REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

1[2, 3, 4].—A. César de Freitas, *Introdução à Análise Numérica*, Vol. I, Universidade de Lourenço Marques, Moçambique, 1968, xii + 194 pp., 23 cm. Price $5.50.

This is the first of two volumes of an introductory textbook on Numerical Analysis. It contains the usual material on errors, finite differences, interpolation, numerical differentiation and quadratures, a very short review of numerical integration of ordinary differential equations, and a fairly extensive (relatively speaking) treatment of basic numerical linear algebra. This is probably one of the few books on the subject originally written in the Portuguese language and in that sense has special interest. The printing and general presentation are good.

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2[2, 3, 4].—I. P. Mysovskih, *Lectures on Numerical Methods*, Wolters-Noordhoff Publishing, Groningen, 1969, iii + 344 pp., 23 cm. Price $12.50.

The Russian version of this book appeared in 1962 and the subject matter appears to predate 1960. This is an excellent book when considered as a mathematical text. However, the author imposes a guideline which the computational scientist will find very restrictive. This is:

> The only computational problems considered seriously are those for which it is possible to place a rigorous numerical bound on the accuracy of the result.

For example, to illustrate how to locate a zero, the problem treated is one for which it is known that $f(a) \cdot f(b)$ is negative and $f'(x)$ is positive between a and b. The numerical quadrature section is extensively illustrated by integrating $\sin x/x$ between 0 and 1. It happens here that the nth derivative is bounded by $(n + 1)^{-1}$. Using this information, one starts by deciding which quadrature rule to use on the basis of the standard bound on the discretisation error. In the section on numerical differentiation, the words 'round-off error' do not occur.

This reviewer is a stranger to the world where numerical problems are so tractable, but I did enjoy reading about it. The book has four chapters. These treat numerical solution of equations, algebraic interpolation, numerical quadrature and initial value ordinary differential equations respectively. In each chapter a few methods are chosen and are given a careful, clear and rigorous treatment. The section on divided differences, for example, succeeded in being thorough without being excessively long. The Hermite osculating polynomial is treated by means of Cauchy's Theorem. This treatment happened to be new to this reviewer and seems much more suitable for sufficiently qualified students than the more familiar long-winded discussion. In the quadrature section, the relations between the trapezoidal rules and periodic integrands is introduced early on and a treatment of Bernoulli functions and polynomials is included.

I would recommend this book for instructors who will find excellent descriptions and proofs of various standard theorems in these fields. But students should be exposed to a more realistic view of computing than the one which might be inferred from reading this book.

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3[2, 4, 12].—Ralph H. Pennington, *Introductory Computer Methods and Numerical Analysis*, 2nd ed., The Macmillan Co., New York, 1970, xi + 497 pp., 24 cm. Price $10.95.

The first edition of this textbook was published in 1965. (For a review thereof, see *Math. Comp.*, v. 20, 1966, pp. 198–199, RMT **43**.) This second edition incorporates a number of changes, revisions, and modernizations without at the same time altering the basic character of the original work. More explicitly than before, the author stresses that—to paraphrase his words—"he is drawn toward the needs of departments devoted to science, engineering, business administration, and so on", which "find it necessary to include some computer courses for their own purposes". In line with this, fundamental concepts and rigor "were allowed to suffer to some degree in order to include a sufficient number of descriptions of algorithms to leave the reader with a reasonably versatile beginner's kit of problem-solving tools". The background prerequisites have remained at the integral calculus level.

The overall organization of the book is essentially the same as before, although the arrangement of the material and its subdivision into chapters has been improved in places. The first five chapters now present the introduction to the fundamentals of computers and to FORTRAN programming (using ASA standards), and the remaining eight chapters cover the basic numerical methods traditional for this level.

In recognition of the growing importance of interactive computing, a rather novel change has been the introduction of a FORTRAN variation for remote-terminal operation. Also, the artificial hypothetical machine language used before has been modernized and modified as to resemble somewhat that of, say, the IBM 360 series. The discussion of several numerical topics has been added or enlarged. Newly included are Chebyshev series, Romberg integration, and in the differential equations chapter, the Euler-Romberg method, and the Adams-Moulton formulas (instead of Milne's method). The coverage of error propagation, Gaussian quadrature, and the Runge-Kutta method has been expanded. The abandonment of Sturm sequences in favor of Graeffe's method (called erroneously Graeff's method in the text and the index) should be a matter of opinion; rather questionable appears to be the complete replacement of Gaussian elimination by the Gauss-Jordan method for the sole stated reason that the latter gives "shorter and more understandable FORTRAN subroutines". As before, there are many illustrative examples, and the number of exercises appears to be increased.

Altogether the second edition has brought the book more in line with the present-day computer environment, and, in balance, it will probably allow it to continue serving the needs to which it was addressed by the author.

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4[2, 12].—ALEXANDRA I. FORSYTHE, THOMAS A. KEENAN, ELLIOTT I. ORGANICK & WARREN STENBERG, *Computer Science: A First Course*, John Wiley & Sons, Inc., New York, 1969, xviii + 553 pp., 24 cm. Price $9.95.

This book is a development of the School Mathematics Study Group (SMSG) text *Algorithms, Computation and Mathematics*, designed for high school students. It is a carefully written introductory text, covering fully most of the topics suggested for Course B1: Introduction to Computing, of Curriculum 68, the report of the ACM Curriculum Committee on Computer Science. In this reviewer's opinion, it is very suitable for a beginning college or junior college course in Computer Science, and is one of the best among the many texts available for courses at this level.

The work is unusual for an elementary text in that it concentrates from the outset on abstract algorithms and flowcharts for them and uses no programming language other than the author's own "flowchart language." Three supplementary programming texts are available for FORTRAN, BASIC, and PL/1. This unique method of organizing the text allows the authors to concentrate on the essential ideas of computing and algorithms unencumbered by the technical details of a particular language, which the neophyte often finds the most difficult to learn. It also has the advantage of allowing the instructor to choose whichever programming language is available to him or which he considers best for beginning students.

The book is divided into three parts. Part I introduces the student to computing and covers algorithms and flowcharts. Part II covers elementary numerical analysis and applications to computing including quadrature, simultaneous linear equations, and linear regression. Part III covers some of the newer areas of Computer Science including trees, lists, strings, and compiling. A second version of the text, *Computer Science: A Primer*, contains Parts I and II only and may be more suitable for high school use. Also published with the text is *Computer Science: Teacher's Commentary*, a very detailed supplement which seems intended for mathematics teachers with no previous experience in Computer Science. It contains complete flowcharts with comments for all of the problems in the text and additional problems, explanations and suggestions for the teacher.

The book is mathematically oriented and pays particular attention to careful development of mathematical concepts. It is well coordinated with the SMSG mathematics texts.

This reviewer has only a few complaints about the book. One is the violation of the ANSI standard for flowcharts, somewhat disturbing in a book with over

300 flowcharts. Another is a rather strange use of the word 'round' to mean both round and truncate. The authors assume that the reader is familiar with binary arithmetic, not necessarily true in this reviewer's experience with college students. The explanation of computer hardware, especially on core storage, is confusing and sketchy. The book has several misprints which may confuse the beginner. However, these are all minor and should be corrected in the second printing.

The exercises are ample and excellent and should serve students with a wide range of aptitudes and interests. In combination with one of the programming supplements, or with any programming text, the book should be very successful in classroom use.

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5[3].—ROBERT T. GREGORY & DAVID L. KARNEY, *A Collection of Matrices for Testing Computational Algorithms*, John Wiley & Sons, Inc., 1969, ix + 154 pp., 28 cm. Price $9.95.

A much needed collection of matrices for testing algorithms, which arise in numerical linear algebra, is provided by this book. The authors provide both well-conditioned and ill-conditioned test matrices for algorithms concerning: (1) inverses, systems of linear equations, determinants and (2) eigensystems of real symmetric, real nonsymmetric, complex, and tridiagonal matrices. The construction of test matrices is discussed, and a large number of references and a table of symbols is provided.

The authors do not discuss the perplexing problem concerning how a user must choose the appropriate test matrices. In particular, test matrices must be chosen so that all parts of the algorithm are tested. It may not be clear to a user by looking at the examples which matrix (if any) will go through a particular part of his algorithm. Then, he must construct his own examples by working backwards through his algorithm.

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6[5].—DALE U. VON ROSENBERG, *Methods for the Numerical Solution of Partial Differential Equations*, American Elsevier Publishing Co., Inc., New York, 1969, xii + 128 pp. Price $9.50.

This book serves as a good introduction for anyone interested in finite difference methods. In the preface, the author states "This book is written so that a senior undergraduate or first-year graduate student in engineering or science can learn to use these methods in a single semester course, and so that an engineer in industry can learn them by self-study." The book succeeds admirably. The style is very readable and the physical background of each equation is discussed. Each chapter ends with a specific problem which is completely worked out including a program and computer time.

Specifically, in chapter one, the example of heat conduction in an insulated tapered rod with various boundary conditions is used to illustrate some methods for linear ordinary differential equations.

In chapter two, linear parabolic partial differential equations are treated. The forward difference, backward difference and Crank-Nicolson schemes are derived and the stability analysis for each is carried out.

In chapter three linear hyperbolic equations, including systems, are dealt with.

Chapter four is concerned with alternate forms of the coefficient matrices generated by the difference schemes.

Chapters five and six deal with nonlinear parabolic and hyperbolic equations, and chapter seven describes nonlinear boundary conditions.

Chapters two through seven deal with equations with one space variable.

Chapter eight describes parabolic and elliptic equations in two and three space dimensions and includes Alternating-Direction-Implicit schemes.

Chapter nine mentions some examples of complications which can arise in the problems treated earlier, for example shock waves in hyperbolic equations.

There is an appendix of algorithms useful in solving the types of equations generated by the difference schemes.

One slight drawback for use as a text is that no exercises are provided. More serious is the fact that function space approximation methods are not mentioned at all, and the title of the book might lead a student to believe that finite differences are the only known methods for obtaining numerical solutions.

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7[7].—J. W. WRENCH, JR., *The Converging Factor for the Modified Bessel Function of the Second Kind*, NSRDC Report 3268, Naval Ship Research and Development Center, Washington, D. C., January 1970, ii + 56 pp., 26 cm.

This report, which follows the pattern of two earlier reports [1] and [2] (see *Math. Comp.*, v. 20, 1966, pp. 457–458, RMT 71), is concerned with the development of methods and provision of auxiliary tables for the high-precision calculation of the converging factor in the asymptotic expansion of the modified Bessel function of the second kind, $K_p(x)$. While much of the analytical discussion is quite general, the practical application is restricted to functions of integer order and positive real argument.

The converging factor is defined as "that factor by which the last term of a truncated series (usually asymptotic) approximating the function must be multiplied to compensate for the omitted terms." Thus in the case of the function $K_p(x)$ we have

$$K_p(x) = (\pi/2x)^{1/2}e^{-x}\left\{1 + \frac{a_1(p)}{x} + \frac{a_2(p)}{x^2} + \cdots + \frac{a_n(p)}{x^n}\,\sigma_{n,p}(x)\right\},$$

where the $a_i(p)$ are known coefficients and $\sigma_{n,p}(x)$ is the converging factor. This may itself be expanded in a series of the form

$$\sigma_{n,p}(x) \sim \sum_{s=0}^{\infty} b_s(n, p)C_{n-p}^{(s)}(2x)(-2x)^s.$$

It is shown that the function $C_{n-p}(2x)$ is closely related to the converging factor for the probability integral, considered by Murnaghan in [2], where $C_m(m)$ is tabulated to 63D for $m = 2(1)64$.

It is advantageous to choose a value of n such that $2x$ is close to $n - p$; then, writing $m = n - p$ and $2x = m + h$, $C_m(2x)$ may be computed from the Taylor series

$$C_m(m + h) = C_m(m) + d_1(m)h + d_2(m)h^2 + \cdots .$$

An appendix to the report contains 30D values of $C_m(m)$ and its reduced derivatives $d_i(m)$ for $m = 10(1)40$. Once $C_m(2x)$ is known, its reduced derivatives $C_m^{(s)}(2x)$ can be calculated with the aid of a three-term recurrence relation, and hence the series for $\sigma_{n,p}(x)$ can be summed.

As examples of the use of the procedure described, the values of $K_0(2\pi)$ and $K_1(10)$ are evaluated to an accuracy of approximately 17D and 26D, respectively. This represents a substantial improvement on the accuracy of 10D and 14D, respectively, obtainable from the asymptotic series without the converging factor.

Attention may be drawn to an alternative method of computation; namely, the use of a Chebyshev series representation, which appears less laborious than the foregoing and permits higher accuracy to be attained over a considerably extended range of the argument. (Indeed, there is no theoretical limit to the attainable accuracy.) The corresponding coefficients are easily calculated by backward recurrence. In particular, Luke [3] has tabulated to 20D the coefficients for the auxiliary function $(2x/\pi)^{1/2}e^x K_p(x)$ as a function of the reciprocal argument $5/x$ in the range $x \geqq 5$, for $p = 0$ and 1 and several fractional values of p; in each case approximately 20S are obtainable throughout the relevant range with the use of 21 terms.

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1. F. D. Murnaghan & J. W. Wrench, Jr., *The Converging Factor for the Exponential Integral*, DTMB Report 1535, David Taylor Model Basin, Washington, D.C., January 1963.
2. F. D. Murnaghan, *Evaluation of the Probability Integral to High Precision*, DTMB Report 1861, David Taylor Model Basin, Washington, D.C., July 1965.
3. Y. L. Luke, *The Special Functions and Their Approximations*, Vol. II, Academic Press, New York, 1969.

8[9].—Hans Riesel, *En Bok om Primtal*, Studentlitteratur, Denmark, 1968, 174 pp., (Swedish), 23 cm. Price \$6.95 equivalent. (Paperback.)

This monograph on prime numbers will be of special interest to readers of *Math. Comp.* since its number theory is very close to that which appears here. In fact, many of its references are to papers that have appeared here.

There are four chapters and five appendices, the titles of which (translated into English) are: I, "The exact number of primes less than a given limit." II, "The approximate number of primes." III, "How to identify large primes." IV, "Factorization." 1, "Some algebraic theorems." 2, "Some number theoretic theorems." 3, "Quadratic residues." 4, "Arithmetic of quadratic fields." 5, "Algebraic factors of $a^n \pm b^n$."

Chapter I deals in such topics as Meissel's formula and Lehmer's formula. Chapter II discusses distribution questions: the Gauss, Legendre, and Riemann approximations; the remainder term in the prime number theorem; twins, large gaps, etc. The last two chapters are concerned with primality tests and factorization methods. The appendices have background material.

There are 12 extensive tables.

1. All primes $p < 10^4$.
2. All primes $10^n < p < 10^n + 1000$ for $n = 4(1)15$.
3. $\pi(x)$ and the Gauss and Riemann approximations to $x = 10^{10}$.
4. Composites $104395289 < c < 2 \cdot 10^8$, satisfying $2^{c-1} \equiv 1 \pmod c$ and having no prime divisor $\leqq 317$.
5. Known factors of Fermat numbers.
6. Complete factorization of $2^n - 1$. All $n < 137$ and others $\leqq 540$.
7. Factors of $(10^n - 1)/9$ and $10^n + 1$.
8. Primes of form $h \cdot 2^n + 1$ for $h = 1(2)99$. [Note, $h = 1$, $n = 8$ is missing.]
9. Primes of form $h \cdot 2^n - 1$ for $h = 1(2)151$.
10. Primes of form $n^4 + 1$ for $n \leqq 4002$.
11. Miscellaneous.
12. Quadratic residues: for square-free a, $|a| < 100$, all k and l such that primes $p = kx + l$ have $(a \mid p) = +1$.

An English translation may be published in the future, but even readers with no Swedish will be able to grasp much of the present text. Try this:

> SATS B1.3 *Om A är element i en ändlig grupp G med n element, är $A^n = I$.*

The tables are in Arabic numerals and will cause no difficulty at all.

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9[10].—John Leech, Editor, *Computational Problems in Abstract Algebra*, Pergamon Press, Ltd., Oxford, 1970, x + 402 pp., 23 cm. Price $18.50.

This book consists of thirty-five papers, most of which were presented at a conference on the use of computers in solving problems in algebra, held in 1967 at the University of Oxford under the auspices of the Science Research Council Atlas Computer Laboratory.

Over one-half of the papers are concerned with the application of computers to problems in group theory. The balance of the book includes papers on the use of computers in such areas as word problems in universal algebras, nonassociative algebras, latin squares, Galois theory, knot theory, algebraic number theory, algebraic topology and linear algebra. The first paper, a survey of the methods used and the results obtained by computers in the investigation of groups, contains an extensive bibliography on applications of computers to problems in algebra.

Many of the leading experts in the use of computers in algebra are among the authors of these papers, and the book gives a fairly comprehensive picture of the quite diverse activity in the subject.

The number of available papers concerning applications of computers to problems in algebra is relatively meager considering the potential of such applications. This volume is a welcome and a significant addition to the literature on the subject.

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10[12].—R. D. PARSLOW, R. W. PROWSE & R. E. GREEN, Editors, *Computer Graphics: Techniques and Applications*, Plenum Press, London and New York, 1969, xiv + 247 pp., 23 cm. Price $16.00.

This book consists primarily of a collection of papers that were presented at the International Computer Graphics Symposium held in July 1969 at Brunel University, Uxbridge, England. The book, which was designed to cover the field of computer graphics, is organized into four parts.

Part 1 contains nine papers that deal with basic hardware/software concepts in computer graphics. This section of the book is intended to serve as an introduction to the field for the novice; unfortunately, only the first paper, "What has Computer Graphics to Offer?", is suitable for this purpose.

The paper "Computer Graphics Hardware Techniques," although well written, suffers from a lack of visual aids to guide the novice. The companion paper "Computer Graphics Software Techniques" provides a rather sketchy coverage of its subject matter. The highlight of this paper is an extremely good treatment of the concept of a graphic data structure and its relationship to a display file. A third techniques-oriented paper, "Interactive Software Techniques," suffers because of poor organization.

The paper entitled "Computer Display System Tradeoffs" provides an interesting discussion of the relative merits of the so-called buffered display, which contains its own refresh memory, and a display which uses the computer memory as the refresh memory. This discussion concentrates on the hardware aspects of the question and ignores the software considerations. This paper, although well written, may be lost on the novice.

The two papers "Computer Graphics in the United States" and "The U. K. Scene" are intended to provide an overview of computer graphics in the United States and in England. The poor presentation in the second paper is in very striking contrast to the better organized presentation in the first.

The paper "Low Cost Graphics" provides a very good coverage of the tradeoffs inherent in the three most commonly used types of CRT displays; random-scan refresh, sequential-scan refresh, and the direct-view storage tube (DVST). A strong case was made for the use of the DVST in those applications which are not highly interactive.

The final paper in Part 1, entitled "Remote Display Terminals," deals with the various types of display terminals suitable for use in a remote on-line interactive environment. The so-called low-speed terminals (e.g., teletype, incremental plotters, and alphanumeric CRT's) receive the bulk of the attention, while CRT's providing the full range of graphics receive very sketchy treatment.

Part 2 contains nine papers, eight of which describe graphic applications. The graphical discussions contained in these eight papers are, for the most part, not of a general nature and hence would be of little interest to anyone not familiar with the rather specialized applications covered. The final papers in this section consist of a collection of remarks and observations which were recorded during a discussion session at the Symposium.

There is only one paper in Part 3. This paper, "Present Day Computer Graphics Research," gives a good coverage of research efforts in low-cost terminals and in graphic software.

The reference section of the book, Part 4, consists of a series of advertisements for hardware manufacturers of graphics equipment, a glossary of computer graphics terms, and a consolidated bibliography for all of the papers appearing in the book. The glossary provides a comprehensive coverage of graphics-related terms, and is perhaps the most useful portion of the book.

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11[12].—A. VAN WIJNGAARDEN, Editor, B. J. MAILLOUX, J. E. L. PECK & C. H. A. KOSTER, *Report on the Algorithmic Language ALGOL* 68, Second Printing MR 101, Mathematisch Centrum, Amsterdam, 1969, v + 134 pp., 24 cm. Price $4.50. Also available as offprint from *Numerische Mathematik*, v. 14, 1969, pp. 79–218, Springer-Verlag, New York.

The report is the culmination of a five-year effort by Working Group 2.1 of the International Federation for Information Processing (IFIP) to design a successor to ALGOL 60. Section 0 of the report describes the aims and principles of design, the first of which is "completeness and clarity of description." By Section 1, the clarity has disappeared. Embedded among the specifications of syntax and semantics are comments, called pragmatics, intended "to help the reader understand the implications of the definitions." However, they are often little more than verbalization of the syntax rules. The reader well-versed in the language may find the report useful as an authoritative reference manual (provided he has learned all the new terminology), but even the reader experienced in programming languages is advised to seek other expositions to learn the language. (Such documents are beginning to be available in computer science literature.)

As for the language, although it is not strictly an extension of ALGOL 60, it is in the same tradition. (A brief comparison of ALGOL 60 and ALGOL 68 is given in Section 0 of the report.) The ALGOL 60 notion of type is generalized to the concept of mode, of which there are an infinite variety and which include structured values and names (pointers, here called "references"). Subscripting is generalized to include slices. There is a more general conditional statement. (Statements are called "clauses" in ALGOL 68.) The language has facilities for formatted input-output (called "transput") and for environment inquiries.

The opinion has been expressed (cf. Minority Report, ALGOL Bulletin 31, p. 7) that ALGOL 68 is not sufficiently advanced to facilitate the reliable creation of today's more sophisticated programs. Since no implementation of the language has been completed (although several efforts are well advanced), experience is not available to confirm or deny this view.

The offprint from *Numerische Mathematik* has exactly the same material as the Mathematisch Centrum edition but is much easier on the eye because of better choices of type and better spacing.

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12[13.35].—ARTO SALOMAA, *Theory of Automata*, International Series of Monographs in Pure and Applied Mathematics, Vol. 100, Pergamon Press Ltd., Oxford, 1969, xii + 263 pp., 22 cm. Price $12.00.

This is a well organized introduction to finite automata theory. It deals with the mathematical foundations, and not with the practical applications to sequential switching circuits or nerve networks. The construction and programming of actual computers is not treated. Instead, the "machines" considered are certain theoretical models which have been intensively investigated during the last fifteen years. These include the finite deterministic automaton, the finite nondeterministic and probabilistic automata, and the pushdown and linear bounded automata.

Attention centers on the languages which are representable in the various machines. Regular languages (those representable in finite deterministic automata) and stochastic languages (those representable in probabilistic automata) are studied in the first two chapters. The third chapter is devoted to the algebra of regular expressions; here the author presents some of his own results concerning axiomatizations of this algebra.

The fourth and last chapter, entitled "Formal languages and generalized automata," introduces the notion of generation of languages by grammars. It includes proofs that the context-free languages are those which are representable in nondeterministic pushdown automata and that the context-sensitive languages are those which are representable in nondeterministic linear bounded automata. Recursively enumerable sets and Turing machines are mentioned in this chapter; however, the full study of recursive function theory is outside the scope of the book. This chapter also includes a section on the abstract pushdown automata of Letichevskiĭ. The author has prepared the reader for this section by including earlier sections on the analysis of finite automata by means of characteristic equations and on the solution of equation systems in which the unknowns are languages. There are many other items of special interest which cannot be mentioned in a brief review.

Algebraic structure theories, such as those developed by Krohn and Rhodes, by Hartmanis and Stearns, and by Eilenberg and Wright, are not considered in this book. The author has, however, provided a collection of historical and bibliographical remarks, which, together with an extensive list of references, make the book a useful guide to the literature on automata theory, in both the English and Russian languages. Many exercises which explain and extend the theory are listed, and some statements of unsolved problems are given as well, so that the book can be conveniently used as a text in a seminar or in a beginning graduate course.

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