REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

37[2.10].—FRANK STENGER, *Tabulation of Certain Fully Symmetric Numerical Integration Formulas of Degree* 7, 9 *and* 11. Paperbound xeroxed report having 8 typewritten pages and 45 pages of tables prepared from computer output; see microfiche card, this issue.

There are three regions considered, these being (i) the hypercube $|x_i| < 1$, (ii) the hypersphere $\sum x_i^2 < 1$, and (iii) the integral over all space with weight function $\exp[-\sum x_i^2]$, respectively. For each (d, n), $d = 7, 9, 11$, $n = 1, 2, \cdots, 20$, there are listed the weights and abscissas of two n-dimensional rules of degree d for each region. Sixteen-significant-decimal figures are listed and the author claims fourteen-place accuracy. In addition, simple explicit expressions are given for the coefficients and evaluation points of the formulas of degree $d = 3$ and 5.

The underlying theory corresponding to these rules is given in [1] and the place of that theory in the current state of the art of multidimensional quadrature is discussed in Haber [2].

The rules are fully symmetric. That is, the same weight is attached to the function value $f(x_1, x_2, \cdots, x_n)$ as is attached to all distinct function values of the form $f(\pm x_{i_1}, \pm x_{i_2}, \cdots, \pm x_{i_n})$ where $i_1, i_2, \cdots, i_n$ is a permutation of $1, 2, \cdots, n$, any combination of signs being allowed.

For $d = 7, 9$ (or 11), the values of x_i are restricted to five (or seven) values 0, $\pm u$, $\pm v$ (and $\pm w$). In the cases of the hypercube and the infinite integral these include the Gaussian abscissas of that degree. Thus, the degree 9 rules involve only a subset of the points which would be required by the product Gaussian rule of the same degree. For the hypersphere u, v (and w) depend also on n.

These tables fill a gap in the literature and Dr. Stenger has performed a useful service in preparing them.

W. G.

1. J. MCNAMEE & F. STENGER, "Construction of fully symmetric numerical integration formulas," *Numer. Math.*, v. 10, 1967, pp. 327–344.

2. S. HABER, "Numerical evaluation of multiple integrals," *SIAM Rev.*, v. 12, 1970, pp. 481–526.

38[2.10, 2.25].—P. C. CHAKRAVARTI, *Integrals and Sums, Some New Formulae for their Numerical Evaluation*, The Athlone Press, University of London, 1970, x + 89 pp., 24 cm. Price $7.25.

This monograph reports the results of the author's research over a period of ten years.

The author's primary interest seems to be in constructing quadrature formulas which use finite-difference expansions. A prototype is Gregory's quadrature formula.

He has constructed a large number of variants which deal with different situations. These include those in which there is a weighting function e^{px}, those in which end-points do not coincide with grid points and those in which endpoint connections at these inconvenient points are expressed in terms of derivatives. The weighting function $(x - x_0)^\alpha$ is also treated but on a less ambitious scale. The coefficients in this case involve Riemann zeta functions and related functions. These are tabulated and in general the user has to resort to numerical interpolation to calculate the coefficients. The finite-difference enthusiast will surely find many new and complicated expansions here.

As is conventional in this field there is no discussion about the convergence of these expansions. Numerical examples are chosen from the rather limited subset of functions for which the expansions happen to converge.

This reviewer found one part of the book to be of more general interest. The author derives a generalization of the Euler-Maclaurin summation formula. When $F(x) = e^{px}f(x)$, the asymptotic expansion on the right-hand side in the conventional form

$$h \sum_{r=0}^{n-1} F(x_r + sh) - \int_{x_0}^{x_n} F(t)\,dt \simeq \sum_{r=0}^{N} h^{r+1} \frac{B_{r+1}(s)}{(r+1)!} \{F^{(r)}(x_n) - F^{(r)}(x_0)\} + O(h^{N+2})$$

may be transformed into

$$\sum_{r=0}^{N} h^{r+1} \frac{e^{sph}\ D_r(s, ph)}{r!} \{e^{px_n}f^{(r)}(x_n) - e^{px_0}f^{(r)}(x_0)\} + O(h^{N+2}),$$

where the function $D_r(s, q)$ has many interesting elementary properties, some of which are quite tricky. Some of these are generalizations of analogous properties of the Bernoulli functions and the Euler functions. The author presents an excellent account of this function. In an appendix the author establishes the close connection between this function and the higher transcendental function

$$\phi(z, -r, s) = \sum_{j=0}^{\infty} (s + j)^r z^j .$$

The section about the D-functions would have certainly reached a wider and responsive audience if it had been published as a paper in a journal. However, the bulk of the book serves mainly as a repository for finite-difference expansions.

J. N. L.

39[2.20].—J. H. Garcia Rodriguez & F. Reveron Osio, *Tablas para la Resolucion de Ecuaciones de Tercero y Quinto Grado* (Tables for the Solution of Equations of the Third and Fifth Degree), Universidad de los Andes, Facultad de Ingenieria, Escuela de Ingenieria Electrica, Merida, Venezuela, 1968, 201 pp.

For solving the general cubic $Ay^3 + By^2 + Cy + D = 0$, after it is reduced to

$$X^3 + pX + q = 0 \tag{1}$$

by the transformation $y = X - B/3A$, the authors give, on pp. 17–176, tables for the roots of (1) for $p = -100(1)100$, $q = 0(1)100$, to 5D (6S for about 70 percent of the entries). For negative q, let $X = -X'$ in (1). When (1) has a pair of complex roots $R \pm I$, the authors tabulate R and I on opposite sides of a page, leaving the third root $-2R$ for the reader to find. When (1) has three real roots, the authors tabulate two, X_1 and X_2, on opposite sides of a page, leaving the third, $X_3 = -(X_1 + X_2)$, for the reader to find.

For solving the general quintic $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$, expressible as $ay^5 + gy^3 + hy^2 + ky + m = 0$ after the transformation $x = y - b/5a$, the authors make the further transformation $y = z(-m/a)^{1/5}$ to obtain

$$z^5 = pz^3 + qz^2 + rz + 1. \tag{2}$$

The authors tabulate, on pp. 181–201, just a single real root, namely, the smaller or larger positive root of (2) according as $p + q + r$ is negative or positive, for each of p, q, $r = -10(1)10$, to 5D (6S for over 60 percent of the entries). When $p + q + r = 0$, $z = 1$ is a positive root of (2).

For interpolation in (1) for nonintegral p and q, the authors give a 4-point bilinear formula in the fractional portions of p and q, with two examples. For interpolation in (2) for nonintegral p, q and r, the authors give an 8-point trilinear formula in the fractional portions of p, q and r, with one example. However, no mention is made of the accuracy attained by those interpolation formulas.

The introductory text consists of eight small pages for (1), two pages for (2), and a prefatory note by A. Zavrotsky.

The computation was performed on an IBM 1620 system in the Electronics Center of Computation of the University of the Andes, using FORTRAN programs for calculation and printout. For the cubic, Cardan's formulas were employed; for the quintic, Horner's method. Altogether, 115 hours were required for the computation.

The text contains a brief historical note mentioning 16 other tables for solving cubics, just by author, place, and year.

In view of the statement in the preface that it is believed that there is not a single error in the thousands of digits comprising the table, the defective page 142, where there is no printout of the imaginary part of the complex root of (1) for $p = 55, 56, 57, 58$ and $q = 0(1)50$, *may* be only in the reviewer's copy.

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40[2.30, 2.45, 8, 9, 12].—D. E. Knuth, *The Art of Computer Programming—Errata et Addenda*, Report STAN-CS-71-194, Stanford University, January 1971.

Since Volumes I and II of Knuth's *The Art of Computer Programming* have been so well received, (see our reviews RMT **81**, v. 23, 1969, pp. 447–450, and RMT **26**, v. 24, 1970, pp. 479–482), it seems desirable to call attention to the extensive changes offered here. A July 1969 "second printing" of Vol. I already included "about 1000 minor improvements". A list of these changes is available from the author.

The present 28 pages of small print include $7\frac{1}{2}$ more pages of changes in Vol. I, but the main changes are in his second volume. Some of these are quite minor: spelling, middle names, notation, small numerical errors, and they would not justify this review. But, besides many other changes of greater significance, two sections of the book have been extensively revised to bring in recent research.

The first of these topics is Euclid's algorithm and now includes some new research by Philipp, Dixon and Heilbronn.

The largest change is an account of the new Strassen-Schönhage "fast Fourier multiplication" algorithm including

THEOREM S. *It is possible to multiply two n-bit numbers in* $O(n \log n \log \log n)$ *steps.*

It is clear that the Mersenne prime and related industries will have to completely retool to take advantage of this new technology. With such retooling, and the faster machines now available, we can expect a new prime $2^p - 1$ to show itself one of these days.

To make room for this new material while keeping the same pagination, certain topics in the first printing, such as gaps between successive primes, are deleted—through no fault of their own.

Naturally, Knuth is driving his publisher bananas with all these changes, and he begins his compilation here, amusingly, with "The author wishes to encourage everyone to stop doing any further research, so that he may finish writing the other volumes." Why such a mild way of trying to halt the tide of research; why not simply command it, as did his ancestor, the King?

D. S.

41[4].—JAMES W. DANIEL & RAMON E. MOORE, *Computation and Theory in Ordinary Differential Equations*, W. M. Freeman & Co., San Francisco, 1970, viii + 172 pp., 24 cm. Price $7.50.

Because of their training, physicists and engineers tend to stick to analytic methods as long as possible, before submitting their problems to a computer. In their treatment of problems in differential equations, they may sometimes end up by offering them to the computer in a form less suitable than the original version. On the other hand, the present generation of numerical analysts suffers from the opposite extreme: as computers grow faster and memories become larger, numerical analysts spend less and less effort on preparing a problem in differential equations analytically. They simply grind it through the mill. The book under review attempts to counteract this tendency and to resubstitute mathematical analysis, at least in part, for computing by brute force.

For this purpose, the authors present a variety of analytic techniques which may be used to transform a given system of differential equations into one which is better suited for numerical solution. Their main criterion for that "suitability" is the straightness of the "flow" defined by the system through its vector field. Thus, the transformations they study are designed to straighten out large local variations or curvatures in this "flow". This common strategy unifies the treatment in this book which is subdivided into sections on "a priori global transformations" and "a posteriori transformations". The last term denotes efforts to facilitate the computation of a neighboring solution of a given (or computed) solution. The techniques range from old classical approaches to the use of local or moving coordinate systems which combine analytic and computational methods. The authors state no theorems nor do they give rigorous derivations; instead, they demonstrate the rationale of the various lines of attack and try to illuminate them by well-chosen examples. Naturally, they refer to the relevant literature for a more thorough treatment.

The third part of the book is preceded by a section on basic facts about differential equations (selected so as to prepare for the later discussion) and a section on numerical methods which, very briefly, gives some of the principal concepts. In this section, a variety of classes of numerical methods is discussed (including some not so well-known ones). The authors make an interesting attempt at an evaluation of their relative merit. Again, the reader has to resort to the quoted literature for any hard-core facts, if he does not already know them. Throughout the book, the main emphasis rests on initial-value problems. In the first two parts of the book, some attention is given to boundary-value problems.

While these parts may serve as a welcome guide to the literature, it is the third part on transformations which makes the book a very valuable contribution to numerical analysis as well as to scientists who have to solve differential equations. Though many of the techniques described will need some further development and computational experience (as the authors freely admit), it is hoped that their exhibition will stimulate worthwhile efforts in that direction.

H. J. S.

42[7].—Jacques Dutka, *The Square Root of* 2 *to* 1,000,000 *Decimals*, ms. of 200 computer sheets + 1 supplementary page, deposited in the UMT file.

The decimal value of the square root of 2 to one million places is herein presented on 200 computer sheets, each containing 5000 decimal digits. A supplementary page gives the succeeding 82 decimal places.

This carefully checked calculation required a total of about 47.5 hours of computer time on the IBM 360/91 system at Columbia University.

Further details and pertinent background information have been given by the author in a paper [1] appearing elsewhere in this journal.

J. W. W.

1. Jacques Dutka, "The square root of 2 to 1,000,000 decimals," *Math. Comp.*, v. 25, 1971, pp. 927–930.

43[8].—H. Leon Harter & D. B. Owen, Editors, *Selected Tables in Mathematical Statistics*, Markham Publishing Co., Chicago, 1970, vii + 405 pp., 29 cm. Price $5.80 cloth.

This book is Volume I of a projected series of volumes of mathematical tables prepared under the aegis of the Institute of Mathematical Statistics. A list of the tables and their authors follows:

1. Cumulative Noncentral Chi-Square Distribution—Haynam, Govindarajulu, and Leone.

2. Exact Sampling Distribution of the Two-Sample Kolmogorov-Smirnov Criterion D_{mn}—Kim and Jennrich.

3. Critical Values and Probability Levels for the Wilcoxon Rank Sum Test and the Wilcoxon Signed Rank Test—Wilcoxon, Katti, and Wilcox.

4. Null Distribution of the First Three Product-Moment Statistics for Exponential, Half-Gamma, and Normal Scores—Lewis and Goodman.

5. Tables to Facilitate the Use of Orthogonal Polynomials for Two Types of Error Structures—Stewart.

The first table is misnamed, as it consists of two tables, one giving the power of the chi-square test for levels of significance .001, .005, .01, .025, .05, and .1, for degrees of freedom ν = 1(1)30(2)50(5)100, and for noncentrality parameter λ = 0(.1)1(.2)3(.5)5(1)40(2)50(5)100, and the other giving the noncentrality parameter for the aforementioned degrees of freedom and levels of significance and for cumulative noncentral chi-square distribution levels of .1(.02).7(.01).99. Thus, the table should be renamed "Tables related to the cumulative noncentral chi-square distribution."

Table 2 is in two parts. Part 1 gives initial values C for the Kolmogorov-Smirnov test for $m \leqq n$ = 1(1)25, corresponding to all levels of significance ranging from .001 to .14, and includes the largest C corresponding to a level of significance below .001 and the smallest C corresponding to a level of significance above .1. Part 2 gives critical values for α = .001, .005, .01, .025, .05, .1.

Part 1 of Table 3 gives the critical values (and exact level of significance) which have smallest significance level greater than and largest significance level less than the one-tail nominal levels .005, .01, .025, and .05 for all combinations of m and n ranging from 3 to 50 for the Wilcoxon rank sum test. Part 2 gives exact probability levels for all possible rank totals which have probability between .0001 and .5 for sample sizes n = 5(1)50.

Table 4 contains the results of a Monte Carlo investigation to obtain the sampling distribution of the product-moment statistic for samples drawn from the unit normal, mean 1 exponential, and half-gamma (mean 1 and shape parameter .5) distribution. They are tabulated for probability levels .001, .002, .005, .01, .02, .025, .05, .1, .9, .95, .975, .98, .99, .995, .998, .999, for lags 1, 2, and 3, for sample sizes 11(1)40 for the normal and half-gamma data and 11(1)174(2)250(10)500, 900, 2500, 5000, 9000 and for exponential data.

Table 5 is in two parts. For n = 3(1)20, where n is the number of data points, Part 1 gives the orthonormalizing factor and transformation matrix T for transforming from an orthogonal polynomial fit of degree k, k = 1(1) min(n, 4), to a fit for the "natural" independent variables of the problem. Also tabulated are TT' and standard deviations and upper 97.5% confidence limit for (1) an estimated dependent variable from the set of data used to produce the fit, (2) an estimated new random observation of a dependent variable for a given set of independent variables, and (3) the residual between observed and estimated dependent variable.

Part 2 considers a nonstandard error model, wherein the error in an observation is the sum of the errors of its predecessors plus a contribution for error in this observation also. As an aid in using orthogonal polynomials to fit polynomials to such data, the following are tabulated: (1) the transformation matrix T as before, (2) two alternative transformation matrices S and M, used to determine the "natural" parameters directly from the data without using an orthogonal polynomial fit first, and (3) tables to aid in determining the predicted value of the independent variable and its variance. These tables also cover the range $n = 3(1)20$, $k = 1(1) \min(n, 4)$.

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44[8].—CHARLES E. LAND, *Tables of Critical Values for Testing Hypotheses about Linear Functions of the Normal Mean and Variance*, Department of Statistics, Oregon State University, Corvallis, Oregon 97331, ms. of 4 typewritten pp. and 59 computer sheets deposited in the UMT file.

These unpublished tables constitute an appendix to a paper that is published elsewhere [1].

The critical values define the uniformly most powerful unbiased level α tests of hypotheses H_θ: $\mu + \lambda\sigma^2 = \theta$, where λ and θ are arbitrary numbers and μ and σ^2 are the mean and variance of a normal distribution, against one-sided (Table 1) and two-sided (Table 2) alternatives.

The critical value $t(\nu, \xi, \alpha)$ given in Table 1 is the αth quantile of the distribution with density proportional to

$$f_\nu(t \mid \xi) = (\nu + t^2)^{-(\nu+1)/2} \exp\{(\nu + 1)\xi t/(\nu + t^2)^{1/2}\}.$$

The tabulation is to 3D for $\alpha = .0025, .005, .01, .025, .05, .1, .25, .50, .75, .90, .95, .975, .99, .995$, and $.9975$; $\xi = 0(.1).5(.25)2(.5)5(1)10(2)20(5)50(10)100$; and $\nu = 2(1)10(2)20(5)50(10)100(20)200(50)500(100)1000$.

The two-sided critical values $t_1(\nu, \xi, \alpha)$ and $t_2(\nu, \xi, \alpha)$ satisfy the two equations

$$\int_{t_1}^{t_2} f_\nu(t \mid \xi)\, dt = (1 - \alpha) \int_{-\infty}^{\infty} f_\nu(t \mid \xi)\, dt,$$

$$\int_{t_1}^{t_2} t(\nu + t^2)^{-1/2} f_\nu(t \mid \xi)\, dt = (1 - \alpha) \int_{-\infty}^{\infty} t(\nu + t^2)^{-1/2} f_\nu(t \mid \xi)\, dt.$$

They are given to 3D in Table 2 for $\alpha = .005, .01, .02, .05, .1, .2$, and $.5$; $\nu = 2(2)20(10)100$; and the same range of ξ as in Table 1 except for increasing omissions of small positive ξ, beginning at $\nu = 6$.

Details of the methods used in calculating these tables on a CDC 3300 system are supplied by the author in a four-page introduction.

J. W. W.

1. C. E. LAND, "Confidence intervals for linear functions of the normal mean and variance," *Ann. Math. Statist.*, v. 42, 1971. (To appear.)

45[8].—Madan Lal Puri, Editor, *Nonparametric Techniques in Statistical Inference*, Cambridge University Press, New York, 1970, xiv + 623 pp., 24 cm. Price $32.50.

This book is a collection of 35 papers, and discussions on 15 of these papers, presented at the first international symposium on nonparametric techniques, held at Indiana University in June 1969. The subjects covered include general theory of nonparametric inference, specific nonparametric tests and estimates, theory of order statistics, ranking and selection procedures, decision theoretic and empirical Bayes procedures, and teaching of nonparametric statistics. No computational problems are discussed.

Albert Madansky

46[12].—Henry Mullish, *Modern Programming*: *Fortran* IV, Ginn-Blaisdell, Waltham, Massachusetts, 1968, xii + 132 pp., 28 cm. Price $5.25.

This text provides the student with an excellent set of examples for the use of the Fortran language. It is a problem-oriented text more than it is a grammar-oriented one. The author's strategy seems to be to present the reader with a multitude of programming problems and solutions and have him learn the language by using it. This is similar to the total immersion procedure in foreign language schools.

All the sample programs are well documented and cover almost the entire spectrum of Fortran computing. The range is so broad that there is at least one problem in the book to interest every student. Basic concepts are presented in a fashion that stimulates learning, by posing problems in a down-to-earth manner. In addition to standard textbook problems such as matrix manipulation (summing rows, columns, etc.), there are telephone number coding problems and a license plate lottery problem. Some of the examples are introduced with a short story to make them even more palatable. The pleasantness of this text does not lower its level. The writing is easily readable and should be equally interesting for a high school senior, a graduate student or anyone wanting to learn the language and its applications.

The text is not complete in its coverage of the facilities of the Fortran language. It does present a clear picture that carries the reader through all types of arithmetic to subroutines. Extended I/O facilities and features peculiar to FORTAN G, H; WATFOR; and WATFIV are not touched. This text can be used solely in an introductory course, but should be supplemented by another text or instructor's notes in a more advanced course. This is the finest set of examples I have seen on this level and the book is a clear and thorough introduction to the basic language and its uses.

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