REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers is brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

19[65-02, 65G10, 65J10].—EDGAR W. KAUCHER & WILLARD L. MIRANKER, *Self-Validating Numerics for Function Space Problems, Computation with Guarantees for Differential and Integral Equations*, Academic Press, Orlando, Fla., 1984, xiii + 255 pp., $23\frac{1}{2}$ cm. Price \$28.00.

This report concerns an area which is situated half-way between classical numerical analysis and symbolic computation (computer algebra). On the one hand, functions and function space operators constitute explicit objects in the suggested algorithms, on the other hand all relations are eventually projected into some $\mathbf{R}^N$ so that only classical computational processes are executed. With a suitable programming system at hand, this projection should naturally be carried out automatically in the spirit of computer algebra; however, it seems that expert human interaction and guidance will be necessary for the achievement of reasonable results.

The authors treat two aspects of function space problems: The generation of an approximate solution, and the "validation" of such an approximate solution, i.e., the generation of a function strip about it such that the exact solution lies in this strip. The process is called self-validation because it may run automatically on a digital computer and because the successful completion of the algorithm is equivalent to a mathematical proof of the existence of the exact solution within the strip.

The basic ideas are straightforward: The coefficients of approximate solutions in a finite-dimensional subspace are found as solutions of algebraic problems which arise by "rounding" the infinite-dimensional problem into the subspace. The dimension of the approximation may be recursively increased in an iterative defect correction way. The great variety of possible roundings (Taylor rounding, Chebyshev rounding, spline rounding, etc.) offers a high flexibility. The defect-correction formulation is mandatory in the validation process which uses generalizations of classical fixed point theorems: The strip must be mapped into itself by a finite-dimensional interval extension of a mapping whose fixed point is the analytic solution. With a proper choice of the mapping and with sufficient effort, a narrow inclusion of the analytic solution may be obtained. Note that the into-ness of the mapping on a suitably generated strip can be discovered *by the computer* via the comparison of coefficient intervals.

The authors have initiated a direction of research and algorithmic design in numerical analysis which should become increasingly important (e.g., for use on high performance work-stations). They present the fundamental mechanisms of their approach quite clearly with the aid of a good number of well-chosen examples. Unfortunately, they have not always been successful in making the details of their processes transparent; a closer study of the report is made unnecessarily difficult by a notation which is often clumsy and inconsistent, by an excess of detail in some

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places coupled with a complete absence of analysis of important aspects, and a multitude of misprints and little errors. The crucial problem of how to start the validation process so that it may be successful is not analyzed. (The recipe given does not work in the elaborated examples, a fact which is covered up by "short cuts".) Also the choice of the approximate inverse in the validation iteration is left open. Nearly all interesting details are missing in the illustrative examples of Chapter 7.

The reviewer does not share some views of the authors: their analogy with floating-point computation may be more misleading than helpful, and their "block-relaxation" mechanism appears clearly inferior to simply solving larger-dimensional systems. However, he strongly feels that this volume points the way to important developments in numerical analysis; the many open questions and the interesting implementation problems should attract the attention of numerical and computational researchers.

H. J. S.

20[45-01].—ABDUL J. JERRI, *Introduction to Integral Equations with Applications*, Marcel Dekker, New York, Basel, 1985, x + 254 pp., $23\frac{1}{2}$ cm. Price $39.75.

This little text is simply written and easy to read, and it could easily be used as a one- or a two-semester undergraduate course for an introduction to integral equations. It could possibly also be used to teach a one-semester engineering mathematics course on integral equations, along with the one-semester ordinary and partial differential equations courses that are being taught at many universities. The subject matter is motivated with some applications, as well as with connections to ordinary and partial differential equations. The majority of the text is written in a nonrigorous style, the exception being Chapter 6, where the contraction mapping principle is presented; even there, an attempt is made at simplifying the functional analysis concepts. Only the simplest numerical methods of solution are discussed; for example, the only numerical integration techniques used are the midordinate, the trapezoidal and Simpson's rule.

In its table of contents one finds the following chapter headings:

F. S.

21[33-04, 41A60, 42C10, 65Q05].—JET WIMP, *Computation with Recurrence Relations*, Pitman, Boston, 1984, xii + 310 pp., 24 cm. Price $50.00.

This monograph is a timely and welcome addition to the literature. The modern numerical theory of difference equations began with an inspiration of the late J. C. P. Miller while he was engaged upon the last table-making project to be completed by the British Association for the Advancement of Science. It proved to be impossible to compute the modified Bessel function $I_n(x)$ by forward recurrence on the order n owing to severe instability. Miller realized, however, that this function could be generated easily and very accurately by backward recurrence on n, *with almost arbitrary starting values*, provided that the results were multiplied subsequently by an easily-found normalizing factor. This procedure, published eventually in the Introduction to the B. A. tables in 1952, is known now as Miller's algorithm. Since then, a steadily increasing stream of research papers on the whole subject has been issuing from many writers. Jet Wimp himself has made important contributions, and he now becomes the first author to attempt a book that treats all of these developments. The attempt is very successful. Virtually all of the more significant developments are described clearly and concisely; equally important, they are illustrated by carefully chosen examples drawn from various applications areas.

This book is not restricted to numerical methods, however. One of the appendices supplies a concise account of the main results in the asymptotic theory of linear difference equations, results that are illustrated by abundant applications in the main text. Almost anyone who has struggled with the operational version of this theory given in the textbook of Milne-Thomson, or has been discouraged by the massive papers of Birkhoff and Trjitzinsky, will be pleased by this easily-understood and easily-applied account by Wimp.

Another useful appendix—also included to make the book as self-contained as possible—sketches the principal results in the general analytic theory of linear difference equations.

Three-quarters of the main text, eleven chapters in all, is devoted to linear equations. The main topics covered are: Miller's algorithm, with extensions by Gautschi and others, for computing minimal solutions of inhomogeneous first-order equations and homogeneous second-order equations; an algorithm of the reviewer, with variations and extensions by other investigators, for computing intermediate solutions of inhomogeneous second-order equations; generalizations of these algorithms for equations of higher order; computations with orthogonal polynomials, especially Clenshaw's algorithm and its extensions for summing series expansions; the methods of Clenshaw, Elliott and Thacher for expanding solutions of ordinary differential equations in series of powers or orthogonal polynomials. Most of the algorithms are accompanied by proofs of convergence, under appropriate conditions. For some algorithms, particularly that of Miller, numerical and/or asymptotic methods are used to estimate the effects of truncation errors and demonstrate stability with respect to rounding errors. Particularly valuable, because of their inaccessibility in the published literature, are the accounts of the very general algorithm of Lozier for the stable computation of any solution of a broad class of linear difference equations, and the method of Lewanowicz for the construction of recurrence relations for the coefficients of series expansions in Gegenbauer polynomials.

Except for the expansions of solutions of ordinary differential equations, most of the applications are to difference equations satisfied by higher transcendental functions, particularly those belonging to the hypergeometric family. Another immense source of linear difference equations is the numerical discretization of ordinary differential equations. This area is not touched by Wimp, yet it is difficult to quarrel with this exclusion. The discretization process introduces its own convergence problems and sources of error, and a comprehensive treatment would have taken the subject matter well away from the main thrust of this reasonably-sized volume. Moreover, some aspects of this application have been treated in a recent book by J. R. Cash [1].

The remaining part of the main text, Chapters 12 to 14, treats systems of nonlinear difference equations. The aspects here are somewhat different. The emphasis is on convergence to fixed points of the corresponding operators, rather than on error analysis and stability. These chapters provide a brief introduction to convergence questions, invariants and divergence theory (strange attractors). Examples include arithmetic-harmonic means, arithmetic-geometric means, infinite products and generalizations of the algorithm of Gauss and Landen. Much of the applications centers on the computation of definite and indefinite integrals of elliptic type. Some aspects, for example, methods for the acceleration of convergence, are complemented by results given in an earlier book by the same author [2].

On the cover, the publisher claims "This book will be of interest to computer scientists, applied mathematicians, physicists and engineers. It contains a comprehensive, state-of-the-art, account of computational techniques based on the general recurrence relation $\mathbf{x}(n+1) = \mathbf{f}[\mathbf{x}(n), n]$". This reviewer endorses this claim wholeheartedly.

F. W. J. O.

1. J. R. Cash, *Stable Recursions, with Applications to the Numerical Solution of Stiff Systems*, Academic Press, London, 1979.

2. J. Wimp, *Sequence Transformations and Their Applications*, Academic Press, New York, 1981.

22[46-01, 65J05].—R. E. Moore, *Computational Functional Analysis*, Ellis Horwood Series, Mathematics and its Applications (G. M. Bell, Editor), Halsted Press, Wiley, Chichester, New York, 1985, 156 pp., $23\frac{1}{2}$ cm. Price $34.95.

Most areas of mathematics have their roots in the sciences. In fact, entire branches of mathematics arose from attempts to understand and analyze certain physical phenomena. As a discipline matures, however, it can often stray from its origins until it reaches a point in its evolution where it becomes self-sufficient. Its subsequent development can become so esoteric that students (even experts) have no inkling of its practical origins and applications, and indeed are amazed when informed of its usefulness in solving "real-world" problems. A frequently mentioned example is functional analysis.

During the past several years, there has been a trend in mathematics textbooks which parallels the recent emphasis on applied mathematics. This trend takes the form of an increased awareness of the origins of a subject and is evidenced by the growing number of textbooks with the words "applied" and "computational" in their titles. Although these efforts are commendable, unfortunately they are all too often thinly disguised attempts to repackage familiar material in popular terminology.

The first impression one gets of this textbook is that it is unexpectedly short (about 150 pages) for such an ambitious undertaking. How is it possible in so small a book to "taste the flavor of numerical functional analysis" as the author suggests? Perhaps that is possible, but it has not been done in this book.

The book actually fails on two counts. First, it fails as a traditional textbook in functional analysis. Moreover, it does not convey the essence of *computational* functional analysis. Let's consider these two points in greater detail.

In the first several chapters and scattered throughout the remainder of the book, many of the usual functional-analytic topics are presented in an extremely terse machine-gun style. Topics such as linear spaces, topological spaces, metric spaces, Banach and Hilbert spaces, linear functionals, convergence, operators, compact operators, contraction mappings, and Fréchet differentiation are zipped through with whirlwind speed with little or no motivation. Few results are proved in the textbook; rather, the author prefers to leave most of the standard theorems as exercises, and the book does contain many exercises. A reader who is not already familiar with functional analysis would get little out of the presentations and become frustrated and discouraged with attempts to do the exercises. Chapter 8 on types of convergence in function spaces provides a typical example. After defining strong, weak, pointwise, uniform, *, and weak-* convergence, the author immediately gives a series of exercises on the relationships among those various types of convergence. The entire treatment occupies a mere two pages!

The author's terse style does not lend itself to enlightening discussion of the *applications* of functional analysis either. Although most of the applications in the book should appeal to anyone with an interest in how functional analysis can be used, enthusiasm is bound to be replaced again by frustration and discouragement while struggling through the often sketchy presentations. For example, Newton's method in Banach spaces and its variants are introduced briefly in Chapters 17 and 18 (19 pages total), some general results are mentioned, and then the reader is referred to the literature for details. The discussion of homotopy and continuation methods in Chapter 19 includes a fair general description of those powerful techniques, but all the cited references are old, the computational aspects are dismissed in vague terms, and no mention is made of recent work.

The author states in the preface that this textbook is designed for a one-semester, first-year graduate introductory course which can be expanded into two semesters. It is claimed that the only prerequisites are some knowledge of linear algebra and differential equations. That seems to be much too optimistic. At the very least, a student should have a solid grounding in advanced calculus and the corresponding mathematical maturity *before* tackling the material in this book. Otherwise, it simply would not be accessible. In particular, how could anyone without advanced calculus experience be able to handle the exercises on convergence in Chapter 8 or have a chance of understanding the difficult and more advanced notion of Fréchet differentiation in Chapter 16? Even with good backgrounds in advanced calculus, most students must struggle to absorb the concepts of functional analysis.

In addition, a more extensive knowledge of applied mathematics is necessary before a student can appreciate the powerful methods of functional analysis and how they aid in the understanding and solution of applied problems. Undergraduate courses in differential equations usually do not include applications of sufficient complexity to require functional-analytic techniques in their solutions. A complex real-world problem, such as the fluid-flow problem discussed in Chapter 20, is probably beyond the grasp of a student whose applied mathematical experience consists of a single course in differential equations.

I am always attracted by analysis textbooks, and especially by those which purport to explore the rich and fruitful relationships between analysis and the applications. My on-going search will not end with this book.

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23[65B05, 65J05].—K. Böhmer & H. J. Stetter, Editors, *Defect Correction Methods—Theory and Applications*, Springer-Verlag, Wien, New York, 1984, vi + 242 pp., 24½ cm. Price $20.00.

Numerical analysis is rich in iterative methods of diverse types, for example, Newton-Raphson, Gauss-Seidel, multigrid, iterative refinement, and deferred correction. A talk given at the 1973 Dundee Conference by P. E. Zadunaisky, which proposed estimating errors in the numerical solution of ODEs by determining the errors in the numerical solution of a neighboring problem with a known analytical (piecewise polynomial) solution, stimulated H. J. Stetter to propose yet another iterative method. It became apparent that this new method shared with so many other iterative methods the idea of computing a correction based on the computation of a relatively accurate residual, and hence Stetter formulated the "defect correction principle" in a paper which appeared in 1978. The use of the distinctive term "defect" for "residual" had been introduced by R. Frank and C. W. Ueberhuber and was probably helpful in attracting interest to this novel approach to iterative processes. Indicative of its rapid acceptance is the inclusion of a section entitled "Splitting methods and defect corrections" in the 1984 report of the NRC Committee on Applications of Mathematics.

This book is the proceedings of a 1983 Oberwolfach working conference on "Error Asymptotics and Defect Corrections." It is not an attempt to compile a book on defect correction. Rather it is a heterogeneous collection of papers tied together by the common thread of defect correction. Authors and titles follow:

Böhmer, Hemker, Stetter: Introduction: the defect correction approach.
Frank, Hertling, Lehner: Defect correction algorithms for stiff ODEs.

Reinhardt: On a principle of direct defect correction based on a posteriori error estimates.
Chatelin: Simultaneous Newton's iteration for the eigenproblem.
Mandel: On some two-level iterative methods.
Hackbusch: Local defect correction method and domain decomposition techniques.
McCormick: Fast adaptive composite grid methods.
Hemker: Mixed defect correction iteration for the solution of a singular perturbation problem.
Rump: Solution of linear and nonlinear algebraic problems with sharp, guaranteed bounds.
Kaucher, Miranker: Residual correction and validation in functoids.
Böhmer, Gross, Schmitt, Schwarz: Defect corrections and Hartree-Fock method.
Pereyra: Deferred corrections software and its application to seismic ray tracing.
Schönauer, Schnepf, Raith: Numerical engineering: experiences in designing PDE software with selfadaptive variable stepsize/variable order difference methods.

A number of these papers are condensations or revisions of earlier work. Some of them are very difficult to penetrate; others are highly readable, for example, the paper by Hemker, which brings together interesting results from previous papers on the use of alternating defect correction to create hybrid difference schemes for convection-dominated flow problems. Among the more novel papers were those of Hackbusch and McCormick on the construction of discretizations for composite grids. (Equation (3.10a) of the Hackbusch paper seems to have the inequality backwards.)

The introductory paper, written especially for the book, describes defect correction. It is not a popularized treatment of the subject useful to the nonspecialist, but rather a fairly precise and complete technical discussion. Unfortunately, the basic idea is to some extent obscured because of the several versions and extensions that are presented. Because these various manifestations of defect correction are so loosely related, it is better to regard it not as a method but as a pattern (a word twice used by the authors) for iterative methods for solving equations in vector and function spaces. The sole unifying theoretical concept is that of a contraction mapping, and even this idea must be modified in certain applications where a strictly limited number of iterations are performed. Such is sometimes the case for deferred correction, where one has at best a "pseudo-contraction" involving a sequence of progressively weaker, but more relevant, norms. (In this connection the reviewer would like to remark that the "crucial role" attributed to asymptotic expansions of the global error is an overstatement. They are useful but not necessary.) In defect correction a correction is computed from a residual, using a cheap approximation to the inverse of the operator. This may mean replacing a matrix by a "nearby" matrix which is easier to factor, or a high-order discretization by a low-order discretization, or a fine-grid discretization by a coarse-grid discretization, or an exact inverse by a finite-precision inverse. This simplified inverse may be linear or nonlinear. If it is linear, then defect correction amounts to nothing more than simplified Newton-Raphson, which is a pattern well established in numerical analysis. It is the possibility of doing nonlinear simplification that makes the defect correction principle interesting and worthwhile. Significant examples that come to mind are deferred correction and A. Brandt's FAS extension of the multigrid idea to nonlinear problems.

Simplified Newton iteration (including iterative refinement) is of especial importance in interval analysis because of the considerable pessimism of interval extensions of direct methods such as Gaussian elimination. It is better to do the initial computation using point values and to use intervals to compute corrections. Of crucial importance is the very accurate calculation of residuals. Often these residuals are inner products and in most other cases they can be so expressed by rewriting the problem. For this reason, U. Kulisch, W. Miranker, and others have advocated that in addition to the four arithmetic operations, there ought to be a built-in (microprogrammed) operation that delivers an inner product to the full precision of the computer. The paper by Rump describes algorithms for the solution of linear and nonlinear systems of equations based on this Kulisch/Miranker arithmetic, and these are implemented in the IBM program product ACRITH, on the market since March 1984. The paper of Kaucher and Miranker goes beyond this and considers the solution of equations in function space. The development in their paper is guided by an analogy between the digit-by-digit decimal expansion of a number and the term-by-term Chebyshev series expansion (for example) of a function. Both papers provide impressive examples, and together they seem to form a definitive condensation of the Kulisch/Miranker approach. However, the unfamiliar notation and terminology and the excessive formalisms are likely to deter any reader other than an interval analysis enthusiast. (This is typical of work in interval analysis and may be partly responsible for its unfortunate isolation from mainstream numerical analysis.) In addition, the substance of the Kulisch/Miranker approach has been criticized. The calculation of an inner product to full precision can be quite time-consuming because of the need for a Super-Accumulator in order to store the intermediate results to whatever precision is necessary. Also, examples have been given by W. Kahan/E. LeBlanc showing the ill effects of having to rewrite the problem so that the residuals are expressible as inner products; one such example is the rewriting of a continued fraction as the ratio of polynomials. Finally, it remains to be demonstrated that the goals of reliability and high accuracy could not be achieved instead with the use of double-precision interval arithmetic for selected intermediate results.

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24[65-02].—GENE H. GOLUB & CHARLES F. VAN LOAN, *Matrix Computations*, The Johns Hopkins University Press, Baltimore, Md., 1983, xvi + 476 pp., $23\frac{1}{2}$ cm. Price \$49.50 hardcover, \$24.95 paperback.

The authors admit to having taken 6 years to write this book. Those who have experienced the energy and enthusiasm which Professor Golub brings to everything will expect that a project which has occupied him for so long must lead to something special. They will not be disappointed.

One of the aims of the book was to synthesize a number of modern developments since the publication of Wilkinson's "Algebraic Eigenvalue Problem" in 1965. This new book is very different from this illustrious predecessor; it has a different purpose, and a distinctive individual style of its own. It is intended as a text for teaching, and of course for self-instruction, and is therefore copiously supplied with examples and exercises. Choosing an example at random, Section 5.2 discusses the use of Cholesky decomposition in the solution of symmetric positive definite systems. In 4 pages it includes 3 theorems, a detailed algorithm and 2 numerical examples. There follows a set of 9 problems for the reader, and a usefully annotated list of 8 references for going deeper into the topic; there is also a reference to the relevant FORTRAN programs in the LINPACK Library. The same style of presentation runs right through the book.

The layout of the book is fairly conventional, beginning with three chapters on basic matrix algebra, including norms, rounding errors and the elementary transformations. Then follow two chapters on elimination methods, and their application in special cases, such as band matrices, and a chapter on orthogonalization and least squares.

The next two chapters deal with the eigenvalue problem; rather unusually, the unsymmetric case comes first, and the specialization of the QR algorithm is then used as the main method for the symmetric case in the next chapter. Chapter 9 is concerned with the Lanczos method, and Chapter 10 with various iterative methods for solving systems of linear equations. The treatment here is quite condensed; the classical iterative methods are covered in only 8 pages, and the conjugate gradient method in 13 pages.

Finally, there are two chapters on functions of matrices, such as the computation of the matrix exponential e^A, and on a number of assorted special topics. And then, a 24-page bibliography and a useful index.

The theoretical and practical aspects of the Singular Value Decomposition are a recurrent theme throughout the book; the SVD is defined on page 16 and is fittingly very nearly the last entry on the last page of the index. With a concept of such all-pervading importance it may seem odd to the uninitiated that in a book on Matrix Computations the reader has to wait for nearly 300 pages before discovering how the SVD is actually computed.

It is suggested that the book could be used as a text for a 2-semester course in matrix computation; that would be a rather strenuous course. There are more than 450 tightly packed pages, complete with problems, and the pace is often quite brisk. It would be strong meat for an average student with no previous knowledge of the subject. But as a reference book, for one who knows something about the subject and wants to know more, it has no equal. The information it contains is accurate, easy to find and very well illustrated; and there are plenty of examples and problems to keep the reader's attention from wandering.

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25[92-01].—R. BELLMAN, *Mathematical Methods in Medicine*, Series in Modern Applied Mathematics, Vol. 1, World Scientific Publishing Company, Singapore, 1983, xiv + 252 pp., $21\frac{1}{2}$ cm. Price $33.00 hardcover, $18.00 paperback.

From the many applications of mathematics to medicine the author covers a selection centered around pharmacokinetics and radiation biophysics. The book starts with one-dimensional compartment models in continuous and discrete time, multicompartment models and the related ordinary differential equations and matrix problems, problems of parameter identification, positivity and conservation laws (Ch. 1–4). Then follows a general introduction to computers and basic numerical methods (Ch. 5, 6). Ch. 7 and 8 introduce calculus of variations and control theory; Ch. 9 and 10, the theory of dynamic programming and related analytic and computational aspects. A short Chapter 12 treats scanning procedures and tumor detection, while Ch. 13 introduces radiation dosimetry and ends with new results of the author on scattering and transmission functions. Though much of the material is elementary and well known (also at 1978), one will find many interesting facts and views, in particular on those topics (Ch. 7–10) which are typically not presented in elementary biomathematics text books. On the other hand, the book as a whole has serious deficiencies.

1. The presentation is arbitrary, eclectic, and inconsistent (e.g., on p. 31 the problem of multiple eigenvalues and associated eigenvectors is suppressed, but on p. 26 matrix power series, and on p. 37, Stieltjes integrals, appear *en passant*. The transpose of the matrix is used on p. 36, the adjoint (w.r. to the usual inner product) is introduced on p. 49 and is "formed by the interchange of rows and columns". After the reader has just learned about round-off he is told that present (1978) large computers are far too small for most medical problems.

2. There are almost no examples or exercises, in particular no relation to real problems, real biological phenomena, real experiments or data.

3. The bibliography (the end of each chapter) refers almost exclusively to the author's own work or books edited by him.

The book has some merits as a complementary reference. It can be used for the training of medical students or young mathematicians only after careful preparation.

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26[94B05, 05B40, 20D08].—THOMAS M. THOMPSON, *From Error-Correcting Codes through Sphere Packings to Simple Groups*, The Carus Mathematical Monographs No. 21, The Mathematical Association of America, Washington, D. C., 1983, xiv + 228 pp., 19 cm. Price $21.00.

Undergraduate mathematics students may never see a theorem that was proved after their birth, prompting the question "Do people still do mathematics?" Moreover, one such student may have no idea how mathematicians work. Thompson's book is an excellent solution to this problem. Using extensive interviews, he has traced Conway's discovery of three new simple groups [2] from Leech's work on sphere packings [4]. Leech's work was related to Golay's (23, 12) 3-error correcting code [3]. Thompson uses a hands-on approach, and assumes that the reader has had advanced calculus and a first course in algebra. The book is also unusually clear, because one of Thompson's main goals is to give the evolution of the mathematics.

It would be a lively choice for an upper level topics course. There are several interesting historical observations. Here are two facts that the reviewer did not know. It was Cocke [1], not Hamming or Golay, who found the infinite family of 1-error correcting codes over a general finite field $GF(q)$ (the so-called Hamming 1-codes). Bell Labs was able to patent (in 1951) Hamming's original (7, 4) 1-error correcting code. This led to a delay in its publication which caused a priority dispute.

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1. J. COCKE, "Lossless symbol coding with nonprimes," *IRE(IEEE) Trans. Inform. Theory*, v. 5, 1959, pp. 33–34.

2. J. CONWAY, "A perfect group of order 8,315,553,613,086,720,000 and the sporadic simple groups," *Proc. Nat. Acad. Sci. U.S.A.*, v. 61, 1968, pp. 398–400.

3. M. GOLAY, "Notes on digital coding," *Proc. IRE(IEEE)*, v. 37, 1949, p. 657.

4. J. LEECH, "Some sphere packings in higher space," *Canad. J. Math.*, v. 16, 1964, pp. 657–682.

27[65–01, 65–04].—WEBB MILLER, *The Engineering of Numerical Software*, Prentice-Hall Series in Computational Mathematics, Prentice-Hall, Englewood Cliffs, N. J., 1984, viii + 167 pp., $23\frac{1}{2}$ cm. Price $27.95.

This book is primarily a textbook suitable for the senior undergraduate or first-year graduate level. It ought to appeal to a greater audience, however: anyone likely to develop or use computer programs for serious scientific computation. Thus, it should interest engineers, mathematicians, and scientists, as well as computer scientists.

The author presents material related to the production and testing of numerical software that has never before been gathered together. In the Preface he states, "My goal is to present principles for writing numerical software. The ideal textbook about the production of numerical software remains to be written, but I hope that I have verified its worth and feasibility and hastened its arrival." I believe that the author has succeeded admirably in verifying worth and feasibility. While still not the ideal textbook, this book is a valuable first effort to organize and codify principles and concepts that have hitherto only been found scattered through the literature. The text is supplemented with exercises and programming assignments, some quite challenging, designed to enhance the reader's understanding of fundamental issues.

The book contains six chapters. Chapter 1 introduces terminology and illustrates concepts that will be used throughout the book. The distinction between similar terms, particularly those related to programming "mistakes" of various kinds, is sometimes subtle. Fortunately, examples in later chapters make the distinctions, and the reasons for them, understandable.

Chapter 2 is an introduction to those details of floating-point arithmetic necessary for understanding the design and testing of numerical software. Without introducing unnecessary complications, the author outlines the important differences between floating-point arithmetic systems and the mathematical real number system. He introduces and briefly discusses parameters that characterize the former, and surveys various schemes for making these parameters available. The chapter concludes with a lucid discussion of uncertainty diagrams and their use in approximate error analysis.

Chapters 3 through 6 proceed in an orderly way from discussions of software whose behavior can be completely analyzed, and is therefore well understood, to discussions of software whose behavior can only be discovered empirically. Chapter 3 discusses the design and testing of software for the sine function, drawing heavily from material in [1]. The author takes time to explore the nuances and numerics of critical algorithmic details, using the material to illustrate broad principles of software design and implementation.

In Chapter 4, software for the solution of linear equations forms the necessary backdrop for an escalated discussion of testing procedures. Early sections of the chapter concentrate on general implementation issues for linear algebra software, using material from [2], and describe three well-known algorithms that provide fodder for the subsequent discussion of testing methodology. The main topic is the reliability of testing as a means for determining whether or not a program meets design specifications. The chapter concludes with the important point that an algorithm may be a practical success even though theoretically it is unreliable. Only extremely sophisticated tests will detect the unreliability in such cases.

Chapter 5, dealing with software for the solution of a nonlinear equation, concentrates on the principles of measuring the performance of, as opposed to merely testing, numerical software. Loosely speaking, the distinction is that testing determines compliance with a specification, while performance measurements are descriptive. It is shown, however, that the results of carefully selected performance measurements can suggest improvements to an algorithm. The discussion revolves around methods for root determination based on bisection and linear interpolation. Curiously, nowhere in the discussion of bisection is a continuity condition imposed. Thus, the bisection scheme described can converge on a binary machine to consecutive floating-point arguments that bracket a singularity of the function. It is not clear whether this omission is deliberate or an oversight.

Chapter 6 discusses performance measurement in greater detail in the context of software for automatic quadrature. The algorithm discussed is a simple automatic scheme using Simpson's rule. This chapter is the least satisfying of those in the book, because it contains little that is definitive. In that sense, it faithfully reflects its subject. Good algorithms and good software for automatic quadrature exist, but all are demonstrably fallible. Software that sparkles on one integrand will fail with a slightly perturbed integrand. Because acceptable performance specifications do not exist, it is not possible to conduct meaningful tests of such software. Thus, we are reduced to amassing data from experiments in an attempt to determine performance characteristics. The proper design of such experiments, and proper data reduction techniques are research areas. The author presents a clear picture of how little we know about these matters.

In summary, this book is a useful, understandable introduction to the engineering and testing of numerical software that faithfully and fairly reflects the present status of the field. While it necessarily includes some numerical analysis, it is not a numerical analysis text. I recommend it to anyone either interested in working on numerical software or simply curious about what is going on.

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1. W. J. Cody & W. Waite, *Software Manual for the Elementary Functions*, Prentice-Hall, Englewood Cliffs, N. J., 1980.

2. J. J. Dongarra, J. R. Bunch, C. B. Moler & G. W. Stewart, *LINPACK Users' Guide*, SIAM, Philadelphia, Pa., 1979.

28[60K05, 60K10, 65D07, 65D20, 65D30].—Laurence A. Baxter, Ernest M. Scheuer, Wallace R. Blischke & Denis J. McConalogue, *Renewal Tables: Tables of Functions Arising in Renewal Theory*, Technical Report, Graduate School of Business Administration, University of Southern California; 22 pages of typewritten text + 312 pages of tables xeroxed and reduced from computer printout sheets, deposited in the UMT file.

Let $\{N(t),\ t \geqslant 0\}$ denote an ordinary renewal process with inter-renewal distribution function F. In many applications of renewal theory, knowledge of the renewal function

$$(1) \qquad H(t) = E[N(t)] = \sum_{n=1}^{\infty} F^{(n)}(t),$$

the variance function

$$(2) \qquad V(t) = \operatorname{var}[N(t)] = 2H^{(2)}(t) + H(t) - [H(t)]^2,$$

and $\int_0^t H(u)\,du$, where $P^{(n)}$ denotes the n-fold recursive Stieltjes convolution of P, are required. With the exception of the Poisson process, exact expressions do not usually exist and numerical evaluation is quite difficult [2] so, other than the partial tabulations of Soland [6] and White [7], numerical values are not readily available.

The Cléroux-McConalogue cubic spline algorithm [3], [4] partially resolves the numerical problems; this algorithm generates very accurate piecewise polynomial approximations to convolutions of the form $F^{(n)}(t)$ where $F \in C^2[0, \infty)$ is a distribution function whose density is bounded. McConalogue [5] (see also [1]) generalized this algorithm, permitting its application to a subclass of those distribution functions F for which F' exhibits a singularity at the origin.

The generalized algorithm was used to compute $H(t)$, $V(t)$ and $\int_0^t H(u)\,du$ for $t = 0(.05)20$ for the five probability distributions most commonly encountered in applications of renewal theory: the Weibull, gamma, lognormal, inverse Gaussian, and truncated normal distributions. Each of these was tabulated to 4 decimal places

(3 decimal places in the case of $\int_0^t H(u)\,du$) for a unit scale parameter and the following ranges of the shape parameters:

Gamma, Weibull	.55(.05)1(.25)7
Inverse Gaussian	.5(.05)1(.2)2(.5)9, 10, 12, 15, 20
Lognormal	.1, .2(.05).3(.1).7(.05).8, 1.0(.2)1.4(.1)1.6 (.2)2.4(.1)2.6(.2)3.4(.1)3.6(.2)4.0
Truncated normal	−2(.25)4.

The algorithm yields values of $F^{(n)}(t)$, and hence the variance and renewal functions were computed directly from (1) and (2); values of $\int_0^t H(u)\,du$ were obtained by integration of the spline representation.

AUTHOR'S SUMMARY

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1. L. A. BAXTER, "Some remarks on numerical convolution," *Comm. Statist. B—Simulation Comput.*, v. 10, 1981, pp. 281–288.

2. L. A. BAXTER, E. M. SCHEUER, D. J. MCCONALOGUE & W. R. BLISCHKE, "On the tabulation of the renewal function," *Technometrics*, v. 24, 1982, pp. 151–156.

3. R. CLÉROUX & D. J. MCCONALOGUE, "A numerical algorithm for recursively-defined convolution integrals involving distribution functions," *Management Sci.*, v. 22, 1976, pp. 1138–1146.

4. D. J. MCCONALOGUE, "Convolution integrals involving probability distribution functions (Algorithm 102)," *Comput. J.*, v. 21, 1978, pp. 270–272.

5. D. J. MCCONALOGUE, "Numerical treatment of convolution integrals involving distributions with densities having singularities at the origin," *Comm. Statist. B—Simulation Comput.*, v. 10, 1981, pp. 265–280.

6. R. M. SOLAND, "Availability of renewal functions for gamma and Weibull distributions with increasing hazard rate," *Oper. Res.*, v. 17, 1969, pp. 536–543.

7. J. S. WHITE, "Weibull renewal analysis," in *Proceedings of the Aerospace Reliability and Maintainability Conference, Washington, D. C., 29 June–1 July 1964*, Society of Automotive Engineers, New York, pp. 639–657.

29[65-06].—J. G. VERWER (Editor), *Colloquium Topics in Applied Numerical Analysis*, Vols. 1 and 2, CWI Syllabus 4 and 5, Centre for Mathematics and Computer Science, Amsterdam, 1984, vi + 483 pp., 24 cm. Price Dfl. 70.20.

In order to encourage interaction between researchers in academe and scientists in industry, and to draw attention to the widespread use of numerical techniques in most diverse application areas, the Department of Numerical Mathematics of the Centre for Mathematics and Computer Science in Amsterdam held a colloquium on "Topics in Applied Numerical Analysis" during the academic year 1983/84. The two volumes under review contain the 24 lectures presented during this colloquium. Most of them describe the use of existing, or improved, numerical methods in one particular application area. In line with the objectives of the colloquium, the applications are drawn from a wide variety of research activities in science and engineering. Two contributions also deal with vectorizing algorithms for use on a parallel computer.

W. G.

30[65-04, 65M60, 65N3].—C. A. BREBBIA (Editor), *Variational Methods in Engineering*, Springer-Verlag, Berlin, 1985, x + 538 pp., 23 cm. Price $89.00.

These are the proceedings of the 2nd International Conference on Variational Methods in Engineering held at the University of Southhampton in July of 1985. They contain 46 contributions arranged in 12 sections entitled: Basic variational principles; Mixed models and their applications; Non-linear formulations; Fluid dynamics applications; Shell and plate analysis; Variational techniques in contact and crack mechanics; Time dependent formulations; Material non-linear problems; Finite element techniques; Boundary integral equations and boundary elements; Computational techniques; Geotechnics.

W. G.

31[65-02, 65N30].—J. R. WHITEMAN (Editor), *The Mathematics of Finite Elements and Applications V, MAFELAP 1984*, Academic Press, London and Orlando, Fla., 1985, xviii + 650 pp., $23\frac{1}{2}$ cm. Price $59.00.

This volume contains 11 invited lectures, 34 contributed papers, and 35 abstracts of poster sessions presented at the fifth conference on The Mathematics of Finite Elements and Applications held at Brunel University, England, May 1–4, 1984. Special topics featured at this conference, in addition to traditional themes, include boundary element techniques and the finite element/computer-aided design interface.

W. G.

32[65-02, 65N30].—DAVID F. GRIFFITHS (Editor), *The Mathematical Basis of Finite Element Methods*, The Clarendon Press, Oxford Univ. Press, New York, 1984, x + 189 pp., 24 cm. Price $24.95.

This volume evolved from lectures given at a short expository conference on the Mathematical Basis of Finite Element Methods with Applications to Partial Differential Equations, held at the Imperial College of Science and Technology, University of London, January 5–7, 1983. The emphasis is on recent developments, but basic background material is also briefly summarized. The authors and their titles are: R. Wait, "Function spaces"; R. Wait, "Conforming methods for self-adjoint elliptic problems"; T. Dupont, "A short survey of parabolic Galerkin methods"; D. F. Griffiths & A. R. Mitchell, "Nonconforming elements"; O. C. Zienkiewicz & A. W. Craig, "A-posteriori error estimation and adaptive mesh refinement in the finite element method"; K. W. Morton, "Finite element methods for non-self-adjoint elliptic and for hyperbolic problems: Optimal approximations and recovery techniques"; P. A. Raviart, "Mixed finite element methods"; A. R. Mitchell, "Curved elements"; J. R. Whiteman & K. T. Schleicher, "Introduction to the treatment of singularities in elliptic boundary value problems using finite element techniques".

W. G.

33[65-06].—WOLFGANG HACKBUSCH (Editor), *Efficient Solutions of Elliptic Systems*, Notes on Numerical Fluid Mechanics, Vol. 10, Vieweg, Braunschweig, 1984, iii + 154 pp., $22\frac{1}{2}$ cm. Price $20.00.

This book contains 11 papers presented at the GAMM-seminar "Efficient solutions of elliptic systems" at the University of Kiel, January 27–29, 1984. The emphasis is on finite element methods and multi-grid methods.

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34[65F05, 65F10, 65F15, 65H10, 65K05].—DAVID J. EVANS (Editor), *Sparsity and its Applications*, Cambridge Univ. Press, Cambridge, 1985, x + 338 pp., $23\frac{1}{2}$ cm. Price $39.50.

This volume originates from a series of lectures given at the University of Technology, Loughborough, England, in April of 1983. The 13 lectures published here, in varying degree of detail, address a number of questions related to sparse matrix problems. Topics receiving special attention are data structures for storing and manipulating sparse matrices, pivotal strategies, the use of the multigrid principle, parallel algorithms and systolic networks. Direct methods as well as iterative techniques are discussed. While most of the contributions deal with the problem of solving systems of linear algebraic equations, there are brief accounts also of sparse eigenvalue problems, linear programming, and systems of nonlinear equations. Applications include those to network theory, geodesy and photogrammetry.

W. G.

35[65-06, 68-06, 76-06, 82-06].—R. GLOWINSKI & J. -L. LIONS (Editors), *Computing Methods in Applied Sciences and Engineering, VI*, North-Holland, Amsterdam, 1984, xiii + 728 pp., 23 cm. Price $77.00/Dfl. 200.00.

These are the proceedings of the Sixth International Symposium on Computing Methods in Applied Sciences and Engineering, held in Versailles, France, December 12–16, 1983. One of the focal themes of this symposium was to explore the interplays between numerical methods, mathematical software, computer architectures, and modern technology. There are a total of 50 contributions, addressing topics in numerical algebra and software, nonlinear analysis, multigrid methods, parallel computing, asymptotic expansion and homogenization, particle and spectral methods, structural mechanics, fluid flow, reservoir engineering, and semiconductor technology.

W. G.