Mathematics of Computation

Volume 25, Number 116 October, 1971

CODEN: MCMPAF

Published by the American Mathematical Society

PROVIDENCE, RHODE ISLAND

Editorial Committee

EUGENE ISAACSON, Chairman, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012
Assistant to the Chairman: CHARLOTTE W. JOHN

Board of Associate Editors

James H. Bramble, Department of Mathematics, Cornell University, Ithaca, New York 14850

James W. Daniel, Department of Mathematics, University of Texas at Austin, Austin, Texas 78712

WALTER GAUTSCHI, Computer Sciences Department, Purdue University, Lafayette, Indiana 47907

DONALD GOLDFARB, Department of Computer Sciences, School of Engineering, The City College of the City University of New York, 139th Street & Convent Avenue, New York, New York 10031

ALSTON S. HOUSEHOLDER, Department of Mathematics, Ayres Hall, The University of Tennessee, Knoxville, Tennessee 37916

HEINZ-OTTO KREISS, Computer Science Department, University of Uppsala, Uppsala, Sturegaten 4, Sweden

YUDELL L. LUKE, Department of Mathematics, University of Missouri at Kansas City, Kansas City, Missouri 64110

JAMES N. LYNESS, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439

Beresford Parlett, Department of Computer Science, University of California, Berkeley, California 94720

PHILIP RABINOWITZ, Department of Applied Mathematics, The Weizmann Institute of Science, Rehovot, Israel

Daniel Shanks, Naval Ship Research and Development Center, Washington, D.C. 20034

HANS J. STETTER, Institut für Numerische Mathematik, Technische Hochschule Wien, A-1040 Wien, Karlsplatz 13, Austria

JOHN W. WRENCH, Jr., Naval Ship Research and Development Center, Washington, D.C. 20034

Information for Subscribers

The journal is published quarterly in one volume per year, with issues numbered serially since Volume 1, Number 1. The subscription price is \$20.00. All back volumes are available. For Volumes 1–19 (1943–1965), prices are \$20.00 per volume, subsequent volumes are \$24.00.

Unpublished Mathematical Tables

The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section *Reviews and Descriptions of Tables and Books*. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost per page.

Subscriptions, address changes, business communications and payments should be sent to:

AMERICAN MATHEMATICAL SOCIETY
P. O. Box 6248
Providence, Rhode Island 02904

Mathematics of Computation TABLE OF CONTENTS

OCTOBER 1971

Parallel Computation John R. Rice	639
Convergent Generalized Monotone Splitting of Matrices	033
O. L. Mangasarian	649
The Dirichlet Problem for a Class of Elliptic Difference Equations	0.17
G. T. McAllister	655
A Finite-Difference Method for Parabolic Differential Equations with Mixed	000
Derivatives Jan Krzysztof Kowalski	675
On the Convergence Rates of Variational Methods. I. Asymptotically Diag-	0.0
onal Systems L. M. Delves & K. O. Mead	699
Matricial Difference Schemes for Integrating Stiff Systems of Ordinary Differ-	
ential Equations W. L. MIRANKER	717
Convergence of Difference Methods for Initial and Boundary Value Problems	
with Discontinuous Data BRUCE CHARTRES & ROBERT STEPLEMAN	729
On the Computation of Some Grunsky Coefficients Relevant to the Bieber-	
bach Conjecture George G. Ross	733
A Numerical Determination of the Modulus of Doubly Connected Domains	
by Using the Bergman Curvature J. BURBEA	743
Roundoff Error Analysis of the Fast Fourier Transform	
George U. Ramos	757
Infinite Sums of Roots for a Class of Transcendental Equations and Bessel	
Functions of Order One-Half N. LIRON	769
Series Expansions of $W_{k,m}(z)$ Involving Parabolic Cylinder Functions	
R. Wong & E. Rosenbloom	783
Miniaturized Tables of Bessel Functions. II YUDELL L. LUKE	789
Table for Third-Degree Spline Interpolation Using Equi-Spaced Knots	
W. D. Hoskins	797
Obtaining Cubatures for Rectangles and Other Planar Regions by Using	
Orthogonal Polynomials RICHARD FRANKE	803
On the Remainder in Quadrature Rules P. D. TUAN	819
Some Polynomials for Complex Quadrature DAVID K. KAHANER	827
Minimal Quadratures for Functions of Low-Order Continuity	831
L. W. Johnson & R. D. Riess	931
Gauss's Ternary Form Reduction and the 2-Sylow Subgroup Daniel Shanks	837
Pseudo-Random Numbers: The Exact Distribution of Pairs U. DIETER	855
Covering the Set of Integers by Congruence Classes of Distinct Moduli	055
S. L. G. Choi	885
Algorithms for Hermite and Smith Normal Matrices and Linear Diophan-	000
tine Equations Gordon H. Bradley	897
Large Intervals Between Consecutive Primes J. H. CADWELL	909
Unitary Amicable Numbers Peter Hagis, Jr.	915
Some Numerical Evidence Concerning the Uniqueness of the Markov Num-	-
bers D. Rosen & G. S. Patterson, Jr.	919
A Note on Chowla's Function M. Lal & A. Forbes	923
The Square Root of 2 to 1,000,000 Decimals JACQUES DUTKA	927
Obituary—Charles Brown Tompkins	931

A Note on Chowla's Function M. Lal & A. Forbes	923
The Square Root of 2 to 1,000,000 Decimals JACQUES DUTKA	927
Obituary—Charles Brown Tompkins	931
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS	935
Chakravarti 38, Daniel & Moore 41, Dutka 42, Harter & Owen,	
Editors 43, Knuth 40, Land 44, Mullish 46, Puri, Editor 45, Rod-	
RIGUEZ & OSIO 39, STENGER 37.	
TABLE ERRATA	943
HART, CHENEY, LAWSON, MAEHLY, MESZTENYI, RICE, THACHER &	
Witzgall 482, Lehmer 483, Lehmer 484, Poulet 485.	
CORRIGENDUM	947
Berkowitz & Garner	
MICROFICHE SUPPLEMENT	
Tabulation of Certain Fully Symmetric Numerical Integration Formulas	
of Degree 7, 9 and 11 Frank Stenger	
On the Computation of Some Grunsky Coefficients Relevant to the Bie-	
berbach Conjecture George G. Ross	
Nodes for Complex Chebyshev Quadrature David K. Kahaner	
Coefficients for "Miniaturized Tables of Bessel Functions. II"	
YUDELL L. LUKE	
Indices to Volume XXV	949
Index of Papers by Authors	949
Subject Classification System for Index of Reviews	953
Index of Reviews by Author of Work Reviewed	956
Index of Reviews by Subject of Work Reviewed	958
Index of Table Errata	964
Index of Corrigenda	964
Index of Microfiche Supplements	965

Information for Contributors

Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see the latest *Mathematical Reviews* volume index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "Information for Contributors to Mathematics of Computation" and "Manual for Authors," both of which are available upon request from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to Eugene Isaacson, Chairman, Editorial Committee, Mathematics of Computation, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. Institutions sponsoring research reported in the journal are assessed page and microfiche charges.

Each article submitted for publication must be accompanied by a brief and reasonably self-contained abstract, and by AMS (MOS) subject classification numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the Index to Mathematical Reviews, Volume 39 (June 1970).

Microcard Edition

Volumes 1-14 (1943-1960) are available on Microcards at \$39.00 for the complete set and may be purchased from Microcard Editions, Inc., 901 26th Street, N. W., Washington, D.C. 20037.

Proceedings of Symposia in Pure Mathematics

Representation Theory of Finite Groups and Related Topics

Edited by Irving Reiner

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS, VOLUME XXI 184 pages; List Price \$11.70; Member Price \$8.78

This volume constitutes the proceedings of a symposium on Representation Theory of Finite Groups and Related Topics which was held in Madison, Wisconsin, April 14-16, 1970, in conjunction with a sectional meeting of the American Mathematical Society. The symposium was held in honor of Professor Richard Brauer whose fundamental work in representation theory lies at the heart of most of the further developments in this topic. These proceedings contain articles by the participants, based on their symposium lectures. The articles range from brief surveys of results to detailed outlines of proofs, and are intended to indicate the scope of current research in representation theory. Professor Irving Reiner of the University of Illinois, Urbana, was chairman of the Organizing Committee and editor of these proceedings.

Algebraic Topology

Edited by Arunas Liulevicius

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS, VOLUME XXII 290 pages; List Price \$17.80; Member Price \$13.35

This volume constitutes the proceedings of the seventeenth summer research institute of the American Mathematical Society, held at the University of Wisconsin, Madison, from June 29 to July 17, 1970. The program of the institute was divided into four parts: (1) survey talks on recent developments in the field of algebraic topology; (2) invited one-hour talks on important recent work; (3) sessions on problems; and (4) seminars organized by the participants. Appearing in this volume are the lecture notes of the survey talks presented by J. Frank Adams, Edgar H. Brown, Jr., Samuel Gitler, Richard K. Lashof, Franklin P. Peterson, Larry Smith, and James D. Stasheff. The remainder of the volume consists of the texts of the invited one-hour talks and a list of research problems. The list of research problems was edited by R. James Milgram.

Something new has been

The Audio Recordings of Mathematical Lectures are recordings of lectures presented at meetings of the American Mathematical Society: Gibbs Lectures, Colloqium Lectures, and invited hour addresses. These audio recordings are now being prepared on cassettes as well as tapes. Each lecture is accompanied by a manual which contains displays referred to in the lecture, and the recordings contain editorial comments referring the listener to these displays.

The lectures are recorded on tape at a speed of 1%"/second (4.75cm/second) and can be played on standard tape recorders; tapes run at 3%"/second (9.5cm/second) are available if requested. Cassettes and tapes may be purchased at \$6.00 each for hour addresses and \$10.00 for Colloquium Lectures. Additional copies of

American Mathematical Society

SIAM - AMS Proceedings

Mathematical Aspects of Electrical Network Analysis

Edited by Herbert S. Wilf and Frank Harary

SIAM-AMS PROCEEDINGS, VOLUME III 214 pages; List Price \$10.60; Member Price \$7.95

This volume constitutes the proceedings of the SIAM-AMS symposium on Mathematical Aspects of Electrical Network Analysis held in New York City in April 1969. Most of the papers apply various mathematical methods to electrical network problems, but a few of them accomplish the reverse "application," and help to solve purely mathematical problems using electrical network theory technique. The authors include R. K. Brayton, R. W. Brockett, D. A. Calahan, Shu-Park Chan, R. J. Duffin, R. J. Leake, R. Liu, Ronald Rohrer, J. Paul Roth, R. Saeks, I. W. Sandberg, R. A. Skoog, Pravin Varaiya, Dan H. Wolaver, Dante C. Youla, and J. W. T. Youngs.

Computers in Algebra and Number Theory

Edited by Garrett Birkhoff and Marshall Hall, Jr.

SIAM-AMS PROCEEDINGS, VOLUME IV 208 pages; List Price \$12.70; Member Price \$9.53

This volume constitutes the proceedings of the SIAM-AMS symposium on Computers in Algebra and Number Theory held in New York City in March 1970. Part 1 consists of papers devoted to applications of algebraic ideas to computing with especial attention to problems of optimizing computer algorithms. The five papers in Part 2 consider number theory and combinatorial theory. The final section, Part 3, deals with the application of computers to finite groups. The authors include L. D. Baumert et al., B. J. Birch, Garrett Birkhoff, John J. Cannon, J. H. Conway, Marshall Hall, Jr., M. D. Hestenes, D. G. Higman, John Mckay, J. Neubüser, Charles C. Sims, H. P. F. Swinnerton-Dyer, J. H. van Lint, Shmuel Winograd, and Hans Zassenhaus.

added: cassettes

the manual may be ordered for \$0.30 each. At the present time forty-six lectures are available, including three Colloquium Series. Lecturers include Masani, Treves, Boone, Fadell, Bott, Harish-Chandra, Kirby, Baum, Tate, Bass, Feferman, Friedman, Shah, Rosenblum, Heerema, Shult, Hirsch, McMillan, Glimm, Auslander, Atiyah, Kohn, McLaughlin, Mahler, Rubel, Curtis, Blackwell, Gangolli, Billingsley, Ax, Varga, Hofmann, Bing, Hemmingsen, Ornstein, Gorenstein, Kesten, Zariski, Hopf, Nachbin, Ellis, Sullivan, O'Meara, Blattner, Fan and Knopp. For additional information, or to place an order, please write to the Promotion Department, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904.

P.O. Box 6248, Providence, R.I. 02904

"automaticomputation"

four new titles in the Automatic Computation Series, edited by George Forsythe

NUMERICAL INITIAL VALUE PROBLEMS IN ORDINARY DIFFERENTIAL EQUATIONS

C. William Gear, University of Illinois

New—Discusses all classes of step by step methods suitable for the automatic numerical integration of general problems. Covers the derivation of methods; the theory of error and convergence, and the practical implementation on a computer. Includes theoretical and computational end-of-chapter problems, many numerical examples showing error behavior, numerical comparisons of different methods, and a large bibliography.

1971 304 pp. \$12.95 (62660 6)

COLLECTIVELY COMPACT OPERATOR APPROXIMATION THEORY AND APPLICATIONS TO INTEGRAL EQUATIONS

Philip Anselone, Michigan State University

New—Provides an introduction to a currently active field of research, presenting a recently developed abstract operator approximation theory and various applications to numerical integration approximations of integral operators. The theory is based on a new concept, that of a collectively compact (completely continuous) operator. Treats both linear and nonlinear operators. Valuable for courses and seminars at the graduate level.

1971 128 pp. \$12.50 (14067-3)

APPROXIMATE CALCULATION OF MULTIPLE INTEGRALS

A. H. Stroud, State University of New York at Buffalo

New—A research monograph that can serve as text or reference book. Provides the reader with a well-written exposition of all theoretical and practical aspects of the subject. Contains recent results relating integration formulas and orthogonal polynomials. Includes an excellent discussion of error estimates and a complete bibliography.

1972 432 pp. \$16.50 (04389·3)

DISCRETE OPTIMIZATION: INTEGER PROGRAMMING AND NETWORK ANALYSIS FOR MANAGEMENT DECISIONS

Donald R. Plane and Claude McMillan, both of the University of Colorado

New—Management Decision-making through an easy to understand treatment of integer programming and network analysis. Offers extensive discussions of problem formulation, methods of solution, and computational experience. Introduces the concepts of constrained optimization through enumeration algorithms. Emphasizes problem formulation so that the user of operations research techniques can understand the power of discrete optimization. 1971 288 pp. \$13.50 (21604-4)

For further information write: Box 903, Prentice-Hall, Englewood Cliffs, N.J. 07632

from Prentice-Hall

Reviews and Descriptions of Tables and Books	935
Table Errata	943
HART, CHENEY, LAWSON, MAEHLY, MESZTENYI, RICE, THACHER & WITZGALL 482, LEHMER 483, LEHMER 484, POULET 485.	
Corrigendum	947
Berkowitz & Garner	
MICROFICHE SUPPLEMENT	
Tabulation of Certain Fully Symmetric Numerical Integration Formulas of Degree 7, 9 and 11	
INDICES TO VOLUME XXV	949
Index of Papers by Authors	949
Subject Classification System for Index of Reviews	953
Index of Reviews by Author of Work Reviewed	956
Index of Reviews by Subject of Work Reviewed	958
Index of Table Errata	964
Index of Corrigenda	964
Index of Microfiche Supplements	965

Mathematics of Computation

TABLE OF CONTENTS OCTOBER 1971

Parallel Computation John R. Rice	639
Convergent Generalized Monotone Splitting of Matrices	
O. L. Mangasarian	649
The Dirichlet Problem for a Class of Elliptic Difference Equations	
G. T. McAllister	655
A Finite-Difference Method for Parabolic Differential Equations with Mixed	
Derivatives JAN KRZYSZTOF KOWALSKI	675
On the Convergence Rates of Variational Methods. I. Asymptotically Diagonal Systems L. M. Delves & K. O. Mead	699
Matricial Difference Schemes for Integrating Stiff Systems of Ordinary Differential Equations W. L. MIRANKER	717
Convergence of Difference Methods for Initial and Boundary Value Problems with Discontinuous Data BRUCE CHARTRES & ROBERT STEPLEMAN	729
On the Computation of Some Grunsky Coefficients Relevant to the Bieberbach Conjecture	733
A Numerical Determination of the Modulus of Doubly Connected Domains by Using the Bergman Curvature J. BURBEA	743
Roundoff Error Analysis of the Fast Fourier Transform	143
George U. Ramos	757
Infinite Sums of Roots for a Class of Transcendental Equations and Bessel	
Functions of Order One-Half N. LIRON	769
Series Expansions of $W_{k,m}(z)$ Involving Parabolic Cylinder Functions R. Wong & E. Rosenbloom	783
Miniaturized Tables of Bessel Functions, II YUDELL L. LUKE	
Table for Third-Degree Spline Interpolation Using Equi-Spaced Knots	789
W. D. Hoskins	797
Obtaining Cubatures for Rectangles and Other Planar Regions by Using	171
Orthogonal Polynomials RICHARD FRANKE	803
On the Remainder in Quadrature Rules P. D. TUAN	819
Some Polynomials for Complex Quadrature DAVID K. KAHANER	827
Minimal Quadratures for Functions of Low-Order Continuity	
L. W. Johnson & R. D. Riess	831
Gauss's Ternary Form Reduction and the 2-Sylow Subgroup	
DANIEL SHANKS	837
Pseudo-Random Numbers: The Exact Distribution of Pairs U. DIETER	855
Covering the Set of Integers by Congruence Classes of Distinct Moduli	
S. L. G. Choi	885
Algorithms for Hermite and Smith Normal Matrices and Linear Diophantine Equations	897
Large Intervals Between Consecutive Primes J. H. CADWELL	909
Unitary Amicable Numbers Peter Hagis, Jr.	915
Some Numerical Evidence Concerning the Uniqueness of the Markov Num-	13
bers D. Rosen & G. S. Patterson, Jr.	919