REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

42 [1].—JOSEPH A. SCHATZ, *A Mathematics Citation Index*, Sandia Laboratories, Albuquerque, New Mexico, Research Report SC-RR-70-910, xi + 580 pp., $8\frac{1}{2}'' \times 11''$, December, 1970.

About a decade ago, the Institute for Scientific Information, Inc., Philadelphia, Pa., a private concern, began the publication of a Science Source and Citation Index (SSCI for short). A source index is essentially an author index of research articles. Consider the references given in a source article. These are the items cited by the source document. A citation index is a directory of cited references, each of which is accompanied by a list of citing source documents. Whereas most retrieval systems are retroactive in nature, a citation index provides a method of moving forward in time and so gives a technique for updating aging information.

The SSCI covers many fields of science. In 1964, it covered about 31 mathematics journals and, in 1971, this number had grown about fourfold. An examination of the 1972 index for *Mathematical Reviews* indicates that it covers about 1100 publications. It is apparent that the SSCI covers only a small fraction of the mathematical literature surveyed by *Mathematical Reviews*.

The Mathematics Citation Index (MCI for short) was compiled from the bibliographies of over 25,000 source papers which appeared in 48 serial publication during the period 1950–1965. According to the author this represents about 20 per cent of the serial literature for the period covered.

The manner of obtaining and using a citation index is described in the introduction. The author's discussion of techniques for using a citation index is somewhat abbreviated. He states "it is a matter of observation that after about ten minutes of use, mathematicians are able to devise novel ways of using a citation index." Basically, the statement is true. However, in the present instance, it took some elementary but time consuming detective work to figure out the meaning of abbreviations used in the index. All this and other valuable information necessary to facilitate use of the index is conspicuous by its absence. We next turn our attention to these items.

The volume under review obviously covers much ground. Since only a fraction of the literature was examined, it would be helpful to have a ready list of the names of the journals surveyed so that an individual can readily estimate the bias of the index with respect to his field of interest. I found a few references to the journal *Numerische Mathematik* but only one reference to the journal *Mathematics of Computation*. It appears that the index is of very limited use to a numerical analyst.

Only initials of journals are used in the entries and there is no glossary. The reader might try his hand at deciphering PRSESA, PMIHAS, CRASP, PNAS and PNASUSA. Where applicable, information is given where an item is reviewed in *Mathematical Reviews*. Thus, the source journal can usually be determined. Obviously, this is no excuse for not providing the reader with adequate information. MR 21-2891 means *Math. Rev.*, v. 21, No. 2891, while MR 19P428(5) means *Math. Rev.*, v. 19, p. 428, item 5 on that page.

Another source of irritation is that only the first five letters of an author's surname are given. That one may have a qualm as to the wisdom of such a procedure for the ostensible purpose to save space is secondary. Most certainly, the reader is entitled to the courtesy of being informed of such a practice.

How valuable MCI will be to a research worker remains to be seen. I can offer no opinion on the subject since so little of my interests are covered.

If the journals surveyed thoroughly cover a particular segment, say algebraic topology, then it would seem that MCI will be an asset. Whether the concept is the complete answer to the problem of information retrieval is another matter.

The construction of an MCI is simple and requires no mathematical sophistication. It can be accomplished with clerical help only. This is in contrast to the recently published volume by Y. L. Luke, J. Wimp and W. Fair, *Cumulative Index to Mathematics of Computation—Volumes 1–23, 1943–1969*, American Mathematical Society, Providence, R. I., 1972, which is a systematic classification by subject matter and by author of all the contents in the journal *Mathematics of Computation* since its inception in 1943 when it was known as *Mathematical Tables and other Aids to Computation.* The importance of the volume under review and of the one mentioned above is that they break new ground for information retrieval of mathematical literature. The direction of future efforts along these lines will be conditioned by the usefulness of the indices. I am sure all authors will welcome comments and suggestions from users.

Y. L. L.

43 [2, 3, 4, 5, 7.105, 8.50].—R. S. BURINGTON, *Handbook of Mathematical Tables and Formulas*, Fifth edition, McGraw-Hill Book Co., New York, 1973, x + 500 pp., 21 cm. Price $5.50.

This new edition of a widely used mathematical handbook differs from the fourth edition [1] in the considerable enlargement of Part One (Formulas, Definitions, and Theorems from Elementary Mathematics) through the addition of new sections on linear algebra, numerical analysis, differential equations, Legendre polynomials and Bessel functions, Fourier series and transforms, Laplace transforms, and functions of a complex variable.

The table of indefinite integrals remains unchanged; however, the table of definite integrals has been extended to include several integrals involving the Dirac δ-function, formulas for the derivative of a definite integral, and statements of the Law of the Mean for integrals and of Green's Theorem.

In Part Two (Tables) we find fewer changes. Six of the 39 tables in the fourth edition have not been retained. Those omitted include 7D mantissas of the common logarithms of integers between 10^4 and $12 \cdot 10^3$, 10D common logarithms of primes less than 1000, natural secants and cosecants to 5S for every minute of the quadrant, natural trigonometric functions to 5S for decimals of degrees, factors for computing probable errors, and 4D common antilogarithms. On the other hand, additions to this part of the handbook include a table of the summed Poisson distribution function and an extension of the range of the annuity tables to include higher interest rates.

A final new feature of the present edition is the inclusion of a valuable list of 29 references to relevant treatises, textbooks, tables, and general guides for table-users.

The author appears indeed to have taken great pains to make this edition an especially useful and reliable one.

J. W. W.

1. R. S. Burington, *Handbook of Mathematical Tables and Formulas,* Fourth edition, McGraw-Hill, New York, 1965. (See *Math. Comp.*, v. 19, 1965, p. 503, RMT **72.**)

44 [2.20].—H. P. Robinson, *Roots of* tan $x = x$, Lawrence Berkeley Laboratory, University of California, Berkeley, California, December 1972, ms. of 10 typewritten pp. deposited in the UMT file.

This table consists of the first 500 nonnegative roots of the equation stated in the title, all to 40D. The underlying computations were performed on a Wang 720C programmable calculator, and a partial check was provided by a preliminary calculation of the first 300 roots to 40D by means of a different program.

The results of the present calculations clearly supersede in precision and extent those of all previous ones [1] of the roots of this important equation in applied mathematics.

As an example of the use of the table, the author has applied it to the evaluation of the DuBois Reymond constant C_3, using a formula originally developed by Watson [2].

J. W. W.

1. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables,* 2nd ed., Addison-Wesley, Reading, Mass., 1962, v. 1, p. 144.
2. G. N. Watson, "DuBois Reymond constants," *Quart. J. Math.,* v. 4, 1933, pp. 140–146.

45 [2.20, 3, 4].—M. P. Cherkasova, *Problems on Numerical Methods,* translated from the Russian by G. L. Thomas and R. S. Anderssen, Wolters-Noordhoff Publishing, Groningen, The Netherlands, 1972, vii + 210 pp., 23 cm. This book is available from International Scholarly Book Services, Inc., P. O. Box 4347, Portland, Oregon 97208. Price $8.50.

This book is intended to serve as an educational aid in elementary numerical analysis courses by providing the student with a comprehensive set of numerical problems (with answers) upon which he can cut his computational teeth. Chapter 1, "The approximate solution of nonlinear algebraic and transcendental equations," contains 301 problems; Chapter 2, "Numerical methods in linear algebra," contains 146 problems; and Chapter 3, "Numerical solution of ordinary differential equations," contains 28 problems, many containing 5 or 6 parts. Each chapter contains brief summaries of the methods intended for use on the problems.

The idea of such a book is a good one; carefully chosen illustrative problems could help experienced teachers of the area as well as those inexperienced in the area but called upon to teach such courses. Unfortunately, the present text falls far short of this ideal. Firstly, the choice of methods to be described is disappointing; one finds no mention, for example, of bisection, regula falsi (as opposed to the secant method), partial pivoting, QR, trapezoid rule, et cetera. Of course, the problems can be used independently of the explanatory material. Secondly, the problems are purely computational with no attempt to illustrate important aspects of theory such as convergence rates, error estimates, instability, ill-conditioning, et cetera; the typical problem is: "Use method M to solve problem P." Again, one can select the problems for these purposes oneself, but the question is how to find a particular problem to illustrate a particular point. It is just as easy to make up one's own problem.

Thus, the book probably can serve mainly as a source of numerical problems with numerical answers. It is almost as easy, however, to make up one's own problems for assignment and deduce the correct answer from students' computer output.

I cannot attest to the accuracy of the answers given. On four problems (31, 70, 72 from Chapter 2; 25 from Chapter 3) the answers appeared to be correctly rounded to all figures given, as compared at least to answers produced on the CDC 6600 at the University of Texas using some of the best methods available.

J. W. D.

46 [2.35, 3].—S. L. S. Jacoby, J. S. Kowalik & J. T. Pizzo, *Iterative Methods for Nonlinear Optimization*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1972, xi + 274 pp., 24 cm. Price $17.30.

An introductory chapter contains examples, definitions of convexity, first-order optimality conditions, discussion of the meaning of rate of convergence, and the traditional test problems that algorithmists use to try out their optimization codes. Chapter 2 gives ways of transforming problems and variables to facilitate obtaining solutions. Chapter 3 discusses methods for optimizing along a line, concentrating on those methods requiring only function evaluations. The usual direct search methods for solving unconstrained problems are found in Chapter 4, and Chapter 5 contains discussions of steepest descent, conjugate directions, quasi-Newton, modified Newton, and other methods for minimizing unconstrained functions. Chapter 6 discusses how general constrained problems can be solved by solving unconstrained (or merely linearly constrained) problems. Chapter 7 discusses direct methods for constrained problems including MAP, the cutting plane method, the gradient projection method, and the reduced gradient method. A concluding appendix on auxiliary techniques, such as the SIMPLEX method, is followed by a useful glossary.

Most chapters have a "Computing Systems" section which is unique in books on optimization. It contains references to codes implementing the algorithms developed in the book. Another area of strength is the integration of the numerical analysis point of view into solving the systems of linear equations required by many of the algorithms. There is a lack of precision in the definitions, and some of the material should have been deleted or further explained (such as the suggestion to eliminate two constraints $b_i \leqq x_i \leqq u_i$ by the variable transformation $x_i = b_i + (u_i - b_i)\sin^2(y_i))$. There is a wealth of material; much neglected work done by engineer optimizers is included. It is not a text book. It has no problems and no small numerical examples. The authors have achieved their main aim, to synthesize and explain the vast amount of algorithmic material now extant in the optimization area.

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47 [2.35, 5].—DAVID M. YOUNG, *Iterative Solution of Large Linear Systems*, Academic Press, New York, 1971, xxiv + 570 pp., 24 cm. Price $25.00.

In his 1950 Harvard thesis, David Young laid a solid theoretical foundation for the successive overrelaxation (SOR) method. Overall, this method perhaps remains the most useful method for the solution of large sparse systems of algebraic equations and, in particular, those which arise in the numerical solution of elliptic partial differential equations. The main topic of the present book is the study of the rate of convergence of the SOR method, its many variants and various semi-iterative methods. Much of the material is already quite familiar from Richard Varga's well-known textbook *Matrix Iterative Analysis* (1962). However, in recent years David Young and his coworkers have systematically explored many important aspects of the theory. Of perhaps greatest general interest are some new results on the use of a combination of the symmetric successive overrelaxation (SSOR) method with semi-iteration. For the standard second-order finite difference approximation to Laplace's equation and the natural ordering, good values for acceleration parameters can be found which lead to an order-of-magnitude gain in the rate of convergence (i.e., $R \sim h^{-1/2}$) compared to that of the optimal SOR method (i.e., $R \sim h^{-1}$).

The use of a line version of SSOR is shown to give further gains. It appears that further study of these potentially very powerful methods applied to more general elliptic problems could be very profitable.

This book requires only a background corresponding to a standard undergraduate mathematics program. The book is self-contained and admirably clearly written. The theory is illustrated by well-chosen examples worked out in sufficient detail. The usefulness of the book is further enhanced by many exercises. It is a most welcome addition to both the textbook and handbook literature.

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48 [7].—L. N. KARMAZINA, *Tablit͡sy funkt͡siĭ Lezhandra ot mnimogo argumenta* (*Tables of Legendre functions of imaginary argument*), Akad. Nauk SSSR, Moscow, 1972, x + 391 pp., 27 cm. Price 3.86 rubles.

This book consists mainly of a table of 7S values of the real and imaginary parts of $P_{-1/2+i\tau}(ix)$, in floating-point format, for $\tau = 0(0.01)15$ and $x = 0(0.1)2(0.2)5(0.5)10(10)60$.

As noted in the preface, the present table constitutes a natural sequel to a series of four earlier Russian tables [1]–[4] of the same function, wherein the tabular arguments were limited to real numbers.

A valuable introduction includes a set of formulas permitting the extension of the tables to corresponding negative values of τ and x. Also included therein are asymptotic series for Re $P_{-1/2+i\tau}$ and Im $P_{-1/2+i\tau}$, as well as expressions for these functions in terms of hypergeometric functions.

The introductory text contains a description of the tables and a brief discussion of their construction. Interpolation with respect to τ is shown to be feasible to full tabular precision by means of Lagrange formulas for three and four points. On the other hand, it is noted that such interpolation with respect to x is not practical in these tables, and direct calculation by the appropriate formula in the introduction is recommended by the author.

As noted in the introduction, these functions are encountered in applications of the Mehler-Fock transformation; in particular, they are useful in the solution of certain mixed boundary value problems in mathematical physics, such as the distribution of electricity on hyperboloids of revolution. Reference to such applications is included in the appended bibliography of nine titles.

J. W. W.

1. M. I. ẐHURINA & L. N. KARMAZINA, *Tablitsy funktsiĭ Lezhandra $P_{-1/2+i\tau}(x)$*, Tom I, Akad. Nauk SSSR, Moscow, 1960. [See *Math. Comp.*, v. 16, 1962, pp. 253–254, RMT **22**.]

2. M. I. ẐHURINA & L. N. KARMAZINA, *Tablitsy funktsiĭ Lezhandra $P_{-1/2+i\tau}(x)$*, Tom II, Akad. Nauk SSSR, Moscow, 1962. [See *Math. Comp.*, v. 18, 1964, pp. 521–522, RMT **79**(a); *ibid.*, v. 19, 1965, p. 692, RMT **123**, for a brief review of English translations of volumes 1 and 2.]

3. M. I. ẐHURINA & L. N. KARMAZINA, *Tablitsy i formuly dlia sfericheskikh funktsiĭ* $P^m_{-1/2+i\tau}(z)$, Akad. Nauk SSSR, Moscow, 1962. [See *Math. Comp.*, v. 18, 1964, pp. 521–522, RMT **79**(b); *ibid.*, v. 21, 1967, pp. 508–509, RMT **66**, for a review of an English translation.]

4. M. I. ẐHURINA & L. N. KARMAZINA, *Tablitsy funktsiĭ Lezhandra*, Akad. Nauk SSSR, Moscow, 1963.

49 [7].—HENRY E. FETTIS, JAMES C. CASLIN & KENNETH R. CRAMER, *An Improved Tabulation of the Plasma Dispersion Function and Its First Derivative*—Part I—*Argument with Positive Imaginary Part*; Part II—*Argument with Negative Imaginary Part: Zeros and Saddle Points*, Reports ARL 72-0056 and 72-0057, Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio, July 1972, Part I, IV + 408 pp., 28 cm. and Part II, IV + 434 pp., 28 cm. Copies obtainable from the National Technical Information Service, Operations Division, Springfield, Virginia 22151.

Let

$$(1) \qquad z(\rho) = \pi^{-1/2} \int_{-\infty}^{\infty} \frac{e^{-t^2}\, dt}{t - \rho}, \qquad \rho = x + iy,\ y > 0.$$

Then

(2) $$z(\rho) = 2ie^{-\rho^2} \int_{-i\rho}^{\infty} e^{-t^2}\, dt = 2ie^{-\rho^2} \operatorname{Erfc}(-i\rho),$$

with no restrictions on ρ. We also have

(3) $$z(\rho) = i\pi^{1/2}\omega(\rho)$$

where $\omega(\rho)$ is the function considered by Fried and Conte [1], and Faddeeva and Terent'ev [2]. Let $\mu = t - x$ in (1). Then

(4) $$z(\rho) = z(x, y) = \pi^{-1/2} \int_0^{\infty} \frac{ue^{-u^2} \sinh 2ux\, du}{u^2 + y^2} + iy\pi^{-1/2} \int_0^{\infty} \frac{e^{-u^2} \cosh 2ux\, du}{u^2 + y^2},$$

and these integrals are essentially the so-called Voigt functions. Both of the latter integrals are easily evaluated using the trapezoidal rule. For details on this technique, see Hunter and Regan [3], and the references given therein. Using this procedure, the present authors tabulate in Part I, $z(\rho)$ and $z'(\rho) = -2 - 2\rho\, z(\rho)$ for $x = 0(0.1)20$, $y = 0(0.1)10$, 11S. The Part II tables are as above, except that $-y = 0(0.1)10$. Part II also contains the first 200 zeros of $z(\rho)$ and $z'(\rho)$, 11S, and the first 200 zeros of Erf (ρ), 11S. Each part has an errata insert which pertains only to the introduction and has no bearing on the numerical data. In the tables, the first and second columns listed, for example, under $z(x, y) \equiv z(\rho)$ are the real and imaginary parts of $z(\rho)$, respectively. The authors remark that the Fried and Conte [1] tables were found to contain inaccuracies, particularly when $y < 0$, and that the present work was done to fill the need for a more accurate tabulation. Certainly, this is the most extensive tabulation of the error function available.

Y. L. L.

1. B. D. Fried & S. D. Conte, *The Plasma Dispersion Function: The Hilbert Transform of the Gaussian,* Academic Press, New York, 1961. (See *Math. Comp.*, v. 17, 1963, pp. 94–95.)
2. V. N. Faddeeva & N. M. Terent'ev, *Tables of Values of the Function* $\omega(z) = e^{-z^2}(1 + 2i\pi^{-1/2} \int_0^z e^{t^2}\, dt)$, *for Complex Argument*, Pergamon Press, New York, 1961. (See *Math. Comp.*, v. 16, 1962, p. 384.)
3. D. B. Hunter & T. Regan, "A note on the evaluation of the complementary error function," *Math. Comp.*, v. 26, 1972, pp. 539–542.

50 [7].—Henry E. Fettis & James C. Caslin, *A Table of the Inverse Sine-Amplitude Function in the Complex Domain*, Report ARL 72-0050, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, April 1972, iv + 174 pp., 28 cm. Copies available from the Defense Documentation Center, Cameron Station, Alexandria, Virginia 22151.

The Jacobian elliptic functions with complex argument arise in numerous applications, e.g., conformal mapping and tabular values are available in [1] and [2]. Often, one desires the inverse function. This could be accomplished by inverse interpolation in the above tables. However, such a procedure is inconvenient and of doubtful accuracy, especially in some regions where a small change in the variable produces a large change in the function. Charts are available in [1] from which qualitatively correct values of the inverse could be deduced, but no prior explicit tabulation is known.

Consider

$$z = \mathrm{sn}(\omega, k), \qquad z = a + ib, \qquad \omega = u + iv,$$
$$u + iv = \sin^{-1}(a + ib) = F(\psi, k),$$
$$a + ib = \sin \psi = \sin(\theta + i\varphi),$$

where $F(\psi, k)$ is the incomplete elliptic integral of the first kind, and k is the usual notation for the modulus. Let C, D, E, and F stand for certain ranges on the parameters. Thus:

$$C\colon 0(0.1)1; \qquad D\colon 0.9(0.01)1;$$
$$E\colon 0.01(0.01)0.1; \qquad F\colon 0.1(0.1)1.$$

Let K and K' be the complete elliptic integrals of the first kind of modulus k and $k' = (1 - k^2)^{1/2}$, respectively. Then the tables give 5D values of $u/k + iv/k'$ for

$$k = \sin \theta, \qquad \theta = 5°(5°)85°(1°)89°,$$

and the ranges

$$a = C, b = C; \quad a = D, b = C; \quad a = C, b^{-1} = E; \quad a = C, b^{-1} = F;$$
$$a^{-1} = E, b = C; \quad a^{-1} = F, b = C; \quad a^{-1} = E, b^{-1} = E;$$
$$a^{-1} = F, b^{-1} = F.$$

The headings for each page were machine printed and here no confusion should arise provided one understands that $K = \sin 5$, for example, should read $k = \sin 5°$.

The method of computation and other pertinent formulas are given in the introduction.

Y. L. L.

1. H. E. Fettis & J. C. Caslin, *Elliptic Functions for Complex Arguments*, Report ARL 67-0001, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, January, 1967. (See *Math. Comp.*, v. 22, 1968, pp. 230–231.)

2. F. M. Henderson, *Elliptic Functions with Complex Arguments*, Univ. of Michigan Press, Ann Arbor, Mich., 1960. (See *Math. Comp.*, v. 15, 1961, pp. 95, 96.)

51 **[9]**.—Bryant Tuckerman, *Odd Perfect Numbers*: *A Search Procedure*, *and a New Lower Bound of* 10^{36}, IBM Research Paper RC-1925, October 20, 1967, original report (marked "scarce") and one Xerox copy deposited in the UMT file, 59 pages.

This is the original (1967) much more detailed version of Tuckerman's paper printed elsewhere in this issue. It established the lower bound of 10^{36}. See the following review for a description of the UMT supplement to his present paper.

D. S.

52 **[9]**.—Bryant Tuckerman, *Odd-Perfect-Number Tree to* 10^{36}, IBM, Thomas J. Watson Research Center, Yorktown Heights, New York, 1972, ms. of 9 computer sheets, deposited in the UMT file.

The paper [3] which describes the algorithm used to generate this tree appears elsewhere in this issue. Each node of the tree corresponds to a restriction on the canonical decomposition of an odd perfect number (hereafter denoted by n); and since these restrictions exhaust the logical possibilities, all odd perfect numbers are accounted for. The tree is finite, since no branching is permitted from a node at which it can be determined that the "associated" odd perfect numbers all exceed 10^{36}. Thus, there are only two "least prime divisor" nodes (of level 1) from which branching is permitted. For if the smallest prime divisor of n is neither 3 nor 5, then it follows easily from the tables to be found in [2] that $n > 10^{41}$. Also, as soon as it is known that $p^{2\alpha} \mid n$ and $p^{2\alpha} > 10^{18}$ the tree is truncated, since then $n \geqq p^{2\alpha} \cdot \sigma(p^{2\alpha}) > 10^{36}$. Truncation nodes of the latter type have not been printed out here, and the reviewer would like to suggest that if and when similar trees are generated the program be modified so that such nodes are presented explicitly. Truncation also occurs when the numbers associated with a node can be shown to be either abundant or to possess a prime divisor less than the "least prime divisor." Such nodes *are* printed out here.

Since the algorithm would detect any odd perfect number which did not satisfy the restrictions at a truncation node this tree shows that (i) $n > 10^{36}$, (ii) if neither 3 nor 5 divides n, then $p^{2\alpha} \mid n$ where $p^{2\alpha} > 10^{18}$. ((i) has been improved by the reviewer [1].)

In general, the branching process is dependent on the determination of the prime factors of $F_q(p)$ where p is a known prime divisor of n, q is a prime and $F_q(x)$ is the qth cyclotomic polynomial. (For if $p^\alpha \| n$ it follows that $F_q(p) \mid n$ if $q \mid (\alpha + 1)$.) The complete factorizations of all of the relevant $F_q(p)$ are given here (except when a "least prime" contradiction occurs); and it was the large expenditure of time and effort required for this phase in the execution of the algorithm that necessitated the truncation at 10^{36}. With the steady development of faster computers and more efficient tests of primality it is obviously only a matter of time until Tuckerman's algorithm is utilized to establish a *much better* lower bound for n. Unless, of course, the smallest odd perfect number is discovered in the process.

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1. P. HAGIS, JR., "A lower bound for the set of odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 951-953.
2. K. K. NORTON, "Remarks on the number of factors of an odd perfect number," *Acta Arith.*, v. 6, 1961, pp. 365–374.
3. B. TUCKERMAN, "A search procedure and lower bound for odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 943-949.

53 [9].—PETER HAGIS, JR., *If n is Odd and Perfect, then* $n > 10^{45}$. *A Case Study Proof with a Supplement in which the Lower Bound is Improved to* 10^{50}, Temple University, Philadelphia, Pennsylvania, 1972, ms. of 83 pp. deposited in the UMT file.

This manuscript comprises mainly the detailed case study that supports the author's paper [1] appearing elsewhere in this issue. Here the author first assembles twelve criteria (1)–(12), of which the first eight are classical (such as results of Euler and others, and properties of $\sigma(\cdot)$); the ninth and tenth are proved in the present manuscript; the eleventh is due to Muskat. The twelfth is due to Hagis and McDaniel [2], also appearing in this issue.

The author then subdivides the set of odd perfect numbers n (if any) into cases (or subcases), repeatedly branching and drawing conclusions, until a lower bound $\geq 10^{45}$ is derived in each case. Each such lower bound is a product (or a minimum of such products) of known factors and/or known underbounds for unknown necessary factors of every n (if any) in this case. The tools used are judiciously chosen in each case from the aforementioned criteria (1)–(12), results of Kanold and Norton, properties of $\sigma(\cdot)$, deductions from incomplete factorizations and about sources for 3's, etc.

The branching is done first on divisibility by various combinations of 3, 5, 7; then, primarily, on powers or groups of powers, first of 7 or 3, and then of other primes successively generated by $\sigma(\cdot)$ and branching. Some of the above tools are used to pre-exclude certain branches. At times, presumably for greater efficiency, this pattern is varied by the use of other branchings, such as on $11 \mid n$ versus $11 \nmid n$. A computer was used to find factors, generally the ones $< 10^5$, of the relevant $\sigma(p^\beta)$.

This case study is followed on page 47 by a useful outline which gives, for each case and subcase, its name, its defining restrictions, remarks (in some cases), and the deduced lower bound.

The supplement (pages 64–81), which raises the lower bound to 10^{50}, uses two additional tools, (14) due to Tuckerman and (15) due to Robbins and to Pomerance, together with further application of the previous methods.

The author's paper [1] includes 11 typical cases and subcases selected and edited from this manuscript. These illustrate most of the methods used; however, the specialist will want to consult the complete manuscript also.

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1. PETER HAGIS, JR., "A lower bound for the set of odd perfect numbers," *Math. Comp.*, v. 27, 1973, pp. 951-953.
2. PETER HAGIS, JR., & WAYNE L. MCDANIEL, "On the largest prime divisor of an odd perfect number," *Math. Comp.*, v. 27, 1973, pp. 955-957.

54 **[10]**.—P. A. MORRIS, *A Catalogue of Trees*, University of the West Indies, St. Augustine, Trinidad, West Indies, October 1972. Ms. of 10 pp. + 46 computer sheets deposited in the UMT file.

This catalogue lists all unlabeled mathematical trees, without duplication, up to 13 nodes, inclusive. The trees are described by their node pairs, preceded by a code giving the number of edges; thus, for example, the tabular entry 04003, 0102, 0203, 0204 refers to the tree on 4 nodes with the 3 edges (1, 2), (2, 3), (2, 4).

AUTHOR'S SUMMARY

55 [12].—A. COLIN DAY, *Fortran Techniques, with Special Reference to Non-Numerical Applications*, Cambridge Univ. Press, Cambridge, England and New York, 1972, viii + 96 pp., 22 cm. Price $3.95 paper bound, $10.95 cloth.

For a great number of students who have taken a somewhat thorough course in Fortran—no matter how good the course—the result is often successful only from the point of view that the student has been exposed (at long last!) to the fundamental principles of computer programming and has hopefully learned most of the repertoire of the language. For many, if not most, instructors, this is sufficient. But there invariably are those students who have no difficulty whatever absorbing the material and who find themselves hampered from progressing further because of two reasons: (a) the lack of time for the instructor to cover more advanced material and (b) the unavailability of suitable textbooks to carry the students to the next plateau.

This pocket-size, paperback booklet by A. Colin Day is an excellent attempt to help fill this void. With hardly a single superfluous word, it covers a wide range of advanced topics—including packing numbers, printing histograms, open-coded subroutines, binary searches, hashing, stacks, recursion, various sorting techniques—to name just a few of the topics.

At the end of each of the nine chapters is a group of suitable exercises based on the material covered.

Many would justifiably consider this "must" reading for all students specializing in Computer Science.

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