REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1 [14].—H. A. LAUWERIER, *Asymptotic Analysis*, Mathematisch Centrum, Amsterdam, 1974, i + 145 pp., 25 cm. Price Dfl. 16.

This little book contains the expanded lecture notes of courses on asymptotic methods given for students in mathematical physics at the University of Amsterdam. The book is entirely devoted to the study of asymptotic series and asymptotic evaluation of integrals. The standard topics are covered, e.g., the methods of Laplace and Kelvin, the saddle point method of stationary phase, etc. Each topic is illustrated with examples which are worked out in detail. In addition, various special topics are discussed such as the asymptotic behavior of Cauchy integrals and asymptotics in the theory of probability.

The book is clearly and carefully written and would, with some supplementary material on differential equations, make an excellent text.

R. W. DICKEY

Mathematics Department
University of Wisconsin-Madison
Van Vleck Hall
480 Lincoln Drive
Madison, Wisconsin 53706

2 [2].—L. COLLATZ & W. WETTERLING, Editors, *Numerische Methoden bie Optimerungsaufgaben*, Band 2, ISNM International Series of Numerical Mathematics, Vol. 23, Birkhäuser Verlag, Basel, 1974, 165 pp., 25 cm. Price $13.00.

This volume contains papers presented at a meeting organized by L. Collatz and W. Wetterling on "Numerical Methods in Optimization Problems" held November 18–24, 1973 at the Mathematical Research Institute at Oberwolfach.

J. B.

3 [3.15], [5.15].—L. COLLATZ & K. P. HADELER, Editors, *Numerische Behandlung von Eigenwertaufgaben*, ISNM International Series of Numerical Mathematics, Vol. 24, Birkhäuser Verlag, Basel, 1974, 142 pp., 25cm. Price $12.00.

This volume contains papers presented at a meeting organized by L. Collatz and K. P. Hadeler on "Numerical Treatment of Eigenvalue Problems" held November 19–24, 1972 at the Mathematical Research Institute at Oberwolfach.

J. B.

4 [2.05].—P. L. BUTZER & B. SZ.-NAGY, *Linear Operators and Approximation*. II, ISNM International Series of Numerical Mathematics, Vol. 25, Birkhäuser Verlag, Basel, 1974, 585 pp., 25 cm. Price $33.00.

This volume contains papers presented at a meeting organized by P. L. Butzer and B. Sz.-Nagy on "Linear Operators and Approximation. II" held March 30 to April 6, 1974, at the Mathematical Research Institute at Oberwolfach.

J. B.

5 [3.15].—B. T. SMITH, J. M. BOYLE, B. S. GARBOW, Y. IKEBE, V. C. KLEMA & C. B. MOLER, *Matrix Eigensystem Routines–EISPACK Guide*, Springer-Verlag, Berlin, 1974, 387 pp., 24cm. Price $11.50.

This text is part of the lecture notes in computer science series of Springer-Verlag. It is a user guide to EISPACK and to a control program EISPAC.

EISPACK is a systematized collection of Fortran subroutines which compute the eigenvalues and eigenvectors for matrices in one of six classes: complex general, complex Hermitian, real general, real symmetric tridiagonal, and special real tridiagonal.

The subroutines are based mainly upon Algol procedures published in the Handbook series of Springer-Verlag by Wilkinson and Reinsch [1].

The text is divided into seven sections. Section 1 is an introduction that discusses briefly the development of the codes from their Algol base to the systematized collection of certified FORTRAN subroutines that resulted.

Section 2 is a guide to using EISPACK. It describes how one can link EISPACK subroutines together to solve various eigenproblems. Such linkages are called paths in the discussion.

The user first selects one of twenty-two categories of the eigenproblem. A table gives the corresponding subsection within the text that discusses the recommended path for the given category. Each of these subsections gives clear instructions on how to use EISPACK subroutines or the EISPAC control program to solve the problem. Other subsections discuss variations of the recommended EISPACK paths, and give additional information and examples.

Sections 3, 4, 5, and 6 discuss validation of EISPACK, execution times for EISPACK, certification and availability of EISPACK, and the differences between EISPACK subroutines and Handbook Algol procedures, respectively.

Section 7 contains complete documentation and FORTRAN source listings for EISPACK subroutines. Documentation is also given for the EISPAC control program.

This is an excellent text valuable not only for potential users but also as a reference text for persons in the mathematical software area. It does attain the goal that so many seek, of producing a well-documented, thoroughly tested, easy-to-use collection of subroutines.

T. J. AIRD

1. J. H. WILKINSON & CHRISTIAN REINSCH, *Handbook for Automatic Computation*, vol. 2, *Linear Algebra*, Part II, Die Grundlehren der math. Wissenschaften, Bd. 186, Springer-Verlag, New York, 1971.

International Mathematical & Statistical Libraries, Inc.
GNB Building
7500 Bellaire Boulevard
Houston, Texas 77036

6 [9].—ROBERT BAILLIE, *Table of* $\phi(n) = \phi(n + 1)$, Computer-Based Education Lab., University of Illinois, Urbana, 1975. Thirty-eight computer output pages deposited in the UMT file.

With $\phi(n)$ being Euler's function, there are listed here all 306 solutions of

$$\phi(n) = \phi(n + 1)$$

for $1 \leqslant n \leqslant 10^8$. The factorizations of n, $n + 1$, and $\phi(n)$ are included. This extensive computation goes far beyond Ballew, Case & Higgins [1] who gave the eighty-nine solutions $<28 \cdot 10^5$. If $E(m)$ is the number of solutions with $n \leqslant 10^m$, one has

m	$E(m)$	m	$E(m)$	m	$E(m)$
0	1	3	10	6	68
1	2	4	17	7	142
2	3	5	36	8	306

Since $E(m)$ approximately doubles each line, *empirically* the number of solutions with $n \leqslant N$ can be expected to increase something like $N^{0.3}$. There is little doubt that there are infinitely many solutions, but this has not been proven.

However, a $N^{0.3}$ growth does suggest that there are only finitely many triples:

$$\phi(n) = \phi(n + 1) = \phi(n + 2);$$

and, in fact, none is known besides the single triple $n = 5186$ discovered long ago.

Of these three hundred and six solutions, I find that there are only twenty-three where the $\phi(n)$ residue classes prime to n have the same abelian group under multiplication (mod n) as the $\phi(n + 1)$ classes have (mod $n + 1$). These twenty-three are determined from the listed factorizations as in [2, Theorem 43, p. 93]. The twenty-three are $n =$

1	3	15	104
495	975	22935	32864
57584	131144	491535	2539004
3988424	6235215	7378371	13258575
17949434	25637744	26879684	29357475
32235735	41246864	48615735	

For example, for the largest n here, the $\phi(n) = \phi(n + 1)$ residue classes both have the abelian group

$$C(2) \times C(2) \times C(2) \times C(12) \times C(230208).$$

It is not unlikely that there are infinitely many solutions even with this much more stringent requirement, but note that there are none at all in the second half of the range of n here.

D. S.

1. DAVID BALLEW, JANELL CASE & ROBERT N. HIGGINS, *Table of* $\phi(n) = \phi(n + 1)$, UMT 2, *Math. Comp.*, v. 29, 1975, pp. 329–330.

2. DANIEL SHANKS, *Solved and Unsolved Problems in Number Theory.* Vol. I, Spartan, Washington, D. C., 1962.

7 [10].—LOUIS COMTET, *Advanced Combinatorics,* D. Reidel Publishing Co., Dordrecht, Holland; Boston, Mass., 1974, translated from the French by J. W. Nienhuys, xi + 343 pp. Price $34; $19.50 paperback.

The original French edition, *Analyse Combinatoire*, appeared in 1970 in two pocketsize paper-covered volumes of modest price; a review, by the present reviewer, is published in *Math. Rev.*, v. 41, 1971, #6697. The current edition, as advertised, is revised and enlarged; the most evident revision is the absence of footnotes, now absorbed in the text, and it is also apparent that there are many additional "supplements and exercises", the author's variation on the conventional problem section. While much of my earlier review is still relevant, the book comes to me now in a new light, probably because I am more at home in English than in French.

Incidentally, the translation has a Dutch accent and seems to have lost the author's French elegance; one oddity is the use of figured for figurate (numbers).

In the new light, the book appears as a continuation of traditional combinatorial analysis, made current by greater use of the vocabulary of set theory. To some combinatorialists this is a great step forward; it supplies a sorely lacking mathematical respectability. To me, the prescribed use of any particular vocabulary is a Procrustean bed, the more irrelevant as combinatorial mappings proliferate.

However, the author's uses of his vocabulary are both elegant and uninhibited, with lively appreciation of the protean aspects of his subject. One nice example is his treatment of bracketing problems (Chapter I, p. 52) where the familiar Catalan problem is exposed both in the traditional manner and in the equivalent form of trivalent plane trees, the latter derived *ab ovo*. Moreover, two other representations, triangulations of a convex polygon, and majority paths (also known as weak-lead ballots) are mentioned, strangely without reference to H. G. Forder's elegant mappings in a paper included in the book's bibliography. The new settings discovered in the last five years, many unpublished, convey the strong feeling that the last word on this subject will be a long time coming.

In content, the emphasis is on variety rather than depth; paraphrasing a remark in the introduction, the book is fairly described as various questions of elementary combinatorial analysis. The variety is impressive; it extends from the inevitable material on permutations and combinations to necklaces, set coverings, Steiner triple systems, Ramsey numbers, Sperner systems, postage stamp foldings, polynomials, permanents, tournaments, full sets, geometries, T_0-topologies, and so on. Most of these appear as exercises, and in compressed exposition, though with references to more expanded treatments. It should be noted that a number of exercises are devoted to enumerations in which the structure is so elusive that no general numerical results are known. A celebrated example is postage stamp folding. The reference list is extensive, often up to the minute, but for me especially valuable in spelling out the abbreviated early references in Netto. One remaining abbreviation bothers me. Authors are identified only by surnames, which seems dangerous. Maybe it works; I have not found a single case where different authors are confounded. However, the Newman appearing alone is Morris, the one with collaborators is Donald J.

I omit further consideration of combinatorial aspects of these problems, in order to give attention to the profuse numerical tables, which may be of greater interest to readers of this journal.

The following are of general interest: Fibonacci numbers, 0 (1) 25; Lucas numbers, 1 (1) 12; Bernoulli, Euler and Genocchi numbers, trinomial and quadrinomial numbers, generated by $(1 + t + t^2)^n$ and $(1 + t + t^2 + t^3)^n$; respectively; partitions with distinct parts 1 (1) 22; sums of multinomial coefficients; D'Arcais numbers, generated by powers of the generating function for partitions of numbers; Euler's totient function, 1 (1) 28; tangent numbers, and numbers $T(n, k)$ generated by $(\tan t)^k/k!$; Salie's numbers, generated by $\cosh t/\cos t$; Leibniz numbers, associated with the harmonic triangular array, defined by $(n + 1)\binom{n}{k}L(n, k) = 1$ (the author uses a Gothic L); the number of terms in expressions for derivatives of implicit functions; postage stamp foldings 2 (1) 28; coefficients in the expansion of gamma functions of large arguments (given a combinatorial setting) 1 (1) 7, borrowed from [1]; Cauchy numbers (integrals over the unit interval of falling and rising factorials) 0 (1) 10.

Of course, in most cases, these tables may seem of trivial interest to the experts in their computation.

A few remarks on the section, Fundamental Numerical Tables, which concludes the book, may be helpful. First, the Bell polynomials $Y_n(x_1, x_2, \ldots, x_n)$ are exhibited as the sum $Y_n = B_{n1} + B_{n2} + \cdots + B_{nn}$, with B_{nk} the collection of terms corresponding to partitions with k parts; B_{nk} is called a partial Bell polynomial. This terminology seems to me unfortunate; apparently, it is dictated by the need to write inverse Bell polynomials, which are called logarithmic polynomials, as

$$L_n(x_1, \ldots, x_n) = \sum f_k B_{nk}, \quad f_k = (-1)^{k-1}(k-1)!$$

and so avoid the compressed notation (proscribed in France?):

$$L_n(x_1, \ldots, x_n) = Y_n(fx_1, \ldots, fx_n), \qquad f^k \equiv f_k = (-1)^{k-1}(k-1)!$$

which I have been using for years.

The third kind of Bell polynomial, with the nondescriptive title Partial Ordinary Bell polynomial, may be written

$$\hat{B}_n(x_1, \ldots, x_n) = \sum [k; k_1, \ldots, k_n] x_1^{k_1} \ldots x_n^{k_n},$$

with summation over partitions of $n = k_1 + 2k_2 + \cdots + nk_n$, $k = k_1 + \cdots + k_n$ and $[k; k_1, \ldots, k_n]$ a multinomial coefficient. It stays in the memory as a consequence of the generating function identity, dropping the arguments in $\hat{B}_n$:

$$1 = (1 - x_1 y - x_2 y^2 - \cdots)(1 + \hat{B}_1 y + \hat{B}_2 y^2 + \cdots).$$

Finally, I notice that the definition of coefficients a_{ms} in Exercise 27 of Chapter III (p. 166) does not agree with the table on p. 167. Indeed, the table gives values of $\binom{m}{s} a_s a_{m-s}$, with a_n the double factorial (for odd factors): $a_0 = 1$, $a_n = (2n-1)a_{n-1}$. The simplest recurrence seems to be $(s+1)a_{m,s} = m(2s+1)a_{m-1,s}$.

Also, many of the number sequences in the first edition appear in [2].

JOHN RIORDAN

1. J. W. WRENCH, JR., "Concerning two series for the gamma function," *Math. Comp.*, v. 22, 1968, pp. 617–626.

2. NEIL J. A. SLOANE, *A Handbook of Integer Sequences*, Academic Press, New York, 1973.

Rockefeller University
New York, New York 10021

8 [8].—E. S. PEARSON & H. O. HARTLEY, Editors, *Biometrika Tables for Statisticians*, Vol. 2, Cambridge Univ. Press, 1972, xvii + 385 pp., 29 cm. Price $17.50.

Abramowitz [*MTAC*, v. 9, 1955, pp. 205–211; see Savage, *Math. Comp.*, v. 21, 1967, pp. 271–273] reviewed *Biometrika Tables for Statisticians*, Volume 1, with remarks which in large part remain appropriate to Volume 2: this is a major continuing project of fine table making. Again there is an extensive introduction of 149 pages to the tables, 230 pages for 69 tables. The authors point out that Volume 2 "is one of many possible companions" to Volume 1. The main difference between Volume 1 and Volume 2 is the computer revolution. The rationalization for Volume 2 deserves full quotation and careful consideration:

"It seems appropriate to comment briefly on the relevance of statistical tables vis à vis the advent of high-speed computers. Indeed it has been argued by some that there is no need for a new volume of statistical tables since any desired numerical value of the mathematical functions involved can be readily computed with the help of fast subroutines loaded into a high-speed computer. Tables, it is argued, will in due course be superseded by a library of algorithms for mathematical functions.

"Whilst we do not wish to underrate the growing importance of the latter, we believe the need for printed tables will be with us for a good time to come, both in the area of (a) data analysis and (b) research in statistical methodology. With regard to (a) there is a real danger that automated, stereotype 'processing' of data may discourage intelligent examination of observations for unexpected features which may suggest new results and interpretations. Such intelligent inspection, besides being often assisted by graphical means, will generally be accompanied by the computation of test criteria, the need to apply which evolves in the course of the examination of the observations; this process will require the use of appropriate pre-tabled functions. Moreover, the immediate access to a high-speed computer permitting the permanent storage of computer codes for all statistical criteria is not likely to be universal for some time longer.

"With regard to (b), research in statistical methodology, there is no doubt that systematic evaluations of the properties of new statistical functions are today being performed to an increasing extent with the help of special algorithms implemented as computer subroutines. However, the efficient planning of such computations invariably requires pilot studies for which pre-worked numerical values are invaluable. Indeed it is an essential feature of research that new ideas should be tested in small pilot computations which will provide feed-backs to the researcher leading to modifications and improvements in method. It would clearly be foolish to invest in large systematic computations before a reasonable chance of success is indicated by such pilot studies.

"On a more personal level, those of us who have learnt to get the 'feel' of our data or gain fresh sidelights on our research by work done at home at the end of the day or at weekends, or even on vacation, find it hard to believe that there is not still a place for the desk computer and appropriate books of printed tables. There is surely something lost if a new generation of students is taught to think that the proper thing is to hand everything over to computerized subroutines."

pp. xiv–xv.

Following is the list of sections and table titles. After the titles are parenthetical remarks regarding: (1) range and accuracy, (2) overlap with Volume 1, (3) origin of table. These notes are rough in that complete descriptions would in many cases defy the reviewer and make the review excessively long.

I. The Normal Probability Function and Certain Derived Tables.

1. Values of X and Z in terms of P. (1. $P = .500$ (.001) .9990 (.0001) .9999, 10D. 2. T. 3, 4, 5 have 5D and include X values for $P = .9800$ (.0001) .9990 but do not include Z values for $P = .9990$ (.0001) .9999. 3. *Biometrika* (1931) with corrections.)

2. Differential coefficients $D^nZ(X)$. (1. $X = .00$ (.02) 4.00 (.05) 6.20 from 6D for P to 1D for $n = 9$. 2. none. 3. new.)

3. Percentage points of the χ^2 distribution for integral and fractional degrees of freedom. (1. $P = .0001$, .0005, .001, .005, .010, .025, .050, .10 (.10) .50 and $1 - P$ and $\nu = .1$ (.1) 3.0 (.2) 10.0 (1) 100, 6 SF. 2. T. 8 does not have the fractional ν values and covers only $P = .001$, .005, .010, .025, .050, .100, .250, .500, and $1 - P$. T. 7 gives the probability integral for the chi-square distribution or Poisson sums. 3. *Biometrika* (1964).)

4. Percentage points of the F-distribution for certain fractional degrees of freedom. (1. $\nu_1 = .1$ (.1) 1.0 (.2) 2.0 (.5) 4.5, $\nu_2 = .5$ (.1) 1.0 (.2) 3.0 (.5) 7.0, $P = .950$, .975, .990, .995, .999, 5 SF. 2. none. 3. new.)

5. Percentage points of the F-distribution (variance ratio). Integral degrees of freedom only. (1. $\nu_1 = 1$ (1) 10, 12, 15, 20, 24, 30, 40, 60, 120, ∞, $\nu_2 = 1$ (1) 30, 40, 60, 120, ∞, $P = .50$, .75, .90, .95, .975, .99, .995, .9975, .9990, 5 SF. 2. T. 18 has same ν_1 and ν_2 values, $P = .75$, .90, .95, .975, .99, .995, .999, usually 2D but 4 or 5 SF for large entries. 3. *Biometrika* (1943), recomputed, enlarged, corrected.)

6. Probability integral of the extreme standardized deviate from the population mean, $X_n = (x_{(n)} - \mu)/\sigma$ or $X_1 = (\mu - x_{(1)})/\sigma$. (1. $n = 3$ (1) 25 (5) 60, 100 (100) 1000, $X_n = -\infty$ (.1) ∞, 7D. 2. T. 24. $n = 1$ (1) 30, $P = .001$, .005, .010, .025, .050, .100, and $1 - P$, 3D. 3. *Biometrika* (1925) with some newly computed values.)

7. Probability integral of the extreme standardized deviate from the sample mean, $u_n = (x_{(n)} - \bar{x})/\sigma$ or $u_1 = (\bar{x} - x_{(1)})/\sigma$. (1. $n = 3$ (1) 25, $u_n = .00$ (.01) ∞, 6D for

$n \leqslant 9$, 5D for $10 \leqslant n \leqslant 19$, 4D for $20 \leqslant n \leqslant 25$. 2. T. 25. $n = 3$ (1) 9; P as in T. 24, see 6, 2D. 3. *Biometrika* (1948), *Ann. Math. Statist.* (1950).)

8. Probability integral of the mean deviation, m, from the sample mean. (1. $n = 2$ (1) 10, $m = .00$ (.01) ∞, 5D. 2. T. 34. $n = 11$ (5) 51 (10) 101 (100) 1001, $P = .01$, .05, .10, and $1 - P$, 4D. 3. *Biometrika* (1945).)

II. **Tables for Procedures Based on the Use of Order Statistics.**

9. Expected values of normal order statistics, $\xi(i|n)$. (1. $n = 2$ (1) 99, 5D. 2. T. 28. $n = 2$ (1) 26 (2) 50, 3D for $n \leqslant 20$, 2D for $n \geqslant 21$. 3. *Biometrika* (1961).)

10. Variances and covariances of normal order statistics. (1. $n = 2$ (1) 20, 6D. 2. none. 3. *Ann. Math. Statist.* (1956).)

11. Coefficients for estimating mean and standard deviation as linear functions of k normal order statistics.

11a. With minimum variance $\sigma^2(\breve{\mu})$. (1. $k = 2$ (2) 10, 4D. 2. none. 3. *J. Amer. Statist. Assoc.* (1965).)

11b. With minimum $\sigma^2(\breve{\mu}) + \sigma^2(\breve{\sigma})$. (1. $k = 2$ (2) 12. 2. and 3. same as 11a.)

11c. With minimum variance and $p_1 \geqslant 0.025$. (1. $k = 10$ (2) 14. 2. and 3. same as 11a.)

11d. With minimum variance $\sigma^2(\breve{\sigma})$. (Same as 11b.)

11e. With minimum $\sigma^2(\breve{\mu}) + \sigma^2(\breve{\sigma})$. (Same as 11b.)

11f. With minimum variance and $p_1 \geqslant 0.025$. (1. $k = 4$ (2) 12. 2. and 3. same as 11a.)

12. Moments and moment ratios of the extreme values, $x_{(1)}$ and $x_{(n)}$, in normal samples. (1. $n = 1$ (1) 50, μ_2 and μ_3 have 8D, μ_4 has 7D. 2. none. 3. *Biometrika* (1954).)

13. Sums of squares of expected values of normal order statistics. (1. $n = 2$ (1) 100, 5D. 2. none. 3. new from 9.)

14. Conversion factors to be applied to Table 15 to derive a best linear estimate of σ. (1. $n = 2$ (1) 50, $n \leqslant 20$ has 4D and $n \geqslant 21$ has 3D. 2. none. 3. new from 15.)

15. Test for departure from normality: Coefficients $a_{i,n}$ to use in the W-test. (1. $n = 2$ (1) 50, 4D. 2. none. 3. *Biometrika* (1965).)

16. Test for departure from normality: percentage points of W. (1. $n = 3$ (1) 50, $P = .01$, .02, .05, .10, .50, and $1 - P$. 2. none. 3. *Biometrika* (1965).)

17. Test for departure from normality: coefficients for converting W to a standardized normal variate, $n = 7$ (1) 50. (1. 4 SF. 2. none. 3. *Technometrics* (1968).)

18. Test for departure from normality: values of G for argument v, for normal conversion of W, $n = 3$ (1) 6. (1. $v = -7.0, -5.4$ (.4) 9.8, 2D. 2. none. 3. *Technometrics* (1968).)

19. Expected values of negative exponential order statistics, $\eta(i|n)$. (1. $n = 1$ (1) 60, 5D. 2. none. 3. Aerospace Research Labs. (1964).)

20. Expected values of order statistics, $\eta(i|n, m)$, in samples from certain gamma distributions. (1. $n = 1$ (1) 20, $m = .5$ (.5) 3.5, 3D. 2. none. 3. Aerospace Research Labs. (1964).)

21. Expected values of order statistics in samples from a half-normal distribution, $\zeta(j|n)$. (1. $n = 1$ (1) 30, 4D. 2. none. 3. Case Institute of Technology (1964).)

III. **Mean Slippage Tests Based on Ranks.**

22. Lower tail critical values, W_1, for the Wilcoxon two-sample rank-sum test. (1. $n = 2$ (1) 25, $m = 1$ (1) n, $P = .001$, .005, .010, .025, .05. 2. none. 3. *Biometrika* (1963).)

23. The Wilcoxon paired rank test.

23A. Probability integral, $P(T|N)$ for $3 \leqslant N \leqslant 15$. (1. $T = 0$ (1) ∞. 2. none. 3. Lederle Labs. (1964) and Kraft and van Eeden (1968).)

23B. Lower percentage points, $T_1(\alpha|N)$ for $5 \leqslant N \leqslant 50$. (1. $P = .005, .01, .025, .05$. 2. none. 3. same as 23A.)

IV. **Tables and Charts for Non-Central Distributions.**

24. Percentage points of the non-central χ distribution. (1. $\nu = 1$ (1) 12, 15, 20, $\sqrt{\lambda} = .0$ (.2) 6.0, $P = .005, .01, .025, .05$ and $1 - P$, 4 SF. 2. none. 3. *Biometrika* (1969).)

25. Non-central χ^2. Values of the non-central parameter, λ, for given degrees of freedom, ν, and power, β. (1. $\nu = 1$ (1) 30 (2) 50 (5) 100, $P = .95, .99$, $\beta = .25, .50, .60, .70$ (.05) .95, .97, .99, 3D. 2. none. 3. Case Institute of Technology (1962).)

26. Non-central t. Factors, l, for determination of percentage points of t'. (1. $\nu = 2$ (1) 9, 10, 36, 144, $u = -1.00$ (.05) $-.80$ (.1) 1, $P = .5, .75, .95, .975, .99, .995$, 4D. 2. T. 10 is a power function chart for $\nu = 6$ (1) 10, 12, 15, 20, 30, 60, ∞, $P = .95, .99$. 3. Sandia Corp. (1963).)

27. Non-central t. Factors, l, for determination of confidence limits for the non-central parameter, Δ. (1. $\nu = 2$ (1) 9, 16, 36, 144, $y = -1.0$ (.1) .80 (.05) 1.00, 4D. 2. none. 3. Sandia Corp. (1963).)

28. Coefficients to assist the determination of the moments of non-central t. (1. $f = 2$ (1) 25 (5) 80 (10) 100 (50) 200 (100) 1000, 6 SF. 2. none. 3. *Biometrika* (1961).)

29. Percentage points of non-central χ. Extension of Table 24 for $\sqrt{\lambda} = 8, 10$. (1. $\nu = 1$ (1) 12, 15, 20, $P = .005, .01, .025, .05$, and $1 - P$. 2. none. 3. *Biometrika* (1969).)

30. Charts for determining the power of the t and F tests: fixed effects model. (1. $\nu_1 = 1$ (1) 8, 12, 24, $\nu_2 = 6$ (1) 10, 12, 15, 20, 30, 60, ∞, $P = .95, .99$. 2. T. 10 corresponds to $\nu_1 = 1$. 3. *Biometrika* (1951) with additions.)

V. **Systems of Univariate Frequency Distributions.**

31. Pearson curves: parameters a and b against $\sqrt{\beta_1}$, β_2 for J and U–Type I distributions included in Table 32. (1. $\sqrt{\beta_1} = .0$ (.1) 2.0, β_2 in increments of .2, 4D. 2. none. 3. new.)

32. Pearson curves: percentage points for given $\sqrt{\beta_1}$, β_2 expressed in standard measure. (1. $\sqrt{\beta_1} = 0.0$ (.1) 2.0, β_2 in increments of .2, $P = 0.000, .0025, .005, .01, .025, .05, .10, .25, .50$, and $1 - P$, 4D. This is a major table. 2. T. 42, $\beta_1 = .00, .01, .03, .05$ (.05) .20 (.10) 1.00, $\beta_2 = 1.8$ (.2) 5.0, $P = .005, .01, .025, .05$, and $1 - P$, 2D. 3. *Biometrika* (1963).)

33. Pearson curves: extension of Table 32 into J and U distribution region. (1. $\sqrt{\beta_1} = .2, .4$ (.1) 2.0, β_2 in increments of .2, P as in 32, 4D. 2. none. 3. new.)

34. Johnson S_U system: parameter $-\gamma$ in terms of $\sqrt{\beta_1}$, β_2. (1. $\sqrt{\beta_1} = .05$ (.05) 2.0, $\beta_2 = 3.2$ (.2) 15.0, 4 SF. 2. none. 3. *Biometrika* (1965).)

35. Johnson S_U system: parameter δ in terms of $\sqrt{\beta_1}$, β_2. (see 34.)

36. Johnson S_B system: parameters in terms of $\sqrt{\beta_1}$, β_2. (1. $\sqrt{\beta_1} = .05$ (.05) 2.0, $\beta_2 = 1.1$ (.1) 10.7, 4 SF. 2. none. 3. Univ. N. Carolina (1968).)

37. Maximum likelihood estimator of p in the gamma (Type III) distribution, start assumed known. (1. $v = .00$ (.01) 1.40, 6D and $v = 1.4$ (.2) 18.0, 5D. 2. none. 3. *Technometrics* (1960).)

38. Coefficients in expansions for bias and variance of the maximum likelihood estimator $\hat{p}$ in a gamma distribution, derived from Table 37. (1. $p = .1$ (.1) 1, 2, 5, 10, 25, 50, 5 SF. 2. none. 3. Union Carbide Corp. (1968).)

VI. Tables to Use in Applying Techniques of Quantal Assay.

39. Minimum normit χ^2 procedure: weights for arguments r and $n \leqslant 50$. (1. 5D. 2. none. 3. *Biometrika* (1957).)

40. Minimum normit χ^2 procedure: weights for arguments $p = r/n$. (1. $p = .000$ (.001) 1.000, 5D. 2. none. 3. *Biometrika* (1957).)

41. Logits, $l = \log[P/(1 - P)]$ for argument P. (1. $P = .500$ (.001) 1.000, 5D. 2. none. 3. *J. Amer. Statist. Assoc.* (1953).)

42. Antilogits: table giving P for argument l. (1. $l = 0.00$ (.01) 4.99, 5D. 2. and 3. same as 41.)

43. Minimum logit χ^2 procedure: weights for argument P. (1. $P = .000$ (.001) 1.000, 4D. 2. and 3. same as 41.)

44. Nomograms to assist in fitting the logistic function, using maximum likelihood (equally spaced doses) $\hat{g}$ = estimate of γ, (ED 50), 3 doses. (1. 8 charts. 2. none. 3. *Biometrika* (1960).)

45. Logistic function fitted by maximum likelihood: standard errors of estimators derived using charts of Table 44. (1. Too involved for concise description. 2. none. 3. *Biometrika* (1960).)

46. Maximum likelihood solution for the logistic (general case): weights $w = PQ$ for argument l. (1. $l = .00$ (.01) 4.99, 5D. 2. none. 3. *Biometrics* (1957).)

VII. **Tables for Multivariate Analysis.**

47. Wilks' likelihood criterion, $W = |A|/|A + B|$. Factors $C(p, \nu_2, M)$ to adjust to $\chi^2_{p\nu_2}$. (1. $M = 1$ (1) 10, 12, 15, 20, 30, 60, ∞, $p = 3$ (1) 10, $\nu_2 = 2$ (2) 22, $P = .95$, .99, 3D. 2. none. 3. *Biometrika* (1966 and 1969).)

48. Percentage points of the largest characteristic root of the determinantal equation $|B - t(A + B)| = 0$ (after Pillai et al.). (1. $n = 5$ (5) 50, 48, 60, 80, 120, 240, ∞, $m = 0$ (1) 5, 7, 10, 15, $p = 2$ (1) 10, $P = .95, .99$, 4D. 2. none. 3. *Biometrika* (1967).)

49. Percentage points of the largest characteristic root of the determinantal equation $|B - t(A + B)| = 0$ (after Foster & Rees). (1. $P = .80$ (.05) .95, .99, 4D, $p = 2$, $\nu_1 = 5$ (2) 41 (10) 101, 121, 161, $\nu_2 = 2, 3$ (2) 21, $p = 3$, $\nu_1 = 4$ (2) 46 (4) 70, 98, 194, $\nu_2 = 3$ (1) 10, $p = 4$, $\nu_1 = 5$ (2) 51 (4) 71, 99, 195, $\nu_2 = 4$ (1) 11. 2. none. 3. *Biometrika* (1957).)

50. Test for equality of k covariance matrices. (1. 4 SF, $k = 2$ (1) 10, $p = 2$, $\nu_0 = 3$ (1) 10, $p = 3$, $\nu_0 = 5$ (1) 13, $p = 4$, $\nu_0 = 6$ (1) 15, $p = 5$, $k = 2$ (1) 7, $\nu_0 = 8$ (1) 16, $p = 6$, $k = 2$ (1) 5, $\nu_0 = 10$ (1) 20. 2. none. 3. *Biometrika* (1969).)

51. Percentage points of the extreme roots of $|S\Sigma^{-1} - cI| = 0$. (1. $p = 2$ (1) 10, $\nu = 2$ (1) 12, 15 (5) 30 (10) 100 (20) 200, $P = .95, .99$, 4 SF. For $P = .01$ and .05 there is less detail. 2. none. 3. *Biometrika* (1968) and *Ann. Inst. Statist. Math.* (1968).)

52. Percentage points of the multiple correlation coefficient R. (1. $R = .0$ (.1) .9, $\nu_1 = 2$ (2) 12 (4) 24, 30, 34, 40, $\nu_2 = 10$ (10) 50, $P = .01, .05, .95, .99$, 3D. 2. none. 3. new.)

53. Test of the hypothesis that a covariance matrix $\Sigma = \Sigma_0$. Percentage points of L. (1. $p = 2$ (1) 10, ν irregular, 3 SF on 2D, $P = .95, .99$. 2. none. 3. *Biometrika* (1968).)

VIII. **Goodness of Fit Tests Based on the Empirical Distribution Function. Tests of Uniformity.**

54. Modifications yielding approximate percentage points for the statistics D, V, W^2, U^2 and A in finite samples of n observations. (1. $P = .85$ (.05) .95, .975, .99, 3D. 2. none. 3. *J. R. Statist. Soc. B* (1970).)

55. The Kolmogorov two-sample test. Upper critical values of $c = mnD_{m,n}$. (1. $1 \leqslant m \leqslant n \leqslant 25$, $P = .9, .95, .975, .99, .995, .999$. 2. none. 3. new.)

IX. **Analysis of Directions on a Circle and Sphere.**

56. Percentage points of R/N (on circle), for given N and κ. (1. $N = 5$ (1) 10, 12, 16, 20, 30, 40, 60, 100, 200, ∞, $\kappa = .0$ (.5) 5.0, 3D. 2. none. 3. *Biometrika* (1969).)

57. Charts to determine percentage points of R (on circle), for given N and X. (1. 2 charts. 2. none. 3. *Biometrika* (1962).)

58A. Critical values of Z for test of equality of two modal vectors (on circle): equal sample sizes, $N_1 = N_2 = \frac{1}{2}N$. (1. $W = .05$ (.05) .70, $N = 12$ (4) 24, 30, 40, 60, 120, 240, ∞, $P = .9$, .95, .975, .99, 3D. 2. none. 3. *J. Amer. Statist. Assoc.* (1972).)

58B. Critical values of Z for test of equality of two modal vectors (on circle): unequal sample sizes, $N_1 \neq N_2$. (Like 58A with $N_1 = 2N_2$ or $N_1 = 4N_2$.)

59. Percentage points for R/N (on sphere), for given N and κ. (1. $N = 4$ (1) 10, 12 (4) 20 (10) 40, 60, 100, ∞, $\kappa = .0$ (.5) 5.0, $P = .01$, .05, .95, .99, 4D. 2. none 3. *Biometrika* (1967).)

60. Charts to determine percentage points of R (on sphere), for given N and X. (1. 2 charts. 2. none. 3. *Biometrika* (1962).)

61A. Critical values of Z for test of equality of two modal vectors (on sphere): equal sample sizes, $N_1 = N_2 = \frac{1}{2}N$. (1. Like 58A. 2. none. 3. *Biometrika* (1969).)

61B. Critical values of Z for test of equality of two modal vectors (on sphere): unequal sample sizes, $N_1 \neq N_2$. (1. like 58B. 2. none. 3. *Biometrika* (1969).)

62. Estimation of κ for dispersion on circle. (1. $a = .10$ (.05) .70 (.02) .86, 4 SF. 2. none. 3. *J. Amer. Statist. Assoc.* (1953).)

63. Estimation of κ for dispersion on sphere. (1. $a = .10$ (.05) .60 (.02) .80, 4 SF. 2. none. 3. Toronto thesis (1962).)

64. Percentage points of $S = \Sigma_i (\cos^2 \theta_i)/N$ (on sphere). (1. $N = 3$ (1) 10 (2) 20 (5) 50 (10) 100, $P = .005$, .01, .025, .05, .10 and $1 - P$, 3D. 2. none. 3. *Biometrika* (1965).)

65. Lower tail percentage points for $S_{\min}$ and upper tail for $S_{\max}$ (on sphere). (1. $N = 5$ (1) 10 (2) 20 (5) 30 (10) 80, 100, ∞, $P = .01$, .025, .05, .10 and $1 - P$, 3D. 2. none. 3. Stanford Report, no date.)

X. **Tables to Aid Interpolation.**

66. Coefficients B_2, B_3, B_4 for Bessel interpolation formula. (1. $p = .000$ (.001) .500, B_2 6D, B_3 5D, B_4 4D. 2. none. 3. *Interpolation and Allied Tables* (1956).)

67. Miscellaneous four-point Lagrangian interpolation coefficients. (new.)

68. Lagrangian coefficients for use with harmonic arguments in certain tables of percentage points. (*Biometrika* (1941).)

69. Five-point Lagrangian coefficients, L_i ($i = 1, 2, \ldots, 5$) for interpolation between tabled percentage points. (*Biometrika* (1968).)

A striking aspect of this project is the loving care for detail and accuracy. The final product shows this. The only "error" I noted is that on p. 3 (bottom) and p. 4 (top) $D(X)$ should be replaced by $DZ(X)$. Professor Pearson showed me an obvious error in Table 1. Also Table 34 will need substantial revision; see *Biometrika*, v. 61, 1974, pp. 203–205.

I. RICHARD SAVAGE

Department of Statistics
Yale University
New Haven, Connecticut 06520