Computing the Irreducible Characters of the Group $GL_6(2)$

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Abstract. All the sixty ordinary irreducible characters of the group of six by six nonsingular matrices over a field with two elements are found. To do this we use methods of Steinberg and also characters induced from certain subgroups which makes it possible to calculate the whole character table by hand.

1. Introduction. In [5] and [6] Steinberg gives methods for obtaining a number of irreducible representations of the general linear group $GL_n(q)$. In particular, using geometric properties of the underlying vector space $V = V_n(q)$, he obtains $p(n)$ irreducible representations of $GL_n(q)$ where $p(n)$ denotes the number of partitions of n . These $p(n)$ irreducible representations are related to those of the symmetric group Σ_n .

Let $n = \lambda_1 + \lambda_2 + \cdots + \lambda_n \equiv (\lambda)$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$, be a partition of n . Let a k -dimensional subspace of V be denoted by $V(k)$. Let $\Delta^{(\lambda)}$ denote the set of all the sequences of subspaces $V(\lambda_1), V(\lambda_1 + \lambda_2), \dots, V(\lambda_1 + \lambda_2 + \cdots + \lambda_n)$ of V such that each one is contained in the next one. Then $GL_n(q)$ acts transitively on $\Delta^{(\lambda)}$. Let $\chi^{(\lambda)}$ denote the permutation character obtained in this way. Let the letters from 1 to n be partitioned into sets consisting of $\lambda_1, \lambda_2, \dots, \lambda_n$ letters. If we set $H_{(\lambda)} = \Sigma_{\lambda_1} \times \Sigma_{\lambda_2} \times \cdots \times \Sigma_{\lambda_n}$ and denote by $1_{H_{(\lambda)}} \uparrow^{\Sigma_n}$ the character induced by the identity character of $H_{(\lambda)}$, then by [6] $\chi^{(\lambda)}$ and $1_{H_{(\lambda)}} \uparrow^{\Sigma_n}$ split in the same manner.

Using this fact and properties of elements of $G = GL_6(2)$ we can find 11 irreducible characters of G corresponding to 11 partitions of 6 which will be described later. To obtain the rest of the irreducible characters we use the method of inducing characters from certain subgroups of G .

Throughout this paper $H \cdot K$ denotes the extension of group H by group K . If $H \leq G$ and χ is a character of H , then $\chi_H \uparrow^G$ denotes the character induced by χ . For $x \in G$ then $C(x)$ denotes the centralizer of x in G .

2. Conjugacy Classes of $G = GL_6(2)$. $G = GL_6(2)$ has order $|G| = 2^{15} \cdot 3^4 \cdot 5 \cdot 7^2 \cdot 31 = 20,158,709,760$. Using irreducible polynomials of degrees up to six over $GF(2)$ [2] we find that G has 60 conjugacy classes. In Table I (Table I and all subsequent tables appear at the end of this paper) a representative from each class in its matrix form together with its centralizer order and the dimension of its fixed space, d , when acting upon V are given. Here (m) denotes a conjugacy class of

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elements of order m and when there are more than one class of elements of order m then subscripts (m_i) are used to differentiate classes. ($\overline{m}$) denotes one of the pair of classes such that elements of a class are inverses of elements in the other class. x^2 , x^3 , x^5 , x^7 define the power map. Finally, ($\widetilde{31}$) denotes one of the pair of classes of elements of order 31 such that elements in a class are third powers of elements in the other class.

3. Conjugacy Classes of Certain Subgroups of $G = \text{GL}_6(2)$. Let $6 = \lambda_1 + \lambda_2 + \dots + \lambda_6 \equiv (\lambda)$, $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_6 \geq 0$, be a partition of 6. Let $\Delta^{(\lambda)}$ be the set mentioned in Section 1. For an element of $\Delta^{(\lambda)}$ the set of elements of G stabilizing it (as a whole), $G_{(\lambda)}$, consists of matrices of the form:

$$\begin{bmatrix} A_1 & & & & & \\ & A_2 & & & & \\ & & \ddots & & & \\ & & & \ddots & & \\ & \mathbf{0} & & & & \\ & & & & & A_6 \end{bmatrix},$$

where $A_i \in \text{GL}_{\lambda_i}(2)$, $1 \leq i \leq 6$. Therefore $G_{(\lambda)}$ is a group of the form $T \cdot (\text{GL}_{\lambda_1}(2) \times \dots \times \text{GL}_{\lambda_6}(2))$, where T is an elementary Abelian 2-group of order 2^N , $N = \sum_{1 \leq i < j \leq 6} \lambda_i \lambda_j$, and will be denoted by 2^N from now on.

If χ_i is a character of $\text{GL}_{\lambda_i}(2)$, $1 \leq i \leq 6$, then $\chi = \chi_1 \chi_2 \dots \chi_6$ is a character of $G_{(\lambda)}$. To evaluate $\chi \uparrow^G$, by [3] we have

$$\chi \uparrow^G(g) = |C(g)| \sum_{i=1}^m \frac{\chi(g_i)}{|C_{G_{(\lambda)}}^{(g_i)}|},$$

where $g = g_1, g_2, \dots, g_m$ are representatives of those classes of $G_{(\lambda)}$ which are conjugate to g in G . We can write the expression above as

$$\chi \uparrow^G(g) = \sum_{i=1}^m \frac{|C(g)|}{|C_{G_{(\lambda)}}^{(g_i)}|} \chi(g_i).$$

Therefore, to evaluate $\chi \uparrow^G(g)$ we need to know the ratios $|C(g)|/|C_{G_{(\lambda)}}^{(g_i)}|$ together with the image of g_i under the canonical homomorphism $G_{(\lambda)} \rightarrow G_{(\lambda)}/T$.

We shall call these images the coset representatives.

In our work, most of the irreducible characters of G are obtained from inducing characters from the groups $G_{(4,2)}$ and $G_{(3^2)}$. These groups have the form $G_{(4,2)} \cong 2^8 \cdot (\text{GL}_4(2) \times \text{GL}_2(2))$ and $G_{(3^2)} \cong 2^9 \cdot (\text{GL}_3(2) \times \text{GL}_3(2))$. As $\text{GL}_4(2) \cong A_8$, $\text{GL}_2(2) \cong \Sigma_3$, $\text{GL}_3(2) \cong \text{PSL}_2(7)$, we have $G_{(4,2)} \cong 2^8 \cdot (A_8 \times \Sigma_3)$, $G_{(3^2)} \cong 2^9 \cdot (\text{PSL}_2(7) \times \text{PSL}_2(7))$. Now the conjugacy classes of the groups A_8 , Σ_3 , $\text{PSL}_2(7)$ are given in [4], and using these, together with the analysis of each coset, we find conjugacy classes of the groups $G_{(4,2)}$ and $G_{(3^2)}$. In Tables II and III we give the ratios $|C(g)|/|C_{G_{(\lambda)}}^{(g_i)}|$ together with the coset representatives and the fusion of elements of the groups $G_{(4,2)}$ and $G_{(3^2)}$ in G . In giving the coset representatives we use the notations of [4].

In these tables (17) and (35) denote one of the pairs of classes of elements of order 7 and 15 respectively of A_8 such that elements in one class are inverses of elements in the other class. (7_1) and (7_2) denote two classes of elements of order 7 in $\text{PSL}_2(7)$ and in this case elements in one class are inverses of elements in the other class.

4. Computation of the Irreducible Characters of G . Let $6 = \lambda_1 + \lambda_2 + \cdots + \lambda_6$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_6 \geq 0$, be a partition of 6. As we mentioned in the introductory section, $1_{H(\lambda)} \uparrow^{\Sigma_6}$ and $\chi^{(\lambda)}$ split in the same manner [6]. Now the character table of Σ_6 is given in [4], and using this we find the irreducible constituents of $1_{H(\lambda)} \uparrow^{\Sigma_6}$ for each partition (λ) of 6. By comparison we get the following relations:

$$\begin{aligned}\chi^{(6)} &= \chi_1, \quad \text{the identity character of } G, \\ \chi^{(5,1)} &= \chi_1 + \chi_2, \\ \chi^{(4,2)} &= \chi_1 + \chi_2 + \chi_3, \\ \chi^{(3^2)} &= \chi_1 + \chi_2 + \chi_3 + \chi_4, \\ \chi^{(4,1^2)} &= \chi_1 + 2\chi_2 + \chi_3 + \chi_5, \\ \chi^{(3,2,1)} &= \chi_1 + 2\chi_2 + 2\chi_3 + \chi_4 + \chi_5 + \chi_6, \\ \chi^{(3,1^3)} &= \chi_1 + 3\chi_2 + 3\chi_3 + \chi_4 + 3\chi_5 + 2\chi_6 + \chi_7, \\ \chi^{(2^3)} &= \chi_1 + 2\chi_2 + 3\chi_3 + \chi_4 + \chi_5 + 2\chi_6 + \chi_8, \\ \chi^{(2^2,1^2)} &= \chi_1 + 3\chi_2 + 4\chi_3 + 2\chi_4 + 3\chi_5 + 4\chi_6 \\ &\quad + \chi_7 + \chi_8 + \chi_9, \\ \chi^{(2,1^4)} &= \chi_1 + 4\chi_2 + 6\chi_3 + 3\chi_4 + 6\chi_5 + 8\chi_6 \\ &\quad + 4\chi_7 + 2\chi_8 + 3\chi_9 + \chi_{10}, \\ \chi^{(1^6)} &= \chi_1 + 5\chi_2 + 9\chi_3 + 5\chi_4 + 10\chi_5 + 16\chi_6 \\ &\quad + 10\chi_7 + 5\chi_8 + 9\chi_9 + 5\chi_{10} + \chi_{11},\end{aligned}$$

where in the above, χ_i , $1 \leq i \leq 11$, are distinct irreducible characters of G . Using the formula given for the size of the sets $\Delta^{(\lambda)}$ in [6], we are able to find the degrees of χ_i , $1 \leq i \leq 11$. We have: $\chi_1(1) = 1$, $\chi_2(1) = 62$, $\chi_3(1) = 588$, $\chi_4(1) = 744$, $\chi_5(1) = 1240$, $\chi_6(1) = 6480$, $\chi_7(1) = 9920$, $\chi_8(1) = 5952$, $\chi_9(1) = 18816$, $\chi_{10}(1) = 31744$, $\chi_{11} = 32768$. A glance at the above relations shows that if $\chi^{(\lambda)}$, for each partition (λ) of 6, is known, then we are able to evaluate χ_i , $1 \leq i \leq 11$, on the whole of G . Now we know that $\chi^{(5,1)}(g)$, $g \in G$, is equal to the number of nonzero vectors of V , fixed by g , which can be computed from Table I. Therefore, it is straightforward to calculate χ_2 on the whole of G . Using Table I and properties of elements of G , together with combinatorial arguments, we computed $\chi^{(\lambda)}$ for each element of G , which are tabulated in Table IV.

Now using relations written at the beginning of this section, together with Table IV, we evaluate χ_i , $1 \leq i \leq 11$ on the whole of G , which are tabulated in Table V.

At this stage, as 31 divides $|G|$ to the first power, the consequences of [1] yield six more irreducible characters of G . G has exactly one block of the lowest type, namely the principal block $B_0(13)$. $B_0(13)$ contains 5 ordinary irreducible characters and 6 exceptional characters which are 31-conjugate. Incidentally, the 5 ordinary irreducible characters of $B_0(31)$ are χ_1 , χ_3 , χ_6 , χ_9 and χ_{11} . By [1], if φ denotes one of the 6 exceptional characters in $B_0(31)$, then we have the following relation for all

31-regular elements x of G :

$$\varphi(x) = \chi_1(x) - \chi_3(x) + \chi_6(x) - \chi_9(x) + \chi_{11}(x).$$

Thus φ can easily be evaluated on all 31-regular elements of G , in particular we have $\varphi(1) = 19845$. Let these 6 exceptional characters be denoted by χ_{12} , χ_{13} , χ_{14} , χ_{15} , χ_{16} and χ_{17} . Then using [1], we evaluated χ_i , $12 \leq i \leq 17$ on the 31-singular elements of G , which are tabulated in Table V.

Now, we use Tables II and III to induce certain characters of the subgroups $G_{(4,2)}$ and $G_{(3^2)}$. Let the irreducible characters of Σ_3 , $\text{PSL}_2(7)$ and A_8 be denoted as follows: Irreducible characters of Σ_3 are denoted by θ_1 , θ_2 and θ_3 , where θ_1 is the identity character of Σ_3 , with the degrees: $\theta_1(1) = 1$, $\theta_2(1) = 1$ and $\theta_3(1) = 2$. Irreducible characters of $\text{PSL}_2(7)$ are denoted by φ_1 , φ_2 , φ_3 , φ_4 , φ_5 and φ_6 with the degrees: $\varphi_1(1) = 1$, $\varphi_2(1) = \varphi_3(1) = 3$, $\varphi_4(1) = 6$, $\varphi_5(1) = 7$, $\varphi_6(1) = 8$. Finally, irreducible characters of A_8 are denoted by ψ_i , $1 \leq i \leq 14$, with the degrees: $\psi_1(1) = 1$, $\psi_2(1) = 7$, $\psi_3(1) = 14$, $\psi_4(1) = 20$, $\psi_5(1) = 21$, $\psi_6(1) = \psi_7(1) = 21$, $\psi_8(1) = 28$, $\psi_9(1) = 35$, $\psi_{10}(1) = \psi_{11}(1) = 45$, $\psi_{12}(1) = 56$, $\psi_{13}(1) = 64$ and $\psi_{14}(1) = 70$. In the above, ψ_6 and ψ_7 are a pair of 5-conjugate characters while ψ_{10} and ψ_{11} are a pair of 7-conjugate characters.

Now we proceed to describe irreducible characters of G obtained from inducing certain characters of $G_{(4,2)}$ and $G_{(3^2)}$ which was described in Section 3. We start with $G_{(4,2)} \cong 2^8 \cdot (A_8 \times \Sigma_3)$. Recall that if ψ and θ are characters of A_8 and Σ_3 , respectively, then $\chi = \psi\theta$ is a character of $G_{(4,2)}$. χ can be evaluated easily using Table II, and if

$$(\chi, \chi) = \frac{1}{|G|} \sum_{g \in G} \chi(g) \overline{\chi(g)} = 1,$$

then χ is an irreducible character of G . Various irreducible characters of G are obtained in this way, which are listed below.

$\psi_1\theta_2$	induces	an	irreducible	character	χ_{18}	of G	with	degree	651,
$\psi_3\theta_2$	"	"	"	"	χ_{19}	"	"	"	9114,
$\psi_5\theta_1$	"	"	"	"	χ_{20}	"	"	"	13671,
$\psi_5\theta_3$	"	"	"	"	χ_{21}	"	"	"	27342,
$\psi_4\theta_2$	"	"	"	"	χ_{22}	"	"	"	13020,
$\psi_{12}\theta_2$	"	"	"	"	χ_{23}	"	"	"	36456,
$\psi_2\theta_1$	"	"	"	"	χ_{24}	"	"	"	4557,
$\psi_8\theta_1$	"	"	"	"	χ_{25}	"	"	"	18228,
$\psi_6\theta_1$	"	"	"	"	χ_{26}	"	"	"	13671,
$\psi_7\theta_1$	"	"	"	"	χ_{27}	"	"	"	13671,
$\psi_2\theta_3$	"	"	"	"	χ_{28}	"	"	"	9114,
$\psi_{10}\theta_2$	"	"	"	"	χ_{29}	"	"	"	29295,
$\psi_{11}\theta_2$	"	"	"	"	χ_{30}	"	"	"	29295,
$\psi_6\theta_3$	"	"	"	"	χ_{31}	"	"	"	27342,
$\psi_7\theta_3$	"	"	"	"	χ_{32}	"	"	"	27342,
$\psi_8\theta_3$	"	"	"	"	χ_{33}	"	"	"	36456,
$\psi_{13}\theta_1$	"	"	"	"	χ_{34}	"	"	"	41664,
$\psi_5\theta_2$	"	"	"	"	χ_{35}	"	"	"	13671,
$\psi_6\theta_2$	"	"	"	"	χ_{36}	"	"	"	13671,
$\psi_7\theta_2$	"	"	"	"	χ_{37}	"	"	"	13671.

No other irreducible characters of G are obtained in this manner and what is obtained are either repeated earlier or compound characters of G . Similarly, we induce certain characters of $G_{(3^2)}$. Below are irreducible characters obtained by inducing certain characters of $G_{(3^2)}$ which have not appeared earlier. Note that these characters are evaluated on G using Table III.

$\varphi_2\varphi_1$	induces	an	irreducible	character	χ_{38}	of G	with	degree	4185,
$\varphi_3\varphi_1$	"	"	"	"	χ_{39}	"	"	"	4185,
$\varphi_2\varphi_4$	"	"	"	"	χ_{40}	"	"	"	25110,
$\varphi_3\varphi_4$	"	"	"	"	χ_{41}	"	"	"	25110,
$\varphi_2\varphi_6$	"	"	"	"	χ_{42}	"	"	"	33480,
$\varphi_3\varphi_6$	"	"	"	"	χ_{43}	"	"	"	33480,
$\varphi_2\varphi_3$	"	"	"	"	χ_{44}	"	"	"	12555.

No other irreducible character of G is obtained in this way. At this point we take tensor products of certain characters of G which are already known and obtain various relations between the unknown characters of G . After solving these equations we obtain the remaining 16 characters of G . We omit the details and tabulate the irreducible characters of G in Table V.

Note that in Table V we have:

$$\begin{aligned}\rho &= \frac{1}{2}(-1 + i\sqrt{15}), & \omega &= \frac{1}{2}(-1 + i\sqrt{7}), \\ a &= \varepsilon + \varepsilon^2 + \varepsilon^4 + \varepsilon^8 + \varepsilon^{16}; & b &= \varepsilon^3 + \varepsilon^6 + \varepsilon^{12} + \varepsilon^{17} + \varepsilon^{24}, \\ c &= \varepsilon^5 + \varepsilon^9 + \varepsilon^{10} + \varepsilon^{18} + \varepsilon^{20}, & \varepsilon &= \cos \frac{2\pi}{31} + i \sin \frac{2\pi}{31}.\end{aligned}$$

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TABLE I (continued)

[illegible]

TABLE II. *Conjugacy classes of $G_{(4,2)}$*

$\rightarrow \text{GL}_6(2)$	$A_8 \times \Sigma_3$ -Coset	Ratio	$\rightarrow \text{GL}_6(2)$	$A_8 \times \Sigma_3$ -Coset	Ratio
1	$(1^8, 1^3)$	651		$(2^4, 3)$	1
2_1	$(1^8, 1^3)$	15	6_3	$(1^2 3^2, 12)$	4
	$(1^8, 12)$	16		$(1^2 3^2, 1^3)$	3
	$(2^4, 1^3)$	140		$(26, 1^3)$	4
2_2	$(1^8, 1^3)$	1	6_4	$(26, 3)$	1
	$(1^4 2^2, 1^3)$	16		$(12^2 3, 12)$	1
	$(2^4, 1^3)$	18	6_5	$(1^5 3, 12)$	1
	$(2^4, 12)$	24		$(26, 3)$	5
2_3	$(1^4 2^2, 12)$	28	6_6	$(1^2 3^2, 3)$	1
	$(2^4, 1^3)$	7		$(12^2 3, 1^3)$	1
4_1	$(1^8, 12)$	1	12_1	$(12^2 3, 3)$	1
	$(2^4, 1^3)$	7	12_2	$(1^2 24, 3)$	1
	$(2^4, 12)$	7		$(26, 12)$	1
	$(4^2, 1^3)$	28	12_3	$(1^2 3^2, 12)$	1
4_2	$(1^4 2^2, 1^3)$	2		$(4^2, 3)$	1
	$(1^4 2^2, 12)$	2		$(26, 12)$	1
	$(2^4, 1^3)$	1		$(26, 1^3)$	1
	$(2^4, 12)$	2	5	$(1^3 5, 1^3)$	1
	$(4^2, 12)$	8	10	$(1^3 5, 12)$	1
	$(4^2, 1^3)$	4	$\overline{15}_1$	$(\overline{35}, 1^3)$	1
4_3	$(1^4 2^2, 1^3)$	1	$\overline{15}_1$	$(\overline{35}, 1^3)$	1
	$(4^2, 12)$	6	$\overline{15}_2$	$(\overline{35}, 3)$	1
4_4	$(2^4, 12)$	1	$\overline{15}_2$	$(\overline{35}, 3)$	1
	$(1^2 24, 1^3)$	4	15_3	$(1^3 5, 3)$	1
	$(4^2, 1^3)$	3	$\overline{30}$	$(\overline{35}, 12)$	1
	$(4^2, 12)$	3	$\overline{30}$	$(\overline{35}, 12)$	1
4_5	$(1^4 2^2, 12)$	1	$\overline{7}_2$	$(\overline{17}, 1^3)$	7
	$(1^2 24, 12)$	4	$\overline{7}_2$	$(\overline{17}, 1^3)$	7
	$(4^2, 12)$	1	$\overline{14}_2$	$(\overline{17}, 1^3)$	1
	$(4^2, 1^3)$	1		$(\overline{17}, 12)$	2
8_1	$(1^2 24, 1^3)$	1	$\overline{14}_2$	$(\overline{17}, 1^3)$	1
	$(1^2 24, 12)$	1		$(\overline{17}, 12)$	2
8_2	$(4^2, 12)$	1	$\overline{28}$	$(\overline{17}, 12)$	1
	$(1^2 24, 12)$	1	$\overline{28}$	$(\overline{17}, 12)$	1
3_1	$(1^5 3, 3)$	21	$\overline{21}_1$	$(\overline{17}, 3)$	1
3_2	$(1^8, 3)$	1	$\overline{21}_1$	$(\overline{17}, 3)$	1
	$(1^2 3^2, 1^3)$	35			
3_3	$(1^5 3, 1^3)$	1			
	$(1^2 3^2, 3)$	5			
6_1	$(1^5 3, 3)$	1			
	$(12^2 3, 3)$	4			
6_2	$(1^4 2^2, 3)$	1			
	$(1^2 3^2, 1^3)$	1			
	$(26, 12)$	6			

TABLE III. *Conjugacy classes of $G_{(3^2)}$*

$\rightarrow GL_6(2)$	$PSL_2(7) \times PSL_2(7)$ -Cosets	Ratio	$\rightarrow GL_6(2)$	$PSL_2(7) \times PSL_2(7)$ -Cosets	Ratio
1	$(1^7, 1^7)$	1395		$(1^7, 13^2)$	1
2_1	$(1^7, 1^7)$	35	6_3	$(1^3 2^2, 13^2)$	6
	$(1^7, 1^3 2^2)$	120		$(13^2, 1^7)$	1
	$(1^3 2^2, 1^7)$	120		$(13^2, 1^3 2^2)$	6
2_2	$(1^7, 1^7)$	3	6_4	$(13^2, 13^2)$	1
	$(1^7, 1^3 2^2)$	12	6_5	$(13^2, 13^2)$	5
	$(1^3 2^2, 1^3 2^2)$	72	6_6	$(13^2, 13^2)$	3
	$(1^3 2^2, 1^7)$	12			
2_3	$(1^7, 1^7)$	1	12_2	$(13^2, 124)$	1
	$(1^3 2^2, 1^3 2^2)$	42		$(124, 13^2)$	1
4_1	$(1^7, 124)$	8	12_3	$(1^3 2^2, 13^2)$	1
	$(1^7, 1^3 2^2)$	7		$(13^2, 124)$	2
	$(1^3 2^2, 1^3 2^2)$	21		$(13^2, 1^3 2^2)$	1
	$(1^3 2^2, 1^7)$	7		$(124, 13^2)$	2
	$(124, 1^7)$	8	$\overline{7}_1$	$(7_1, 7_1)$	9
4_2	$(1^7, 1^3 2^2)$	1	$\overline{7}_1$	$(7_2, 7_2)$	9
	$(1^3 2^2, 124)$	8			
	$(1^3 2^2, 1^3 2^2)$	9	$\overline{7}_2$	$(1^7, 7_1)$	1
	$(1^3 2^2, 1^7)$	1		$(7_1, 1^7)$	1
	$(124, 1^3 2^2)$	8			
4_3	$(1^3 2^2, 1^3 2^2)$	3	$\overline{7}_2$	$(1^7, 7_2)$	1
	$(124, 124)$	12		$(7_2, 1^7)$	1
4_4	$(1^7, 124)$	1	7_3	$(7_1, 7_2)$	1
	$(1^3 2^2, 124)$	3		$(7_2, 7_1)$	1
	$(1^3 2^2, 1^3 2^2)$	3	$\overline{14}_1$	$(7_1, 7_1)$	1
	$(124, 1^3 2^2)$	3	$\overline{14}_1$	$(7_2, 7_2)$	1
	$(124, 1^7)$	1			
			$\overline{14}_2$	$(1^3 2^2, 7_1)$	1
4_5	$(1^3 2^2, 1^3 2^2)$	1		$(7_1, 1^3 2^2)$	1
	$(1^3 2^2, 124)$	2	$\overline{14}_2$	$(1^3 2^2, 7_2)$	1
	$(124, 124)$	2		$(7_2, 1^3 2^2)$	1
	$(124, 1^3 2^2)$	2			
8_1	$(1^3 2^2, 124)$	1	$\overline{28}$	$(124, 7_1)$	1
	$(124, 124)$	1		$(7_1, 124)$	1
	$(124, 1^3 2^2)$	1			
8_2	$(124, 124)$	1	$\overline{28}$	$(124, 7_2)$	1
3_2	$(1^7, 13^2)$	15		$(7_2, 124)$	1
	$(13^2, 1^7)$	15			
3_3	$(13^2, 13^2)$	15	$\overline{21}_1$	$(13^2, 7_1)$	1
6_2	$(13^2, 1^3 2^2)$	3		$(7_1, 13^2)$	1
	$(1^3 2^2, 13^2)$	3	$\overline{21}_1$	$(13^2, 7_2)$	1
				$(7_2, 13^2)$	1

TABLE V. The character table of $GL_6(2)$

x	1	2_1	2_2	2_3	4_1	4_2	4_3	4_4	4_5	8_1	8_2	3_1	3_2	3_3	6_1	6_2	6_3	6_4	6_5	6_6
$ C(x) $	$ G $	$2^{15} \cdot 3^2 \cdot 5 \cdot 7$	$2^{14} \cdot 3^2$	$2^{12} \cdot 3 \cdot 7$	$2^{11} \cdot 3 \cdot 7$	$2^{11} \cdot 3 \cdot 7$	$2^9 \cdot 3$	$2^8 \cdot 3$	2^8	2^6	2^5	$2^6 \cdot 3^4 \cdot 5 \cdot 7$	$2^6 \cdot 3^3 \cdot 5 \cdot 7$	$2^3 \cdot 3^3 \cdot 5$	$2^6 \cdot 3^3 \cdot 5$	$2^5 \cdot 3^2$	$2^6 \cdot 3^2$	$2^3 \cdot 3$	$2^3 \cdot 3^2 \cdot 5$	$2^3 \cdot 3^2$
X_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X_2	62	30	14	6	14	6	2	6	2	2	2	-1	14	2	-1	2	6	1	1	2
X_3	588	140	44	28	28	12	4	4	4	4	4	21	21	3	5	5	5	1	5	-1
X_4	744	104	40	8	8	8	8	8	8	8	8	-21	-6	9	-5	-2	2	-1	-1	1
X_5	1240	280	56	8	56	8	8	8	8	8	8	-20	55	-5	-4	-1	7	-1	-5	-1
X_6	6480	720	144	48	48	16	8	8	8	8	8	20	50	-10	4	-2	-6	1	1	-2
X_7	9920	960	64	64	64	64	64	64	64	64	64	84	-36	-3	4	4	-4	1	5	1
X_8	5952	320	64	64	64	64	64	64	64	64	64	-84	-84	6	-4	-4	-4	-4	-4	2
X_9	18816	896	128	128	128	128	128	128	128	128	128	64	-56	4	-	-8	-8	-	4	-
X_{10}	31744	1024	1024	1024	1024	1024	1024	1024	1024	1024	1024	-64	-64	8	-	-	-	-	-	-
X_{11}	32768	1024	1024	1024	1024	1024	1024	1024	1024	1024	1024	-64	-64	8	-	-	-	-	-	-
X_{12}	19845	-315	-27	21	21	5	-3	-3	-3	1	1	1	1	1	1	1	1	1	1	1
X_{13}	19845	-315	-27	21	21	5	-3	-3	-3	1	1	1	1	1	1	1	1	1	1	1
X_{14}	19845	-315	-27	21	21	5	-3	-3	-3	1	1	1	1	1	1	1	1	1	1	1
X_{15}	19845	-315	-27	21	21	5	-3	-3	-3	1	1	1	1	1	1	1	1	1	1	1
X_{16}	19845	-315	-27	21	21	5	-3	-3	-3	1	1	1	1	1	1	1	1	1	1	1
X_{17}	19845	-315	-27	21	21	5	-3	-3	-3	1	1	1	1	1	1	1	1	1	1	1
X_{18}	651	139	11	-21	27	-5	-5	3	-5	-1	-1	21	36	6	5	-4	4	1	4	2
X_{19}	9114	826	10	-14	42	-14	-10	-6	-2	-2	-2	-21	84	9	-5	4	4	1	1	1
X_{20}	27342	231	-89	7	7	7	7	-1	-1	-1	-1	126	21	6	-2	1	-3	-2	6	-2
X_{21}	13020	540	60	-84	-20	-4	4	-4	-4	-4	-4	-126	-21	12	2	-1	3	-	-	-4
X_{22}	36456	1064	8	56	-56	-8	-8	-8	-8	-8	-8	105	-15	-9	9	-3	9	-	-	-
X_{23}	4557	77	13	77	-35	-3	-3	-3	5	1	1	-84	21	-9	-4	5	5	-1	-1	-1
X_{24}	18228	308	-76	84	-28	4	4	-4	4	-	-	84	42	6	5	-2	2	-1	-1	2
X_{25}	13671	231	-89	7	7	7	7	-1	-1	-1	-1	126	21	6	-2	1	-3	-2	6	-2
X_{26}	13671	231	-89	7	7	7	7	-1	-1	-1	-1	-63	21	-3	1	1	-3	1	-3	1
X_{27}	9114	-70	74	-14	-70	2	6	2	-2	2	2	-84	63	3	-4	-1	-1	1	5	-1
X_{28}	29295	-465	15	63	-17	-1	-9	7	-1	-1	-1	-	45	-	-	-3	-3	-	-	-

TABLE V (continued)

x	12_1	12_2	12_3	9	5	10	$\overline{15}_1$	$\overline{15}_1$	$\overline{15}_2$	$\overline{15}_2$	15_3
$ C(x) $	$2^4 \cdot 3$	$2^3 \cdot 3$	$2^4 \cdot 3$	$3^2 \cdot 7$	$2 \cdot 3^2 \cdot 5$	$2 \cdot 3 \cdot 5$	$2 \cdot 3^2 \cdot 5$	$2 \cdot 3^2 \cdot 5$	$3^2 \cdot 5$	$3^2 \cdot 5$	$3^2 \cdot 5$
X_1	1	1	1	1	1	1	1	1	1	1	1
X_2	-1	.	2	-1	2	.	2	2	-1	-1	-1
X_3	1	1	1	.	-2	.	-2	-2	1	1	.
X_4	-1	.	2	.	-1	-1	-1	-1	-1	-1	-1
X_5	.	-1	-1	1	.	.	.	.	.	.	.
X_6	.	.	.	.	.	.	.	.	.	.	.
X_7	.	.	-2	-1	.	.	.	.	.	.	.
X_8	.	.	.	.	2	.	2	2	-1	-1	-1
X_9	.	.	.	.	1	1	1	1	1	1	1
X_{10}	.	.	.	1	-1	-1	-1	-1	-1	-1	-1
X_{11}	.	.	.	-1	-2	.	-2	-2	1	1	1
X_{12}	.	.	.	.	.	.	.	.	.	.	.
X_{13}	.	.	.	.	.	.	.	.	.	.	.
X_{14}	.	.	.	.	.	.	.	.	.	.	.
X_{15}	.	.	.	.	.	.	.	.	.	.	.
X_{16}	.	.	.	.	.	.	.	.	.	.	.
X_{17}	.	.	.	.	.	.	.	.	.	.	.
X_{18}	1	.	.	.	1	-1	1	1	1	1	1
X_{19}	-1	.	.	.	-1	1	-1	-1	-1	-1	-1
X_{20}	-2	-1	1	.	1	1	1	1	1	1	1
X_{21}	2	1	-1	.	2	.	2	2	-1	-1	-1
X_{22}	1	-1	1	.	.	.	.	.	.	.	.
X_{23}	.	1	1	.	1	-1	1	1	1	1	1
X_{24}	.	.	-2	.	2	2	-1	-1	-1	-1	2
X_{25}	1	-1	-1	.	-2	-2	1	1	1	1	-2
X_{26}	1	-1	1	.	1	1	$\bar{\rho}$	$\bar{\rho}$	ρ	ρ	1
X_{27}	1	-1	1	.	1	1	$\bar{\rho}$	$\bar{\rho}$	ρ	ρ	1
X_{28}	.	-1	-1	.	4	.	-2	-2	1	1	-2
X_{29}	.	1	1	.	.	.	.	.	.	.	.

TABLE V (continued)

[illegible]

TABLE V (continued)

x	1	2_1	2_2	2_3	4_1	4_2	4_3	4_4	4_5	8_1	8_2	3_1	3_2	3_3	6_1	6_2	6_3	6_4	6_5	6_6
$ C(x) $	$ G $	$2^{15} \cdot 3^2 \cdot 5 \cdot 7$	$2^{14} \cdot 3^2$	$2^{12} \cdot 3 \cdot 7$	$2^{11} \cdot 3 \cdot 7$	$2^{11} \cdot 3 \cdot 7$	$2^9 \cdot 3$	$2^8 \cdot 3$	2^8	2^6	2^5	$2^6 \cdot 3^4 \cdot 5 \cdot 7$	$2^6 \cdot 3^3 \cdot 5 \cdot 7$	$2^3 \cdot 3^3 \cdot 5 \cdot 7$	$2^6 \cdot 3^2$	$2^5 \cdot 3^2$	$2^6 \cdot 3^2$	$2^3 \cdot 3$	$2^3 \cdot 3^2 \cdot 5$	$2^3 \cdot 3^2$
X ₃₀	29295	-465	15	63	-17	-1	-9	7	-1	-1	-1	.	45	.	.	-3	-3	.	.	.
X ₃₁	27342	-210	-34	-42	14	6	2	-2	2	-2	.	63	-21	-6	-1	-1	3	.	.	2
X ₃₂	27342	-210	-34	-42	14	6	2	-2	2	-2	.	63	-21	-6	-1	-1	3	.	.	2
X ₃₃	36456	-280	40	-56	-56	8	8	.	.	.	.	-21	42	-3	-5	-2	2	1	5	1
X ₃₄	41664	-64	64	.	-64	.	.	.	.	.	.	84	-6	-6	4	-2	2	.	-4	-2
X ₃₅	13671	-441	55	-49	7	-1	-5	-1	3	-1	1	126	21	6	-2	1	-3	2	-6	-2
X ₃₆	13671	-441	55	-49	7	-1	-5	-1	3	-1	1	-63	21	-3	1	1	-3	-1	3	1
X ₃₇	13671	-441	55	-49	7	-1	-5	-1	3	-1	1	-63	21	-3	1	1	-3	-1	3	1
X ₃₈	4185	345	-39	-39	25	-7	9	1	1	1	1	.	45	.	.	-3	-3	.	.	.
X ₃₉	4185	345	-39	-39	25	-7	9	1	1	1	1	.	45	.	.	-3	-3	.	.	.
X ₄₀	25110	630	-90	-66	6	-2	-6	6	2	2	.	.	.	.	.	.	.	.	.	.
X ₄₁	25110	630	-90	-66	6	-2	-6	6	2	2	.	.	.	.	.	.	.	.	.	.
X ₄₂	33480	-120	-24	24	8	-8	.	8	.	.	.	.	-45	.	.	3	3	.	.	.
X ₄₃	33480	-120	-24	24	8	-8	.	8	.	.	.	.	-45	.	.	3	3	.	.	.
X ₄₄	12555	-405	21	51	27	-13	15	3	-1	-1	1	.	-45	.	.	.	.	.	.	.
X ₄₅	9765	-315	21	-3	21	-3	1	-3	1	1	-1	45	.	.	-3	.	.	.	.	.
X ₄₆	217	-7	9	-15	-7	1	5	1	-3	1	-1	19	7	4	3	3	-1	.	-4	.
X ₄₇	4340	-140	52	-76	-28	4	4	4	-4	.	.	65	35	5	1	7	-5	-1	-5	1
X ₄₈	13888	-448	64	-64	.	.	.	.	.	.	.	-44	28	1	4	4	-4	-1	-1	1
X ₄₉	9765	-315	21	-3	21	-3	1	-3	1	1	-1	-90	.	.	6	.	.	.	.	.
X ₅₀	9765	-315	21	-3	21	-3	1	-3	1	1	-1	-90	.	.	6	.	.	.	.	.
X ₅₁	11160	-360	24	24	24	-8	8	.	.	.	.	-45	.	.	3	.	.	.	.	.
X ₅₂	11160	-360	24	24	24	-8	8	.	.	.	.	-45	.	.	3	.	.	.	.	.
X ₅₃	1395	-45	3	27	3	-5	7	3	-1	-1	1	45	.	.	-3	.	.	.	.	.
X ₅₄	1395	-45	3	27	3	-5	7	3	-1	-1	1	45	.	.	-3	.	.	.	.	.
X ₅₅	9765	-315	21	-3	21	-3	1	-3	1	1	-1	45	.	.	-3	.	.	.	.	.
X ₅₆	9765	-315	21	-3	21	-3	1	-3	1	1	-1	45	.	.	-3	.	.	.	.	.
X ₅₇	9765	-315	21	-3	21	-3	1	-3	1	1	-1	45	.	.	-3	.	.	.	.	.
X ₅₈	9765	-315	21	-3	21	-3	1	-3	1	1	-1	45	.	.	-3	.	.	.	.	.
X ₅₉	9765	-315	21	-3	21	-3	1	-3	1	1	-1	45	.	.	-3	.	.	.	.	.
X ₆₀	9765	-315	21	-3	21	-3	1	-3	1	1	-1	45	.	.	-3	.	.	.	.	.

TABLE V (continued)

x	$\overline{63}_1$	$\overline{63}_1$	$\overline{63}_1$	$\overline{63}_2$	$\overline{63}_2$	$\overline{63}_3$	$\overline{63}_3$	$\overline{63}_3$	$\overline{31}_1$	$\overline{31}_1$	$\overline{31}_2$	$\overline{31}_2$	$\overline{31}_3$	$\overline{31}_3$
$ C(x) $	$3^2 \cdot 7$	$3^2 \cdot 7$	$3^2 \cdot 7$	$3^2 \cdot 7$	$3^2 \cdot 7$	$3^2 \cdot 7$	$3^2 \cdot 7$	$3^2 \cdot 7$	31	31	31	31	31	31
χ_{30}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{31}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{32}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{33}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{34}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{35}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{36}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{37}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{38}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{39}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{40}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{41}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{42}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{43}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{44}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{45}	.	.	.	.	.	.	.	.	.	.	.	.	.	.
χ_{46}	1	1	1	1	1	1	1	1	.	.	.	.	.	.
χ_{47}	-1	-1	-1	-1	-1	-1	-1	-1	.	.	.	.	.	.
χ_{48}	1	1	1	1	1	1	1	1	.	.	.	.	.	.
χ_{49}	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	.	.	.	.	.	.
χ_{50}	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	.	.	.	.	.	.
χ_{51}	$-\omega$	$-\overline{\omega}$	$-\omega$	$-\overline{\omega}$	$-\omega$	$-\overline{\omega}$	$-\omega$	$-\overline{\omega}$	.	.	.	.	.	.
χ_{52}	$-\overline{\omega}$	$-\omega$	$-\overline{\omega}$	$-\omega$	$-\overline{\omega}$	$-\omega$	$-\overline{\omega}$	$-\omega$	.	.	.	.	.	.
χ_{53}	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	.	.	.	.	.	.
χ_{54}	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	$\overline{\omega}$	ω	.	.	.	.	.	.
χ_{55}	$2\omega - 1$	$2\overline{\omega} - 1$	$-\omega + 2$	$-\overline{\omega} + 2$	$-\omega + 2$	$-\overline{\omega} + 2$	$-\omega + 2$	$-\overline{\omega} + 2$	.	.	.	.	.	.
χ_{56}	$2\overline{\omega} - 1$	$2\omega - 1$	$-\overline{\omega} + 2$	$-\omega + 2$	$-\overline{\omega} + 2$	$-\omega + 2$	$-\overline{\omega} + 2$	$-\omega + 2$	.	.	.	.	.	.
χ_{57}	$-\omega + 2$	$-\overline{\omega} + 2$	$-\omega - 1$	$-\overline{\omega} - 1$	$-\omega - 1$	$-\overline{\omega} - 1$	$-\omega - 1$	$-\overline{\omega} - 1$	.	.	.	.	.	.
χ_{58}	$-\overline{\omega} + 2$	$-\omega + 2$	$-\overline{\omega} - 1$	$-\omega - 1$	$-\overline{\omega} - 1$	$-\omega - 1$	$-\overline{\omega} - 1$	$-\omega - 1$	.	.	.	.	.	.
χ_{59}	$-\omega - 1$	$-\overline{\omega} - 1$	$2\omega - 1$	$2\overline{\omega} - 1$	$2\omega - 1$	$2\overline{\omega} - 1$	$2\omega - 1$	$2\overline{\omega} - 1$	.	.	.	.	.	.
χ_{60}	$-\overline{\omega} - 1$	$-\omega - 1$	$2\overline{\omega} - 1$	$2\omega - 1$	$2\overline{\omega} - 1$	$2\omega - 1$	$2\overline{\omega} - 1$	$2\omega - 1$	.	.	.	.	.	.