High-Precision Values of the Gamma Function and of Some Related Coefficients

By Arne Fransen and Staffan Wrigge

Abstract. In this paper we determine numerical values to 80D of the coefficients in the Taylor series expansion $\Gamma^m(s+x) = \sum_0^\infty g_k(m, s)x^k$ for certain values of m and s and use these values to calculate $\Gamma(p/q)$ ($p, q = 1, 2, \dots, 10; p < q$) and $\min_{x>0} \Gamma(x)$ to 80D. Finally, we obtain a high-precision value of the integral $\int_0^\infty (\Gamma(x))^{-1} dx$.

1. Introduction. This paper traces its origin from a wish to determine high-precision values of the integrals $F = \int_0^\infty (\Gamma(x))^{-1} dx$ and $\int_n^{n+1} (\Gamma(x))^{-1} dx$ because the distribution defined by $G(x) = F^{-1} \int_0^x (\Gamma(t))^{-1} dt$ may be useful in reliability theory (Fransen [5]). That can also be approximated by a weighted sum of Gamma Distributions or be seen as a special case of using the Fox H -function as a distribution (Carter and Springer [3]). The integrals in question had not been properly studied (to our great astonishment) before. They were not even mentioned in Nielsen's book on the Gamma function [11]. The closest we came in our literature study was to a paper from 1883 by Bourguet [2]. When establishing high-precision values of the integrals Fransen [4] had to calculate the Riemann Zeta function to 80D for integer values, thereby using formulas of Katayama and Ramanujan [8]. These values might be used for many purposes: to determine the coefficients in the Taylor series expansion of $\Gamma^m(s+x)$ for certain interesting values of m and s , the value of $\min_{x>0} \Gamma(x)$, and many other relevant coefficients.

When carrying out the necessary multiple-precision calculation on our DEC-10 computer we have used a Simula Class HIGHPREC developed by a student, Demetre Betsis, at the University of Stockholm.

2. Numerical Values to 80D of the Coefficients in the Taylor Series Expansion of $\Gamma^m(s+x)$ for Certain Values of m and s .

a. *Basic Formulas.* Let m and s be real numbers, $s > 0$. Consider the Taylor series expansion

$$(2.1) \quad \Gamma^m(s+x) = \sum_{k=0}^{\infty} g_k(m, s)x^k,$$

where

$$g_k(m, s) = \frac{1}{k!} \frac{d^k \Gamma^m(s)}{ds^k}.$$

Received April 30, 1979; revised August 14, 1979.

AMS (MOS) subject classifications (1970). Primary 65D20; Secondary 33A15, 33A25, 65A05.

Key words and phrases. Special functions, Gamma function, Riemann Zeta function.

The coefficients $g_k(m, s)$ may be obtained recursively if we apply Leibniz' differentiation formula to the identity

$$(2.2) \quad \frac{d\Gamma^m(s)}{ds} = m\psi(s)\Gamma^m(s),$$

where $\psi(s)$ is the Psi function ($= \dot{\Gamma}(s)/\Gamma(s)$). We state the main result in the following

THEOREM. *Let m and s be real numbers, $s > 0$. The coefficients $g_k(m, s)$ in the Taylor series expansion*

$$\Gamma^m(s+x) = \sum_{k=0}^{\infty} g_k(m, s)x^k$$

are obtained from the recursion formula

$$(n+1)g_{n+1}(m, s) = m \sum_{k=0}^n (-1)^{k-1} g_{n-k}(m, s)h_k(s),$$

where $h_k(s)$ ($k = 1, 2, \dots$) is the Hurwitz Zeta function

$$h_k(s) = \sum_{n=0}^{\infty} \frac{1}{(s+n)^{k+1}}$$

and

$$h_0(s) = -\psi(s) = \lim_{n \rightarrow \infty} \left(\sum_{k=0}^n \frac{1}{s+k} - \ln(n) \right),$$

with the initial value $g_0(m, s) = \Gamma^m(s)$.

A similar result can partly be found in Nielsen's book [11]. We now apply the theorem to some special cases.

b. *Special Cases.* Choosing $s = 1$, we get $g_0(m, 1) = 1$ and $h_0(1) = \gamma$, the Euler constant. Further, we then have $h_k(1) = \zeta(k+1)$, the Riemann Zeta function, for $k = 1, 2, \dots$. The computation of values of $\zeta(k)$ for even values of k is straightforward, while we use the Ramanujan formula as described in [8] to compute them in the odd case. In Table I we present these values to 80D for k up to 51.

Putting $m = -1$ and denoting $a_{k+1} = g_k(-1, 1)$, we compute the coefficients using the recursion formula. In Table II we present the values of a_k obtained to 80D for k up to 52. Note that a_k approaches zero very fast. We prove that $\lim_{k \rightarrow \infty} a_k = 0$ below. We have not been able to derive an approximate expression of a_k or to explain the rather irregular occurrences of plus and minus signs. Already for moderate sizes of s , however, the expansion is properly alternating. Similarly when choosing $m = 1$ and denoting $b_{k+1} = g_k(1, 1)$ we get the values of b_k presented in Table III. We prove also that $\lim_{k \rightarrow \infty} g_{2k}(1, 1) = 1$ and $\lim_{k \rightarrow \infty} g_{2k+1}(1, 1) = -1$ below.

Finally, choosing $s = 3/2$, we get $-h_0(s) = \psi(s) = 2 - \gamma - 2 \ln 2$ and $h_k(s) = (2^{k+1} - 1)\zeta(k+1) - 2^{k+1}$ (see [1]) for $k = 1, 2, \dots$. We denote for $m = 1$, $c_{k+1} = g_k(1, 3/2)$ and for $m = -1$, $d_{k+1} = g_k(-1, 3/2)$, and present the

values of c_k and d_k in Tables IV and V. Note that $g_k(1, 3/2) \approx (-1)^k(2/3)^{k+1}$ and $\lim_{k \rightarrow \infty} g_k(-1, 3/2) = 0$.

Some of the results presented in Tables I–V have partly been published previously. A paper by J. W. Wrench [13] gives the coefficients a_k to 31D for $k = 2(1)41$. Last-figure corrections appeared in *Math. Comp.*, v. 27, 1973, pp. 681–682, MTE 505. A. H. Morris has compiled two unpublished tables, deposited in the UMT file. The first one, [9], gives $\zeta(k)$ to 70D for $k = 2(1)90$. The second one, [10], includes a tabulation of a_k to 70D for $k = 1(1)73$.

c. *A Draft Proof.* In the text (Section 2b) it is remarked that $\lim_{k \rightarrow \infty} a_k = 0$ and that $\lim_{k \rightarrow \infty} g_{2k}(1, 1) = -\lim_{k \rightarrow \infty} g_{2k+1}(1, 1) = 1$.

By using ordinary residue calculus one sees that

$$(2.3) \quad \frac{1}{2\pi i} \int z^{-k} \frac{1}{\Gamma(z)} dz = a_{k-1},$$

where the integration is carried out around $|z| = 1$. We put $z = e^{i\theta}$. Then

$$(2.4) \quad a_k = \frac{1}{2\pi} \int_0^{2\pi} (\cos(k\theta) - i \sin(k\theta)) \frac{1}{\Gamma(e^{i\theta})} d\theta.$$

Using a well-known lemma of Riemann-Lebesgue, one gets $\lim_{k \rightarrow \infty} a_k = 0$. To prove the other results one starts with the identity

$$(2.5) \quad \Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx = R_1(s) + R_2(s),$$

where

$$R_1(s) = \int_0^1 x^{s-1} e^{-x} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(s+k)k!}$$

and

$$R_2(s) = \int_1^\infty x^{s-1} e^{-x} dx.$$

Differentiating Eq. (2.5) n times with respect to s , we get

$$(2.6) \quad \Gamma^{(n)}(s) \frac{1}{n!} = \sum_{k=0}^{\infty} \frac{(-1)^{k+n}}{(s+k)^{n+1} k!} + \frac{R_2^{(n)}(s)}{n!}.$$

When $s = 1$ and $n \rightarrow \infty$ “everything” in the right-hand side of Eq. (2.6) approaches zero except the term $(-1)^n$. Q.E.D.

3. Numerical Values to 80D of $\Gamma(p/q)$; $p, q = 1, 2, \dots, 10, p < q$.

a. *By Taylor Series Expansions.* We shall calculate in all 32 different values $\Gamma(p/q)$, where $p/q \in I$ and $I = I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_5$, where

$$I_1 = \left\{ \frac{1}{10}, \frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6} \right\},$$

$$I_2 = \left\{ \frac{1}{5}, \frac{2}{9}, \frac{1}{4}, \frac{2}{7}, \frac{3}{10} \right\},$$

$$I_3 = \left\{ \frac{1}{3}, \frac{3}{8}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{1}{2}, \frac{5}{9}, \frac{4}{7}, \frac{3}{5}, \frac{5}{8}, \frac{2}{3} \right\},$$

$$I_4 = \left\{ \frac{7}{10}, \frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5} \right\},$$

$$I_5 = \left\{ \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{1}{1} \right\}.$$

We use the Taylor series expansions we have, viz.

$$(3.1) \quad f(x) = \frac{1}{\Gamma(1+x)} = \sum_{k=0}^{\infty} g_k(-1, 1)x^k$$

and

$$(3.2) \quad g(x) = \frac{1}{\Gamma(3/2+x)} = \sum_{k=0}^{\infty} g_k\left(-1, \frac{3}{2}\right)x^k.$$

We have to calculate $xf(x)$ and $(x + \frac{1}{2})g(x)$ with a precision a little greater than $0.5 \cdot 10^{-80}$, whereupon we invert. Therefore, we must have

$$|g_{51}(-1, 1)|x^{52} \lesssim 0.5 \cdot 10^{-80}, \quad |\frac{1}{2}g_{51}(-1, 3/2)||x|^{51} \lesssim 0.5 \cdot 10^{-80}.$$

The corresponding values of x are $0 < x < 0.1970$ and $|x| < 0.1964$. If we also allow negative values of x in $f(x)$, we get $-0.1908 < x < 0$.

We can hereby calculate the following values:

$$(3.3) \quad \text{with } xf(x) \quad \Gamma\left(\frac{p}{q}\right) \quad \text{where } \frac{p}{q} \in I_1: x = \frac{p}{q} \text{ in Eq. (3.1),}$$

$$(3.4) \quad \text{with } (x + \frac{1}{2})g(x) \quad \Gamma\left(\frac{p}{q}\right) \quad \text{where } \frac{p}{q} \in I_3: x + \frac{1}{2} = \frac{p}{q} \text{ in Eq. (3.2),}$$

$$(3.5) \quad \text{with } f(x) \quad \Gamma\left(\frac{p}{q}\right) \quad \text{where } \frac{p}{q} \in I_5: 1+x = \frac{p}{q} \text{ in Eq. (3.1).}$$

The values $\Gamma(p/q)$ where $p/q \in I_2 \cup I_4$ cannot be calculated in this simple way. For these missing values we use the duplication formula

$$(3.6) \quad \Gamma(2x)\sqrt{\pi} = 2^{2x-1}\Gamma(x)\Gamma(x + \frac{1}{2}).$$

The 32 values of $\Gamma(p/q)$ appear in Table VII.

b. *By Elliptic Integrals.* As is shown by Wrigge [14]–[16] and Glasser and Wood [7] and many others, there is a close relationship between certain values of the gamma function and the complete elliptic integral $K(t)$. The easiest way of getting a good value of $K(t)$ is to use the arithmetic-geometric mean $M(t)$ (see [1]), which will give an accurate value of $K(t)$ to at least 80D in less than 10 steps. We have

$$(3.7) \quad K(t) = \frac{\pi}{2M(t)}.$$

It is, e.g., known that (Fransén [4])

$$(3.8) \quad \Gamma^2\left(\frac{1}{4}\right) = 4\sqrt{\pi}K\left(\frac{1}{\sqrt{2}}\right),$$

$$(3.9) \quad \Gamma^2\left(\frac{1}{8}\right) = 16K(\sqrt{2}-1)2^{-3/4}\Gamma\left(\frac{1}{4}\right).$$

By using the duplication and reflection formulas for the Gamma function (see [1]) it is possible to get easy-to-calculate expressions for all $\Gamma(p/8)$, $p = 1, 2, \dots, 8$. Such values are presented to 80D in [4]. Similar results hold for $\Gamma(p/6)$ (see [7]). The values thus calculated agree excellently with the values calculated by the Taylor series method in a. or by other methods as presented in [6].

4. Numerical Values to 80D of x_0 and $\Gamma(x_0) = \min_{x>0} \Gamma(x)$. Instead of $\min_{x>0} \Gamma(x)$ we study $\max_{x>0} (\Gamma(x))^{-1}$ and, thereby, use the Taylor series expansion

$$(4.1) \quad g(x) = \frac{1}{\Gamma(3/2 + x)} = \sum_{k=0}^{\infty} g_k \left(-1, \frac{3}{2}\right) x^k.$$

By making a more and more refined tabulation of $g(x)$ we managed to get a value of x_0 to 37D and of $\min \Gamma(x)$ to 74D. In order to get a better value of x_0 we studied

$$(4.2) \quad \dot{g}(x) = \sum_{k=1}^{\infty} k g_k \left(-1, \frac{3}{2}\right) x^{k-1},$$

and then made use of “repeated” inverse Bessel interpolation. We thus got

$$(4.3) \quad \begin{aligned} x_0 = 1. & \quad 46163 \ 21449 \ 68362 \ 34126 \\ & \quad 26595 \ 42325 \ 72132 \ 84681 \\ & \quad 96204 \ 00644 \ 63512 \ 95988 \\ & \quad 40859 \ 87864 \ 40353 \ 80181 \end{aligned}$$

and

$$(4.4) \quad \begin{aligned} \Gamma(x_0) = 0. & \quad 88560 \ 31944 \ 10888 \ 70027 \\ & \quad 88159 \ 00582 \ 58873 \ 32079 \\ & \quad 51533 \ 66990 \ 34488 \ 71200 \\ & \quad 16587 \ 51362 \ 27417 \ 39635. \end{aligned}$$

5. Numerical Values to 60D of $\int_n^{n+1} (\Gamma(x))^{-1} dx$ ($n = -10, -9, \dots, 48$) and $\int_0^{\infty} (\Gamma(x))^{-1} dx$. The original problem for this paper was to determine a high-precision value of the integral $F = \int_0^{\infty} (\Gamma(x))^{-1} dx$ (named Fransen’s constant in H. P. Robinson’s extensive file of mathematical constants). In [4] Fransen did that using the Euler-Maclaurin formula and got the value of F to 65D. Here we will use the Taylor series methods in Section 2 to calculate $\int_n^{n+1} (\Gamma(x))^{-1} dx$ for $n \geq 0$.

Using routine manipulation, we get

$$(5.1) \quad \begin{aligned} \int_n^{n+1} (\Gamma(x))^{-1} dx &= \int_0^{1/4} (\Gamma(x + n))^{-1} dx \\ &+ \int_{-1/4}^{+1/4} (\Gamma(x + n + \tfrac{1}{2}))^{-1} dx \\ &+ \int_{-1/4}^0 (\Gamma(x + n + 1))^{-1} dx. \end{aligned}$$

But

$$(5.2) \quad \int_v^u (\Gamma(s+x))^{-1} dx = \sum_{k=0}^{\infty} g_k(-1, s) \frac{u^{k+1} - v^{k+1}}{k+1}.$$

Using the methods in Section 2 and/or some simpler recurrence relations, the coefficients $g_k(-1, n)$ and $g_k(-1, 3/2 + n)$ were calculated for the necessary values of n and the integration carried out. The results to 60D appear in Table VI.

To calculate $\int_n^{n+1} (\Gamma(x))^{-1} dx$ for $n < 0$ we used Eq. (5.1) valid also for $n < 0$. Furthermore,

$$(5.3) \quad \frac{1}{\Gamma(-n+x)} = x(x-1)(x-2) \dots (x-n) \frac{1}{\Gamma(1+x)}$$

and

$$(5.4) \quad \frac{1}{\Gamma(-n+\frac{1}{2}+x)} = (\frac{1}{2}+x)(-\frac{1}{2}+x) \dots (-n+\frac{1}{2}+x) \frac{1}{\Gamma(\frac{3}{2}+x)}.$$

Using the Eqs. (5.3), (5.4) and the Taylor series expansions of $(\Gamma(1+x))^{-1}$ and $(\Gamma(3/2+x))^{-1}$ we calculated the integrals also for negative values of n . The results to 60D are presented in Table VI.

6. Tables. In this section we present the tables previously mentioned, i.e.,

I. The values of γ , the Euler constant and $\zeta(k)$, $k = 2, 3, \dots, 51$, the Riemann Zeta function, to 80D.

II. The coefficients $g_k(-1, 1)$ in the Taylor series expansion $(\Gamma(1+x))^{-1} = \sum_0^{\infty} g_k(-1, 1)x^k$ to 80D. Note that $a_{k+1} = g_k(-1, 1)$.

III. The coefficients $g_k(1, 1)$ in the Taylor series expansion $\Gamma(1+x) = \sum_0^{\infty} g_k(1, 1)x^k$ to 80D. Note that $b_{k+1} = g_k(1, 1)$.

IV. The coefficients $g_k(1, 3/2)$ in the Taylor series expansion $\Gamma(3/2+x) = \sum_0^{\infty} g_k(1, 3/2)x^k$ to 80D. Note that $c_{k+1} = g_k(1, 3/2)$.

V. The coefficients $g_k(-1, 3/2)$ in the Taylor series expansion $(\Gamma(3/2+x))^{-1} = \sum_0^{\infty} g_k(-1, 3/2)x^k$ to 80D. Note that $d_{k+1} = g_k(-1, 3/2)$.

VI. The values of $\int_0^{\infty} (\Gamma(x))^{-1} dx$ and $\int_n^{n+1} (\Gamma(x))^{-1} dx$ ($n = -10, -9, \dots, 48$) to 60D.

VII. The values of $\Gamma(p/q)$, $p, q = 1, 2, \dots, 10$: $p < q$, to 80D.

Tabulated values are commonly rounded with a last-figure error not exceeding half a unit.

Acknowledgements. For valuable support during this work we want to thank Dr. John W. Wrench, Jr. and Herman P. Robinson.

TABLE I

Values of the Euler constant, Gamma, and the Riemann Zeta function for integral values.

Gamma	=	0.57721	56649	01532	86060	65120	90082	40243	10421	59335	93992	35988	05767	23488	48677	26777	66467
Zeta(2)	=	1.64493	40668	48226	43647	24151	66646	02518	92189	49901	20679	84377	35558	22937	00074	70403	20087
Zeta(3)	=	1.20205	69031	59594	28539	97381	61511	44999	07649	86292	34049	88817	92271	55534	18382	05786	31309
Zeta(4)	=	1.08232	32337	11138	19151	60036	96541	16790	27747	50951	91872	69076	82976	21544	41206	16186	96885
Zeta(5)	=	1.03692	77551	43369	92633	13654	86457	03416	80570	80919	50191	28119	74192	67790	38035	89786	28148
Zeta(6)	=	1.01734	30619	84449	13971	45179	29790	92052	79018	17490	03285	35618	42408	66400	43321	82901	95790
Zeta(7)	=	1.00834	92773	81922	82683	97975	49849	79675	95998	63560	56523	87064	17283	13657	16014	78317	35574
Zeta(8)	=	1.00407	73561	97944	33937	86852	38508	65246	52589	60790	64985	00203	29110	20265	25829	52574	74881
Zeta(9)	=	1.00200	83928	26082	21441	78527	69232	41206	04856	05851	39488	87565	48596	61590	97850	53390	25840
Zeta(10)	=	1.00099	45751	27818	08533	71459	58900	31901	70060	19531	56447	75172	57788	99463	62914	65151	91295
Zeta(11)	=	1.00049	41886	04119	46455	87022	82526	46993	64686	06435	75820	86171	19141	43610	00540	59798	21981
Zeta(12)	=	1.00024	60865	53308	04829	86379	98047	73967	09604	16088	45800	34045	33040	95213	32520	19681	94091
Zeta(13)	=	1.00012	27133	47578	48914	67518	36526	35739	57142	75105	89550	98451	36702	67162	08967	26829	84421
Zeta(14)	=	1.00006	12481	35058	70482	92585	45105	13533	37474	81696	16915	45494	82755	20225	28629	41023	17742
Zeta(15)	=	1.00003	05882	36307	02049	35517	28510	64506	25876	27948	70685	81775	06569	93289	33322	67156	34228
Zeta(16)	=	1.00001	52822	59408	65187	17325	71487	63672	20232	37388	99047	15311	53105	20358	87870	87027	95315
Zeta(17)	=	1.00000	76371	97637	89976	22736	00293	56302	92130	88249	09426	26790	95379	84397	29356	43290	28246
Zeta(18)	=	1.00000	38172	93264	99983	98564	61644	62193	97304	54697	21895	33311	43174	42998	76300	39542	65005
Zeta(19)	=	1.00000	19082	12716	55393	89256	56957	79510	13532	58571	14483	86302	35933	04676	18239	49705	34131
Zeta(20)	=	1.00000	09539	62033	87279	61131	52038	68344	93459	43794	18741	05957	50056	48985	11375	13731	14390
Zeta(21)	=	1.00000	04769	32986	78780	64631	16719	60437	30459	66446	69478	49376	00207	48737	65968	39087	89816
Zeta(22)	=	1.00000	02384	50502	72773	29900	03648	18675	29949	35041	82177	96582	69849	60311	64744	58935	62291
Zeta(23)	=	1.00000	01192	19925	96531	10730	67788	71888	23263	87254	99778	45198	58603	22579	72362	43730	42744
Zeta(24)	=	1.00000	00596	08189	05125	94796	12440	20793	58012	27503	91883	73027	95864	24697	23217	24495	35547
Zeta(25)	=	1.00000	00298	03503	51465	22801	86063	70506	93660	11844	73091	95433	12398	68133	90133	84460	76746
Zeta(26)	=	1.00000	00149	01554	82836	50412	34658	50663	06986	28864	78816	78859	10547	43596	87899	71296	74486
Zeta(27)	=	1.00000	00074	50711	78983	54294	91981	00417	06041	19454	71903	18825	65829	99323	95783	52147	60627
Zeta(28)	=	1.00000	00037	25334	02478	84570	54819	20401	84024	23232	89305	92958	11519	76933	47061	69604	96030
Zeta(29)	=	1.00000	00018	62659	72351	30490	06403	90994	54169	48061	66533	04692	00665	77489	38055	58091	69327
Zeta(30)	=	1.00000	00009	31327	43241	96681	82871	76473	50212	19813	56795	51368	16185	00861	33604	41960	67294
Zeta(31)	=	1.00000	00004	65662	90650	33784	07298	92332	51220	07106	26918	53369	47307	37297	16933	71175	66989
Zeta(32)	=	1.00000	00002	32831	18336	76505	49200	14559	75940	49502	48298	22845	30311	07760	22583	87912	18939
Zeta(33)	=	1.00000	00001	16415	50172	70051	97759	29738	35456	30951	65224	71727	63593	25651	77399	47029	12462
Zeta(34)	=	1.00000	00000	58207	72087	90270	06892	43685	98910	63054	17312	26046	17215	95507	16881	24163	07140
Zeta(35)	=	1.00000	00000	29103	85044	49709	96869	29425	22788	40464	10698	19874	33032	25621	02548	25640	48890
Zeta(36)	=	1.00000	00000	14551	92189	10419	84235	92963	22453	18420	98380	88941	24038	06913	95422	18571	74587
Zeta(37)	=	1.00000	00000	07275	95983	50574	81014	52086	90123	38059	26485	09255	55466	10770	57969	42638	43837
Zeta(38)	=	1.00000	00000	03637	97954	73786	51190	23723	63558	73273	51264	60283	84897	46994	79515	94042	71425
Zeta(39)	=	1.00000	00000	01818	98965	03070	65947	58483	21007	30085	03058	93096	18664	07053	52512	53356	50932
Zeta(40)	=	1.00000	00000	00909	49478	40263	88928	25331	18386	94908	75386	00009	90878	82850	54797	10112	02537
Zeta(41)	=	1.00000	00000	00454	74737	83042	15402	67991	12029	48857	03390	45299	11438	62808	12340	35905	00260
Zeta(42)	=	1.00000	00000	00227	37368	45824	65251	52268	21577	97869	12138	29821	98915	87258	05336	47882	22296
Zeta(43)	=	1.00000	00000	00113	68684	07680	22784	93491	04838	02590	64374	35902	84251	79989	90122	76309	35911
Zeta(44)	=	1.00000	00000	00056	84341	98762	75856	09277	18296	75240	68553	05715	88993	88351	68064	44440	46359
Zeta(45)	=	1.00000	00000	00028	42170	97688	93018	55455	07370	49426	62074	36882	65309	83382	76290	62781	75669
Zeta(46)	=	1.00000	00000	00014	21085	48280	31606	76983	43071	41739	53767	86986	05633	95195	74517	00244	20901
Zeta(47)	=	1.00000	00000	00007	10542	73952	10852	71287	73544	79956	80002	27420	43593	68768	83638	28887	75095
Zeta(48)	=	1.00000	00000	00003	55271	36913	37113	67329	84695	34059	34299	21456	55503	06261	50125	17934	04758
Zeta(49)	=	1.00000	00000	00001	77635	68435	79120	32747	33490	14400	27957	01555	08575	32695	19787	59241	23327
Zeta(50)	=	1.00000	00000	00000	88817	84210	93081	59030	96091	38639	13863	25608	87146	46446	66449	76989	90084
Zeta(51)	=	1.00000	00000	00000	44408	92103	14381	33641	97770	94026	81213	36459	60307	02441	80285	97831	15341

TABLE II

Values of the coefficients in the expansion of the inverted Gamma function for $s = 1$.

a(1) =	1.00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
a(2) =	0.57721	56649	01532	86060	65120	90082	40243	10421	59335	93992	35988	05767	23488	48677	26777	66467	
a(3) =	-0.65587	80715	20253	88107	70195	15145	39048	12797	66380	47858	43472	92362	44568	38708	38353	72210	
a(4) =	-0.04200	26350	34095	23552	90039	34875	42981	87113	94500	40110	60935	22065	81297	61800	96875	97599	
a(5) =	0.16653	86113	82291	48950	17007	95102	10523	57177	81502	24717	43405	70468	90317	89938	66056	47425	
a(6) =	-0.04219	77345	55544	33674	82083	01289	18739	13016	52684	18982	24863	76918	87327	54590	11185	58900	
a(7) =	-0.00962	19715	27876	97356	21149	21672	34819	89753	62942	25211	30021	05138	86262	73116	73514	46074	
a(8) =	0.00721	89432	46663	09954	23950	10340	44657	27099	04800	88023	83180	01094	78117	36225	94974	15854	
a(9) =	-0.00116	51675	91859	06511	21139	71084	01838	86668	09333	79538	40574	43407	50527	56200	25848	16653	
a(10) =	-0.00021	52416	74114	95097	28157	29963	05364	78064	78241	92337	83387	50350	26748	90856	39463	71678	
a(11) =	0.00012	80502	82388	11618	61531	98626	32816	43233	94892	09969	36772	14900	54583	80412	03552	04347	
a(12) =	-0.00002	01348	54780	78823	86556	89391	42102	18183	82294	83329	79791	15261	16267	09082	29186	18897	
a(13) =	-0.00000	12504	93482	14267	06573	45359	47383	30922	42322	65562	11539	59815	34992	31574	91212	45561	
a(14) =	0.00000	11330	27231	98169	58823	74129	62033	07449	43324	00483	86210	75654	29550	53954	60408	42730	
a(15) =	-0.00000	02056	33841	69776	07103	45015	41300	20572	83651	25790	26293	37945	34683	17253	32456	80371	
a(16) =	0.00000	00061	16095	10448	14158	17862	49868	28553	42867	27586	57197	12320	86732	40292	77235	07435	
a(17) =	0.00000	00050	02007	64446	92229	30055	66504	80599	91303	04461	27424	94481	71895	33788	77374	72132	
a(18) =	-0.00000	00011	81274	57048	70201	44588	12656	54365	05577	73875	95049	32587	59096	18926	31696	43391	
a(19) =	0.00000	00001	04342	67116	91100	51049	15403	32312	25019	14007	09823	12581	21210	87107	39273	47588	
a(20) =	0.00000	00000	07782	26343	99050	71254	04993	73113	60777	22606	80861	81392	93881	94355	07326	92987	
a(21) =	-0.00000	00000	03696	80561	86422	05708	18781	58780	85766	23657	09634	51360	99513	64845	46554	43000	
a(22) =	0.00000	00000	00510	03702	87454	47597	90154	81322	86323	18027	26886	06970	76321	17350	10485	65735	
a(23) =	-0.00000	00000	00020	58326	05356	65067	83222	42954	48552	37419	74609	10808	10147	18805	81964	44349	
a(24) =	-0.00000	00000	00005	34812	25394	23017	98237	00173	18727	93994	89897	15478	12068	21116	80954	93211	
a(25) =	0.00000	00000	00001	22677	86282	38260	79015	88938	46622	42242	81654	55750	45632	13660	11359	99606	
a(26) =	-0.00000	00000	00000	11812	59301	69745	87695	13764	58684	22978	31211	55729	18048	47879	83750	81233	
a(27) =	0.00000	00000	00000	00118	66922	54751	60033	25797	77242	92867	40710	88494	07966	48271	10740	06109	
a(28) =	0.00000	00000	00000	00141	23806	55318	03178	15558	03947	56670	90370	86350	75033	45256	25641	22263	
a(29) =	-0.00000	00000	00000	00022	98745	68443	53702	06592	47858	06336	99260	28450	59314	19036	70148	89830	
a(30) =	0.00000	00000	00000	00001	71440	63219	27337	43338	39633	70267	25706	68126	56062	51743	31746	49858	
a(31) =	0.00000	00000	00000	00000	01337	35173	04936	93114	84678	13951	22268	02287	50594	71761	89478	98583	
a(32) =	-0.00000	00000	00000	00000	02054	23355	17666	72789	32502	53513	55733	79668	20379	35238	73641	27301	
a(33) =	0.00000	00000	00000	00000	00273	60300	48607	99984	48315	09904	33098	20148	65311	69583	63633	70165	
a(34) =	-0.00000	00000	00000	00000	00017	32356	44591	05166	39057	42845	15647	79799	06974	91087	94998	41377	
a(35) =	-0.00000	00000	00000	00000	00000	23606	19024	49928	72873	43450	73542	75310	07926	41355	21453	70486	
a(36) =	0.00000	00000	00000	00000	00000	18649	82941	71729	44307	18413	16187	86668	98945	86842	90736	68232	
a(37) =	-0.00000	00000	00000	00000	00000	02218	09562	42071	97204	39971	69136	26860	37973	17795	00675	67580	
a(38) =	0.00000	00000	00000	00000	00000	00129	77819	74947	99366	88244	14486	33059	41656	19499	86463	91332	
a(39) =	0.00000	00000	00000	00000	00000	00001	18069	74749	66528	40622	27454	15509	97151	85596	84637	84158	
a(40) =	-0.00000	00000	00000	00000	00000	00001	12458	43492	77088	09029	36546	74261	43951	21194	11795	58301	
a(41) =	0.00000	00000	00000	00000	00000	00000	12770	85175	14086	62039	90206	67775	11246	47748	77206	56005	
a(42) =	-0.00000	00000	00000	00000	00000	00000	00739	14511	69615	14082	34612	89330	10855	28237	10568	99245	
a(43) =	0.00000	00000	00000	00000	00000	00000	00001	13475	02575	54215	76095	41652	59469	30639	30086	12196	
a(44) =	0.00000	00000	00000	00000	00000	00000	00004	63913	46410	58722	02994	48049	07952	22846	30579	68680	
a(45) =	-0.00000	00000	00000	00000	00000	00000	00000	53473	36818	43919	88750	77418	19670	98933	20904	88591	
a(46) =	0.00000	00000	00000	00000	00000	00000	00000	03207	99592	36133	52622	86123	72790	82794	39109	01464	
a(47) =	-0.00000	00000	00000	00000	00000	00000	00000	00044	45829	73655	07568	82101	59035	21246	43637	40144	
a(48) =	-0.00000	00000	00000	00000	00000	00000	00000	00013	11174	51888	19887	12901	05849	43899	22190	23663	
a(49) =	0.00000	00000	00000	00000	00000	00000	00000	00001	64703	33525	43813	88681	82593	27906	39414	53996	
a(50) =	-0.00000	00000	00000	00000	00000	00000	00000	00000	10562	33178	50358	12186	00561	07153	82850	49997	
a(51) =	0.00000	00000	00000	00000	00000	00000	00000	00000	00267	84429	82643	04947	83549	63071	89085	19485	
a(52) =	0.00000	00000	00000	00000	00000	00000	00000	00000	00024	24715	49485	17826	89673	03293	83709	21241	

TABLE III

Values of the coefficients in the expansion of the Gamma function for $s = 1$.

b(1) =	1.00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
b(2) =	-0.57721	56649	01532	86060	65120	90082	40243	10421	59335	93992	35988	05767	23488	48677	26777	66467		
b(3) =	0.98905	59953	27972	55539	53956	51500	63470	79391	83520	72821	40904	43195	78368	61366	32049	47877		
b(4) =	-0.90747	90760	80886	28901	65601	67356	27511	49286	11449	07256	37609	41331	15405	04651	82372	23069		
b(5) =	0.98172	80868	34400	18733	63802	94021	85085	03605	73679	72346	54154	04957	45559	38568	39248	69345		
b(6) =	-0.98199	50689	03145	20210	47014	13791	37467	55174	26507	14719	89304	99967	19048	80063	64964	05004		
b(7) =	0.99314	91146	21276	19315	38672	53328	65849	80374	90755	23943	16264	78246	06777	46309	93337	89471		
b(8) =	-0.99600	17604	42431	53397	00784	19664	56668	67352	98809	55457	89153	46292	61939	78976	92730	81415		
b(9) =	0.99810	56937	83128	92197	85754	03088	36723	75239	68524	79018	34093	17824	15586	78509	37181	57519		
b(10) =	-0.99902	52676	21954	86779	46780	59648	88808	85323	03963	52566	09266	60460	49520	89440	53968	52065		
b(11) =	0.99951	56560	72777	44106	70508	77594	37019	44345	03297	99459	96618	87795	53794	16479	85013	58241		
b(12) =	-0.99975	65975	08601	28702	58424	49140	60923	59969	51385	62883	01160	02987	61907	94492	39829	67513		
b(13) =	0.99987	82713	15133	27572	61716	42590	00321	93876	29108	95432	35135	96499	86723	61444	36372	24390		
b(14) =	-0.99993	90642	06444	31683	58522	31368	95513	18579	43502	82804	05438	51445	15253	46144	07783	23981		
b(15) =	0.99996	95177	63482	10449	86114	05091	95350	72655	28042	47987	55488	65763	48316	84486	59036	21443		
b(16) =	-0.99998	47526	99377	04874	37096	31724	44753	83260	83325	77144	87187	73327	14971	24085	59359	28436		
b(17) =	0.99999	23744	79073	21585	53950	94505	10782	58338	16344	69466	23437	59939	15975	39564	68407	77736		
b(18) =	-0.99999	61865	89473	31202	89649	57795	61431	38020	17312	43262	96264	91003	35457	52459	43096	37567		
b(19) =	0.99999	80930	81130	89205	18661	91514	59489	77316	95571	98830	07247	42117	16990	05886	05646	00075		
b(20) =	-0.99999	90464	68911	15771	74868	79470	54372	63246	96163	24955	25356	39485	37951	58126	65559	98870		
b(21) =	0.99999	95232	10605	73957	52392	92991	06456	81680	99689	08595	02411	15566	89661	93859	34343	77032		
b(22) =	-0.99999	97615	97344	38057	09247	01062	58744	74860	97486	06106	79761	15804	89247	54105	43853	93630		
b(23) =	0.99999	98807	96019	16841	66504	18404	24924	05265	35466	12259	91412	90364	30956	36225	64234	55058		
b(24) =	-0.99999	99403	97124	98374	58628	87976	75081	78480	70342	34988	01115	11147	99845	55286	43596	09524		
b(25) =	0.99999	99701	98267	58235	55744	96192	51141	98133	93331	05311	53702	60600	44530	05330	03611	41237		
b(26) =	-0.99999	99850	99035	47504	70871	68476	76946	50623	93258	45458	43468	42905	34506	54146	37210	86400		
b(27) =	0.99999	99925	94984	96246	47047	99253	72366	70398	87448	08553	98289	22009	20621	80220	82484	90296		
b(28) =	-0.99999	99962	74731	55543	69143	33339	28727	57230	56827	55466	89035	41647	03001	13697	93514	95990		
b(29) =	0.99999	99981	37362	13559	46670	62812	73498	73662	16664	36388	53269	11266	06099	68930	66551	38883		
b(30) =	-0.99999	99990	68679	85370	78792	17910	28525	58993	63865	14797	10538	55556	63652	87611	28711	83105		
b(31) =	0.99999	99995	34339	52214	54213	38976	33392	29355	42708	48053	10417	69173	95130	24666	82328	65762		
b(32) =	-0.99999	99997	67169	62616	68609	99055	77699	57113	80146	58538	46642	27035	39958	12151	57182	27574		
b(33) =	0.99999	99998	83584	76811	40614	12144	33725	11380	04193	65578	38013	30269	38598	69010	10603	32961		
b(34) =	-0.99999	99999	41792	36906	70528	49822	71872	51768	40486	60470	81390	48199	76676	58386	67194	70938		
b(35) =	0.99999	99999	70896	17953	68200	96608	90584	08312	49885	84544	42256	21830	34948	51881	62591	16490		
b(36) =	-0.99999	99999	85448	08810	28295	10703	86516	33114	56842	76628	90964	78557	67215	67939	92009	77029		
b(37) =	0.99999	99999	92724	04349	62183	02305	31125	59562	70723	22128	96118	33293	84663	18671	51876	65851		
b(38) =	-0.99999	99999	96362	02156	30429	31646	52931	24475	06762	42713	56100	22386	52560	97617	65315	96127		
b(39) =	0.99999	99999	98181	01071	98325	42192	88049	41138	18322	77943	53591	93637	62116	85866	09849	78422		
b(40) =	-0.99999	99999	99090	50533	93532	50603	07999	67918	15632	65304	67698	65514	79163	38316	60450	38611		
b(41) =	0.99999	99999	99545	25266	28222	73649	38304	56804	97889	34445	69833	14009	39994	14834	16264	43744		
b(42) =	-0.99999	99999	99772	62632	91263	50068	66751	57690	47623	95962	25072	62801	05408	82448	35143	07778		
b(43) =	0.99999	99999	99886	31316	38015	78730	99034	07802	58258	70406	38240	02685	30789	66797	91847	33938		
b(44) =	-0.99999	99999	99943	15658	16469	23751	54501	22513	67242	74506	10189	56126	36214	29442	33318	51727		
b(45) =	0.99999	99999	99971	57829	07388	39959	57983	22160	15192	01567	57545	05116	67538	83008	72142	77634		
b(46) =	-0.99999	99999	99985	78914	53412	12663	17329	62179	85436	85324	88272	58742	76261	92447	85040	86160		
b(47) =	0.99999	99999	99992	89457	26612	03889	90966	13410	35809	53299	72637	68872	93563	02268	98016	63603		
b(48) =	-0.99999	99999	99996	44728	63274	67797	02797	04483	84165	49645	59679	80929	27883	77811	90235	63128		
b(49) =	0.99999	99999	99998	22364	31626	89182	36306	50722	66169	97315	41557	27183	51523	40265	16810	02934		
b(50) =	-0.99999	99999	99999	11182	15809	96352	42073	40154	16512	26935	39198	49178	90362	57498	91079	92032		
b(51) =	0.99999	99999	99999	55591	07903	82096	61247	78598	29551	61102	78137	65474	02255	93051	45294	16835		
b(52) =	-0.99999	99999	99999	77795	53951	52355	10420	34656	87615	95224	82168	26197	72031	72466	04994	22854		

TABLE IV

Values of the coefficients in the expansion of the Gamma function for $s = 3/2$.

$c(1) =$	0.88622	69254	52758	01364	90837	41670	57259	13987	74728	06119	35641	06903	89492	64556	42295	51609
$c(2) =$	0.03233	83974	48885	01382	88698	84268	97030	77813	34788	87050	70206	36641	01945	98595	99162	17310
$c(3) =$	0.41481	34536	88301	16823	00376	23111	35634	28489	09963	37042	23679	77719	75186	72661	53692	42118
$c(4) =$	-0.10729	48045	64772	21168	75419	56389	70966	20545	75923	82129	83009	38639	21109	25010	51470	65111
$c(5) =$	0.14464	53590	44621	54303	83322	10253	88452	40700	26861	53098	14284	13968	13793	81159	21866	83805
$c(6) =$	-0.07752	30522	99854	20344	46773	21416	50897	04742	16125	82748	32689	98953	06131	91086	87003	27711
$c(7) =$	0.05861	03038	17176	28950	41887	37819	14405	71055	54892	49810	41604	63949	88584	17696	43212	84699
$c(8) =$	-0.03800	19355	54865	13025	20510	71015	03415	52379	66692	62041	88995	98331	43979	82709	03702	22051
$c(9) =$	0.02583	76064	55756	20389	37000	08736	64624	62969	62568	25229	61249	47665	54531	45610	51421	42953
$c(10) =$	-0.01722	24431	13464	62506	58306	84260	38043	06974	60083	97127	20406	21622	85099	10670	93449	78632
$c(11) =$	0.01152	25153	92399	22834	77287	32942	17459	05333	95721	40395	14716	17920	32958	91599	29065	18147
$c(12) =$	-0.00769	02113	64241	57866	25887	86617	69250	21593	80742	47981	01565	75780	88757	51219	48279	84295
$c(13) =$	0.00513	16435	01912	38754	09072	03354	32845	98970	30651	46507	49394	36462	31904	81235	66209	27091
$c(14) =$	-0.00342	28024	97359	70609	69796	85004	86505	43079	80807	07929	91521	72606	30447	00181	83299	26725
$c(15) =$	0.00228	25897	63790	26741	39310	80530	31818	45352	76813	90405	82695	37026	75690	83498	77869	82913
$c(16) =$	-0.00152	20100	71124	42832	08129	12968	77465	83302	59316	02021	86016	31910	74780	83475	77592	67488
$c(17) =$	0.00101	47877	42151	47788	22410	83948	50899	46922	74047	58873	39039	37789	10126	35348	11730	99948
$c(18) =$	-0.00067	65708	41060	01236	72918	42178	80653	68956	95772	42457	64648	66473	73239	88956	15111	42475
$c(19) =$	0.00045	10655	25395	65954	88279	05704	94617	89655	74112	98111	94085	78059	18701	98661	53268	32044
$c(20) =$	-0.00030	07176	71200	56376	19066	02886	49784	35376	31145	96634	48783	28920	64831	64974	58934	43688
$c(21) =$	0.00020	04813	77049	05741	94009	82011	47684	33453	36157	87241	63377	88621	21677	06693	48612	50599
$c(22) =$	-0.00013	36554	23459	29399	54756	50331	27853	01242	01755	85915	24419	25690	77270	29904	26756	58796
$c(23) =$	0.00008	91040	84561	05664	04004	21412	26639	38782	39331	08834	30564	32635	48171	94400	32978	93219
$c(24) =$	-0.00005	94029	10632	31535	13012	25167	21857	78346	10450	64335	77096	25310	08342	49411	34066	18863
$c(25) =$	0.00003	96020	15464	87878	36361	72078	33491	09552	06150	62389	95320	37589	33484	14582	24681	89622
$c(26) =$	-0.00002	64013	73662	48702	57551	88087	54784	96532	13206	92752	77718	82657	34241	25067	91753	13501
$c(27) =$	0.00001	76009	27783	22951	64529	72681	33788	27811	77736	52187	06775	10448	67827	67777	76442	37939
$c(28) =$	-0.00001	17339	56658	93706	36684	37940	59902	36069	50655	16376	27388	49518	25769	32793	00087	62999
$c(29) =$	0.00000	78226	39694	04943	19435	33876	71525	53840	47916	30500	55934	03932	23113	18428	07044	40105
$c(30) =$	-0.00000	52150	93897	94887	37143	33551	93104	39418	36686	35542	06709	44825	84931	43113	60123	96866
$c(31) =$	0.00000	34767	29572	73591	15713	22171	61285	64384	05000	41716	85149	38869	57508	51864	30836	35189
$c(32) =$	-0.00000	23178	19838	13297	60470	39693	92435	15019	68400	22859	52331	45919	28968	78368	92847	71196
$c(33) =$	0.00000	15452	13274	61256	09181	90541	25142	92738	80392	18072	17360	28503	82135	41330	24529	89758
$c(34) =$	-0.00000	10301	42202	75135	60058	92014	61242	31274	52613	15377	41648	31494	34995	45801	34496	86703
$c(35) =$	0.00000	06867	61476	37145	43450	64507	45562	17745	85887	55205	77053	80494	04224	73002	52641	18131
$c(36) =$	-0.00000	04578	40987	39586	32763	38171	82045	54054	26493	96367	12796	71874	62121	69597	48888	36072
$c(37) =$	0.00000	03052	27326	18986	83103	68655	64826	08903	33865	47656	61817	11737	72017	02282	81470	17594
$c(38) =$	-0.00000	02034	84884	63029	65547	97896	18473	99005	24248	74575	58748	51090	04102	55916	66547	94125
$c(39) =$	0.00000	01356	56589	95501	82701	91952	37968	76252	71942	70730	71092	90739	28727	86109	37244	82109
$c(40) =$	-0.00000	00904	37726	71727	37863	08497	15425	60565	38098	48512	37762	17889	03701	01031	93282	74830
$c(41) =$	0.00000	00602	91817	84375	38465	01769	93898	55945	06013	35063	94966	85861	49314	14662	85586	25375
$c(42) =$	-0.00000	00401	94545	24206	44303	45524	37362	97737	00774	07221	33401	58542	14733	49092	13827	53339
$c(43) =$	0.00000	00267	96363	49986	77010	42160	90373	56985	36663	90711	69859	40713	98770	03761	39860	11234
$c(44) =$	-0.00000	00178	64242	33530	83666	59711	33835	57223	12310	02056	00685	14945	73265	37316	10901	25641
$c(45) =$	0.00000	00119	09494	89103	08709	13462	69578	41772	91879	71520	63867	90118	24265	02730	17928	86417
$c(46) =$	-0.00000	00079	39663	26101	73645	56822	06567	44089	70669	48459	04685	63050	59462	57210	35342	23790
$c(47) =$	0.00000	00052	93108	84081	02899	57511	08455	25307	28220	19044	74985	66930	35495	06481	64485	88413
$c(48) =$	-0.00000	00035	28739	22725	96787	41570	97129	66874	04484	52906	40263	72065	77619	56691	43907	93965
$c(49) =$	0.00000	00023	52492	81819	42466	69590	37576	20462	03941	44952	09392	91776	48070	49350	64170	91649
$c(50) =$	-0.00000	00015	68328	54547	12821	16644	41689	81494	39758	28853	73731	73620	29860	69147	44477	85036
$c(51) =$	0.00000	00010	45552	36365	09018	12577	64217	21101	19877	12791	34863	83922	62313	52187	57468	01825
$c(52) =$	-0.00000	00006	97034	90910	19533	68991	26839	53340	14203	06814	69181	21993	99273	23125	99544	06742

TABLE V

Values of the coefficients in the expansion of the inverted Gamma function for $s = 3/2$.

d(1) =	1.12837	91670	95512	57389	61589	03121	54517	16881	01258	65799	77136	88171	44342	12849	36882	98683
d(2) =	-0.04117	45264	45283	10145	02472	05115	70419	01750	06113	89637	71286	25112	74615	60795	59704	31681
d(3) =	-0.52665	44355	25544	47926	32079	72841	09288	56032	64837	55682	29889	27550	59489	95837	80514	92687
d(4) =	0.17510	20260	43934	56149	51226	18525	57115	01517	82399	43893	90470	02625	73625	82513	61031	60628
d(5) =	0.05096	68602	47706	07677	46983	49409	75961	84704	59543	20866	62027	58435	19671	90962	96174	66347
d(6) =	-0.04215	51693	68535	60099	31854	35843	46657	85781	06883	49080	02276	12384	49631	63043	85193	72522
d(7) =	0.00661	28978	26824	12727	65662	56030	54543	62835	01817	64006	03074	35963	45731	65790	77870	53574
d(8) =	0.00212	07314	42572	93833	60118	52790	99258	13126	70664	81192	91585	49409	76582	99680	35325	44544
d(9) =	-0.00111	07302	54594	89071	71194	98478	86160	64859	35345	62990	25123	08439	06982	69138	39347	47839
d(10) =	0.00015	23576	20767	47687	21655	65418	67277	37230	10050	44436	30714	55534	29774	16969	00130	22408
d(11) =	0.00002	53552	04923	81416	52782	52816	10103	66226	94458	49751	44008	40150	53067	66890	55699	18599
d(12) =	-0.00001	38968	05717	91375	60219	66503	95717	70842	77581	75530	38754	33480	20611	52146	09153	36819
d(13) =	0.00000	21562	03290	51417	24534	55616	23949	45218	76113	90476	78306	41166	47242	92559	96346	63859
d(14) =	0.00000	00579	42640	54052	67250	42262	33219	53071	59604	91641	74158	79757	64532	20495	75252	98682
d(15) =	-0.00000	00891	35511	18311	11605	40721	02332	51438	73328	32204	81665	68336	39829	28840	42062	90282
d(16) =	0.00000	00171	03469	41591	53737	49320	41168	64490	33182	09015	74846	64867	87406	65644	16245	23533
d(17) =	-0.00000	00009	31368	64452	41901	56847	57121	03992	04377	10204	26376	63538	14457	12126	51131	38061
d(18) =	-0.00000	00002	68047	41033	49662	55650	42604	12811	57723	47782	92912	33769	49097	97790	46589	61604
d(19) =	0.00000	00000	74589	32233	31632	60506	92751	48471	72632	51046	47969	74368	04065	52473	64610	46225
d(20) =	-0.00000	00000	08012	80706	14147	18370	91842	48691	23608	79618	12302	19768	62240	23604	19783	89465
d(21) =	-0.00000	00000	00083	82343	03345	18549	30489	93806	33902	12506	66315	30665	00298	02797	60315	32813
d(22) =	0.00000	00000	00169	46340	90432	05222	67740	94974	38014	13022	65545	63535	59019	78203	07906	97520
d(23) =	-0.00000	00000	00027	87575	67071	25752	08297	53147	96493	29340	82681	07805	36761	79478	59550	19168
d(24) =	0.00000	00000	00001	86703	94695	06530	54191	19188	40589	11440	53485	06186	66982	61684	91953	71864
d(25) =	0.00000	00000	00000	13049	49900	85879	86588	17799	98927	35780	40861	92882	68833	50321	38856	35928
d(26) =	-0.00000	00000	00000	04858	87414	41877	86529	61731	08402	98295	71065	90453	72022	35628	34674	10040
d(27) =	0.00000	00000	00000	00582	95426	92459	46783	18599	84244	21052	15780	29982	96525	21865	05927	44168
d(28) =	-0.00000	00000	00000	00025	92909	41799	37838	70477	84514	25919	18218	95695	57926	68398	65789	88532
d(29) =	-0.00000	00000	00000	00003	32675	40102	85788	59871	42882	20413	88729	54026	03708	80374	40064	46931
d(30) =	0.00000	00000	00000	00000	79449	61635	76810	53924	96881	65457	63873	70190	56982	80506	50196	09257
d(31) =	-0.00000	00000	00000	00000	07755	54328	84373	57293	90253	77073	16772	37834	35449	41906	17538	96607
d(32) =	0.00000	00000	00000	00000	00255	33736	29132	96957	91806	30480	15362	58531	88942	07410	97570	30235
d(33) =	0.00000	00000	00000	00000	00042	74520	16014	71733	79460	02920	72363	42310	60718	16496	48645	13088
d(34) =	-0.00000	00000	00000	00000	00008	26338	13746	68449	50568	79321	72514	28478	54547	49187	06390	08468
d(35) =	0.00000	00000	00000	00000	00000	71081	87657	25339	79736	55066	50482	39651	03124	66698	42696	82293
d(36) =	-0.00000	00000	00000	00000	00000	02074	94638	87704	29612	45013	10690	01248	14732	98561	32832	61884
d(37) =	-0.00000	00000	00000	00000	00000	00328	59544	06994	86048	30125	91386	18242	74687	86599	77266	97282
d(38) =	0.00000	00000	00000	00000	00000	00058	19433	90819	87478	73747	69479	02225	03415	36340	08314	23132
d(39) =	-0.00000	00000	00000	00000	00000	00004	70293	21304	49893	36039	90226	26453	86248	96238	86005	06248
d(40) =	0.00000	00000	00000	00000	00000	00000	14478	55651	58528	64715	71007	47334	16737	14946	90517	77690
d(41) =	0.00000	00000	00000	00000	00000	00000	01597	83150	54786	78867	34059	40543	53327	73835	18743	61494
d(42) =	-0.00000	00000	00000	00000	00000	00000	00287	82471	74761	34448	59106	01443	70749	76498	12799	76317
d(43) =	0.00000	00000	00000	00000	00000	00000	00023	00886	40061	20204	75117	23789	27644	25923	08048	86900
d(44) =	-0.00000	00000	00000	00000	00000	00000	00000	81524	69850	72319	69959	00845	41422	71516	81071	83108
d(45) =	0.00000	00000	00000	00000	00000	00000	00000	04842	01452	13897	81552	19553	33904	52797	10271	67115
d(46) =	0.00000	00000	00000	00000	00000	00000	00000	01018	82590	41957	85489	64669	06836	98274	81330	98481
d(47) =	-0.00000	00000	00000	00000	00000	00000	00000	00084	12953	04954	03489	52788	55482	99738	59373	63814
d(48) =	0.00000	00000	00000	00000	00000	00000	00000	00003	48870	05836	57446	86098	51780	64378	28045	47036
d(49) =	0.00000	00000	00000	00000	00000	00000	00000	00000	07563	10899	94869	57784	54309	91541	18481	05670
d(50) =	-0.00000	00000	00000	00000	00000	00000	00000	00000	02589	53190	69537	33195	18160	84218	45828	88632
d(51) =	0.00000	00000	00000	00000	00000	00000	00000	00000	00230	70239	97849	78036	96572	53603	35271	82461
d(52) =	-0.00000	00000	00000	00000	00000	00000	00000	00000	00011	09196	43852	26435	46500	35654	57083	62269

TABLE VI

Values of the integral from a to b of the inverted Gamma function.

a	b	Value
-10	-9	2.60547.70793 20506 24303 22753 58692 01437 67668 48219 45086 41142 47077 28908
-9	-8	-27136.66927 95424 98804 74304 17037 11893 49938 94902 74756 63019 48137 25963
-8	-7	3156.81596 18461 08044 20868 33161 53950 15210 52315 75487 85943 49217 51207
-7	-6	-415.89693 20832 07590 12059 97940 79191 40021 67070 76448 45235 77151 79573
-6	-5	63.17186 75843 52479 94427 47404 14317 31537 90134 88805 14586 10927 46851
-5	-4	-11.33036 90334 53643 82938 34047 21839 05995 87457 39828 43339 67321 55875
-4	-3	2.48180 29455 21126 14719 41931 36672 91175 68743 69964 28280 37034 32296
-3	-2	-0.69865 99099 13205 54269 78540 82284 22044 99644 10869 97857 05792 28033
-2	-1	0.27577 30330 41742 06946 05785 09521 11579 20666 80306 05785 13265 54887
-1	0	-0.18372 07119 05075 62303 64064 07517 25872 20736 81031 96338 50692 60152
0	1	0.54123 57343 28670 53014 95373 28879 55201 38992 56206 64997 60973 58961
1	2	1.08514 26643 57470 08432 68666 42788 47468 07913 88749 51824 40151 55964
2	3	0.75184 97084 95638 58030 24034 16281 13389 20045 96553 91095 00942 05304
3	4	0.31183 45669 98537 56114 35447 32465 29676 07627 89771 02522 03021 96225
4	5	0.09201 87627 02450 04602 66032 93339 36466 74236 62051 68938 15300 98004
5	6	0.02104 69087 10163 38860 26792 04887 32958 68536 59161 40934 39519 65930
6	7	0.00392 75453 99260 25069 76775 05698 14214 55084 13884 54217 97648 31450
7	8	0.00061 87196 85282 88900 55916 45690 32071 10020 78927 14522 69898 31581
8	9	0.00008 43101 02376 96441 09833 24979 05141 63306 31927 10260 74513 96317
9	10	0.00001 01204 67243 01888 69534 55520 69096 16485 14307 66301 62907 10441
10	11	0.00000 10854 58619 45183 43671 84949 43201 90384 73051 34683 60422 04483
11	12	0.00000 01052 05646 89034 15165 91603 67371 87103 48315 10961 31278 45146
12	13	0.00000 00093 00396 95135 62725 18603 74292 22185 05312 38377 27314 61803
13	14	0.00000 00007 55703 23632 16225 78532 98253 65890 55909 97059 89052 88909
14	15	0.00000 00000 56809 61723 34546 11600 98577 05081 37985 38393 09207 00059
15	16	0.00000 00000 03973 20013 40782 13593 23653 18664 36230 77679 78222 57915
16	17	0.00000 00000 00259 78223 06816 40553 49907 80379 48734 80667 50933 99216
17	18	0.00000 00000 00015 94657 92540 62504 27622 85991 87165 31961 93750 66759
18	19	0.00000 00000 00000 92243 61105 15693 74263 85586 84367 05432 23009 50137
19	20	0.00000 00000 00000 05044 94534 03530 72798 94698 76182 18409 90637 30357
20	21	0.00000 00000 00000 00261 64749 32751 81510 99875 52328 28011 12536 85480
21	22	0.00000 00000 00000 00012 90255 88783 50632 68074 76806 74769 91081 78224
22	23	0.00000 00000 00000 00000 60643 28149 31565 65824 94574 49989 06280 73563
23	24	0.00000 00000 00000 00000 02722 63781 56648 95646 07305 97409 13802 58658
24	25	0.00000 00000 00000 00000 00116 99491 96378 39095 36558 86531 28603 94208
25	26	0.00000 00000 00000 00000 00004 82070 97057 38511 32772 92431 03110 96429
26	27	0.00000 00000 00000 00000 00000 19078 96094 17036 94107 69823 32783 72517
27	28	0.00000 00000 00000 00000 00000 00726 39787 21487 98659 98742 55453 34646
28	29	0.00000 00000 00000 00000 00000 00026 64382 56603 50770 93457 89274 90625
29	30	0.00000 00000 00000 00000 00000 00000 94276 17992 14809 55078 33636 12144
30	31	0.00000 00000 00000 00000 00000 00000 03222 04589 77308 87021 30986 47037
31	32	0.00000 00000 00000 00000 00000 00000 00106 48553 43188 38011 72724 70892
32	33	0.00000 00000 00000 00000 00000 00000 00003 40683 24048 39273 38385 93189
33	34	0.00000 00000 00000 00000 00000 00000 00000 10562 20763 81559 13715 32268
34	35	0.00000 00000 00000 00000 00000 00000 00000 00317 62744 07275 99815 96835
35	36	0.00000 00000 00000 00000 00000 00000 00000 00009 27324 62655 67454 45996
36	37	0.00000 00000 00000 00000 00000 00000 00000 00000 26306 59850 82825 95610
37	38	0.00000 00000 00000 00000 00000 00000 00000 00000 00725 71289 69607 28736
38	39	0.00000 00000 00000 00000 00000 00000 00000 00000 00019 48325 28602 32404
39	40	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 50940 87870 57866
40	41	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 01298 00257 13773
41	42	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00032 25297 72329
42	43	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 78201 74373
43	44	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 01851 27060
44	45	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00042 81272
45	46	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 96774
46	47	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 02139
47	48	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00046
48	49	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00001
F		2.80777 02420 28519 36522 15011 86557 77293 23080 85920 93019 82912 20055

TABLE VII
Values of Gamma(p/q).

p	q	Value
1	10	9.51350 76986 68731 83629 24871 77265 40219 25505 78626 08837 73430 50000 77043 42653 83322 82101
1	9	8.52268 81392 19475 95051 43922 14439 55975 47588 31469 32202 08985 32701 61790 58870 16992 95138
1	8	7.53394 19587 97611 90469 92298 41215 13362 46104 19588 14907 59409 83127 89777 66636 57198 90641
1	7	6.54806 29402 47824 43771 40933 49428 99626 26211 35187 38413 51489 40168 81914 85762 04738 23914
1	6	5.56631 60017 80235 20425 00968 95207 72611 13987 99114 87285 34616 16744 62632 29075 02817 80231
1	5	4.59084 37119 98803 05320 47582 75929 15200 34341 09998 29340 30177 88853 13623 00392 73106 44500
2	9	4.10657 95667 16061 19664 22907 88918 36054 03210 52485 15292 42469 64982 38258 66838 99761 46476
1	4	3.62560 99082 21908 31193 06851 55867 67200 29951 67682 88006 54674 33377 99956 99192 43538 72912
2	7	3.14911 51177 59936 59097 01136 64680 76889 22297 78611 76625 26847 90761 50003 94279 84532 96946
3	10	2.99156 89876 87590 62831 25165 15904 91779 11128 06024 92171 51127 44119 65095 63887 67876 32022
1	3	2.67893 85347 07747 63365 56929 40974 67764 41286 89377 95730 11009 50428 32759 04176 10167 74382
3	8	2.37043 61844 16600 90864 64735 04176 65250 98874 00803 35892 49877 75126 93467 31615 31358 00179
2	5	2.21815 95437 57688 22305 90540 21907 67945 07705 66501 77146 95822 41977 75264 61851 68123 00474
3	7	2.06751 17265 60229 35302 46124 06308 82694 35592 14211 49238 75280 50717 59023 46033 90293 97673
4	9	1.99289 35227 56922 75771 82035 62637 98751 71413 60387 65003 04062 92209 96814 24773 53781 19038
1	2	1.77245 38509 05516 02729 81674 83341 14518 27975 49456 12238 71282 13807 78985 29112 84591 03218
5	9	1.60071 61184 13983 28967 76129 26406 83369 71616 21038 83996 53728 30023 76424 22664 60581 80870
4	7	1.55858 10329 02475 00827 50092 91245 97392 25208 50472 09453 86922 66736 62932 89725 58387 84230
3	5	1.48919 22488 12817 10239 43333 88321 34228 13205 99038 75992 47353 38679 56404 50801 63121 93494
5	8	1.43451 88480 90556 77563 60197 39456 42313 66322 07772 20666 73307 70679 85809 50941 97302 09691
2	3	1.35411 79394 26400 41694 52880 28154 51378 55193 27266 05679 36983 94022 46796 37829 65401 74254
7	10	1.29805 53326 47557 78568 11711 79152 81161 77841 41170 55394 62479 21645 38825 41681 50818 97580
5	7	1.27599 26754 93444 05848 53056 07789 87494 84545 88992 91105 19162 28146 37620 71014 76123 92985
3	4	1.22541 67024 65177 64512 90983 03362 89052 68512 39248 10807 06112 30118 93828 98228 88426 79836
7	9	1.19015 11869 12872 71460 38590 53883 03526 48713 81437 77011 42320 24031 09342 67621 33617 86587
4	5	1.16422 97137 25303 37363 63209 38268 45869 31419 61768 89118 77529 84894 46786 18354 66078 95374
5	6	1.12878 70299 08125 96126 09010 90258 84201 33267 87441 66475 54517 52083 51433 37705 10987 50399
6	7	1.10576 70723 29567 32661 98492 94247 33752 92315 46976 82003 88489 45380 02358 64184 93347 92056
7	8	1.08965 23574 22896 95125 23767 55102 89297 11478 70067 76756 51205 13704 04325 36264 17465 87950
8	9	1.07775 88331 33495 79725 70063 33077 01632 92059 53740 72476 67863 58975 39844 05959 74420 55203
9	10	1.06862 87021 19319 35489 73053 35694 48077 81698 38785 06097 31790 49370 68398 15721 77025 44757
1	1	1.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000

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