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New Integer Factorizations

By Thorkil Naur

Abstract. New factorizations of Fibonacci numbers, Lucas numbers, and numbers of the form $2^n \pm 1$ are presented together with the strategy (a combination of known factorization methods) used to obtain them.

1. Introduction. This paper presents new complete factorizations of 174 large integers of interest. The factorizations have been obtained by the author over the past four years using a combination of known methods. Our factorization strategy, which has been implemented on a micro-programmable computer, makes extensive use of J. M. Pollard's factorization methods (Pollard [8] and Pollard [9]). The strategy also includes the well-known method of trial division, the continued fraction method (Morrison and Brillhart [6]), and the primality tests of Brillhart, Lehmer and Selfridge [3].

Section 2 contains a summary of our factorization strategy. Since we use known methods, no details of these methods are given. Section 3 contains the actual results and describes their form. We have attacked the cofactors (i.e. the factors remaining after the known factors are removed) of Fibonacci numbers, Lucas numbers, and numbers of the form $2^n \pm 1$, whose factorizations were not previously known.

The paper is a condensed version of Naur [7], which, in addition to the material in this paper, contains summaries of the factorization methods used and full factorization tables.

It should be noted that most of the factorizations presented here have been independently discovered by J. Brillhart, D. H. Lehmer, J. L. Selfridge, B. Tuckerman, S. S. Wagstaff, or their colleagues, possibly using a strategy similar to the one described here (Lehmer [5], Pollard [10], and Wagstaff [12]).

2. Strategy. The factorization strategy combines known methods and may be summarized as follows:

Step 1. Trial divide to 10^6. This step hardly needs further comment (for an efficient implementation of trial division, see Wunderlich and Selfridge [18]).

Step 2. Use Pollard's two methods simultaneously. The $P - 1$ method is able to discover a prime factor p, if the factors of $p - 1$ are small. We use the first stage of the method as described at the end of Pollard [8]. For previous uses of the $P - 1$ method, see Williams and Judd [15] and [16], Williams and Seah [17], and Williams [13], the latter giving an account of the related $P + 1$ method.

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The rho method (or Monte Carlo method, see Pollard [9]) is usually able to find a prime factor p in about $O(p^{1/2})$ steps. Improvements of the rho method (see Brent [1] and Brent and Pollard [2]) have only been recently published and have not been used.

Both the $P - 1$ method and the rho method can be broken into smaller steps, and a step from each method is executed alternately.

Step 3. Having executed a suitable number of steps of Pollard's methods, the continued fraction method (Morrison and Brillhart [6]) is invoked, if the number has 53 or fewer digits.

Step 4. Whenever a factorization is discovered in Steps 2 or 3, both factors are tested for primality using the tests from Brillhart, Lehmer and Selfridge [3] (the number itself is also tested after Step 1). Since these tests require a certain number of factors of $N \pm 1$ to be known before the primality of N can be verified, these auxiliary factorizations are attempted simultaneously using our strategy recursively. When a sufficient number of factors is found, the factoring is stopped and the primality tests are carried out.

The above strategy has been implemented on the micro-programmable computer Mathilda, developed at the Aarhus University (Schriver and Kornerup [11]).

TABLE 1. *Fibonacci numbers*

n	Factorization of U_n
191.	4870723671313 * 757810806256989128439975793
221.	233 * 1597 : 203572412497 * 906574987180246453263929940193
233.	139801 * 25047390419633 * 631484089583693149557829547141
239.	10037 * 62141 * 2228536579597318057 * 28546908862296149233369
251.	582416774750273 * 21937080329465122026187124199656961913
253.	89 * 28657 : 4322114369 * 2201228236641589 * 1378497303338047612061
257.	5653 * 32971978671645905645521 * 123002672171931347136071 4649
259.	13 * 73 * 149 * 2221 : 1553 * 404656773793 * 3041266742295771985148799223649
265.	5 * 953 * 55945741 : 15901 * 2741218753681 * 92691859945746812592082758 1
267.	2 * 1069 * 1665088321800481 : 122887425153289 * 6445587734970304287730 9
269.	5381 * 2517975182669813 * 32170944747810641 * 169360439829648789853
289.	1597 : 577 * 1733 * 98837 * 101232653 * 106205194357 * 658078658277725444483848541
291.	2 * 193 * 389 * 3084989 * 361040209 : 76674415738994499773 * 227993117754975870677
303.	2 * 743519377 * 770857978613 : 8550224389674481 * 960496579172798748513 69421
305.	5 * 4513 * 555003497 : 2441 * 6101 * 20415253966247698801 * 64727767071799824094 3861
319.	89 * 514229 : 1913 * 578029 * 1435522969 * 153541455600361 3 * 186262431846834633482835 29
333.	2 * 17 * 73 * 149 * 2221 * 1459000305513721 : 12653 * 124134848933957 * 930507731557590226767593761
355.	5 * 6673 * 46165371073 : 4261 * 75309701 * 309273161 * 9207609261398081 * 49279722643391864192801
361.	37 * 113 : 6567762529 * 1196762644057 * 31509278278169308781415 97 * 120201265107147347830092 41
367.	733 * 17969789 * 75991753 * 5648966761 * 43397676601 * 114150315493 * 797357235624701499134444201
369.	2 * 17 * 2789 * 59369 * 68541957733949701 : 8117 * 199261 * 84738793193 * 9382599520669 * 117838518633351469
377.	233 * 514229 : 104264251753 * 361575655741 * 6081465853455679816708931999854492020150600942 37

TABLE 2. *Lucas numbers*

n Factorization of V_n

166. 3 : 6464041 * 245329617161 * 10341247759646081
197. 3149858711911339 * 4701907222895068350249889
211. 33128448586319 * 3768695026320506495615952689771
221. 521 * 3571 : 233712704402297302 1 * 35314950421248638631 41
227. 39499 * 5098421 * 4311537234701 * 3173513869617946787973 01
230. 3 * 41 * 4969 * 275449 : 3116523496881881 * 22247004553118573 47241
242. 3 * 43 * 307 : 200872171147 * 3564873012035809 * 1325308602599354 2387
244. 7 : 487 * 5247147754162601 0209 * 5500902230146438151405489047
252. 2 * 7^2 * 23 * 167 * 14503 * 103681 * 65740583 : 503 * 4322424761927
* 571385160581761
253. 139 * 199 * 461 : 1334309745903 7867049 * 43958971527497857699509 7049
256. 34303 * 73327699969 * 125960894984050328038716298487435392001
257. 2107028233569599 * 12509044778250215 9 * 194504226146879075853 1
258. 2 * 3^2 * 3131957115165782 81 : 7772507 * 7325404181 6089 * 258422401920467
269. 13451 * 4909852485573 3491 * 2903410268838131 09 * 860882346042166879
272. 2207 : 4470047 * 7378607647 * 42848407775681 * 2241891649308161061 06049
274. 3 : 547 * 27947 * 86409516719752275209 * 46196393961267734345849 0143601
277. 1109 * 5923369 * 1003666289 * 322458613167451 * 3647646099535497480264359
281. 20567460049 * 4641534315443 4259 * 5567813533108035935034668181 4561
286. 3 * 43 * 307 * 90481 : 5147 * 244112999612024 3
* 130928610356523706566086969 09281
287. 29 * 370248451 : 256579 * 319973431 * 101731310703289
* 10635841025639256246541
288. 2 * 1087 * 4481 * 11862575248703 : 270143 * 25033626656641
* 19747377957460801495 67
292. 7 : 839207 * 121355783 * 2864461601 * 4953066392881
* 1045794092558661358680161
296. 47 : 15400289 * 19088449 * 77894162661647 * 89311781152481
* 7542763303464323 03
299. 139 * 461 * 521 : 599 * 2233531 * 1194215681621 * 143236388738249
* 40197222522537856361
301. 29 * 6709 * 144481 : 39488879317091 * 1050474234583201
* 689529693448123842995171
302. 3 : 70963651961 * 95305716283 * 6411965749391838850095902897691672421 9027
304. 2207 : 607 * 1823 * 20063 * 91807 * 1156984541407 * 12441241017224321
* 52601970578546783
308. 7^2 * 263 * 881 * 967 * 14503 : 7872253927 * 9623520524969002343
* 193529898067277876104 1
309. 2^2 * 619 * 1031 * 5257480026438961 : 1270029990781
* 2216051880587916003268636813231
310. 3 * 41 * 3020733700601 : 1805019110667134254990 01
* 9113162636597558947796254 01
320. 127 * 186812208641 : 623795558318030998672729 61
* 507918025665967543174374400 1
322. 3 * 281 * 4969 * 275449 : 643 * 770867 * 25154641 * 163674763583689
* 83578027239020971306830 89
331. 526291 * 54184296181 * 4386848568249611
* 1195795459010394227506385297803918292 9
334. 3 : 821641 * 7162963 * 50187047747 * 14167898020159929481
* 504752765667203736366779801
338. 3 * 90481 : 2027 * 141283 * 404112157123 * 47806156571279752464 1
* 2892106995173496522201467
339. 2^2 * 41267042784492103747077 1 : 44607276283528829839
* 95442322534604096497886854 9
340. 7 * 2161 * 23230657239121 : 5441 * 897601 * 17276792316211992881
* 38349368324041346449749 61
348. 2 * 7 * 23 * 299281 * 834428410879506721 : 560589524253219666621 83
* 118503104637213751738144 7
350. 3 * 41 * 281 * 401 * 570601 * 12317523121 : 2801 * 28001
* 248773766357061401 * 7358192362316341243805801
374. 3 * 43 * 67 * 307 * 63443 : 2243 * 49369 * 3827019260681
* 1586361987756363049 * 1281280767267251812597555038 7

TABLE 2. (*Continued*)

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n    Factorization of V_n

391. 139 * 461 * 3571 : 10949 * 2476673936041
     * 8344848804983721285378913074459418529255664192499116 16229
393. 2^2 * 1049 * 414988698461 * 5477332620091 : 328451309226 38389
     * 432743702808878906877493417505847418 09
399. 2^2 * 29 * 211 * 229 * 9349 * 95419 * 10694421739 * 2152958650459 : 28729
     * 519499 * 561434197549 * 2521711674572072771367 19
400. 2207 * 23725145626561 : 107420801 * 177736001 * 5179368521466 2401
     * 760158710112872948977300866780 4801
402. 2 * 3^2 * 6163 * 201912469249 * 2705622682163 : 1609 * 2915186157721
     * 362504998543351862072462975494515095 6569
410. 3 * 41^2 * 163 * 800483 * 350207569 : 628774904181521
     * 14386018829678116716 1 * 232242909933669291971829426048 1
411. 2^2 * 541721291 * 78982487870939058281 : 242491 * 104446810724929
     * 1807018626789444918934709207745776824 9
441. 2^2 * 19 * 29 * 211 * 1009 * 31249 * 65269 * 620929 * 8844991 * 599786069
     : 35281 * 80642113181244469 * 1624735075664061773219277075034 9
444. 2 * 7 * 23 * 10661921 * 114087288048701953998401 : 887 * 2663 * 17761
     * 21423730326721 * 175358325117577112755922815666434960 1
450. 2 * 3^3 * 41 * 107 * 401 * 601 * 2521 * 570601 * 10783342081
     * 87129547172401 : 1801 * 186374563189054810201
     * 427694148584338087778220001
457. 143392891 * 48175086409 * 864351271995241
     * 538655620387010089753971464077054421188204623261302859056692 99
474. 2 * 3^2 * 21803 * 5924683 * 14629892449 * 184715524801 : 947 * 10279163
     * 8411395441 * 5922309413062354009 * 3769434424925841309915818 89
478. 3 : 632890270126128456721 * 41461247558242540111975469706627623201 6-
                                             47584436974600916412430373621050140728 81
483. 2^2 * 29 * 139 * 211 * 461 * 691 * 1289 * 1485571 * 1917511
     * 96584086226852975 9 : 9661 * 236235695207989
     * 995264815850055684164903573745584428 9
498. 2 * 3^2 * 6464041 * 245329617161 * 10341247759646081 : 20276569
     * 93750172283 * 2122163146205802445142519991774766393387376957202 83
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3. Results. The factorization results are given in Tables 1 through 6. All factorizations listed are complete, i.e. all factors are prime. The algebraic factors (if any) are separated from the primitive factors by a colon (:). We have not given any details of the actual methods used to obtain the various factorizations.

The author has investigated the Fibonacci numbers U_n, defined by $U_0 = 0$, $U_1 = 1$, $U_{n+2} = U_{n+1} + U_n$, for odd n, $1 \leqslant n \leqslant 399$, and the Lucas numbers V_n, defined by $V_0 = 2$, $V_1 = 1$, $V_{n+2} = V_{n+1} + V_n$, for all n, $0 \leqslant n \leqslant 500$. Tables 1 and 2 contain factorizations of U_n and V_n, respectively, which have not been listed as complete in Jarden [4], Morrison and Brillhart [6], Williams and Judd [15] and [16], Williams and Holte [14], or Williams [13].

The numbers $2^n - 1$ have been investigated for odd n, $1 \leqslant n \leqslant 299$, and Table 3 contains factorizations of numbers of this form which have not been listed as complete in Brillhart, Lehmer and Selfridge [3] or Williams [13]. The numbers $2^n + 1$ have been investigated for $0 \leqslant n \leqslant 300$, $2 \leqslant n \leqslant 598$, if $n = 4k + 2$. Table 4 contains factorizations of $2^n + 1$ with $n \neq 4k + 2$, which have not been listed as

TABLE 3. $2^n - 1$, *n odd*

```
 n   Factorization of 2^n-1

169. 8191 : 4057 * 6740339310641 * 3340762283952395329506327023033
173. 730753 * 1505447 * 70084436712553223 * 155285743288572277679887
185. 31 * 223 * 616318177 : 1587855697992791 * 7248808599285760001152755641
191. 383 * 7068569257 * 39940132241 * 332584516519201 * 87274497124602996457
193. 13821503 * 61654440233248340616559 * 14732265321145317331353282383
199. 164504919713 * 4884164093883941177660049098586324302977543600799
205. 31 * 13367 * 164511353 : 2940521 * 70171342151
     * 3655725065508797181674078959681
213. 7 * 228479 * 48544121 * 212885833 : 66457 * 2849881972114740679
     * 4205268574191396793
215. 31 * 431 * 9719 * 2099863 : 1721 * 731516431 * 514851898711
     * 297927289744047764444862191
217. 127 * 2147483647 : 5209 * 62497 * 6268703933840364033151
     * 378428804431424484082633
219. 7 * 439 * 2298041 * 9361973132609 : 3943 * 671165898617413417
     * 4815314615204347717321
223. 18287 * 196687 * 1466449 * 2916841 * 1469495262398780123809
     * 5962425999871161284215063
235. 31 * 2351 * 4513 * 13264529 : 2391314881 * 72296287361
     * 732023003951580058454735537146974751
237. 7 * 2687 * 202029703 * 1113491139767 : 1423 * 49297 * 23728823512345609279
     * 31357373417090093431
243. 7 * 73 * 2593 * 71119 * 262657 * 97685839 : 487 * 16753783618801
     * 192971705688577 * 3712990163251158343
247. 8191 * 524287 : 15809 * 6459570124697 * 402004106269663
     * 12828161176172650604534969562121 69
267. 7 * 61897001964269013744956211 1 : 78903841 * 28753302853087
     * 2412433243771392408426731 6537353
271. 15242475217
     * 248927757868131890277330541567820045256364273970773286542188386932989391
273. 7^2 * 79 * 127 * 337 * 911 * 8191 * 121369 * 112901153 * 23140471537
     : 108749551 * 4093204977277417 * 869775958019498 44993
279. 7 * 73 * 2147483647 * 658812288653553079 : 16183 * 34039 * 1437967
     * 833732508401263 * 203443983695186729988 8617
283. 9623 * 68492481833
     * 23579543011798993222850893929565870383844167873851502677311057483194673
285. 7 * 31 * 151 * 191 * 32377 * 524287 * 1212847 * 420778751 * 30327152671
     : 1491477035689218775711 * 2534924298663772057 3561
287. 127 * 13367 * 164511353 : 17137716527
     * 51954390877748655744256192963206220919272895548843817842228913
295. 31 * 179951 * 3203431780337 : 4721 * 132751 * 5794391 * 128818831
     * 3812358161 * 452824604065751 * 441097523065082797371 1
```

complete in Brillhart, Lehmer and Selfridge [3] or Williams [13]. For $n = 4k + 2$,

$$2^n + 1 = (2^{2k+1} - 2^{k+1} + 1)(2^{2k+1} + 2^{k+1} + 1),$$

which has been investigated for $1 \leqslant 2k + 1 \leqslant 299$. Tables 5 and 6 contain factorizations of these factors which have not been listed as complete in Brillhart, Lehmer and Selfridge [3] or Williams [13].

TABLE 4. $2^n + 1$

```
 n    Factorization of 2^n+1

151. 3 : 18717738334417 * 5083405082410077967730646062l499
152. 257 : 27361 * 6939446046394048l * 1169955781771735890448l
157. 3 : 15073 * 2350291 * 17751783757817897 * 96833299198971305921
163. 3 : 11281292593 * 1023398150341859 * 3375705470503904150417 69
169. 3 * 2731 : 4929910764223610387 * 1852623864601108673274261404 3
172. 17 : 3855260977 * 64082150767423457 * 1425343275103126327372769
179. 3 : 58745093521 * 434786819066587937349595056277570770714 3803
184. 257 : 43717618369 * 549675408461419937 * 397029956747290287979 1777
188. 17 : 1198107457 * 23592342593 * 4501946625921233 * 181352306852476069537
193. 3 : 6563 * 35679139 * 1871670769 * 7455099975844049
     * 1280761337388845898643
197. 3 : 197002597249 * 1348959352853811313 * 2519515738672530122591 44010843
208. 65537 : 928513 * 18558466369 * 23877647873 * 21316654212673
     * 715668470267111297
209. 3 * 683 * 174763 : 419 * 341062328465463944070 7
     * 160779201878039402409551431700 3
211. 3 : 4643 * 9878177 * 5344743097 * 199061567251
     * 224811275125751758642341851902 99
219. 3^2 * 1753 * 1795918038741070627 : 9070197542196643
     * 327824469015622243413590613 7
221. 3 * 2731 * 43691 : 443 * 4714692062809
     * 450751357540644651584540145836674148752691 3
223. 3 : 219256122131 * 2049349592090504395040765045091817126031830315470840551 3
227. 3 : 297371 * 345463157971421038 7
     * 69982170658265444713117545258712031103399659
235. 3 * 11 * 283 * 165768537521 : 328006342451 * 461797907949997211
     * 2354573745100921150868346 91
243. 3^6 * 19 * 163 * 87211 * 135433 * 272010961 : 1459 * 139483
     * 1042940743191133461 1 * 9181250516025688997 53
245. 3 * 11 * 43 * 281 * 86171 * 4363953127297 : 491 * 15162868758218274451
     * 50647282035796125885000330641
249. 3^2 * 499 * 1163 * 2657 * 155377 * 13455809771 : 9202419446683
     * 338809829056758737705201652562794859 3
253. 3 * 683 * 2796203 : 4049 * 85009 * 31797547 * 81776791273
     * 2822551529460330847604262086149015242689
260. 17 * 61681 * 858001 * 308761441 : 42641 * 5746001 * 2400573761
     * 65427463921 * 1733083439188748105219238 41
261. 3^3 * 19 * 59 * 3033169 * 960767918716136 11 : 523 * 6929826139
     * 3453412901832690553 * 3356385645051570276 1
264. 97 * 257 * 673 * 229153 * 119782433 * 43872038849 : 16875081675650881
     * 869453889972104428282594949923 21
273. 3^2 * 43 * 2731 * 5419 * 224771 * 1210483 * 22366891 * 25829691707 : 547
     * 105310750819 * 292653113147157205779127526827
277. 3 : 25792643401363
     * 313828000939967901734463105154262276920587713495384512820233434582285 7
279. 3^3 * 19 * 529510939 * 715827883 * 2903110321 : 26227 * 119232435043
     * 85384915399027 * 6444365376140611199022187
288. 641 * 6700417 * 18446744069414584321 : 3457 * 816769
     * 156298590135008570995 3 * 1422346738975853644793916289
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TABLE 5. $2^n - 2^{(n+1)/2} + 1$, *n odd*

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n    Factorization of 2^n-2^((n+1)/2)+1

149. 5 : 12961064789 * 11011808951971745915313242336927641
157. 5 : 2790467761 * 5941035366826969 * 22039420334391483439 73
169. 53 * 157 : 677 * 615946323850313 * 2156563293828915509201 92462661
173. 5 : 13625405957 * 17573966531050575296887740350313227534889
179. 5 : 31815461 * 416115013830990336221 * 115757093366365952788 66333
181. 5 : 9413 * 178925762979037 * 3830538323149121 * 950163761355531 73181
187. 5 * 397 * 26317 : 26509131221 * 35155077044989397 * 4029292065629191839853
219. 5 * 293 * 9929 * 649301712182209 : 877 * 1013533 * 704710824913
     * 142406868765525436670617
225. 13 * 37 * 41 * 61 * 101 * 1201 * 8101 * 29247661 * 1182468601
     : 413150254353901 * 319226150421611 2476901
235. 5^2 * 3761 * 7484047069 : 941 * 894434441 * 3357909154141 * 38425816980821
     * 722501809616926841
243. 5 * 109 * 246241 * 106979941 * 168410989 : 3333950193493
     * 1753477469677913202190537606674204157
245. 5^2 * 29 * 197 * 47392381 * 19707683773 : 306178659371201
     * 1372226516822701 * 1008787906424294727221
249. 13 * 997 * 4620219 7673 * 209957719973 : 136453 * 218166829 * 41732461753
     * 579148740542722837871 7709
253. 5 * 397 * 1013 * 1657 : 6994042018866541
     * 6211095415428845718023045687903315012830989259295 29
257. 22988734297
     * 1007381161062241802842574173831975781810739698060547170245057092631 3
267. 5 * 1237940039285450643643301 89 : 3401264941 * 11221454641
     * 1003805584154595697911113729202066 1
269. 5 : 2153 * 3229 * 5381 * 4273873 * 1633401082697
     * 391869517930421432788515 7 * 1853821129478118282760762 81
273. 13^2 * 53 * 113 * 157 * 313 * 1249 * 1429 * 4733 * 556338525912325157
     : 503413 * 467811806281 * 275700717951546566946854497
275. 5^3 * 397 * 268501 * 3630105520141 : 12101 * 35201 * 698617420601
     * 18735216413769901 * 2251172339268843846064 01
289. 137 * 953 : 7698961 * 21886549 * 113478990853
     * 39841016052722109417874918118447229080523618788169942631 3
```

TABLE 6. $2^n + 2^{(n+1)/2} + 1$, *n odd*

n	Factorization of $2^n+2^{(n+1)/2}+1$
137.	5 : 168434085820849 * 206875670104957744917147613
151.	5 : 4373689270176379261201 * 130530323901899210670077
167.	5 : 75005713 * 27395325377910797 * 18208260781190156536114609
169.	5 * 1613 : 180201997 * 1259036730797 * 408946876729703992293841657
173.	7152893721041 * 1673815085186574700322174232069942181681
183.	5 * 733 * 1709 * 368140581013 : 12836737570021 * 414194958733796530899181
187.	137 * 953 * 2113 : 5237 * 551353793 * 1819762572673
	* 135322045917118601273437
191.	5 : 3821 * 89618875387061 * 1833085153842665442652283234165143433597
193.	5 : 3089 * 148997 * 14402030644704405877 * 378791300027089635677652285973
203.	113 * 536903681 : 9810958633253 * 21597468549493958664902504331670645757
209.	5 * 229 * 397 * 457 : 6689 * 2039731321 * 149832750683283097
	* 1937385241416564065603093
217.	5 * 29 * 8681 * 49477 : 31249 * 776729668507005203702993
	* 139335546032913681584758997
219.	13 * 9444732965601851473921 : 371335727233
	* 18478609113710122023550126425157
237.	13 * 604462909806215075725313 : 18890331057055511701
	* 1487840558911519281039078769
239.	5 : 77852679293
	* 2269474963255693085711432948387582114817557263546457947501201
243.	13 * 37 * 279073 * 3618757 * 4977454861 : 2917 * 4861 * 26129603777437
	* 15778453094691989880197773477
249.	5 * 13063537 * 148067197374074653 : 1993 * 80485166514184335373
	* 583117579691967491546961181
253.	277 * 2113 * 30269 : 25301 * 109297 * 756550961 * 2569737193 * 9623862953
	* 156296877661 * 101027360307659633
263.	5 : 119929 * 731141 * 99972364781
	* 338153229347093487293402061645864051641494661202651405269
265.	5^2 * 1801439824104653 : 51941 * 24082141 * 31213331016701
	* 33716583668208510447368101472499412321
285.	13 * 41 * 61 * 761 * 131101 * 160969 * 525313 * 2416923620660807201
	: 1457772869697961 * 64326196787727903551977150861
297.	5 * 109 * 397 * 42373 * 246241 * 4327489 * 15975607282273 : 2377 * 22573
	* 155399494141 * 4712151755917 * 41523259994275786297957
299.	53 * 157 * 277 * 30269 : 20333 * 956801 * 15595841 * 19294368341
	* 6339840806910833 * 393345821366273907459718331839045409

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Department of Computer Science
Odense University
DK-5230 Odense M., Denmark

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