REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification can be found in the December index volumes of Mathematical Reviews.

15[65L00].—WILLARD L. MIRANKER, *Numerical Methods for Stiff Equations and Singular Perturbation Problems*, Mathematics and Its Application/5, Reidel, Boston, 1980, xiii + 202 pp., 23 cm. Price $29.95.

When applied to singularly perturbed problems, standard numerical methods often "fail" in the sense that, as the problem becomes more singular, they demand too much additional work. They might do so in order to stably follow a solution component which quickly becomes negligible or to follow a highly oscillatory component overlaid on a slowly developing part of more interest. Thus, a common principle in numerical analysis of singularly perturbed problems is to devise methods which either overlook the uninteresting components while faithfully tracing the important ones or, based on prior knowledge of the form of the singular components, incorporate them cheaply in the method. (A third possibility is to temporarily disregard the singularities and then recover them a posteriori.)

Numerical analysis of singularly perturbed differential equations is the subject of much recent research. By now, firm foundations have been laid in many problems involving ordinary differential equations but, for partial differential equations, only the case of one space dimension is somewhat advanced. For most partial differential equations, even if the nature and form of singularities is thoroughly analyzed and understood, the behavior of many a clever numerical scheme is known only in computational examples.

Topics for ordinary differential equations where the behavior of numerical schemes is reasonably well understood include:

I. The role of A-stability and its variations in solving stiff initial value problems.

II. The role of upwinding in singularly perturbed second order two-point boundary value problems.

III. Utilization of the matched asymptotic expansion formalism in singularly perturbed initial value problems and two-point boundary value problems in order to construct numerical schemes.

Among books dedicated to these problems we have [2], devoted mainly to no. III, and [1], treating all of the three subjects to some extent. No. I is more "classical" and can be found in many books on numerical solution of ordinary differential equations.

The present volume offers a smorgasbord of interesting problems; in addition to those three above, it includes a detailed description of numerical methods based on averaging in problems with highly oscillatory solutions, where the author and Hoppensteadt have made fundamental contributions.

I shall next describe the contents in more detail.

Chapters 1–3 are introductory. The "classical" theory for numerical solution of stiff initial value problems for ordinary differential equations (a fast decaying component) is given. Following Dahlquist, linear multistep methods and their A-stability, or not, are treated and also procedures for solving the equations in implicit methods. Further, Certaine's and Jain's methods are exposed, and the last of the about 50 pages treat Runge-Kutta methods and their properties in this context.

In Chapters 4 and 5 more specialized methods are applied to problems with fast decaying components. The "Exponential Fitting" idea of Willoughby, Liniger and the author is given in Chapter 4, and in Chapter 5 the method of matched asymptotic expansions (a la Vishik and Lyusternik) is set up, and numerical methods based on it are given. In all this, the author's experience guarantees a very lucid and to the point account, guiding the reader in the details of how to set up the scheme and to what is important and what is not. The interesting case of matched asymptotic expansion without a clearly identifiable parameter is also treated.

Furthermore, exponential fitting in the case of highly oscillatory solutions is briefly discussed, setting the stage for Chapter 6. In a somewhat "off-side" section, the simple hyperbolic equation $u_t = u_x$ in the presence of rough initial data is treated, and an interesting modification of the classical idea of a small local truncation error given.

The highly oscillatory case and numerical implementation of the averaging method of Bogoliubov occupies Chapter 6. Following the two-time method of Hoppensteadt and the author, including its nice algebraic setting, the reader is shown how to carry on computationally. In Section 6.4, again, an entertaining modification of classical finite difference ideas is introduced.

Chapter 7 is concerned with a singularly perturbed recurrence relation, following the ideas of Chapter 6, and with the singularly perturbed second order two-point boundary value problem, also in the presence of turning points. In the latter problem the WKB method is briefly recalled and a semianalytical numerical scheme of upwind type given.

Written by an expert researcher and covering mainly areas where he himself has been actively involved, this book succeeds in treating several interesting problems, lucidly collecting relevant properties of their solutions and then motivating, constructing and (sometimes) analyzing the numerical schemes. Frequently these schemes differ in their rationale from more classical and well-known methods; the author always gives the salient points. In accordance with editorial policy for this series, the treatment is not complete but rather brisk and tentative. E.g., I could not discern any principle as to when a theorem was stated, as opposed to a development given without stated theorems; the guideline appeared to be "when the author felt like it". Computational experiments are given but are not very exhaustive or convincing (the point of the one in Article 6.1.7 escapes one completely). Misprints are few and not serious, although one on page 6, line 17 leaves one momentarily puzzled. The index is very decent; a spot check revealed that the (rather nonstandard) term "matrizant" did not occur on page 10 but instead on pages 5 and 110, being italicized the second time but not the first.

The book is accessible to anyone with a modest background in numerical ordinary differential equations and singular perturbation theory and could be used for a specialized graduate course. While topical, the author has pointed out many interconnections and underlying principles. In surveying broadly important aspects of a growing field in numerical analysis, this volume (a considerable revision of lecture notes by the author from 1975) is a most welcome addition to the library of numerical analysts and applied mathematicians.

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1. E. P. DOOLAN, J. J. H. MILLER & W. H. A. SCHILDERS, *Uniform Numerical Methods for Problems with Initial and Boundary Layers*, Boole Press, Dublin, 1980.

2. P. W. HEMKER, *A Numerical Study of Stiff Two-Point Boundary Problems*, Math. Centre Tracts 80, Amsterdam, 1977.

16[65P05].—LEON LAPIDUS & GEORGE F. PINDER, *Numerical Solution of Partial Differential Equations in Science and Engineering*, Wiley, New York, 1982, 677 pp., 24 cm. Price $44.95.

This volume considers virtually all numerical methods for solving partial differential equations known and widely used in the late seventies. Written as a textbook for a course, it includes hardly any proofs but devotes its considerable number of pages to lucid developments of the methods, often starting with simple examples and building upwards.

The Chapters are as follows:

Ch. 1. Fundamental Concepts (in partial differential equations).

Ch. 2. Basic Concepts in the Finite Difference and Finite Element Methods.

Ch. 3. Finite Elements on Irregular Subspaces.

Ch. 4. Parabolic Partial Differential Equations.

Ch. 5. Elliptic Partial Differential Equations.

Ch. 6. Hyperbolic Partial Differential Equations.

The basics of finite difference and finite element methods (Galerkin, collocation, boundary elements) are treated in each context where it applies. Fast methods for solving relevant linear systems of equations are given thorough consideration: these methods include odd-even reduction, point and line iterative methods (Jacobi, Gauss-Seidel, successive overrelaxation...) and also alternating direction and locally one-dimensional methods.

Certain modern developments are not treated, thus somewhat dating the book and making it less useful as a reference. (The inside cover flap states that the book is aimed to be a reference/textbook, whereas the preface makes no claim that it is a reference book.) These topics include many "fast" methods recently in vogue such as: Spectral methods, node reordering schemes, conformal mapping techniques, random choice methods, multigrid techniques (except classical extrapolation schemes), capacitance matrix techniques, and use of fast methods as preconditioners in iterative schemes. Also, mixed methods are treated very briefly.

Generally, pitfalls are clearly pointed out; as omissions I noted that hardly anything is said about the need for one-to-one mappings in isoparametric elements or about problems with corner singularities in elliptic problems (although sources and sinks are briefly considered).

The only really objectionable statement I found occurs on p. 242 (and recurs on p. 246). It is stated that the problem with implicit finite difference methods in parabolic problems in two space dimensions is that the resulting linear system of equations to solve involves a penta-diagonal matrix, and thus the effective tri-diagonal solver cannot be applied. Of course, there are very effective penta-diagonal solvers, but the matrix is not penta-diagonal.

Given that the book aims for a first course covering "The Classics", the treatment is excellent from a pedagogical point of view. The student is slowly and carefully led towards a thorough understanding of the methods.

Finally, the writing is very polished and I found it a pleasure to read!

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17[65L00, 65M00, 65N00].—JAMES M. ORTEGA & WILLIAM G. POOLE, JR., *An Introduction to Numerical Methods for Differential Equations*, Pitman, Boston, Mass., 1981, ix + 329 pp., 24 cm. Price $24.95.

Although the title of this book is a correct description of its content, it may be a bit misleading. This is really an introductory textbook on Numerical Methods, and it is distinct from other such books mainly by its arrangement of the material; linear equations, e.g., are treated as the predominant part of a chapter on boundary value problems for o.d.e., numerical quadrature is presented as a tool in the context of projection methods, etc. While thus the constructive solution of differential equations (including p.d.e.) serves as the frame of reference, the treatment of the classical material within this frame is just as broad and detailed as is usual for a first introduction to Numerical Mathematics. It is difficult to tell whether this arrangement will be more appealing to students; in any case it permits the use of demonstrative application examples in differential equations throughout the book.

Generally, the authors have avoided formulating and proving theorems; they state many results in a semiformal way and rather provide motivations and explanations. However, the presentation is sufficiently technical and concise that more formal results and proofs may easily be added here and there by a more ambitious instructor; references, supplementary remarks, and a set of well-chosen exercises help the reader to gain a deeper understanding if he wishes. On the other hand, the text may disappoint those who are looking for an easy access to the practical use of numerical methods (say science or engineering students); they will feel diverted by the many mathematically minded discussions and will miss concrete guidelines for the use of relevant library programs.

The introductory chapter on "the world of scientific computing" which includes a discussion of symbolic computation starts the text off nicely, and there are a few chapters which are exemplary in their short and clear presentation of essential aspects. The sections on eigenvalue problems, on sparse linear equations, and on projection methods I found particularly well-composed. Also I liked a number of details (like the "interval of uncertainty" about a zero of a function) and many of the numerous illustrations. (Curiously, the grossly incorrect Figure 5.2(b) for the midpoint rule went unnoticed.) What I missed most is a systematic introduction of the concept of condition, beyond the discussion in the context of linear equations.

It would also have been nice to have a general chapter on numerical software, with detailed references for each subject area. An innocent reader of this text may rather be tempted to program his (or her) own routines than to resort to the well-known software packages.

On the whole, this is a refreshingly written presentation of wide areas of our field and a welcome addition to the textbook literature.

H. J. S.

18[65N30].—J. R. WHITEMAN (Editor), *The Mathematics of Finite Elements and Applications* IV, MAFELAP 1981, Academic Press, London, New York, 1982, xvi + 555 pp., $23\frac{1}{2}$ cm. Price $40.50.

This volume contains 44 papers and 39 abstracts of poster session papers presented at the fourth conference on The Mathematics and Finite Elements and Applications held at Brunel University, England, from April 28–May 1, 1981.

19[65K10].—M. J. D. POWELL (Editor), *Nonlinear Optimization* 1981, Academic Press, London, New York, 1982, xvii + 559 pp., $23\frac{1}{2}$ cm. Price $39.50.

This volume is based on the proceedings of the NATO Advanced Research Institute held at Cambridge from July 13–24, 1981. There are 31 invited papers divided into the following chapters: Unconstrained Optimization, Nonlinear Fitting, Linear Constraints, Nonlinear Constraints, Large Nonlinear Problems, The Current State of Software, and Future Software Testing. Each chapter ends with a discussion of that particular topic.

20[65–00].—R. GLOWINSKI & J. L. LIONS (Editors), *Computing Methods in Applied Sciences and Engineering* V, North-Holland, Amsterdam, New York, 1982, x + 668 pp., 23 cm. Price $95.00.

This is the proceedings of the Fifth International Symposium on Computing Methods in Applied Sciences and Engineering held at Versailles, France, from December 14–18, 1981. It contains 41 papers on the following topics: Numerical Algebra, Stiff Differential Equations, Parallel Computing, Approximation of Eigenvalues and Eigenfunctions-Bifurcation, Wave Propagation, Nonlinear Elasticity, Fluid Mechanics, Plasma Physics, Turbulence, Semiconductors, Biomathematics, and Inverse Problems.

21[12A50].—FRANCISCO DIAZ Y DIAZ, *Tables Minorant la Racine n-ième du Discriminant d'un Corps de Degré n*, Publications Mathématiques d'Orsay, France, 1980, 60 pp., 30 cm. Price—not available.

Let K be a number field of degree n, and let d be the absolute value of its discriminant. Odlyzko [3] showed how to give lower bounds for $d^{1/n}$. Subsequent work was done by Serre and Poitou [5]. In the present work, the author uses Poitou's formulas to calculate such lower bounds in the following cases: Table 1: K totally imaginary, $2 \leqslant n \leqslant 4000$; Table 2, K totally real, $1 \leqslant n \leqslant 2000$; Table 3: all K with

$1 \leqslant n \leqslant 10$. The estimates in Table 3 are better than those listed in the other three tables since certain local contributions are taken into account; Table 4: all K with $1 \leqslant n \leqslant 100$. The numbers in the tables are truncated to eight decimal places.

The calculations needed to obtain the optimal values from Poitou's formulas can be shortened if one accepts slightly inferior values. The accuracy of some of these approximations is investigated in some preliminary Tableaux (not to be confused with the Tables).

Earlier, less extensive tables of lower bounds for $d^{1/n}$ had been given by Odlyzko [4] and Poitou [5]. There are also some unpublished tables of Odlyzko (see [6]). The estimates in the present work are much better than those in [4], slightly better than the limited table of [5], and appear to be slightly better than the unpublished table of Odlyzko (see [6, p. 705]). In [2], some bounds were also computed under the assumption of the Generalized Riemann Hypothesis, and some explicit examples are given, which show that the estimates are reasonably sharp. In [1], the existence of infinite class field towers is used to give an upper bound for these lower bounds for n large.

Lower bounds for $d^{1/n}$ have been used by Masley and van der Linden [6] to calculate the class number of totally real abelian number fields.

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1. J. MARTINET, "Tours de corps de classes et estimations de discriminants," *Invent. Math.*, v. 44, 1978, pp. 65–73.

2. J. MARTINET, *Petits Discriminants des Corps de Nombres*, Journées Arithmétiques 1980 (ed. by J. V. Armitage), Cambridge Univ. Press, 1982, pp. 151–193.

3. A. ODLYZKO, "Some analytic estimates of class numbers and discriminants," *Invent. Math.*, v. 29, 1975, pp. 275–286.

4. A. ODLYZKO, "On conductors and discriminants," in *Algebraic Number Fields* (ed. by A. Fröhlich), Academic Press, London and New York, 1977, pp. 377–407.

5. G. POITOU, *Sur les Petits Discriminants*, Séminaire Delange-Pisot-Poitou, Théorie des Nombres, 18e année 1976/77, exp. no. 6.

6. F. VAN DER LINDEN, "Class number computations of real abelian number fields," *Math. Comp.*, v. 39, 1982, pp. 693–707.