REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

28 [2.35, 3, 4].—JAMES M. ORTEGA, *Numerical Analysis, A Second Course*, Academic Press, New York, 1972, xiii + 201 pp., 24 cm. Price $11.00.

This is a concise account of certain topics in numerical analysis which a student is expected to know when he reaches an advanced course yet may not have been introduced to in his first course on the subject.

The book is organized around the notion of *error*. After the concepts of stability and ill-conditioning (important in gauging the effects of all kinds of error) are elucidated in a first part of the book, discretization error, convergence error and rounding error are each studied separately in a few important situations in the last three parts of the book. A review chapter on the Jordan canonical form and on norms for vectors and matrices precedes all.

Stability (or the lack of it) is described as it occurs in the solution of a linear system, in the estimation of eigenvalues and eigenvectors, and in the solution of the initial value problem for a system of linear first-order differential equations or of linear difference equations. In the first instance, the author relies entirely on the condition number of the matrix of the linear system to measure the system's conditioning. The treatment of *a posteriori* error bounds in eigenvalue-eigenvector calculations seems more detailed; it includes a section for the important special case of a symmetric matrix. *Discretization error* is discussed in connection with the numerical solution of the general first-order initial value problem and of the linear second-order boundary value problem by finite difference methods. For the former, "Consistency plus Stability implies Convergence" is proved; the treatment of the latter relies on the maximum principle, else on the diagonal dominance of the coefficient matrix of the finite difference equations. The iterative solution of systems of linear and of nonlinear equations serves to illustrate *convergence error*. Major topics are the analysis of SOR in the linear case and of Newton's method in the nonlinear case. Finally, the backward error analysis for the triangular factorization of a matrix in finite precision arithmetic makes the discussion of *rounding error* concrete.

This is a textbook (of help in the mathematical analysis of some numerical methods) for first year graduate mathematicians and mathematically inclined computer scientists, written very carefully and with much attention to clean and simple proofs, with many interesting and varied exercises, and very carefully proofread.

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29 [3].—M. LUNELLI, Editor, *Una Biblioteca de Sottoprogrammi dell'Algebra Lineare*, Franco Angeli, Editore, Milano, 1972, 429 pp. Price: Lit. 12000.—.

A collection of Fortran programs derived from *The Algol Collection in Linear Algebra* by Wilkinson and Reinsch.* The first half of the book introduces and discusses the methods and relevant perturbation theory—all in Italian.

B. P.

The following review has been reprinted from SIAM Review, Vol. 14, No. 4, October 1972, p. 658, with the permission of Arthur Wouk, editor of Book Reviews of *SIAM Review*.

30 [3].—J. H. WILKINSON & C. REINSCH, *Handbook for Automatic Computation*. Vol. II, *Linear Algebra*, Springer-Verlag, New York, 1971, ix + 439 pp., 24 cm. Price $20.80.

Those with a strong interest in numerical linear algebra will already be familiar with some of the algorithms given in this book. In this review I shall try to address an imaginary SIAM member who is not very interested in the subject but who wishes to know when something important has happened, which topics are receiving most attention, and which of them are dead.

This important reference book presents 82 procedures written in an official subset of the language ALGOL 60 to perform a variety of well-defined tasks in solving linear systems of equations or in finding eigenvalues and eigenvectors. With each algorithm there is a brief discussion of its scope, the relevant theory, special features, numerical properties and test results. This collection represents continuous efforts by acknowledged experts over more than ten years. The algorithms have been pre-published individually in Numerische Mathematik and thus have been subject to public scrutiny and usage. In a real sense this anthology defines the state of the art in this domain, although Wilkinson hastens to say that he is not claiming that these programs are the last work on the subject.

It is only proper to hand out bouquets to the authors for creating this landmark and for setting such high standards of performance and documentation. One of the pleasant aspects of the effort has been the friendly cooperation on an international scale, a contrast with the intensely individual and competitive atmosphere in the world of mathematics.

The appearance of this book raises a number of interesting questions.

Why has it taken nearly fifteen years to implement decent programs in a subject which was finished off by the beginning of the century and has become a standard part of all undergraduate training in the physical sciences and engineering? It is one thing to learn that you cannot just say Newton's method when the subject of polynomial zeros is raised. It is quite another matter to provide a zero finder which will cope with most eventualities, never lie, and not be too clumsy. When a mathematician

* See following review.

asks to be told the deepest result or the key theorem in numerical linear algebra I feel that he is imposing the wrong framework. The subject is the design and analysis of algorithms which must use noisy arithmetic. This is part of computer science. It is a new field and we are still not sure how to teach it or how to talk about algorithms.

Consider, for example, Rutishauser's chapter presenting an implementation for Jacobi's method for diagonalizing a real symmetric matrix by finding principal axes in successive plane sections. He reorganizes the traditional formulas for effecting a plane rotation so that the new diagonal elements differ from the old ones by a multiple of the tangent of the angle of rotation. This permits an elegant improvement over the traditional version: each diagonal element is updated just once in each "sweep" through the matrix. This is valuable algorithmically but trivial mathematically. This ruse had escaped previous investigations including an intensive one by Murray, Goldstine and von Neumann [2]. Admittedly they were considering only so-called fixed-point calculations, but the trick is no less applicable in that case. Perhaps the most useful function of error analysis is its power to make us seek and judge alternative expressions, all equivalent in exact arithmetic, of the quantities of interest to us. When, if ever, would you write $(x - 0.5) - 0.5$ instead of $x - 1.0$?

In a sense this book does finish off an important part of numerical linear algebra. Frankly, I cannot imagine major improvements to many of these programs. The advance from very little, in 1956, to fast, compact, reliable routines in 1971 is so great that subsequent improvements must look pale in comparison. Of course there is work for a few specialists. We do not really know how to scale our equations. We do not have a rapid, well justified test for neglecting subdiagonal elements of Hessenberg matrices. We do not have standard routines for effecting a posteriori error analyses. Many important tasks still lack reliable algorithms. However, the pioneering days are over. You can no longer present a bad method and rest assured that no one else's routine works either.

My reader may have heard occasional reports on the war between ALGOL and FORTRAN, the two most popular languages for writing numerical programs. The first volume in this Handbook Series was in two parts. Rutishauser gave a lucid *Description of Algol 60* and Grau et al. discussed the *Translation of Algol* (into machine language) and presented a compiler to do it. With this underpinning, the algorithms do constitute unambiguous descriptions of computing processes. They are abstract in the sense that the basic operations of arithmetic and the set of numbers in which variables can take values are not prescribed and may be given different interpretations by different machines. In particular, they may be interpreted with exact arithmetic on the set of extended real numbers. This is the context in which the theories of the various methods are studied.

FORTRAN would not have been so elegant for the purposes of definition, but it is important to say that for nearly all numerical routines a standard ASA FORTRAN version also provides unambiguous definition of the algorithm.

Those who do not have access to good ALGOL compilers will be pleased to hear of the NATS project. Stanford University and the University of Texas at Austin are collaborating with Argonne National Laboratory to produce FORTRAN versions of the eigenvalue programs in this book, helpful documentation and elaborate testing. The procedure for obtaining codes may be obtained from NATS Project, Applied Mathematics Division, Argonne National Laboratory, Argonne, Illinois 60439. FORTRAN translations of some of these handbook algorithms are already available in a rental package from IMSL, Suite 510, 6200 Hillcroft, Houston, Texas.

Of all those with an interest in some numerical method, only a small number wish to carry their investigations as far as a reliable program in FORTRAN or ALGOL. Of these only a few worry about the suitability of the particular arithmetic systems which will in practice interpret the algorithms. For this review it suffices to stress the fact that this level does exist, that sometimes hardware characteristics should determine portions of the algorithm. It surprises many people to learn that the execution of *if* $a \neq b$ *then* $d = c/(b - a)$ can cause the computation to be aborted because of an attempted division by zero! This can occur if the test for a zero divisor is not identical to a test for equality. It is as well to remember such possibilities when there is a discussion about proving that algorithms do what they claim to do.

For certain simple tasks, such as solving a quadratic or iterating on residuals, very precise statements could be made about certain algorithms provided that some quantities could be computed in twice the precision of the rest of the computation. Intentionally, ALGOL was not made to describe such calculations. Consequently the comments and notes in which this information is conveyed form an important part of these procedures.

A great many matrix programs have been written over the last fifteen years. Hopefully many of these will now enjoy a well earned retirement and the new algorithms will become standard equipment. The only other collection of comparable quality of which I know is by Dekker and Hoffman in Holland [1]. They present fewer routines, a brief general discussion for each section, and comments opposite each page of ALGOL.

The engineering, or at least the nonmathematical aspects of the study of algorithms become apparent when we compare the Dekker-Hoffman procedure (D) with the Wilkinson-Martin procedure (W) for reducing an equilibrated real matrix A to Hessenberg form $H\,(h_{ij} = 0, i > j + 1)$. Even though this is a straightforward task, the routines are surprisingly different. (1) D reduces only the full matrix while W can reduce a principal submatrix. (2) D begins by computing $\|A\|_\infty$ whereas W does not use this quantity. (3) To find the maximal subdiagonal element in column $j - 1$, W finds the minimal $i\,(\geqq j)$ such that $|a_{i,j-1}| = \max|a_{k,j-1}|$, $k \geqq j$, and then compares i with j. On the other hand, D finds $s = \max(\text{tol}, \max|a_{k,j-1}|, k > j)$ and then either annihilates these $a_{k,j-1}$ if $s = \text{tol}$ or else compares s with $|a_{j,j-1}|$. Here $\text{tol} = \|A\|_\infty \times \epsilon$, where ϵ is the largest number such that the computed value of $1 + \epsilon$ is 1. The former case is flagged for purposes of later transformations by setting to zero the index which otherwise records an interchange at this step. Thus D effectively says that rounding errors of up to a value of tol will be committed at some places in the reduction, and it is reasonable to apply such a tolerance uniformly throughout. W, on the other hand, does not introduce such machine dependence where it is not necessary. The only tests are on (machine) zero. (4) Another difference between the procedures is that W uses subscripts such as $j - 1$ quite freely, even inside *for* loops, whereas D is careful to define $j1 = j - 1$ and uses $j1$ inside loops. Of course, an intelligent ALGOL compiler will spot that a quantity $j - 1$ may only need to be computed once for a whole loop and act accordingly. D's approach uses more variables but does not have to rely on much optimization in the compiler.

How are we to learn to distinguish between important and unimportant programming details if such things are not discussed somewhere? How this should be done I am not quite sure. The study of Volumes I and II of the *Handbook for Automatic Computation* might be a good way to begin.

B. P.

1. T. J. DEKKER AND W. HOFFMAN, *Algol* 60 *Procedures in Numerical Algebra, Parts I and II*, Mathematisch Centrum, Amsterdam, Holland, 1968.
2. H. H. GOLDSTINE, F. J. MURRAY AND J. VON NEUMANN, "The Jacobi method for real symmetric matrices," *J. Assoc. Comput. Mach.*, v. 6, 1959, pp. 59–96.

31 [4].—C. WILLIAM GEAR, *Numerical Initial Value Problems in Ordinary Differential Equations*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971, xvii + 253 pp., 24 cm. Price $12.95.

"I have tried to gather together methods, mathematics and implementations and to provide guidelines for their use on problems." The author has succeeded admirably in this effort. With a careful selection of illustrative examples, he presents clear discussions of the reasons that various algorithms perform as they do. In each case, he begins with a concrete description of the numerical method and ends with a definite mathematical analysis of the procedure. The reader is masterfully guided through the regions of stability for each method. He explains how to choose an appropriate method (step size and order) for solving the initial value problem; and in particular, discusses the treatment of stiff equations, gives a brief development for handling singular perturbation or singular implicit equations, and shows how to solve for certain parameters that may appear as unknowns in a given system of differential equations. The author only describes those techniques that he has found to be of the most utility; in this way the book is kept slim and its subject matter alive. Three FORTRAN subroutines for the numerical solution of differential equations are listed. As indicated in the preface, the author hoped to repay his debt to society by setting his "thoughts on paper so that the useful among them might benefit others." In this connection, the reviewer believes that Gear's debt has been repaid many times.

E. I.

32 [7].—ALFRED H. MORRIS, JR., *Table of the Riemann Zeta Function for Integer Arguments*, Naval Weapons Laboratory, Dahlgren, Virginia, ms. of 3 pp. +2 computer sheets deposited in the UMT file.

The Riemann zeta function, $\zeta(n)$, is herein tabulated to 70D for $n = 2(1)90$. Confidence in the complete reliability of the tabular entries is inspired by the accompanying description of the details of the underlying calculations, which were carried to 80D.

This carefully prepared tabulation constitutes a valuable supplement to the corresponding 50D table of Lienard [1] and the 41S table of $\zeta(x) - 1$ of McLellan [2].

J. W. W.

1. R. LIENARD, *Tables Fondamentales à 50 décimales des Sommes* S_n, u_n, Σ_n, Centre de Documentation Universitaire, Paris, 1948. (See *MTAC*, v. 3, 1948–1949, p. 358, RMT **589.**)
2. ALDEN MCLELLAN IV, *Tables of the Riemann Zeta Function and Related Functions*, Desert Research Institute, University of Nevada, Reno, Nevada, ms. deposited in UMT file. (See *Math. Comp.*, v. 22, 1968, pp. 687–688, RMT **69.**)

33 [7].—ALFRED H. MORRIS, JR., *Tables of Coefficients of the Maclaurin Expansions of* $1/\Gamma(z + 1)$ *and* $1/\Gamma(z + 2)$, Naval Weapons Laboratory, Dahlgren, Virginia, ms. of 2 pp. + 4 computer sheets deposited in the UMT file.

Using independently the method previously employed by this reviewer [1], the author has calculated and tabulated to 70D the first 71 and 72 coefficients, respectively, of the expansions

$$1/\Gamma(z + 1) = \sum_{n=0}^{\infty} a_n z^n \quad \text{and} \quad 1/\Gamma(z + 2) = \sum_{n=0}^{\infty} b_n z^n.$$

These coefficients are connected by the known relation $a_i = b_{i-1} + b_{i-2}$. The recursive calculation of the b_i's involved the Riemann zeta function for integer arguments, which the author had calculated [2] to more than 70D for this express purpose.

Comparison of these more extended tables with the corresponding 31D tables [1] of this reviewer has revealed a series of erroneous end figures in the latter tables. Detailed corrections therein are listed in the errata section of this issue.

J. W. W.

1. J. W. WRENCH, JR., "Concerning two series for the Gamma function," *Math Comp.*, v. 22, 1968, pp. 617–626.
2. A. H. MORRIS, JR., *Table of the Riemann Zeta Function for Integer Arguments*, ms. deposited in the UMT file. (See *Math. Comp.*, v. 27, 1973, p. 673, RMT **32.**)

34 [7].—RAÚL LUCCIONI, *Tables of Zeros of* $hJ_0(\xi) - \xi J_1(\xi)$, Instituto de Matematica, Facultad de Ciencias Exactas y Tecnologia, Universidad Nacional de Tucuman (R. Argentina), ms. of 10 pp. deposited in the UMT file.

A need for such zeros arises in a variety of physical problems, as noted by Carslaw & Jaeger [1], who have tabulated the first six zeros to 4D for 36 values of h ranging from zero to infinity.

In a recent paper [2] by the author, in collaboration with S. L. Kalla and A. Battig, it was found that more zeros are required to insure sufficient accuracy in the evaluation of certain infinite series.

Accordingly, the present tables have been prepared listing to 10D the first 25 zeros corresponding to $h = 0.1(0.1)6.0$.

Y. L. L.

1. H. S. CARSLAW & J. C. JAEGER, *Conduction of Heat in Solids*, Oxford Univ. Press, New York, 1947, p. 379.
2. S. L. KALLA, A. BATTIG & RAÚL LUCCIONI, "Production of heat in cylinders," *Rev. Ci. Mat. Univ. Lourenço Marques Ser. A*, v. 4, 1973.

35 [7].—Henry E. Fettis & James C. Caslin, *Tables of Toroidal Harmonics*, III: *Functions of the First Kind—Orders* 0—10, Report ARL 70-0127, Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio, July 1970, iv + 391 pp., 28 cm. [Copies obtainable from National Technical Information Service, Springfield, Virginia 22151. Price $3.00.]

The first table in this report consists of 11S values (in floating-point form) of the Legendre function of the first kind, $P^m_{n-1/2}(s)$, for $m = 0(1)10$, $s = 1.1(0.1)10$, and degree n ranging from 35 to 160, as in two earlier companion reports [1], [2], which were devoted to the tabulation of the Legendre function of second kind, $Q^m_{n-1/2}(s)$.

This table is followed by a tabulation, also to 11S, of the same function for similar orders m and for arguments $s = \cosh \eta$, where $\eta = 0.1(0.1)3$. The upper limit for the degree, n, here varies from 34 to 450.

A concluding table gives values of the cross product $P^m_{n+1/2}(s)Q^m_{n-1/2}(s) - Q^m_{n+1/2}(s)P^m_{n-1/2}(s)$ to 16S for $m = 0(1)10$, $n = 0(1)450$. This table evolved from spot-checking the other tables by means of identities that were derived from the known Wronskian relation and that are presented in the introductory section describing the method [3] of calculation by means of IBM 1620 and IBM 7094 systems.

Also included is a discussion of the application of toroidal functions to the determination of the potential field induced by a charged circular torus.

J. W. W.

1. Henry E. Fettis & James C. Caslin, *Tables of Toroidal Harmonics*, I: *Orders* 0–5, *All Significant Degrees*, Report ARL 69-0025, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1969. (See *Math. Comp.*, v. 24, 1970, pp. 489–490, RMT **36.**)

2. Henry E. Fettis & James C. Caslin, *Tables of Toroidal Harmonics*, II: *Orders* 5–10, *All Significant Degrees*, Report ARL 69-0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1969. (See *Math. Comp.*, v. 24, 1970. pp. 989–990, RMT **70.**)

3. Henry E. Fettis, "A new method of computing toroidal harmonics," *Math. Comp.*, v. 24, 1970, pp. 667–670.

36 [8].—Ludo K. Frevel, *Evaluation of the Generalized Binomial Density Function*, Department of Chemistry, The Johns Hopkins University, Baltimore, Maryland, 1972. Ms. of 13 pp. deposited in the UMT file.

The author defines herein a generalized binomial density function by the relation

$$\beta(x;\, n,\, \alpha) = \frac{\Gamma(1 + 2n)(\sin \alpha)^{2(n+x)}(\cos \alpha)^{2(n-x)}}{\Gamma(1 + n + x)\Gamma(1 + n - x)}$$

which reduces to the standard binomial function $b(k;\, m,\, p)$ when $x = m/2 - k$, $n = m/2$, and $\alpha = \arcsin p^{1/2}$.

A table of this function is included for $\alpha = \pi/4$, $x = 0(0.05)3$, and $n = -0.1$, 0, 0.1, 1, 2; it was computed to 10D on a Wang 360 calculator before truncation of the final tabular entries to 8D.

In addition, a probability density function $\phi_n(x)$ is defined in terms of $\beta(x;\, n,\, \alpha)$ by the relation

$$\phi_n(x) = \left[\int_{-n-1}^{n+1} \beta(\xi;\, n,\, \alpha)\, d\xi\right]^{-1}\left[\frac{1}{2} + \frac{1 + n - |x|}{2\,|1 + n - |x||}\right]\beta(x;\, n,\, \alpha),$$

and the normalizing factor $\int_{-n-1}^{n+1} \beta(\xi;\, n,\, \alpha)\, d\xi$ is tabulated to 5D for $n = 0, 0.1, 0.5, 1, 2, \infty$.

Two computer plots are also included: one of $\beta(x;\, n,\, \pi/4)$ for the tabular arguments; the other of $\beta(x;\, 0,\, \alpha)$ for $\alpha/\pi = 0.05(0.05)0.25$ and $-1 \leqq x \leqq 3$.

J. W. W.

37 [9].—M. Lal, C. Eldridge & P. Gillard, *Solutions of* $\sigma(n) = \sigma(n + k)$, Memorial University of Newfoundland, May 1972. Plastic bound set of 88 computer sheets (unnumbered) deposited in the UMT file.

The function $\sigma(n)$ is the sum of all positive divisors of n. Table 2 contains 50 separate tables. The kth of these gives all $n \leqq 10^5$ such that

(1) $\sigma(n) = \sigma(n + k)$.

Also listed here are $n + k$ and $\sigma(n)$.

Table 1 gives the number of solutions above for each k. Thus, $k = 1$ has 24 solutions, the first being $n = 14$ and the last being $n = 92685$.

An earlier table, apparently unpublished, was by John L. Hunsucker, Jack Nebb, and Robert E. Stearns, Jr. of the University of Georgia. This larger table listed all 113 solutions for $k = 1$ and $n \leqq 10^7$. Their last is $n = 9693818$. They had the same 24 solutions $< 10^5$. They also computed (1) for all $1 \leqq k \leqq 5000$ and $n + k \leqq 2\cdot 10^5$, and so should include everything here deposited. I have not seen this larger table.

In their larger range of n there are still only two solutions for $k = 15$: $n = 26$ and $n = 62$. Won't someone please prove that there are only two? Or are there others?

D. S.

38 [9].—Sol Weintraub, *Four Tables Concerning the Distribution of Primes*, 23 pages of computer output deposited in the UMT file, 1972.

Tables 2, 2A and 2B (6 pages each) are very similar to Weintraub's earlier [1]. See that review for the definitions of GAPS, PAIRS, ACTUAL, and THEORY. For the same variable $k = 2(2)600$, Table 2 lists these four quantities for the 11078937 primes in $0 < p < 2\cdot 10^8$; Table 2A for the (unstated number of) primes in $10^{16} < p < 10^{16} + 25\cdot 10^5$; and Table 2B for the 255085 primes in $10^{17} < p < 10^{17} + 10^7$. Nothing extraordinary occurs in these tables that requires special mention. The largest gap here is a case of $k = 432$ in Table 2A. ACTUAL and THEORY agree very well, as expected.

Table A (5 pages) covers the same range as Table 2 does. For $n = 1(1)200$ it first lists

$$\pi(n \cdot 10^6) \quad \text{and} \quad \pi(n \cdot 10^6) - \pi((n-1) \cdot 10^6),$$
$$R(n \cdot 10^6) \quad \text{and} \quad \mathrm{DIF}(n \cdot 10^6),$$

where

$$R(X) = \sum_{m=1}^{\infty} m^{-1}\mu(m) \operatorname{li}(X^{1/m}) \quad \text{and} \quad \mathrm{DIF}(X) = \pi(X) - R(X).$$

Except for rounding differences in $R(X)$, this part of Table A coincides with one-fifth of Mapes' [2] which goes to $n = 1000$. (The two authors are performing very different calculations for their $\pi(n \cdot 10^6)$, since Weintraub counts the actual primes while Mapes is using an elaborate recursive formula.)

Table A continues with the number of twin pairs in these intervals, and cumulatively. These counts agree, where they overlap, with those in [3] and [4]. Table A concludes with the maximal gap in each million—its size and location. Compare [3] and [4].

Which is the first million containing more primes than its predecessor? The thirty-third. Which is the first million with more twins than its predecessor? The eighth.

D. S.

1. Sol Weintraub, *Distribution of Primes between* 10^{14} *and* $10^{14} + 10^8$, UMT 27, *Math. Comp.*, v. 26, 1972, p. 596.
2. David Mapes, UMT 39, *Math. Comp.*, v. 17, 1963, p. 307.
3. D. H. Lehmer, UMT 3, *MTAC*, v. 13, 1959, pp. 56–57.
4. F. Gruenberger & G. Armerding, UMT 73, *Math. Comp.*, v. 19, 1965, pp. 503–505.

39 [13.15].—Norman S. Land, *A Compilation of Nondimensional Numbers*, NASA SP-274, National Aeronautics and Space Administration, Washington, D. C., 1972, 122 pages, softcover. Price $0.70.

All applied mathematicians know of the Mach number, the Reynolds, the Froude. But do you know the Jeffrey, the Jacob, and Jakob, the Hersey, the Hartmann, etc.?

All such technical numbers, together with others not named after investigators, such as "magnetic force number," are listed alphabetically in 97 pages of this booklet in the following format: Name, formula, explanation of symbols, technical field in which it occurs, reference. Usually, there is also a characterization of its nondimensionality as a ratio of like quantities, such as

$$\frac{\text{heat radiated}}{\text{heat conducted}} \quad \text{or} \quad \frac{\text{vibration speed}}{\text{translation speed}}.$$

There are 34 references and a shorter list of books on dimensional analysis, similitude, and units. The five-page index relists these numbers by subject matter; e.g., under *Surface Waves*, one finds the Boussinesq, Froude, Russell, and Weber. *Heat transfer* and others have much longer lists.

Some names mean the same thing: Crocco = Laval; others are related: Cauchy = Mach^2. The reviewer is unfamiliar with most of these numbers and has no comment on the accuracy here. He has been told, for instance, that Ekman should be

$$\frac{\text{viscous force}}{\text{coriolis force}} \text{ not } \frac{\text{viscous force}}{\text{centrifugal force}}.$$

In any case, it appears that this is a useful booklet for those in these fields.

D. S.

40 [13.35].—N. V. Findler & B. Meltzer, Editors, *Artificial Intelligence and Heuristic Programming*, American Elsevier Inc., New York, 1971, viii + 327 pp., 24 cm. Price $17.50.

This book consists of a series of papers based on lectures given at the First Advanced Study Institute on Artificial Intelligence and Heuristic Programming, held in Menaggio, Italy, on August 3–15, 1970. The papers cover a wide range of topics in Artificial Intelligence: theorem proving, problem-oriented languages, game playing, problem solving, heuristic search, question-answering systems, natural language analysis, picture processing, and cognitive learning. Five papers are tutorials dealing with well-established results in Artificial Intelligence. These are well-written, pertinent papers which should appeal to nonspecialists who wish to learn more about a particular area of AI. The other eight papers are descriptions of recent research in the field, and, in general, can be readily assimilated by those with a certain minimal background in Artificial Intelligence.

The 13 papers presented in this volume are listed below. The first two are clear, concise tutorials on theorem proving. Robinson's paper focuses on the deduction problem, i.e., determining whether a given assumption A logically implies a given conclusion C, and shows that resolution is an interesting way to attack this problem. The paper by Meltzer discusses the efficiency of automatic proof procedures, particularly with regard to the resolution method of theorem proving. Related issues like completeness and proof complexity are also considered, and guidelines for the design of efficient proof procedures are suggested.

The next two papers are accounts of recent research related to problem-oriented languages. The paper by Elcock describes ABSYS, a language for writing programs in the form of unordered, declarative statements. When these programs consist of sets of problem constraints, their compilation is a problem-solving task, and, thus, the compiler for ABSYS can be considered a problem-solving compiler. Findler's paper provides brief descriptions of seven AI projects that are being programmed in AMPPL-II, an associative memory, parallel processing language imbedded in FORTRAN IV.

The next three papers discuss recent work in problem solving, with emphasis on heuristic search. Sandewall, in his paper, introduces a number of quite useful concepts for defining heuristic methods in a general, compact way. These concepts are then used to describe the SAINT program and the unit preference strategy in resolution. The paper by Michie contains a discussion of graph searching algorithms and their application in the formation of plans by machine. To fully appreciate this interesting paper, one should be moderately familiar with the POP-2 language and Michie's work on memo functions. The paper by Pitrat discusses a language for describing the rules of board games like chess, go-moku, and tic-tac-toe. General search techniques (for finding a win) based on the rules of the game and the definition of the winning condition are presented.

The next paper is a tutorial on the frame problem in the context of intelligent robot systems. This is the problem of maintaining and updating the current context or "frame of reference" each time new information is created during problem solving. Raphael describes the problem in a clear, informative manner, and presents lucid evaluations of the primary approaches proposed for solving the frame problem.

The next four papers deal with language and picture processing. Lindsay's paper describes a natural language parsing system, JIGSAW1, based on labelled dependency analysis, which uses both syntax and semantics to guide the parsing. An interesting analogy is drawn between the combined use of syntax and semantics to parse a sentence and the combined use of contour information and picture information to put together a jigsaw puzzle. Simmon's paper describes a generative teaching program which has a semantic net data base and is able to use this information to generate and score quizzes. The paper by Palme is an interesting tutorial on question-answering systems. The one by Clowes is a short, provocative tutorial on picture descriptions. Most of the approaches discussed by Clowes rely on syntax-directed analysis of two-dimensional patterns.

The last paper, by Kochen, discusses the problem of formulating a model of cognitive learning. Examples of how a learning system can learn to maximize the utility of a situation when given a series of situation descriptions are presented. Also, a number of definitions and theorems about cognitive learning are introduced and stated in mathematical terms.

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41 [13.35].—F. GÉCSEG & I. PEÁK, *Algebraic Theory of Automata*, Akademiai Kiadó, Budapest, 1972, xiii + 326 pp., 25 cm. Price $13.00.

The intention of this book is to provide an exact and approximately complete algebraic theory of deterministic Mealy-, Moore- and Medvedev-automata. In fact, the authors have been successful in giving a detailed and well comprehensible version of the theory developed in this field up to the year 1966, approximately, especially of authors in Eastern Europe, e.g. V. G. Bodnarčuk, V. M. Gluškov, L. Kalmár, H. Kaphengst, A. A. Letičevskiĭ, V. N. Red'ko, A. Salomaa and, last but not least, the authors of this book. Only a section concerning experiments and a decomposition in the sense of Krohn-Rhodes is omitted, but other decompositions are studied in detail. For nearly all statements and theorems, there are references to the literature and further results are given as "supplements and exercises" at the end of each paragraph. Considering automata as algebraic structures, the development of the theory is similar to that of semigroups and groups.

In Chapters 1 and 2, there are developed the usual well-known concepts concerning homomorphism, reduction, equivalence, minimization, analysis and synthesis of finite automata, i.e., an effective process to determine the input-output mapping and to realize a given behavior, respectively. Furthermore, the algebra of events $E(X)$ is discussed in detail and, in order to deal with general fixed point-equalities, a norm is defined on $E(X)$ making it a complete normed linear space. Commutative, nilpotent, definite, linear and pushdown automata are treated briefly in Chapter 3, whereas general products of automata and their relationships to automaton mappings (i.e., input-output mappings induced by automata) are studied in Chapter 4. A general product in this sense includes feedback, while several other concepts, like the loop-free composition of J. Hartmanis (called R-product), the cascade product, cross product, semidirect product, and direct product, can be obtained as special cases of the general one. The main problem, whether there exists a finite (or minimal) system of automata for a given type of products such that each automaton mapping can be induced by such a product of automata, is treated for the case of the general and the R-product. Furthermore, semigroups and groups of automaton mappings are studied, including methods for metric groups. In Chapter 5, the monoid of transitions, endomorphism semigroups and automorphism groups of automata are treated, especially for the case of cyclic and commutative automata and those having an input-monoid.

Based on monographs by V. M. Gluškov and N. E. Kobrinskiĭ-B. A. Trahtenbrot, the appendix is devoted to structural systems, i.e., systems over a set of automata having powers of $\mathbf{Z}_2$ as states, input and output, such that the system is closed under three operations, which are defined with respect to technical applications. Using methods from universal algebra, it is proved constructively that each automaton can be embedded isomorphically in an automaton belonging to a system containing a memory element and a complete set of logical elements.

Summarizing, it can be said that this book presents an excellent mathematical theory of deterministic automata with special regard to regular events, general products, semigroups and structural systems of automata.

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