + 398 pp., 24 cm. Price $10.50.

In spite of many entertaining historical digressions, this is a tight book. Much ground is covered. It is assumed that the reader knows only elementary Cartesian geometry and trigonometry. So the early chapters start out with set theory, mathematical induction, propositions, the real numbers, order, vectors in three dimensions, groups, isomorphism, and related notions. Chapter 5, "Linear spaces," deals with linear dependence and bases. Chapter 6 is on "Linear transformations," and duality enters in Chapter 7. Determinants arise in Chapter 11. The Jordan canonical form comes in the next chapter, and Chapter 14, the last, deals with "Similar operators on a unitary space." Interspersed are 760 exercises (by the author's count), and at the end are seven pages of bibliography, carefully classified, 31 pages of "Selected answers and hints," three pages for an index of symbols, and an index of eight pages.

The book has developed through in-training courses given at the Gaseous Diffusion Plant in Oak Ridge, and courses given at the University of Tennessee and at New York University. Generally, these have been three-quarter courses. Throughout, the emphasis is on geometry and applications, hence, so far as possible, on coordinate-free representation.

The phraseology is meticulously precise and highly literate (by contrast with much current literature). To teach to the audience intended would be pleasant but possibly demanding. For readers of this journal it should be remarked that little attention is given to computational techniques, but this is a subject in itself.

A. S. H.

**88[G, H, X].**—André Korganoff & Monica Pavel-Parvu, *Éléments de Théorie des Matrices Carrées et Rectangles en Analyse Numérique*, Dunod, Paris, 1967, xx + 441 pp., 25 cm. Price 98 F.

This is the second book of a series, and it must be said at the outset that this book is at a level quite different from that of its predecessor. For a reading of Volume 1, little was required beyond reasonable mathematical maturity. The present volume is divided into two parts, entitled "Vectorial algebras and normal algebras of matrices," and "Inverses of rectangular matrices," and it begins with Chapter 1.1, whose title may be translated as "Recollections of functional analysis" (the first word is "rappels," and there is no strict English equivalent). The point is simply that many theorems are stated without proof. The other two chapters, covering a total of nearly a hundred pages, deal with norms. And still the proofs are minimal or omitted altogether. We are promised a third volume that will, presumably, fill the gaps.

The main theme occurs in the second part, and is easily recognized as the "pseudo-inverse" (as the authors call it), the "generalized inverse" (as it is often designated), or the "general reciprocal" (in the phraseology of E. H. Moore, the inventor). The fact that the treatment extends to more than 250 pages indicates the amplitude of the development.

There are three separate bibliographies, one "general," and one for each "part." There is no index. There are numerical examples. It is a book only for the mathematically mature professional, who will find therein a vast amount of information.

A. S. H.

**89[H, K, M, P, Q, S, T, V, W, X, Z].**—Ben Noble, *Applications of Undergraduate Mathematics in Engineering,* The Mathematical Association of America, The Macmillan Co., New York, 1967, xvii + 364 pp., 24 cm., Price $9.00.

This delightful book goes a long way towards making it clear that "Part of the art of engineering mathematics is to balance the complexity of the engineering problem and the sophistication of the mathematics used against the degree of accuracy and certainty required in the final conclusion." The author, who is an eminent artist in this medium, began with some examples culled by the Advisory Editorial Committee consisting of Rutherford Aris, R. Creighton Buck, Preston R. Clement, E. T. Kornhauser, and H. O. Pollak. He states in his Preface:

". . . This book is based on examples of applications of undergraduate mathematics in engineering, submitted for the most part by members of engineering and mathematics departments of universities, with some contributions from industrial companies. These were requested by the Commission on Engineering Education and the Committee on the Undergraduate Program in Mathematics.

"One of the principal aims has been to write a book for a reader who has no specialized knowledge of any branch of mathematics, the physical sciences, or engineering."

Actually, the author makes reference to college algebra, trigonometry, analytic geometry, calculus, elementary ordinary differential equations, elementary linear algebra, elements of probability theory (Poisson, binomial, and normal distributions), elementary knowledge of computers and flow charts. On the other hand, he "tried to describe physical and engineering situations from first principles, by which we mean basic physical laws such as Newton's laws of motion, the decomposition of forces, Hooke's law in elasticity (extension proportional to force), the basic laws governing electrical networks, and some simple ideas in connection with chemical reactions."

". . . There has been no attempt to provide systematic coverage of topics in either engineering or mathematics. I have merely taken the random sample of examples chosen by the selection committee and added a number of related examples suggested by other sources. It is purely accidental, for example, that the flow charts for computer programs given in the text involve only examples in probability, and that so much attention has been devoted to probability as opposed to statistics."

The author tries in most cases to give:

(1) "Explanation of the engineering motivation of the problem."

(2) "Abstraction, idealization and formulation" (of the mathematical problem).

(3) "Solution of the mathematical problem."

(4) "The relevance of the results to the original problem."

After an introductory chapter, the book is divided into five parts, each having several chapters:

*Part I.* Illustrative Applications of Elementary Mathematics

E. I.

**90[H, P, S, X].**—EUGENE L. WACHSPRESS, *Iterative Solution of Elliptic Systems and Applications to the Neutron Diffusion Equations of Reactor Physics*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xiv + 299 pp., 24 cm. Price $12.95.

This book deals with many aspects of the theory and practice of numerical computations of solutions of the elliptic equations of reactor physics. It also gives some background material on physics, matrix theory, and partial differential equations in general. Following are some comments on the various chapters of the book.

1. *Mathematical Preliminaries* is a survey of elementary matrix theory, the Perron-Frobenius theory of positive matrices, and basic theory for the iterative and direct solution of systems of linear equations. According to the opinion of the reviewer, this chapter, as well as other parts of this book, should have been rewritten before publication. The author could either have worked out a fully self-contained chapter, or limited himself to give references to the well-known book by Varga or some other standard text.

2. *Formulation and Solution of Discrete Boundary Value Problems.* The author describes and discusses various techniques to discretize elliptic differential equations, giving an adequate background for the following chapters. The chapter also contains a short discussion of various types of partial differential equations and the boundary conditions which give rise to well-posed problems. There is also a very

superficial treatment of stability and convergence of difference methods for hyperbolic and parabolic problems, a subject which could as well have been left out completely.

3. *The Group Diffusion Equations of Reactor Physics.* This chapter presents a derivation of these basic equations and some difference approximations to them. Much stress is rightly put on the correspondence between certain laws of physics and properties of the matrices of the discrete problems.

4. *Successive Overrelaxation* gives the standard theory for this iterative method. Also included are methods for the estimation of the optimal parameter and a discussion of the role of eigenvector deficiency.

5. *Residual Polynomials.* The author describes Lanczos' methods, Chebyshev extrapolation, and some combined semi-iterative methods to improve the convergence rate of iterative procedures.

6. *Alternating Direction Implicit Iteration.* The commutative model problem and the selection of optimal parameters are treated in full detail. Available results for general noncommutative problems are surveyed. The author advocates compound iteration techniques for the general case.

7. *The Positive Eigenvector* is a short section on an important aspect of the eigenvalue problem. There is a discussion of several methods and strategies for the determination of the first eigenvalue.

8. *Numerical Studies for the Diffusion Equation* contains results from a series of numerical experiments designed to compare the efficiency of different numerical methods. This is a valuable contribution to the literature. Both practical and theoretical numerical analysts would profit very much from a larger literature on the results of careful numerical experiments.

9. *Variational Techniques for Accelerating Convergence* discusses the periodic application of variational acceleration techniques in linear iterative schemes.

This book undoubtedly contains much interesting material. It can serve as a source for numerical ideas but it should be read with care. The presentation of the material is not very good and part of this book lacks in precision.

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**91[L].**—HENRY E. FETTIS & JAMES C. CASLIN, *Tables of the Modified Bessel Functions $I_0(x)$, $I_1(x)$, $e^{-x}I_0(x)$, and $e^{-x}I_1(x)$*, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, March 1967, ms. of 223 computer sheets deposited in the UMT file.

These impressive manuscript tables consist of 15S approximations to the modified Bessel functions of the first kind of orders 0 and 1, that is $I_0(x)$ and $I_1(x)$, and their respective products with $e^{-x}$, for $x = 0(0.001)10$.

These values were computed by double-precision arithmetic on an IBM 7094 system, using a computer program based on the integral representation

$$e^{-x}I_n(x) = \frac{1}{\pi}\int_0^{\pi} e^{-x(1-\cos\theta)} \cos n\theta d\theta .$$

This reviewer has compared these data with the 18D and 21D values of $I_0$ and $I_1$ given by Aldis [1] and thereby detected several rounding errors in the present tables, none exceeding a unit in the least significant figure.

The authors compared their values of $e^{-x}I_n(x)$, $n = 0, 1$, with the corresponding 10D data in Table 9.8 in the *NBS Handbook* [2] and discovered a number of rounding errors in the latter, which they will enumerate separately in an appropriate errata notice.

Despite the presence of rounding errors, these manuscript tables constitute a valuable addition to the extensive tabular literature relating to modified Bessel functions [3].

J. W. W.

1. W. S. Aldis, "Tables for the solution of the equation $d^2y/dx^2 + 1/x \cdot dy/dx - (1 + n^2/x^2)y = 0$," *Proc. Roy. Soc. London*, v. 64, 1899, pp. 203–223.

2. M. Abramowitz & I. A. Stegun, editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964.

3. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, second edition, Addison-Wesley Publishing Company, Reading, Massachusetts, 1962, v. I, pp. 417–418, 423.

**92[L, X].**—C. Chang & C. Yeh, *The Radial Prolate Spheroidal Functions*, USCEE Report 166, Department of Electrical Engineering, University of Southern California, Los Angeles, California, June 1966, iii + 25 + 990 pp., 28 cm.

The radial prolate spheroidal functions are the two independent solutions of the differential equation

$$\frac{d}{d\xi}\left[(\xi^2 - 1)\frac{d}{d\xi}R_{mn}(c, \xi)\right] - \left[\lambda_{mn} - c^2\xi^2 + \frac{m^2}{\xi^2 - 1}\right]R_{mn}(c, \xi) = 0$$

which occurs in the solution of the time-periodic scalar wave equation in prolate spheroidal coordinates by the method of separation of variables.

The authors have herein tabulated to 8S the functions $R_{mn}^{(1)}(c, \xi)$, $R_{mn}^{(2)}(c, \xi)$ and their first derivatives with respect to $\xi$ for $m = 0(1)9$, $n = m(1)9$, $c = 0.1(0.1)1(0.2)6$, $\xi = \xi_0(0.001)1.01(0.01)1.1, 1.25(0.25)2, 5, 10$, where $\xi_0$ varies from 1.004 when $m = 0$ to 1.009 when $m = 9$. Functional values corresponding to $\xi = 1.044$ and 1.077 are also included; they were calculated to check the corresponding entries in the tables of Flammer [1]. For each of the listed values of $m$, $n$, and $c$ the corresponding eigenvalue $\lambda_{mn}$ is tabulated to 12S except for $m = 0, 1$, where only 8S are given.

All the underlying calculations were carried to 12S at the computing center at the University of Southern California, and the final results were compared with the tables of Flammer, Slepian [2], Slepian & Sonnenblick [3], and Hunter et al. [4]. The computed values were also checked by use of the appropriate Wronskian relation. The belief is expressed by the authors that the tabulated values are accurate to at least 7S.

The computer output was reduced photographically prior to printing in report form, so that the contents of two computer sheets now appear on a single page; however, the numbering of the original sheets has been retained.

In an effort to make this report self-contained, the authors have included a number of formulas and expansions for both the angular and prolate spheroidal functions, following the notation and normalization adopted by Flammer [1]. The various methods used in calculating these tables are also described.

This introductory explanatory text of 21 pages, which includes an enumeration of the diverse fields of application of spheroidal functions, is followed by a list of 15 references. Inadvertently omitted from the list (although referenced in the introduction) is the treatise of Morse & Feshbach [5].

Unfortunately, the photographic reproduction of these elaborate tables has left much to be desired; in fact, on approximately 75 pages of the review copy an appreciable number of the tabular entries are only partially legible. This serious defect in the reproduction of these important tables naturally impairs their usefulness.

J. W. W.

1. C. Flammer, *Spheroidal Wave Functions*, Stanford Univ. Press, Stanford, Calif., 1957. (See *MTAC*, v. 13, 1959, pp. 129–130, RMT **20**.)

2. D. Slepian, "Asymptotic expansions of prolate spheroidal wave functions," *J. Math. and Phys.*, v. 44, 1965, pp. 99–140.

3. D. Slepian & E. Sonnenblick, "Eigenvalues associated with prolate spheroidal wave functions of zero order," *Bell System Tech. J.*, v. 44, 1965, pp. 1745–1759.

4. H. E. Hunter, D. B. Kirk, T. B. A. Senior & H. R. Wittenberg, *Tables of Spheroidal Functions for m = 0*, Vols. I & II, Radiation Laboratory, University of Michigan, Ann Arbor, Michigan, 1965.

5. P. M. Morse & H. Feshbach, *Methods of Theoretical Physics*, Parts I & II, McGraw-Hill Book Co., New York, 1953. (See *MTAC*, v. 12, 1958, pp. 221–225, RMT **87**.)

**93[M, X].**—Carl H. Love, *Abscissas and Weights for Gaussian Quadrature for $N = 2$ to 100, and $N = 125, 150, 175, 200$*, NBS Monograph 98, U. S. Department of Commerce, 1966, iii + 88 pp., 26 cm. Price $.55. Paperbound.

The abscissas $X_k$ and weights $H_k$ are given to 24D and 23D, respectively, for the Gaussian formula:

$$\int_{-1}^{1} F(X)dx \approx \sum_{k=1}^{n} H_k F(X_k)$$

for $n = 2(1)100(25)200$.

There is, therefore, much overlap here with Table 1 of [1], which gives these numbers to 30S for $n = 2(1)64(4)96(8)168, 256, 384, 512$. Clearly, however, that table and this each give some $n$ not given by the other.

The present monograph has an introduction and six references. The recent treatise [1]—see our review RMT **14**, *Math. Comp.*, v. 21, 1967, pp. 125–126—is not mentioned, as such. The introduction here does make the misleading statement: "A. H. Stroud is working on tables for $N = 2, 64, 96, 168, 256, 384$, and 512 but he has not published them at the present time." The omission of the argument differences: (1), (4), etc. changes the meaning entirely.

The nominal cost of the present tables is less than the fractional part of that of [1]—which costs $14.95—but, of course, they really can't be compared.

D. S.

1. A. H. Stroud & Don Secrest, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966.

**94[P, X].**—M. R. HESTENES, *Calculus of Variations and Optimal Control Theory*, John Wiley & Sons, Inc., New York, 1966, xii + 405 pp., 24 cm. Price $12.95.

This book, by an eminent author whose contributions to the calculus of variations date from 1930 and who wrote in 1950 on the Maximum Principle, will be used and cited for many years.

Emphasis is on first-order necessary conditions for a variety of problems leading to the general control problem of Bolza. Other necessary conditions and sufficient conditions are treated for some of the simpler problems. The approach is classical in the sense that admissible state variables $x$ are of class $D'$, admissible control variables $u$ are piecewise continuous and integrands $f$ together with left members $\varphi$ of side-conditions are of at least class $C'$. The Lebesgue integral is used only in the appendix and in occasional brief remarks.

There is an appropriate treatment of fields and Hamilton-Jacobi methods, needed for various manifestations of the Maximum Principle. That this topic, as well as the foundations of Dynamic Programming, as applied to variational problems, is an extension of classical Hamilton-Jacobi theory and not a separate subject becomes clear.

The book is not a survey of the current status of variational theory. There is very little on parametric problems, since control problems are for the most part nonparametric. There is nothing on existence of global extrema, on multiple integral problems or on numerical methods but there is much valuable background for the last. Those topics that the author has chosen are treated in depth and detail. The exposition is largely self-contained and, in view of the introductory chapter and a full treatment of classical fixed endpoint problems in Chapter 2, does not assume previous acquaintance with the calculus of variations. However, it will be read with more ease and appreciation by those who have some prior knowledge.

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**95[P, X].**—C. T. LEONDES, Editor, *Advances in Control Systems, Vol.* 3, Academic Press, New York, 1966, x + 346 pp., 24 cm. Price $14.50.

**[P, X].**—C. T. LEONDES, Editor, *Advances in Control Systems, Vol.* 4, Academic Press, New York, 1966, xiv + 320 pp., 24 cm. Price $14.50.

As the first two volumes in this series were, the present two are collections of papers which aim to acquaint the reader with recent work in the theory of control systems and its applications.

Volume 3 consists of six articles. The first, by Thomas L. Gunckel, II, entitled "Guidance and Control of Reentry and Aerospace Vehicles" reviews the problems of near-earth navigation and orbit determination, rendezvous guidance and control, and reentry guidance in relation to computer technology and, in particular, the requirements of an on-board computer. The second paper, "Two-Point Boundary-Value-Problem Techniques," by P. Kenneth and R. Mc Gill discusses in detail the numerical solution of two-point boundary value problems for systems of nonlinear ordinary differential equations by the generalized Newton-Raphson algorithm. The third article, "The Existence Theory of Optimal Control Systems," by W. W. Schmaedeke gives a fairly elementary exposition of some of the principal existence theorems, in the usual mathematical setting, for linear as well as nonlinear control systems. The fourth paper, by James M. Swiger, entitled "Application of the Theory of Minimum-Normed Operators to Optimum-Control-System Problems," presents a treatment of various typical control problems in the setting of the moment problem of Akhieser and Krein. The fifth paper, "Kalman Filtering Techniques," by H. W. Sorensen discusses the linear estimation theory as developed by Kalman and others, mainly in terms of a time-discrete model. The last paper, by Stanley F. Schmidt, entitled "Application of State-Space Methods to Navigation Problems," uses the navigation problem as an example to demonstrate the usefulness of various simple mathematical concepts and techniques.

Volume 4 also contains six contributions. The first, by David Isaacs, on "Algorithms for Sequential Optimization of Control Systems" reviews various methods for the numerical solution of the optimal control problem and reports on some computational experiments with them. The second paper, "Stability of Stochastic Dynamical Systems," by Harold J. Kushner gives a brief thoroughly mathematical introduction to Lyapunov's second method as it applies to stochastic stability. The third paper, by Richard E. Kopp and H. Gardner Moyer, entitled "Trajectory Optimization Techniques," discusses the computational solution of optimization problems by indirect methods, gradient methods, the second variation and the generalized Newton-Raphson method, and lists some of their relative advantages and disadvantages. The fourth article, "Optimum Control of Multidimensional and Multilevel Systems," by R. Kulikowski, is concerned with the reduction of complex optimization problems to problems of second- and higher-level control by using for first-level control the known results of standard optimum-control theory. The last two papers are both by Donald E. Johansen. In "Optimal Control of Linear Stochastic Systems with Complexity Constraints" he gives a detailed treatment of a linear stochastic system when the estimator is not of the same order as the process being controlled. In "Convergence Properties of the Method of Gradients" he obtains qualitative results using a large amplitude theory for deviations of the control from the optimal solution and quantitative results in the asymptotic region of small perturbation.

Though the level of presentation varies a great deal and requires different degrees of mathematical and engineering sophistication, these collections should nonetheless be of some use to the informed reader interested in particular aspects of control problems.

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**96[P, X, Z].**—Moshe F. Rubinstein, *Matrix Computer Analysis of Structures*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xiv + 402 pp., 24 cm. Price $12.95.

This text is intended for use in a one-semester senior course or beginning graduate course. As the title of the book would indicate, the intention of the author is not to develop the classical methods of structural analysis, but rather to emphasize those techniques which are directly applicable to computers. Thus an introductory chapter is devoted to computers (including a brief description of FORTRAN), and in two succeeding chapters the necessary concepts of linear algebra are developed. The remainder of the book emphasizes matrix methods along with various techniques of solution and can best be described by simply listing the chapter headings: Computers—Fundamental Concepts, Structures—Fundamental Concepts, Characteristics of Structures—Stiffness and Flexibility, Determinants and Matrices, Solution of Linear Equations, Energy Concepts in Structures, Transformation of Information in Structures, The Flexibility Method, The Stiffness Method, Analysis by Substructures and by Recursion, Analysis by Iteration, Analysis of Plates and Shells—Introduction. Each chapter contains a selection of problems with answers given at the back of the text. The book is clearly written, and can be recommended for use in a computer-oriented course in structural analysis.

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**97[Q].**—Milton P. Jarnagin, Jr., *Expansions in Elliptic Motion*, constituting Volume XVIII of *Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac*, U. S. Government Printing Office, Washington, D. C., 1965, xxxvi + 659 pp., 29 cm. Price $4.50 (paperbound).

This volume is, in effect, a repetition and extension of Cayley's classical tables [1], giving the literal expansions as harmonic series, in the mean anomaly, of such functions as $(r/a)^n \exp (i\, m\, f)$, $\log (r/a)$, and the equation of the center. These expansions are carried to the 20th power of the eccentricity, and all the numerical coefficients are rational fractions.

The Introduction is a model both of clarity of exposition and of probity in the care with which this large computing project was planned and programmed. There is no evidence of any hammer-and-tongs approach, even though the most powerful electronic computer of its day, the NORC, was available for the work, performed at irregular intervals in 1961 and 1962.

For the record, the Introduction should have included an explanation of the printing process. Both for reliability and economy, the computer output was recorded on microfilm by means of the NORC cathode ray tube. Judicious programming provided a compact, self-explanatory format. Owing to the extremely high reliability of the NORC, it may be assumed that probably not a single digit in the 659 pages of tables is in error. Unfortunately (and as a sad commentary on these times) the review copy has 16 pages of one whole signature completely illegible, because of careless printing-press workmanship.

The personnel of the Naval Weapons Laboratory and the Nautical Almanac Office are to be commended for their excellent collaboration in producing and publishing this volume. As a desk-type reference for workers in celestial mechanics, it may be expected to serve all needs during the second century of existence of Cayley's Tables.

Paul Herget

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1. ARTHUR CAYLEY, "Tables of the developments of functions in the theory of elliptic motion," *Memoirs of the Royal Astronomical Society*, v. 29, 1861, pp. 191–306.

**98[W].**—MARTIN SHUBIK, Editor, *Essays in Mathematical Economics*, Princeton University Press, Princeton, N. J., 1967, xx + 475 pp., 24 cm. Price $12.50.

This tribute to Oskar Morgenstern by his friends (both old and young) is remarkable for the high level of its articles. A brief biography and a bibliography of Morgenstern, which explains his impact on economics, are presented at the beginning of the volume. The twenty-seven technical articles are grouped into seven areas in which Morgenstern has worked and made his influence felt. To do more than merely list the titles and authors, would take up too much space:

E. I.

**99[X].**—D. C. Handscomb, Editor, *Methods of Numerical Approximation*, Pergamon Press, New York, 1966, ix + 218 pp., 24 cm. Price $9.50.

This volume is a gem! In spite of the fact that it is written by six authors, the articles are remarkably even in style and clarity. The lectures given at a Summer School held at Oxford University in September 1965 formed the basis for this book. Both the theoretical and the practical aspects of approximation methods are developed. A listing of the chapter headings with their authors will serve to indicate the scope of the book:

E. I.

**100[X].**—M. V. Wilkes, F.R.S., *A Short Introduction to Numerical Analysis*, Cambridge University Press, Cambridge, England, 1966, 76 pp., 22 cm. Price $4.75.

As the title indicates, this is a very short (76 pp.) introduction to numerical analysis. It is also reasonably priced, with a paper bound edition available for $1.95 from Cambridge University Press. The author covers the usual topics treated in a first course in numerical analysis. These include iteration, interpolation, numerical integration and differentiation, the solution of ordinary differential equations, and the solution of linear systems. The level of the material is fairly elementary, although the author does not hesitate to use advanced concepts to simplify derivations. The emphasis is on methods rather than proofs.

The chapter on Interpolation is by far the longest chapter, and, considering the length of the book, by far too long, especially since many important concepts and methods are omitted.

This book has some value as a quick reference book. As a textbook it is of questionable value. The brevity of the material, the omission of proofs and the general lack of cohesion make it, in the referee's opinion, unsatisfactory even for an undergraduate course in numerical analysis.

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**101[X, Z].**—T. E. Hull, *Introduction to Computing*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xi + 212 pp., 24 cm. Price $6.95.

This text is divided into three main parts of five chapters each: first, a discussion of the basic properties of computers and of simple algorithms using machine-language code for a hypothetical computer; the second five chapters are a thorough standard introduction to the Fortran language (based on the Fortran IV compiler under IBSYS on the IBM 7090/94 and 7040/44); the final five chapters cover elementary programming techniques and automata theory.

The title of the book would more properly be that of a beginning course, since the text is not intended to provide comprehensive coverage of many topics. Thus the chapters on Numerical and Non-Numerical Methods and on Simulation actually discuss the computational implications of a few simple examples rather than the subjects themselves. A valuable feature of the text is its attempt to indicate what is implied in terms of actual machine operations by the writing of certain Fortran statements. Because the hypothetical machine code discussed is not for the computers on which the Fortran language is implemented, this technique finds only limited application in the chapters on Fortran.

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**102[Z].**—John von Neumann, *Theory of Self-Reproducing Automata,* Edited and completed by Arthur W. Burks, University of Illinois Press, Urbana, Illinois, 1966, xix + 388 pp., 24 cm. Price $10.00.

This volume consists of a meticulously edited version of a series of five lectures on basic computer theory given by von Neumann at the University of Illinois in December 1949, together with an extensive but unfinished manuscript on computer self-reproduction written by von Neumann in 1952–1953. The Illinois lectures are particularly interesting for von Neumann's digressions on the future and general significance of computers as seen by him in 1949. Some of his comments on the problem of complexity are still highly apropos, perhaps more in connection with software than with hardware.

The manuscript forming the second part of the book is a good example of von Neumann's very brilliant mathematical style, but is perhaps somewhat disappointing in the result which it presents. Consider an infinite set of small Turing machines, all but one initially in a wait state and with blank tapes, and each capable of writing onto the tape of its neighbors and of putting its neighbors into an initial active state. It is then reasonably clear that by copying its own tape onto the tape of one of its neighbors and starting this neighbor, a Turing machine is able to initiate a process of self-reproduction of a suitable given set of tapes. Von Neumann's paper expands upon this observation, showing by explicit construction that both the basic Turing machines and their tapes can be simulated in a hypothetical crystal medium, each of whose points is an elementary 29-state automaton.

The editor provides a well-written and instructive historical account of von Neumann's work with computers.

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**103[Z].**—Jagjit Singh, *Great Ideas in Information Theory, Language and Cyber-*

*netics*, Dover Publications, Inc., New York, 1966, ix + 338 pp., 22 cm. Price $2.00.

Computers which were developed primarily for solving long and complicated mathematical problems involving millions of separate calculations have in recent years begun to be treated as general symbol-processing devices capable of performing any well-defined processes for the manipulation and transformation of information.

The purpose of this well-written paperback is to inform the nonspecialist about current research on intelligent behavior by computer. The topics included are: the ideas of information theory, species of information processors, computers and the nervous system and the realization of Artificial Intelligence. The table of contents is informative and interesting:

I. Language and Communication
II. What is Information?
III. Information Flow over Discrete Channels
IV. Coding Theory
V. Reliable Transmission and Redundancy
VI. Continuous Channels
VII. Information and Entropy
VIII. Automatic Computers—Analogue Machines
IX. Automatic Computers—Digital Machines
X. The Computer and the Brain
XI. Neural Networks—McCulloch and Pitts
XII. Neural Networks—von Neumann
XIII. Turing Machines
XIV. Intelligence Amplifiers
XV. Learning Machines or Perceptrons
XVI. Artificial Intelligence—Game Playing Machines
XVII. Artificial Intelligence—Translating Machines
XVIII. Uttley Machines
XIX. Mathematical Theories of the Living Brain

Not much mathematical preparation is required to understand the mathematical parts—high school algebra appears actually to be adequate. All the chapters start out with lots of good, simple, clear, motivating material which places the subject in its historical and scientific context in an engrossing manner. Unfortunately, and perhaps unavoidably, the difficulties of explaining new advanced technical information to the layman in a few pages become prodigious and the actual explanations are frequently not really satisfying. The typical chapter then winds up with a good summary of what has been discussed and its implications.

This book might be useful to teachers of computing science courses for supplying interesting motivation and organization to their lessons. It will certainly be interesting and helpful to students of computing science and technology. Although in a popular treatise one does not expect a bibliography, it would have been most useful if one had been included.

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