Tables of the Roots of the Associated Legendre Function with Respect to the Degree

By Helmut F. Bauer

Abstract. Tables of the numerical value of the roots with respect to the degree of the Legendre functions of the first kind and those of the first derivative are presented.

Determination of the Roots. In a large number of physical and engineering problems involving conical boundaries, the roots with respect to the degree of the associated Legendre function or those of its first derivative are of importance. Liquid oscillations in a conical container [1], wave propagation in conical horns [2], [3], heat and mass transport in conical systems [4], and many other problems require knowledge of these roots. For the associated Legendre function, as well as for its first derivative, there exists an infinite number of simple roots with respect to the degree. Some of these roots have been given by Pal [5] and later by Bauer [6], who presented better accuracy and found, in addition, that Pal did not detect for $m \geq 1$ the lowest root with respect to the degree of the first derivative of the associated Legendre function. In the following tables (see the supplements section at the end of this issue), the roots of $P_\lambda^m(\cos \alpha) = 0$ (Tables 1.1 through 1.35) and those of $\partial P_\lambda^m / \partial \alpha = 0$ (Tables 2.1 through 2.35) are presented for integer m and given α , ranging from 5° in steps of 5° to 175° . From the representation of the associated Legendre function as a hypergeometric series, i.e.,

$$P_\lambda^m(\cos \alpha) = \frac{(-1)^m \Gamma(m + \lambda + 1) \sin^m \alpha}{2^m m! \Gamma(\lambda - m + 1)} {}_2F_1(1 + m + \lambda, m - \lambda, 1 + m; \tfrac{1}{2}(1 - \cos \alpha))$$

one obtains, after introducing the hypergeometric function ${}_2F_1$ as an infinite series, the equation

$$(1) \quad \sum_{n=0}^{\infty} \frac{(m + \lambda + 1)_n (m - \lambda)_n m!}{n! (m + n)!} \sin^{2n} \left(\frac{\alpha}{2} \right) = 0$$

for the determination of the zeros λ_{mn} of $P_\lambda^m(\cos \alpha) = 0$ (Tables 1.1-1.35). In this equation the Pochhammer symbol $(a)_n = a(a+1)(a+2) \cdots (a+n-1)$, $(a)_0 = 1$ has been used. For the determination of the roots of the first derivative of the associated Legendre function

$$\frac{d}{d\alpha} [P_\lambda^m(\cos \alpha)] = 0$$

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one obtains with the recurrence formula

$$(1 - x^2) \frac{dP_{\lambda'}^m}{dx} = (\lambda' + m) P_{\lambda'-1}^m(x) - \lambda' x P_{\lambda'}^m(x)$$

an equation of the form

$$(2) \quad [\lambda' - m - \lambda' \cos \alpha] + \sum_{n=1}^{\infty} \frac{m!(m + \lambda' + 1)_{n-1}(m - \lambda' + 1)_{n-1}}{n!(n + m)!} \sin^{2n} \left(\frac{\alpha}{2} \right) \\ \times [(\lambda'^2 - m^2)(m - \lambda' + n) - \lambda'(m - \lambda')(m + n + \lambda') \cos \alpha] = 0. \\ \text{(Tables 2.1--2.35)}$$

From these equations (1) and (2), the roots λ_{mn} and λ'_{mn} have been obtained respectively for the above given values of α , where for each $m = 0, 1, 2, 3, 4, 5$ the first ten roots ($n = 1, 2, \dots, 10$) are presented. The roots have been obtained by the regula falsi method. The accuracy of the roots is 10^{-8} , but they were rounded to four digits. Their value was always introduced into the function and led always to values of less than 10^{-10} .

It may be mentioned that reference [6] contains a bibliography of tables of Legendre functions, and that to these references more recent ones [7], [8], [9], should be added.

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1. H. F. BAUER, "Flüssigkeitsschwingungen in Kegelbehälterformen," *Acta Mech.*, v. 43, 1982, pp. 185–200.
2. H. BUCHHOLZ, "Die Bewegung elektromagnetischer Wellen in einem kegelförmigen Horn," *Ann. Physik*, v. 37, Heft 3, 1940, pp. 173–225.
3. H. BUCHHOLZ, "Das Schallfeld in einem Kegelhorn unter verschiedenen Anfangsbedingungen," *Akust. Z.*, v. 5, Heft 4, 1940, pp. 169–189.
4. H. S. CARSLAW & J. C. JAEGER, *Conduction of Heat in Solids*, Clarendon Press, Oxford, 1959.
5. B. PAL, "On the numerical calculation of the roots of the equations $P_n^m(\mu) = 0$ and $dP_n^m(\mu)/d\mu = 0$ regarded as equations in n ," *Bull. Calcutta Math. Soc.*, Part I, v. 9, 1917, pp. 85–95; Part II, v. 10, 1918, pp. 187–194.
6. H. F. BAUER, "On the numerical value of the roots of the associated Legendre function with respect to the order," *Z. Angew. Math. Mech.*, v. 61, 1981, pp. 525–527.
7. J. M. SMITH, F. W. J. OLVER & D. W. LOZIER, "Extended-range arithmetic and normalized Legendre polynomials," *ACM Trans. Math. Software*, v. 7, 1981, pp. 93–105.
8. D. W. LOZIER & J. M. SMITH, "Algorithm 567. Extended-range arithmetic and normalized Legendre polynomials," *ACM Trans. Math. Software*, v. 7, 1981, pp. 141–146.
9. F. W. J. OLVER & J. M. SMITH, "Associated Legendre functions on the cut," *J. Comput. Phys.*, v. 51, 1983, pp. 502–518.

Supplement to Table of the Roots of the Associated Legendre Function with Respect to the Degree

By Helmut F. Bauer

$\alpha = 5^\circ$

$\begin{smallmatrix} n \\ m \end{smallmatrix}$	0	1	2	3	4	5
1	27.0558	43.4110	58.3605	72.6312	86.4862	100.0549
2	62.7548	79.8943	95.9610	111.3661	126.3129	140.9192
3	98.6640	116.0804	132.6583	148.6530	164.2131	179.4337
4	134.6207	152.1791	169.0528	185.4151	201.3771	217.0153
5	170.5955	188.2402	205.3074	221.9221	238.1701	254.1134
6	206.5789	224.2817	241.4855	258.2847	274.7472	290.9244
7	242.5673	260.3118	277.6172	294.5572	311.1870	327.5495
8	278.5586	296.3346	313.7186	330.7697	347.5337	364.0467
9	314.5519	332.3525	349.7991	366.9401	383.8141	400.4522
10	350.5466	368.3669	385.8645	403.0799	420.0458	436.7893

Table 1.1: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 5^\circ$

$\alpha = 10^\circ$

$\begin{smallmatrix} n \\ m \end{smallmatrix}$	0	1	2	3	4	5
1	13.2756	21.4598	28.9462	36.0956	43.0385	49.8392
2	31.1264	39.6995	47.7402	55.4527	62.9376	70.2535
3	49.0814	57.7918	66.0862	74.0912	81.3305	89.5013
4	67.0599	75.8408	84.2819	92.4693	100.4581	108.2862
5	85.0474	93.8711	102.4083	110.7209	118.8516	126.8311
6	103.0362	111.8917	120.4967	128.9008	137.1380	145.2335
7	121.0334	129.9066	138.5620	147.0360	155.3562	163.5436
8	139.0291	147.9180	156.6123	165.1415	173.5282	181.7904
9	157.0258	165.9268	174.6522	183.2260	191.6673	199.9916
10	175.0231	183.9340	192.6847	201.2954	209.7823	218.1588

Table 1.2: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 10^\circ$

$\alpha = 15^\circ$

$\begin{smallmatrix} n \\ m \end{smallmatrix}$	0	1	2	3	4	5
1	8.6812	14.1446	19.1486	23.9306	28.5764	33.1284
2	20.5832	26.3023	31.6710	36.8237	41.8265	46.7180
3	32.5535	38.3630	43.8987	49.2439	54.4471	59.5394
4	44.5394	50.3952	56.0275	61.4927	66.8272	72.0559
5	56.5312	62.4152	68.1106	73.6583	79.0862	84.4146
6	68.5258	74.4287	80.1688	85.7767	91.2746	96.6794
7	80.5220	86.4386	92.2118	97.8656	103.4182	108.8835
8	92.5191	98.4460	104.2449	109.9350	115.5314	121.0459
9	104.5170	110.4518	116.2711	121.9907	127.6230	133.1783
10	116.5152	122.4565	128.2925	134.0363	139.6987	145.2884

Table 1.3: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 15^\circ$

$\alpha = 20^\circ$

$n \backslash m$	0	1	2	3	4	5
1	6.3832	10.4895	14.2553	17.8583	21.3609	24.7942
2	15.3112	19.6044	23.6398	27.5159	31.2816	34.9653
3	24.2894	32.6729	40.7386	48.5702	56.1985	63.7617
4	33.2790	46.6876	59.2633	71.0084	82.0291	93.2147
5	42.2729	59.6975	76.0062	91.6215	106.4379	121.4897
6	51.2690	72.7103	89.0378	104.6222	120.4536	135.9559
7	60.2662	84.7048	102.0378	119.0622	135.4536	151.5599
8	69.2641	97.7103	118.0622	136.0622	156.4536	172.1647
9	78.2625	110.7103	131.0622	151.0622	171.4536	192.7695
10	87.2612	123.7103	146.0622	166.0622	186.4536	213.3742

Table 1.4: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 20^\circ$ $\alpha = 25^\circ$

$n \backslash m$	0	1	2	3	4	5
1	5.0038	8.2960	11.3238	14.2234	17.0444	19.8112
2	12.1478	15.5864	18.8237	21.9367	24.9634	27.9260
3	19.3328	22.8213	26.1545	29.3782	32.7282	36.0584
4	26.5226	30.0398	33.4285	36.7212	39.9387	43.0954
5	33.7179	37.2513	40.6763	44.0163	47.2875	50.5015
6	40.9148	44.4591	47.9097	51.2843	54.5959	57.8538
7	48.1126	51.6647	55.1344	58.5355	61.8784	65.1711
8	55.3110	58.8690	62.3534	65.7754	69.1434	72.4644
9	62.5097	66.0723	69.5685	73.0073	76.3961	79.7405
10	69.7087	73.2750	76.7808	80.2335	83.6395	87.0038

Table 1.5: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 25^\circ$ $\alpha = 30^\circ$

$n \backslash m$	0	1	2	3	4	5
1	4.0837	6.8354	9.3733	11.8074	14.1778	16.5041
2	10.0386	12.9083	15.6194	18.2215	20.7588	23.2267
3	16.0248	18.9364	21.7210	24.4170	27.0476	29.5217
4	22.0113	25.9214	27.7807	30.2355	32.3638	34.8654
5	28.0145	30.8606	32.8423	36.0622	39.3638	42.0320
6	34.0170	36.8616	38.9123	41.8913	46.4313	48.1549
7	40.0195	42.8622	44.9213	47.9197	52.4597	54.2149
8	46.0220	48.8622	50.9313	53.9481	58.4881	60.3251
9	52.0245	54.8622	56.9413	60.9765	64.5165	66.3866
10	58.0271	60.8622	62.9445	66.9848	70.5448	72.4377

Table 1.6: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 30^\circ$ $\alpha = 35^\circ$

$n \backslash m$	0	1	2	3	4	5
1	3.4260	5.7930	7.9835	10.0881	12.1400	14.1554
2	8.5318	10.9958	13.3258	15.5727	17.7621	19.9086
3	13.6633	16.1619	18.5556	20.8764	23.1427	25.3664
4	18.8008	21.3171	23.7478	26.1145	28.4312	30.7072
5	23.9405	26.4675	28.9224	31.3207	33.6932	35.9675
6	29.0813	31.6156	34.0875	36.5089	38.8882	41.1237
7	34.2227	36.7622	39.2469	41.6858	44.9592	46.1558
8	39.3645	41.9079	44.4024	46.8532	49.0307	51.1558
9	44.5063	47.0500	49.5024	51.9501	54.0007	56.8513
10	49.6487	52.1977	54.7063	57.1797	59.6219	62.0363

Table 1.7: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 35^\circ$ $\alpha = 40^\circ$

$n \backslash m$	0	1	2	3	4	5
1	2.9323	5.0120	6.9442	8.8045	10.6206	12.4060
2	7.1635	10.5813	13.1602	15.8929	18.7459	21.6189
3	11.3926	14.8813	16.1830	18.2239	20.2192	22.7789
4	15.6217	18.5916	20.7241	22.8033	24.8407	26.8442
5	20.8506	23.0979	25.2507	27.3562	29.4234	31.4588
6	25.3832	27.6022	29.7892	31.8940	33.9837	36.0435
7	29.8820	32.1054	34.2830	36.4224	38.5295	40.6087
8	34.3811	36.6078	38.7935	40.9446	43.0657	45.1606
9	38.8804	41.1096	43.3019	45.4624	47.5945	49.7029
10	43.3799	45.6111	47.8087	49.9769	52.1191	54.2381

Table 1.8: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 40^\circ$ $\alpha = 45^\circ$

$n \backslash m$	0	1	2	3	4	5
1	2.5479	4.4053	6.1388	7.8117	9.4472	11.0569
2	6.5222	8.4471	10.2781	12.0508	13.7831	15.4852
3	10.5143	12.4633	14.3389	16.1634	17.9497	19.7060
4	14.5106	16.4719	18.3734	20.2301	22.0516	23.8447
5	18.5083	20.4773	22.3957	24.2745	26.1211	27.9410
6	22.5069	24.4809	26.4113	28.3062	30.1716	32.0120
7	26.5059	28.4835	30.4228	32.3300	34.2101	36.0668
8	30.5051	32.4855	34.4316	36.3486	38.2404	40.1103
9	34.5045	36.4871	38.4387	40.3635	42.2650	44.1458
10	38.5041	40.4884	42.4444	44.3758	46.2853	48.1753

Table 1.9: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 45^\circ$

$\alpha = 50^\circ$

$n \backslash m$	0	1	2	3	4	5
1	2.2400	3.9207	5.4972	7.0226	8.5165	9.9886
2	5.8186	7.5556	9.3138	10.8313	12.3984	13.9463
3	13.4160	18.1592	22.6849	26.9376	30.9882	34.8882
4	21.0059	29.1735	36.1735	42.9538	49.4638	55.7599
5	28.5958	39.0900	48.1735	55.9374	63.5474	70.8605
6	36.1857	49.5842	59.6637	69.2804	78.6595	87.0805
7	43.7756	59.0745	69.1540	78.8671	88.0654	96.4864
8	51.3655	68.5644	78.6439	88.3522	97.4511	105.8722
9	58.9554	78.0543	88.1338	97.4317	106.8370	115.2580
10	66.5453	87.5442	97.6235	106.8112	116.2228	124.6438

Table 1.10: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 50^\circ$

$\alpha = 55^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.9878	3.5249	4.9748	6.3821	7.7628	9.1252
2	5.2428	6.8267	8.3447	9.8219	11.2707	12.6982
3	8.5100	10.1107	11.6600	13.1734	14.6601	16.1257
4	11.7801	13.3895	14.9568	16.4928	18.0042	19.4956
5	15.0513	16.6659	18.2458	19.7966	21.3256	22.6358
6	18.3220	19.9412	21.5260	23.0916	24.6337	26.1583
7	21.5927	23.2115	24.8028	26.2640	27.5334	29.4952
8	24.8634	26.4823	28.0885	29.4365	31.7572	32.1701
9	28.1341	29.7530	31.3662	32.9099	34.5172	35.1445
10	31.4048	33.0237	34.6429	36.2312	37.8042	39.3655

Table 1.11: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 55^\circ$

$\alpha = 60^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.7773	3.1957	4.5422	5.8532	7.1423	8.4162
2	4.7628	6.2195	7.6220	8.9908	10.3361	11.6638
3	7.7583	9.2288	10.6571	12.0557	13.4322	14.7914
4	10.7548	12.2338	13.6769	15.0942	16.4911	17.8716
5	13.7503	15.2369	16.6988	18.1198	19.5312	20.9272
6	16.7458	18.2390	19.6988	21.1381	22.5604	23.9682
7	19.7413	21.2405	22.7054	24.1519	25.5826	26.9988
8	22.7368	24.2416	25.7105	27.1626	28.6001	30.0250
9	25.7323	27.2426	28.7146	30.1742	31.6145	33.0455
10	28.7278	30.2433	31.7179	33.1783	34.6260	36.0655

Table 1.12: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 60^\circ$

$\alpha = 65^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.5988	2.9178	4.1786	5.4106	6.6249	7.8267
2	4.0080	5.2655	6.4909	7.6939	8.8859	10.0758
3	6.5766	7.8436	9.0843	10.3062	11.5140	12.7109
4	9.1457	10.4167	11.6598	12.8868	14.1025	15.3262
5	11.7148	12.9917	14.2367	15.4598	16.7024	17.8939
6	14.2839	15.5608	16.8058	18.0723	19.3094	20.5362
7	16.8530	18.1369	19.4005	20.6524	21.8945	23.1280
8	19.4221	20.7090	21.9751	23.2306	24.4770	25.7153
9	22.0012	23.2810	24.5491	25.8075	27.0573	28.2996
10	24.5729	25.8529	27.1226	28.3834	29.6361	30.8818

Table 1.13: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 65^\circ$

$\alpha = 70^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.4456	2.6803	3.8695	5.0362	6.1889	7.3318
2	4.0080	5.2655	6.4909	7.6939	8.8859	10.0758
3	6.5766	7.8436	9.0843	10.3062	11.5140	12.7109
4	9.1457	10.4167	11.6598	12.8868	14.1025	15.3262
5	11.7148	12.9917	14.2367	15.4598	16.7024	17.8939
6	14.2839	15.5608	16.8058	18.0723	19.3094	20.5362
7	16.8530	18.1369	19.4005	20.6524	21.8945	23.1280
8	19.4221	20.7090	21.9751	23.2306	24.4770	25.7153
9	22.0012	23.2810	24.5491	25.8075	27.0573	28.2996
10	24.5729	25.8529	27.1226	28.3834	29.6361	30.8818

Table 1.14: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 70^\circ$

$\alpha = 75^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.3124	2.4751	3.6042	4.7167	5.8188	6.9137
2	3.7059	4.8859	6.0407	7.1799	8.3081	9.4280
3	6.1038	7.2902	8.4509	9.6099	10.7526	11.8872
4	8.5028	9.6955	10.8661	12.0277	13.1799	14.2844
5	10.9028	12.0939	13.2721	14.4396	15.5965	16.7202
6	13.3016	14.4942	15.6762	16.8480	18.0420	19.1652
7	15.6994	16.8522	18.0342	19.2545	20.4320	21.5935
8	18.0972	19.2520	20.4340	21.6596	22.8302	23.9952
9	20.4950	21.6518	22.8336	24.0634	25.2370	26.4030
10	22.8928	24.0589	25.2851	26.4667	27.6425	28.8130

Table 1.15: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 75^\circ$

$\alpha = 80^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	1.1956	2.2962	3.3747	4.4422	5.5028	6.5587
2	3.4414	4.5532	5.6466	6.7335	7.8113	8.8836
3	5.6900	6.8061	7.9092	9.0032	10.0905	11.1725
4	7.9394	9.0576	10.1652	11.2650	12.3585	13.4470
5	10.1890	11.3085	12.4191	13.5228	14.6207	15.7139
6	12.4387	13.5591	14.6719	15.7783	16.8796	17.9765
7	14.6885	15.8096	16.9239	18.0325	19.1364	20.2361
8	16.9384	18.0600	19.1755	20.2858	21.3917	22.4938
9	19.1883	20.3102	21.4267	22.5384	23.6461	24.7500
10	21.4382	22.5604	23.6777	24.7906	25.8996	27.0052

Table 1.16: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 80^\circ$

 $\alpha = 85^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	1.0922	2.1390	3.1748	4.2052	5.2322	6.2567
2	3.2078	4.2601	5.3042	6.3432	7.3786	8.4113
3	5.3248	6.3792	7.4271	8.4706	9.5107	10.5482
4	7.4421	8.4976	9.5478	10.5941	11.6373	12.6780
5	9.5596	10.6157	11.6674	12.7156	13.7610	14.8040
6	11.6771	12.7336	13.7864	14.8360	15.8830	16.9279
7	13.7946	14.8515	15.9050	16.9557	18.0041	19.0503
8	15.9122	16.9693	18.0234	19.0750	20.1244	21.1716
9	18.0298	19.0871	20.1417	21.1939	22.2441	23.2925
10	20.1474	21.2049	22.2598	23.3127	24.3636	25.4128

Table 1.17: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 85^\circ$

 $\alpha = 90^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000
2	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
3	5.0000	6.0000	7.0000	8.0000	9.0000	10.0000
4	7.0000	8.0000	9.0000	10.0000	11.0000	12.0000
5	9.0000	10.0000	11.0000	12.0000	13.0000	14.0000
6	11.0000	12.0000	13.0000	14.0000	15.0000	16.0000
7	13.0000	14.0000	15.0000	16.0000	17.0000	18.0000
8	15.0000	16.0000	17.0000	18.0000	19.0000	20.0000
9	17.0000	18.0000	19.0000	20.0000	21.0000	22.0000
10	19.0000	20.0000	21.0000	22.0000	23.0000	24.0000

Table 1.18: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 90^\circ$

 $\alpha = 95^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	0.9172	1.8764	2.8465	3.8221	4.8011	5.7825
2	2.8139	3.7677	4.7357	5.7069	6.6816	7.6596
3	4.7093	6.6551	8.6133	10.5829	12.5586	14.5367
4	6.6048	8.5693	10.5404	12.5188	14.4939	16.4681
5	8.5093	10.4838	12.4692	14.4618	16.4326	18.4037
6	10.3891	12.3831	14.3792	16.3738	18.3618	20.3422
7	12.2890	14.2883	16.2972	18.2838	20.2729	22.2429
8	14.1838	16.1910	18.1910	20.1838	22.1742	24.1517
9	16.0786	18.0924	20.0849	22.0736	24.0543	26.0262
10	17.9733	19.9973	21.9790	23.9533	25.9267	27.8989

Table 1.19: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 95^\circ$

 $\alpha = 100^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	0.8423	1.7659	2.7114	3.6679	4.6313	5.5994
2	2.6462	3.5592	4.4886	5.4284	6.3754	7.3278
3	4.4475	5.3564	6.2782	7.2050	8.1466	9.0896
4	6.2482	7.1549	8.0722	8.9974	9.9289	10.8653
5	8.0485	8.9540	9.8683	10.7897	11.7167	12.6484
6	9.8488	10.7534	11.6656	12.5841	13.5078	14.4360
7	11.6490	12.5529	13.4636	14.3799	15.3011	16.2263
8	13.4491	14.3525	15.2620	16.1767	17.0957	18.0187
9	15.2492	16.1523	17.0608	17.9740	18.8914	19.8124
10	17.0493	17.9521	18.8598	19.7719	20.6878	21.6072

Table 1.20: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 100^\circ$

 $\alpha = 105^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	0.7741	1.6667	2.5925	3.5347	4.4873	5.4471
2	2.4942	3.3710	4.2728	5.1902	6.1185	7.0551
3	4.2105	5.0811	5.9714	6.8753	7.7895	8.7117
4	5.9258	6.7932	7.6766	8.5720	9.4769	10.3894
5	7.6406	8.5061	9.3850	10.2745	11.1727	12.0781
6	9.3553	10.2194	11.0951	11.9804	12.8736	13.7734
7	11.0699	11.9330	12.8064	13.6883	14.5776	15.4731
8	12.7843	13.6467	14.5183	15.3977	16.2838	17.1758
9	14.4988	15.3606	16.2307	17.1080	17.9915	18.8806
10	16.2132	17.0745	17.9435	18.8190	19.7004	20.5870

Table 1.21: Roots λ_{mn} of associated Legendre Functions with respect to the degree for $\alpha = 105^\circ$

$\alpha = 110^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.7118	1.5775	2.4678	3.4203	4.3666	5.3223
2	2.3559	3.2006	4.0791	4.9788	5.8934	6.8190
3	3.9949	4.8314	5.6759	6.5758	7.4715	8.3780
4	5.6326	6.4647	7.3185	8.1886	9.0715	9.9648
5	7.2697	8.0991	8.9469	9.8091	10.6829	11.5665
6	8.9066	9.7342	10.5777	11.4340	12.3011	13.1773
7	10.5433	11.3696	12.2099	13.0617	13.9235	14.7938
8	12.1800	13.0053	13.8430	14.6914	15.5489	16.4144
9	13.8165	14.6411	15.4768	16.3223	17.1763	18.0379
10	15.4531	16.2770	17.1111	17.9543	18.8054	19.6635

Table 1.19: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 110^\circ$

$\alpha = 115^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.6545	1.4969	2.3959	3.3279	4.2669	5.2226
2	2.2292	3.0456	3.9052	4.7917	5.6971	6.6165
3	3.7978	4.6038	5.4437	6.3073	7.1868	8.0842
4	5.3647	6.1651	6.9933	7.8423	8.7078	9.5865
5	6.9309	7.7279	8.5484	9.3873	10.2442	11.1075
6	8.4968	9.2915	10.1064	10.9379	11.7851	12.6400
7	10.0625	10.8553	11.6693	12.5048	13.3621	14.2329
8	11.6285	12.4235	13.2393	14.0767	14.9107	15.7649
9	13.1935	13.9843	14.7894	15.6070	16.4355	17.2716
10	14.7590	15.5489	16.3520	17.1666	17.9913	18.8252

Table 1.20: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 115^\circ$

$\alpha = 120^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.6015	1.4241	2.3157	3.2410	4.1865	5.1455
2	2.1129	2.9043	3.7490	4.6268	5.5277	6.4458
3	3.6169	4.3957	5.2164	6.0664	6.9384	7.8276
4	5.1190	5.8910	6.6973	7.5294	8.3817	9.2505
5	6.6202	7.3880	8.1843	9.0044	9.8426	10.6961
6	8.1210	8.8860	9.6760	10.4864	11.3138	12.1557
7	9.6216	10.3845	11.1694	11.9728	12.7919	13.6245
8	11.1221	11.8833	12.6643	13.4622	14.2745	15.0995
9	12.6224	13.3824	14.1603	14.9536	15.7604	16.5791
10	14.1227	14.8817	15.6570	16.4466	17.2488	18.0622

Table 1.21: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 120^\circ$

$\alpha = 125^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.5523	1.3583	2.2462	3.1734	4.1236	5.0886
2	2.0055	2.7753	3.6090	4.4626	5.3638	6.3053
3	3.4503	4.2050	5.0103	5.8511	6.7184	7.6065
4	4.8927	5.6933	6.4275	7.2468	8.0906	8.9543
5	6.3342	7.0757	7.8524	8.6567	9.4888	10.3269
6	7.7752	8.5133	9.2817	10.0749	10.8883	11.7111
7	9.2155	9.9515	10.7178	11.5095	12.3232	13.1523
8	10.6569	11.3915	12.1577	12.9585	13.7911	14.6321
9	12.0968	12.8240	13.5828	14.3555	15.1441	15.9475
10	13.5372	14.2681	15.0188	15.7868	16.5700	17.3649

Table 1.22: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 125^\circ$

$\alpha = 130^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.5063	1.2988	2.1867	3.1192	4.0766	5.0492
2	1.9059	2.6571	3.4842	4.3583	5.2646	6.1943
3	3.2961	4.0298	4.8237	5.6599	6.5277	7.4201
4	4.6837	5.4077	6.1813	6.9923	7.8325	8.6965
5	6.0700	6.7880	7.5481	8.3412	9.1612	10.0036
6	7.4558	8.1697	8.9200	9.6999	10.5045	11.3301
7	8.8413	9.5522	10.2952	11.0649	11.8574	12.6696
8	10.2265	10.9351	11.6725	12.4342	13.2170	14.0182
9	11.6116	12.3185	13.0513	13.8064	14.5812	15.3733
10	12.9966	13.7020	14.4311	15.1809	15.9490	16.7334

Table 1.23: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 130^\circ$

$\alpha = 135^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.4631	1.2451	2.1366	3.0771	4.0435	5.0243
2	1.8132	2.5490	3.3738	4.2534	5.1697	6.1118
3	3.1530	3.8685	4.6554	5.4921	6.3661	7.2687
4	4.4898	5.1940	5.9572	6.7646	7.6068	8.4772
5	5.8251	6.5223	7.2695	8.0559	8.8744	9.7197
6	7.1599	7.8522	8.5878	9.3587	10.1590	10.9844
7	8.4942	9.1830	9.9099	10.6688	11.4548	12.2643
8	9.8282	10.5144	11.2346	11.9839	12.7583	13.5548
9	11.1622	11.8462	12.5610	13.3025	14.0674	14.8531
10	12.4959	13.1783	13.8887	14.6238	15.3807	16.1572

Table 1.24: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 135^\circ$

$\alpha = 140^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.4223	1.1968	2.0954	3.0462	4.0220	5.0103
2	1.7266	2.4501	3.2775	4.1677	5.0985	6.0060
3	3.0197	3.7200	4.5045	5.3477	6.2341	7.1532
4	4.3093	4.9967	5.7537	6.5632	7.4140	8.2980
5	5.5974	6.2765	7.0148	7.7999	8.6231	9.4783
6	6.8848	7.5581	8.2829	9.0496	9.8515	10.6934
7	8.1717	8.8408	9.5554	10.3079	11.0928	11.9059
8	9.4582	10.1242	10.8308	11.5720	12.3432	13.1408
9	10.7446	11.4081	12.1063	12.8403	13.6002	14.3849
10	12.0309	12.6924	13.3873	14.1117	14.8621	15.6359

Table 1.25: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 140^\circ$

 $\alpha = 145^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.3834	1.1537	2.0626	3.0250	4.0096	5.0036
2	1.6452	2.3598	3.1952	4.1012	5.0497	6.0271
3	2.8950	3.5834	4.3711	5.2874	6.2320	7.1971
4	4.1409	4.8142	5.7014	6.6810	7.6520	8.6155
5	5.3850	5.9837	6.8007	7.7723	8.7891	9.8227
6	6.6283	7.1523	8.0294	9.0321	10.0301	11.0367
7	7.8715	8.3233	9.2294	10.2720	11.2967	12.3277
8	9.1135	9.5618	10.4586	11.4970	12.5174	13.5464
9	10.3555	10.8010	11.6903	12.7176	13.7886	14.8195
10	11.5977	12.0406	12.9237	13.9419	14.9915	15.1693

Table 1.26: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 145^\circ$

 $\alpha = 150^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.3462	1.1156	2.0378	3.0117	4.0035	5.0010
2	1.5683	2.2780	3.1274	4.0536	5.0207	6.0074
3	2.7777	3.4658	4.2556	5.1328	6.0633	7.0277
4	3.9829	4.6458	5.4078	6.2438	7.1361	8.0702
5	5.1862	5.8378	6.5743	7.3780	8.2361	9.1384
6	6.3884	7.0321	7.7498	8.5282	9.3577	10.2306
7	7.5901	8.2279	8.9312	9.6896	10.4955	11.3428
8	8.7913	9.4246	10.1166	10.8590	11.6454	12.4708
9	9.9923	10.6220	11.3049	12.0341	12.8042	13.6111
10	11.1930	11.8199	12.4953	13.2135	13.9699	14.7610

Table 1.27: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 150^\circ$

 $\alpha = 155^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.3101	1.0825	2.0203	3.0046	4.0010	5.0002
2	1.4951	2.2045	3.0744	4.0235	5.0066	6.0017
3	2.6670	3.3435	4.1596	5.0652	6.0222	7.0073
4	3.8342	4.4909	5.2672	6.1311	7.0568	8.0216
5	4.9994	5.6428	6.3897	7.2160	8.1102	9.0495
6	6.1633	6.7973	7.5220	8.3422	9.1824	10.0939
7	7.3265	7.9535	8.6220	9.4566	10.2707	11.1353
8	8.4883	9.1069	9.8017	10.6663	11.4707	12.3233
9	9.6518	10.2690	10.9518	11.8199	12.5832	13.3202
10	10.8140	11.4278	12.1014	12.8281	13.6026	14.4207

Table 1.28: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 155^\circ$

 $\alpha = 160^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.2745	1.0544	2.0092	3.0014	4.0002	5.0000
2	1.4247	2.1396	3.0367	4.0078	5.0014	6.0002
3	2.5615	3.2405	4.0853	5.0243	6.0057	7.0011
4	3.6934	4.3496	5.1521	6.0547	7.0161	8.0040
5	4.8229	5.4635	6.2328	7.1000	8.0357	9.0106
6	5.9511	6.5804	7.3236	8.1592	9.0668	10.0235
7	7.0785	7.6993	8.4217	9.2301	10.1100	11.0448
8	8.2053	8.8194	9.5252	10.3104	11.1646	12.0761
9	9.3317	9.9406	10.6327	11.3983	12.2293	13.1176
10	10.4579	11.0624	11.7431	12.4920	13.3027	14.1691

Table 1.29: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 160^\circ$

 $\alpha = 165^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.2387	1.0316	2.0031	3.0003	4.0000	5.0000
2	1.3560	2.0844	3.0137	4.0017	5.0002	6.0000
3	2.4599	3.1499	4.0349	5.0058	6.0008	7.0001
4	3.5588	4.2231	5.0674	6.0147	7.0024	8.0003
5	4.6550	5.3011	6.1103	7.0302	8.0062	9.0010
6	5.7498	6.3822	7.1621	8.0534	9.0133	10.0026
7	6.8436	7.4656	8.2210	9.0847	10.0250	11.0057
8	7.9368	8.5505	9.2855	10.1237	11.0425	12.0113
9	9.0295	9.6365	10.3544	11.1697	12.0663	13.0204
10	10.1219	10.7233	11.4267	12.2217	13.0966	14.0339

Table 1.30: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 165^\circ$

$\alpha = 170^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.2012	1.0145	2.0007	3.0000	4.0000	5.0000
2	1.2869	2.0405	3.0031	4.0002	5.0000	6.0000
3	2.3598	3.0748	4.0086	5.0006	6.0000	7.0000
4	3.4277	4.1149	5.0181	6.0018	7.0001	8.0000
5	4.4928	5.1592	6.0324	7.0040	8.0004	9.0000
6	5.5584	6.2065	7.0515	8.0080	9.0008	10.0001
7	6.6268	7.2560	8.0753	9.0132	10.0018	11.0002
8	7.6982	8.3060	9.1032	10.0222	11.0055	12.0008
9	8.7718	9.3592	10.1322	11.0353	12.0082	13.0008
10	9.8027	10.4130	11.1708	12.0511	13.0104	14.0015

Table 1.31: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 170^\circ$

$\alpha = 175^\circ$

$n \backslash m$	0	1	2	3	4	5
1	0.1581	1.0037	2.0000	3.0000	4.0000	5.0000
2	1.2121	2.0109	3.0002	4.0000	5.0000	6.0000
3	2.2551	3.0211	4.0006	5.0000	6.0000	7.0000
4	3.2936	4.0338	5.0014	6.0000	7.0000	8.0000
5	4.3295	5.0487	6.0027	7.0001	8.0000	9.0000
6	5.3638	6.0655	7.0047	8.0002	9.0000	10.0000
7	6.3970	7.0837	8.0076	9.0003	10.0000	11.0000
8	7.4295	8.1032	9.0114	10.0006	11.0000	12.0000
9	8.4613	9.1238	10.0162	11.0010	12.0000	13.0000
10	9.4927	10.1454	11.0222	12.0016	13.0001	14.0000

Table 1.32: Roots λ_{nm} of associated Legendre Functions with respect to the degree for $\alpha = 175^\circ$

$\alpha = 5^\circ$

$n \backslash m$	0	1	2	3	4	5
1	53.1110	20.6155	34.5252	47.6795	60.4841	73.0795
2	75.8919	30.5871	54.3271	91.3458	105.9858	120.0818
3	115.0864	97.3227	113.7087	129.5273	144.8033	159.8083
4	158.1791	133.6431	150.4266	166.6513	182.4502	197.9123
5	188.2402	165.8259	186.8330	203.3520	219.4830	235.2463
6	224.2817	205.9442	223.1051	239.8337	256.2078	272.2847
7	260.3118	242.0270	259.3002	276.1860	292.7466	309.0292
8	296.3346	278.0883	295.4473	312.4554	329.1636	345.6116
9	332.3525	314.1356	331.5621	348.6682	365.4963	382.0802
10	368.3669	350.1730	367.6542	384.8407	401.7682	418.4659

Table 2.1: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 5^\circ$

$\alpha = 10^\circ$

$n \backslash m$	0	1	2	3	4	5
1	21.4598	10.0835	17.0520	23.6459	30.0663	36.3829
2	39.6995	30.0567	37.9444	45.4600	52.7375	59.8465
3	57.7918	48.4155	56.6349	64.5326	72.2007	79.6947
4	75.8408	66.5748	74.9712	83.0903	90.9983	98.7388
5	93.8711	84.6655	93.1729	101.4380	109.5106	117.4256
6	111.8917	102.7242	111.3079	119.6770	127.8703	135.9161
7	129.9066	120.7653	129.4047	137.8519	146.1376	154.2855
8	147.9180	138.7957	147.4777	155.9855	164.3446	172.5745
9	165.9268	156.8192	165.5346	174.0911	182.5097	190.8070
10	183.9340	174.8378	183.5803	192.1767	200.6445	208.9984

Table 2.2: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 10^\circ$

$\alpha = 15^\circ$

$n \backslash m$	0	1	2	3	4	5
1	14.1446	6.8842	11.2452	15.6600	19.9605	24.1926
2	26.3023	19.8793	25.1472	30.1702	35.0364	39.7915
3	38.3630	32.1154	37.6016	42.8762	47.9996	53.0083
4	50.3924	44.2202	45.8230	55.2433	60.5246	65.6956
5	62.4739	56.2739	61.9527	67.4721	72.8618	78.4702
6	74.5127	68.3162	74.0845	79.5915	85.9945	91.4707
7	86.5186	80.3422	86.1093	91.7444	97.9955	103.7132
8	98.4460	92.3626	98.1553	103.8324	109.4106	114.9038
9	110.4518	104.3810	110.1937	115.9019	121.5192	127.0568
10	122.4565	116.3932	122.2238	127.9582	133.6080	139.1828

Table 2.3: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 15^\circ$

$\alpha = 20^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	10.4885	4.8432	8.3553	11.6863	14.9332	18.1296
2	19.6044	14.7931	18.7541	22.5345	26.1995	29.7825
3	28.6492	23.9659	28.0886	32.0545	35.9090	39.6790
4	37.6729	33.0440	37.2516	41.3248	45.2956	49.1852
5	46.6876	42.0880	46.3493	50.4933	54.3439	58.5181
6	55.6975	51.1164	55.4147	59.6090	63.7182	67.7558
7	64.7048	60.1363	64.4616	68.6938	72.8477	76.9368
8	73.7103	69.1510	73.4369	77.7565	81.9480	86.0749
9	82.7158	78.1514	82.3265	86.8057	91.0280	95.1875
10	91.7180	87.1714	91.3467	95.8511	100.0533	104.2803

Table 2.4.: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 20^\circ$ $\alpha = 25^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	8.2960	3.8056	6.6322	9.3179	11.9318	14.5182
2	15.5264	11.7430	16.9266	22.5658	28.2686	33.9524
3	22.8213	19.0790	22.3817	27.9788	32.6658	37.0568
4	30.0598	26.3392	29.7110	35.9798	40.5886	46.7481
5	37.2513	33.5735	36.9872	40.7093	46.5586	52.8066
6	44.4591	40.7957	44.2383	47.5987	50.8066	54.1336
7	51.6647	48.0113	51.4749	54.8658	58.1957	61.4274
8	58.8690	55.2228	58.7055	62.1164	65.4740	68.7827
9	66.0723	62.4316	65.9240	69.3563	72.7364	76.0707
10	73.2750	69.6387	73.1413	76.5887	79.9874	83.3431

Table 2.5.: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 25^\circ$ $\alpha = 30^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	6.8354	3.1196	5.4928	7.7524	9.9589	12.1334
2	12.9083	9.7121	12.3720	14.9180	17.3810	19.8121
3	18.9364	15.8215	18.5810	21.2461	23.8390	26.3784
4	24.9514	21.8702	24.6858	27.4166	30.0829	32.6979
5	30.9606	27.8979	30.7474	33.5229	36.2395	38.9079
6	36.9669	33.9158	36.7886	39.5959	42.3494	45.0579
7	42.9715	39.9283	42.8182	45.6493	48.4311	51.1708
8	48.9749	45.9376	48.8405	51.6902	54.4944	57.2591
9	54.9777	51.9448	54.8579	57.7224	60.5448	63.3302
10	60.9798	57.9505	60.8718	63.7485	66.5860	69.3886

Table 2.6.: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 30^\circ$ $\alpha = 35^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	5.7930	2.6347	4.6872	6.6463	8.5616	10.4504
2	10.9958	8.2626	10.5536	12.7505	14.8870	16.9807
3	16.1619	13.4957	15.8704	18.1640	20.3996	22.5911
4	21.3171	18.6787	21.0980	23.4475	25.7437	27.9976
5	26.4675	23.8444	26.2917	28.6780	31.0158	33.3137
6	31.6156	29.0020	31.4686	33.8811	36.2493	38.5802
7	36.7622	34.1552	36.6338	39.0680	41.4595	43.8162
8	41.9079	39.3057	41.7970	44.2445	46.6545	49.0319
9	47.0530	44.4545	46.9542	49.4140	51.8389	54.2334
10	52.1977	49.6020	52.1086	54.5784	57.0158	59.4244

Table 2.7.: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 35^\circ$ $\alpha = 40^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	5.0120	2.2754	4.0905	5.8278	7.5284	9.2069
2	9.5619	7.1770	9.1928	11.1301	13.0168	14.8677
3	14.0813	11.7522	13.8381	15.8561	17.8257	19.7585
4	18.5916	16.2857	18.4088	20.4735	22.4938	24.4787
5	23.0979	20.8047	22.9512	25.0467	27.1016	29.1234
6	27.6022	25.3170	27.4795	29.5969	31.6773	33.7266
7	32.1054	29.8257	31.9999	34.1337	36.2335	38.3044
8	36.6078	34.3321	36.5152	38.6618	40.7771	42.8651
9	41.1096	38.8370	41.0272	43.1840	45.3118	47.4141
10	45.6111	43.3409	45.5366	47.7020	49.8401	51.9543

Table 2.8.: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 40^\circ$ $\alpha = 45^\circ$

$\frac{n}{m}$	0	1	2	3	4	5
1	4.4053	2.0000	3.6332	5.2014	6.7388	8.2575
2	8.4471	6.3339	8.1373	9.8747	11.5696	13.2344
3	12.4653	10.3970	12.2592	14.0645	15.8291	17.5628
4	16.4773	14.4251	16.3186	18.1631	19.9702	21.7477
5	20.4773	18.4410	20.3541	22.2245	24.0608	25.8692
6	24.4808	22.4514	24.3779	26.2667	28.1243	29.9559
7	28.4895	26.4586	28.3750	30.2575	32.1785	34.0211
8	32.4935	30.4650	32.3750	34.2510	36.2080	38.0732
9	36.4971	34.4681	36.4179	38.2397	40.2372	42.1170
10	40.4884	38.4714	40.4260	42.2358	44.2610	46.1470

Table 2.9.: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 45^\circ$

$\alpha = 50^\circ$

$n \backslash m$	0	1	2	3	4	5
1	3.9207	1.7834	3.2740	4.7102	6.1204	7.5109
2	7.5556	5.6607	7.2956	8.8750	10.4188	11.9372
3	11.1692	9.3136	10.9979	12.6345	14.2369	15.8132
4	14.7765	12.9371	14.6477	16.3173	17.9554	19.5686
5	18.3809	16.5505	18.2776	19.9688	21.6314	23.2706
6	21.9840	20.1592	21.8976	23.6042	25.2847	26.9434
7	25.5862	23.7653	25.5119	27.2301	28.9243	30.5981
8	29.1879	27.3698	29.1227	30.8499	32.5550	34.2410
9	32.7892	30.9732	32.7311	33.7794	36.1794	37.8754
10	36.3902	34.5760	36.3379	38.0782	39.7994	41.5038

Table 2.10: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 50^\circ$

$\alpha = 55^\circ$

$n \backslash m$	0	1	2	3	4	5
1	3.5215	1.6096	2.9863	4.3177	5.6274	6.9240
2	6.8257	5.1110	6.6095	8.0617	9.4840	10.8851
3	10.1107	8.4279	9.9677	11.4677	12.9390	14.3886
4	13.3895	11.7203	13.2820	14.8095	16.3107	17.7911
5	16.6559	15.0042	16.5797	18.1252	19.6469	21.1490
6	19.9412	18.2841	19.8691	21.4275	22.9642	24.4825
7	23.2157	21.5619	23.1538	24.7218	26.2699	27.8010
8	26.4899	24.8384	26.4355	28.0111	29.5683	31.1094
9	29.7637	28.1140	29.7153	31.2969	32.8614	34.4109
10	33.0373	31.3891	32.9936	34.5802	36.1508	37.7073

Table 2.11: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 55^\circ$

$\alpha = 60^\circ$

$n \backslash m$	0	1	2	3	4	5
1	3.1957	1.4680	2.7526	4.0000	5.2295	6.4480
2	6.4288	4.6582	6.0404	7.3884	8.7117	10.0774
3	9.7238	7.9505	9.3109	10.9584	12.2682	13.6253
4	12.9338	11.2067	12.5445	13.8623	15.2959	16.6353
5	16.1369	14.4160	15.7628	17.5594	18.3939	20.4256
6	19.3390	17.6219	18.9795	19.2762	21.5963	23.6256
7	22.5405	20.8261	22.1834	22.6371	24.0803	26.8256
8	25.7426	24.0316	24.1963	25.6467	27.0814	28.5029
9	28.9447	27.2362	27.2036	28.6575	30.0982	31.5266
10	32.1468	30.4433	30.2073	31.6662	33.1120	34.5461

Table 2.12: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 60^\circ$

$\alpha = 65^\circ$

$n \backslash m$	0	1	2	3	4	5
1	2.9178	1.3513	2.5608	3.7405	4.9056	6.0618
2	5.7062	4.2688	5.5614	6.8231	8.0649	9.2924
3	8.4829	7.0874	8.3876	9.6813	10.9559	12.2160
4	11.2562	9.8497	11.1846	12.4966	13.7912	15.0719
5	14.0279	12.6264	13.9704	15.2945	16.6028	17.8979
6	16.7988	15.4004	16.7508	18.0835	19.4017	20.7077
7	19.5693	18.1730	19.5280	20.8671	22.1929	23.5074
8	22.3394	20.9447	22.3032	23.6473	24.9792	26.3004
9	25.1094	23.7159	25.0771	26.4252	27.7620	29.0888
10	27.8792	26.4867	27.8501	29.2015	30.5424	31.8738

Table 2.13: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 65^\circ$

$\alpha = 70^\circ$

$n \backslash m$	0	1	2	3	4	5
1	2.6803	1.2543	2.4025	3.5274	4.6410	5.7476
2	5.2665	3.9396	5.1535	6.3431	7.5172	8.6801
3	7.8438	6.5340	7.7694	8.9841	10.1839	11.3723
4	10.4184	9.1156	10.3625	11.5915	12.8069	14.0115
5	12.9917	11.6928	12.9469	14.1854	15.4114	16.6273
6	15.5645	14.2680	15.5270	16.7722	18.0060	19.2394
7	18.1369	16.8421	18.1046	19.3548	20.5947	21.8256
8	20.7090	19.4155	20.6808	21.9349	23.1794	24.4156
9	23.2810	21.9884	23.2558	24.5131	25.7614	27.0020
10	25.8529	24.5610	25.8302	27.0900	28.3415	29.5857

Table 2.14: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 70^\circ$

$\alpha = 75^\circ$

$n \backslash m$	0	1	2	3	4	5
1	2.4751	1.1731	2.2712	3.3523	4.4250	5.4925
2	4.8859	3.6556	4.8026	5.9318	7.0495	8.1590
3	7.2982	5.6354	7.2354	8.3831	9.5198	10.6481
4	9.7053	7.6152	9.4759	10.6096	11.5578	13.0981
5	12.0939	10.0842	12.0609	13.2261	14.3822	15.5306
6	14.4625	12.4528	14.4203	15.4719	16.5179	17.9631
7	16.8352	14.8255	16.8713	18.0450	19.1217	20.3716
8	19.2361	17.0904	19.2753	20.5427	21.6317	22.7853
9	21.6955	20.4915	21.6780	22.9570	24.0295	25.1963
10	24.1369	22.8923	24.0802	25.2611	26.4359	27.6054

$\alpha = 95^\circ$

n	m	0	1	2	3	4	5
1	1	1.8764	0.9602	1.9416	2.9283	3.9178	4.9089
2	2	3.7677	2.8303	3.7951	4.7658	5.7402	6.7173
3	3	5.6611	4.7195	5.6790	6.6436	7.6117	8.5825
4	4	7.5551	6.6118	7.5685	8.5296	9.4939	10.4608
5	5	9.4494	8.5052	9.4602	10.4189	11.3806	12.3448
6	6	11.3438	10.3990	11.3528	12.3099	13.2698	14.2319
7	7	13.2383	12.2931	13.2460	14.2019	15.1604	16.1209
8	8	15.1328	14.1874	15.1396	16.0946	17.0519	18.0112
9	9	17.0274	16.0817	17.0335	18.0441	19.0023	20.9623
10	10	18.9221	17.9762	18.9275	19.8811	20.8367	21.7941

Table 2.19: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 95^\circ$

 $\alpha = 100^\circ$

n	m	0	1	2	3	4	5
1	1	1.7659	0.9276	1.8962	2.8756	3.8604	4.8405
2	2	3.5592	2.6792	3.6146	4.5617	5.5164	6.4764
3	3	5.3564	4.4682	5.3927	6.3273	7.2691	8.2165
4	4	7.1549	6.2632	7.1821	8.1096	9.0438	9.9832
5	5	8.9540	8.0604	8.9758	9.8987	10.8277	11.7616
6	6	10.7534	9.8586	10.7716	11.6912	12.6164	13.5462
7	7	12.5529	11.6573	12.5685	13.4858	14.4080	15.3346
8	8	14.3525	13.4564	14.3663	15.2816	16.2015	17.1255
9	9	16.1523	15.2556	16.1645	17.0783	17.9964	18.9183
10	10	17.9521	17.0550	17.9631	18.8756	19.7922	20.7123

Table 2.20: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 100^\circ$

 $\alpha = 105^\circ$

n	m	0	1	2	3	4	5
1	1	1.6667	0.9013	1.8629	2.8404	3.8259	4.8162
2	2	3.3710	2.5444	3.4556	4.3846	5.3252	6.2740
3	3	5.0811	4.2419	5.1364	6.0463	6.9671	7.8963
4	4	6.7932	5.9487	6.8345	7.7336	8.6427	9.5598
5	5	8.5061	7.6587	8.5392	9.4312	10.3324	11.2411
6	6	10.2194	9.3702	10.2471	11.1341	12.0295	12.9319
7	7	11.9330	11.0825	11.9567	12.8401	13.7311	14.6286
8	8	13.6467	12.7954	13.6675	14.5480	15.4355	16.3291
9	9	15.3606	14.5085	15.3792	16.2573	17.1419	18.0324
10	10	17.0745	16.2220	17.0913	17.9675	18.8496	19.7376

Table 2.21: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 105^\circ$

 $\alpha = 80^\circ$

n	m	0	1	2	3	4	5
1	1	2.2962	1.1050	2.1626	3.2090	4.2498	5.2873
2	2	4.5532	3.4083	4.4963	5.5767	6.6475	7.7128
3	3	6.8691	5.6955	6.7700	7.8685	8.9438	10.0216
4	4	9.1785	7.9323	9.0302	10.1389	11.1899	12.2946
5	5	11.4899	10.1753	11.2688	12.3966	13.4885	14.5861
6	6	13.7997	12.4289	13.5868	14.6863	15.7862	16.8814
7	7	16.1096	14.6802	15.7940	16.9018	18.0042	19.1030
8	8	18.4060	16.9311	18.0463	19.1559	20.2610	21.3620
9	9	20.7022	19.1819	20.2980	21.4092	22.5161	23.6193
10	10	23.0044	21.4325	22.5494	23.6619	24.7703	25.8752

Table 2.16: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 80^\circ$

 $\alpha = 85^\circ$

n	m	0	1	2	3	4	5
1	1	2.1390	1.0478	2.0730	3.0927	4.1096	5.1248
2	2	4.2601	3.1914	4.2328	5.2685	6.3006	7.3295
3	3	6.3792	5.3145	6.3612	7.4030	8.4411	9.4765
4	4	8.4976	7.3346	8.4841	9.5293	10.5712	11.6105
5	5	10.6157	9.5537	10.6049	11.6523	12.6968	13.7388
6	6	12.7336	11.6722	12.7246	13.7737	14.8200	15.8641
7	7	14.8515	13.7905	14.8438	15.8940	16.9418	17.9875
8	8	16.9693	15.9086	16.9625	18.0138	19.0627	20.1096
9	9	19.0871	18.0266	19.0811	20.1330	21.1829	22.2308
10	10	21.2049	20.1446	21.1994	22.2520	23.3026	24.3514

Table 2.17: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 85^\circ$

 $\alpha = 90^\circ$

n	m	0	1	2	3	4	5
1	1	2.0000	1.0000	2.0000	3.0000	4.0000	5.0000
2	2	4.0000	3.0000	4.0000	5.0000	6.0000	7.0000
3	3	6.0000	5.0000	6.0000	7.0000	8.0000	9.0000
4	4	8.0000	7.0000	8.0000	9.0000	10.0000	11.0000
5	5	10.0000	9.0000	10.0000	11.0000	12.0000	13.0000
6	6	12.0000	11.0000	12.0000	13.0000	14.0000	15.0000
7	7	14.0000	13.0000	14.0000	15.0000	16.0000	17.0000
8	8	16.0000	15.0000	16.0000	17.0000	18.0000	19.0000
9	9	18.0000	17.0000	18.0000	19.0000	20.0000	21.0000
10	10	20.0000	19.0000	20.0000	21.0000	22.0000	23.0000

Table 2.18: Roots λ'_{nm} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 90^\circ$

$\alpha = 110^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.5775	0.8809	1.8407	2.8215	3.8124	4.8094
2	3.2006	2.4240	3.3161	4.2326	5.1643	6.1084
3	4.8314	4.0375	5.0667	6.1016	7.1468	8.2018
4	6.4647	5.6637	6.8210	7.9862	9.1588	10.3488
5	8.0991	7.2942	8.5442	9.8088	11.0765	12.3601
6	9.7342	8.9268	10.1819	11.4428	12.7805	14.0935
7	11.3696	10.5605	11.8199	13.1199	14.4728	15.6106
8	13.0053	12.1949	13.0336	14.8834	16.3685	17.2319
9	14.6411	13.8298	14.6663	16.5130	17.9962	18.8559
10	16.2770	15.4650	16.2997	18.1438	19.6244	20.4818

Table 2.22: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 110^\circ$

$\alpha = 115^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.4969	0.8660	1.8291	2.8177	3.8180	4.8248
2	3.0456	2.3165	3.1948	4.1044	5.0345	5.9795
3	4.6038	3.8525	4.7007	5.5757	6.4699	7.3790
4	6.1651	5.4046	6.2374	7.0933	7.9667	8.8541
5	7.7279	6.9623	7.7858	8.6292	9.4887	10.3612
6	9.2915	8.5227	9.3397	10.1745	11.0238	11.8853
7	10.8558	10.0845	10.8959	11.7252	12.5667	13.4197
8	12.4206	11.6472	12.4561	13.2793	14.1148	14.9609
9	13.9863	13.2105	14.0166	14.8358	15.6663	16.5068
10	15.5469	14.7742	15.5780	16.3939	17.2204	18.0562

Table 2.23: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 115^\circ$

$\alpha = 120^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.4241	0.8563	1.8274	2.8275	3.8397	4.8571
2	2.9043	2.3205	3.1908	4.1000	5.0352	5.9896
3	4.3857	3.7849	4.6508	5.5719	6.4708	7.3987
4	5.8610	5.1684	6.0398	6.9219	7.8448	8.7651
5	7.3380	6.6591	7.5298	8.4019	9.3107	10.2023
6	8.8160	8.1531	8.9458	9.8252	10.7222	11.6411
7	10.2945	9.6489	10.4358	11.3626	12.2655	13.2077
8	11.7733	11.1458	11.9283	12.7266	13.6844	14.6429
9	13.2524	12.6434	13.4225	14.2128	15.1672	16.1479
10	14.7317	14.1415	14.9178	15.7090	16.5133	17.5293

Table 2.24: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 120^\circ$

$\alpha = 125^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.3583	0.8517	1.8351	2.8491	3.8729	4.8986
2	2.7753	2.1364	3.0041	3.9206	4.8696	5.8425
3	4.2050	3.5326	4.3526	5.2136	6.1046	7.0191
4	5.6393	4.9527	5.7489	6.5799	7.4377	8.3171
5	7.0757	6.3814	7.1631	7.9751	8.8112	9.6672
6	8.5133	7.8141	8.5860	9.3847	10.2052	11.0443
7	9.9515	9.2490	10.0139	10.8026	11.6114	12.4375
8	11.3901	10.6852	11.4447	12.2258	13.0254	13.8411
9	12.8290	12.1223	12.8776	13.6526	14.4447	15.2519
10	14.2681	13.5600	14.3119	15.0820	15.8679	16.6660

Table 2.25: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 125^\circ$

$\alpha = 130^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.2988	0.8520	1.8512	2.8791	3.9112	4.9389
2	2.6277	2.0520	2.9385	3.8684	4.8410	5.8408
3	4.0298	3.3968	4.2886	5.2428	6.1940	7.1605
4	5.4077	4.7256	5.5999	6.5426	7.4928	8.4642
5	6.7580	6.1266	6.9932	7.9464	8.8988	9.8732
6	8.1697	7.5025	8.3572	9.3041	10.2595	11.2320
7	9.5522	8.8810	9.6272	10.4021	11.3612	12.3910
8	10.9351	10.2610	11.0008	11.7655	12.8544	13.9616
9	12.3195	11.6422	12.3768	13.1352	14.2141	15.3111
10	13.7020	13.0240	13.7546	14.5069	15.5784	16.6668

Table 2.26: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 130^\circ$

$\alpha = 135^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.2451	0.8572	1.8743	2.9130	3.9466	4.9694
2	2.5490	2.0000	2.8868	3.8464	4.8504	5.8776
3	3.8685	3.2709	4.0850	4.9612	5.8850	6.8476
4	5.1940	4.5756	5.3532	6.1822	7.0520	7.9560
5	6.5223	5.8927	6.6486	7.4486	8.2842	9.1501
6	7.8522	7.2155	7.9570	8.7371	9.5492	10.3887
7	9.1830	8.5415	9.2727	10.0382	10.8329	11.6527
8	10.5144	9.8694	10.5928	11.3472	12.1283	12.9327
9	11.8462	11.1986	11.9159	12.6615	13.4316	14.2236
10	13.1783	12.5287	13.2410	13.9794	14.7406	15.5222

Table 2.27: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 135^\circ$

$\alpha = 140^\circ$

$\frac{m}{n}$	0	1	2	3	4	5
1	1.1968	0.8672	1.9019	2.9449	3.9728	4.9874
2	2.4501	1.9487	2.8602	3.8556	4.8897	5.9300
3	3.7200	3.1605	3.9837	4.8851	5.8472	6.8543
4	4.9967	4.4119	5.1893	6.0303	6.9237	7.8631
5	6.2765	5.6781	6.4285	7.2328	8.0815	8.9695
6	7.5581	6.9512	7.6840	8.4635	9.2822	10.1352
7	8.8408	8.2281	8.9483	9.7099	10.5069	11.3347
8	10.1242	9.5074	10.2181	10.9662	11.7464	12.5550
9	11.4081	10.7881	11.4915	12.2288	12.9957	13.7890
10	12.6924	12.0699	12.7674	13.4960	14.2520	15.0327

Table 2.28: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 140^\circ$ $\alpha = 145^\circ$

$\frac{m}{n}$	0	1	2	3	4	5
1	1.1537	0.8818	1.9307	2.9700	3.9885	4.9958
2	2.3598	1.9100	2.8580	3.8905	4.9383	5.9709
3	3.5634	3.0643	3.9088	4.8528	5.8660	6.9095
4	4.8142	4.2642	5.0504	5.9181	6.8557	7.8520
5	6.0487	5.4819	6.2340	7.0533	7.9311	8.8632
6	7.2853	6.7081	7.4360	8.2284	9.0628	9.9461
7	8.5211	7.9389	8.6531	9.4181	10.2270	11.0757
8	9.7618	9.1754	9.8751	10.6231	11.4107	12.2340
9	11.0010	10.4178	11.1015	11.9362	12.6072	13.4108
10	12.2406	11.6445	12.3510	13.0550	13.8125	14.6002

Table 2.29: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 145^\circ$ $\alpha = 150^\circ$

$\frac{m}{n}$	0	1	2	3	4	5
1	1.1156	0.8005	1.9567	2.9862	3.9960	4.9989
2	2.3280	1.8827	2.8796	3.8555	4.8745	5.9213
3	3.5358	3.0510	3.9059	4.8374	5.8220	6.8496
4	4.7458	4.2640	5.0535	5.8220	6.8221	7.8496
5	5.9578	5.4813	6.2340	6.8221	7.8496	8.8631
6	7.1701	6.7081	7.4360	7.8496	8.8631	9.8521
7	8.3824	7.9389	8.6531	8.8631	9.8521	10.8866
8	9.5946	9.1754	9.8751	10.8866	11.8992	12.9106
9	10.8068	10.4178	11.1015	11.8992	12.9106	13.9215
10	12.0189	11.6445	12.3510	13.0550	13.8125	14.6002

Table 2.30: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 150^\circ$ $\alpha = 155^\circ$

$\frac{m}{n}$	0	1	2	3	4	5
1	1.0825	0.9223	1.9768	2.9949	3.9989	4.9998
2	2.2045	1.8783	2.9177	3.9716	4.9924	5.9982
3	3.3435	2.9229	3.8673	4.9220	5.9713	6.9914
4	4.4909	4.0219	4.8763	5.8704	6.9287	7.9728
5	5.6428	5.1466	5.9407	6.8582	7.8783	8.9361
6	6.7973	6.2843	7.0390	7.8959	8.8527	9.8882
7	7.9535	7.4292	8.1569	8.9707	9.8698	10.9543
8	9.1109	8.5784	9.2867	10.0694	10.9245	11.9552
9	10.2690	9.7305	10.4239	11.1634	12.0056	12.9925
10	11.4273	10.8846	11.5863	12.3076	13.1048	13.9578

Table 2.31: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 155^\circ$ $\alpha = 160^\circ$

$\frac{m}{n}$	0	1	2	3	4	5
1	1.0544	0.8453	1.9898	2.9985	3.9988	5.0000
2	2.1356	1.8897	2.9572	3.9711	4.9985	5.9998
3	3.2405	2.8876	3.8669	4.9705	5.9955	6.9967
4	4.3436	3.9332	4.8667	5.9687	6.9904	7.9955
5	5.4436	5.0332	5.8667	6.9687	7.9904	8.9955
6	6.5436	6.1332	6.8667	7.9687	8.9904	9.9955
7	7.6436	7.2332	7.8667	8.9687	9.9904	10.9955
8	8.7436	8.3332	8.8667	9.9687	10.9904	11.9955
9	9.8436	9.4332	9.8667	10.9687	11.9904	12.9955
10	10.9436	10.5332	10.8667	11.9687	12.9904	13.9955

Table 2.32: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 160^\circ$ $\alpha = 165^\circ$

$\frac{m}{n}$	0	1	2	3	4	5
1	1.0316	0.9671	1.9966	2.9997	4.0000	5.0000
2	2.0844	1.9189	2.9844	3.9982	4.9998	5.9998
3	3.1499	2.8871	3.9689	4.9935	5.9992	6.9999
4	4.2231	3.8879	4.9224	5.9826	6.9973	7.9989
5	5.3011	4.9171	5.8872	6.9627	7.9929	8.9989
6	6.3822	5.9656	6.8680	7.9336	8.9840	9.9971
7	7.4656	7.0264	7.8712	8.9003	9.9685	10.9934
8	8.5505	8.0951	8.8951	9.8726	10.9453	11.9863
9	9.6365	9.1691	9.9348	10.8596	11.9160	12.9742
10	10.7233	10.2467	10.9861	11.8644	12.8863	13.9558

Table 2.33: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 165^\circ$

$\alpha = 170^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.0145	0.9849	1.9993	3.0000	4.0000	5.0000
2	2.0405	1.9572	2.9967	3.9998	5.0000	6.0000
3	3.0748	2.9250	3.9905	4.9993	6.0000	7.0000
4	4.1149	3.8988	4.9790	5.9981	6.9999	8.0000
5	5.1592	4.8859	5.9615	6.9958	7.9996	9.0000
6	6.2065	5.8877	6.9386	7.9908	8.9991	9.9999
7	7.2560	6.9024	7.9133	8.9831	9.9980	10.9998
8	8.3072	7.9271	8.8903	9.9714	10.9961	11.9996
9	9.3596	8.9591	9.8742	10.9554	11.9928	12.9991
10	10.4130	9.9967	10.8677	11.9354	12.9678	13.9983

Table 2.34: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 170^\circ$

$\alpha = 175^\circ$

$n \backslash m$	0	1	2	3	4	5
1	1.0037	0.9962	2.0000	3.0000	4.0000	5.0000
2	2.0109	1.9886	2.9998	4.0000	5.0000	6.0000
3	3.0211	2.9774	3.9994	5.0000	6.0000	7.0000
4	4.0338	3.9634	4.9985	6.0000	7.0000	8.0000
5	5.0487	4.9477	5.9971	6.9999	8.0000	9.0000
6	6.0655	5.9315	6.9949	7.9998	9.0000	10.0000
7	7.0837	6.9163	7.9916	8.9997	10.0000	11.0000
8	8.1032	7.9035	8.9871	9.9994	11.0000	12.0000
9	9.1238	8.8939	9.9812	10.9989	12.0000	13.0000
10	10.1454	9.8860	10.9738	11.9982	12.9999	14.0000

Table 2.35: Roots λ'_{mn} of the derivative of the associated Legendre Functions with respect to the degree for $\alpha = 175^\circ$