Mathematics of Computation

Coden: MCMPAF
Volume 27, Number 121

Pages 1–220
January 1973

Published by the American Mathematical Society
PROVIDENCE, RHODE ISLAND

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A Quarterly Journal
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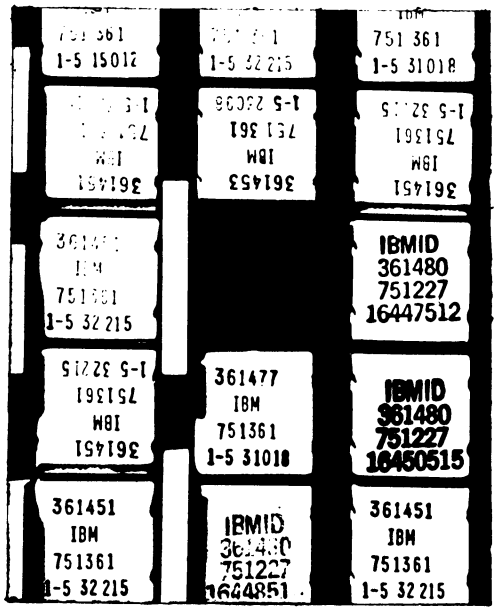

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