REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

9[73–02, 73C50].—PHILIPPE G. CIARLET, *Mathematical Elasticity*, Volume I: *Three-Dimensional Elasticity*, Studies in Mathematics and its Applications, Vol. 20, North-Holland, Amsterdam, 1988, xl + 451 pp., 23 cm. Price \$107.25/Dfl.220.00.

While many important advances in the theory of elasticity date from the first half of the previous century, the subject has been pursued with renewed vigor in the past few decades. A deep and coherent mathematical theory has been developed, although many fundamental open problems remain to challenge researchers. Several monographs—this being the latest but surely not the last—have appeared recently with the aim of introducing mathematically schooled readers to the theory, and preparing them to appreciate, or contribute to, contemporary research. For many readers, particularly those trained in partial differential equations, Ciarlet's *Mathematical Elasticity* will be among the most useful.

The book is strictly limited to static finite elasticity. (A second volume will be devoted to the more specialized topic of plate and rod theories.) Dynamic problems are not mentioned, and the linear theory appears only when required to prove something about the nonlinear model. The central problem considered is the determination of the deformations and corresponding stress fields of a three-dimensional body in equilibrium with imposed forces. The equations of equilibrium, common to all models in continuum mechanics, are three first-order PDE's in the six independent components of stress and three components of deformation. The assumption of elasticity means that the value of the stress field at a point depends only on the deformation gradient there. This constitutive assumption enables one to close the system, giving rise to a quasi-linear system of three second-order PDE's in the deformation. This system must be supplemented by boundary conditions and supplementary conditions such as injectivity or orientation-preservation of the deformation.

For a very important class of constitutive laws, these PDE's are the Euler-Lagrange equations for the minimization of a certain functional of the deformation, called the stored energy. For such *hyperelastic materials*, the boundary value problems of equilibrium may be formally viewed as problems in the calculus of variations. This alternate formulation of elasticity is dominant in much contemporary research. Ciarlet gives the boundary value problem formulation and the calculus of variations formulation equal time.

The book is divided into two parts. The first part describes the two formulations of the theory, determines the restrictions on the constitutive equations which can be derived rigorously from simple mechanical principles such as frame-indifference, and classifies the various types of constitutive laws and boundary conditions. The second part presents the two major approaches to existence. The first of these, initiated by F. Stoppelli in 1954, begins with the boundary value problem formulation and uses the implicit function theorem and the theory of the linearized elastic equations to obtain existence under the assumption of small data. The second approach, initiated by Ball in 1977, shows the existence of a minimizer of the stored energy function. Both approaches establish existence without uniqueness, which is essential, since many simple physical examples of nonuniqueness are known. Regularity of solutions remains, in large part, an open question.

Ciarlet's writing is clear and his notation consistent and carefully thought out. Complete proofs are generally given, in a style which is concise but not telegraphic. Many of the results needed from real analysis, differential geometry, functional analysis, and matrix theory are included, so that the book is surprisingly self-contained. Topics departing from, or extending the main line of the exposition, are presented in exercises, and there are extensive pointers to the literature and to open problems. In all, this book is an excellent introduction to the modern mathematical theory of elasticity.

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10[70–02, 70–08].—Leslie Greengard, *The Rapid Evaluation of Potential Fields in Particle Systems*, An ACM Distinguished Dissertation, The MIT Press, Cambridge, Massachusetts, 1988, ix + 91 pp., $23\frac{1}{2}$ cm. Price $25.00.

This book presents recent and important results obtained by V. Rokhlin and the author concerning the fast evaluation of the interactions in a system of particles governed by Coulombian forces.

In a situation where a naive computation would require an $O(N^2)$ work, it proposes an $O(N)$ algorithm based upon the remark that the logarithmic potential (in 2D) created by one particle can be replaced, away from this particle, by an inverse power expansion. For a group of particles, by simple additivity, this yields a multipole expansion valid in the external region; that is where most of the other particles are expected to live.

The author explains how to translate those multipole expansions and provide truncation estimates which indicate how many terms are needed in the expansion to reach a given accuracy. He then very clearly describes an algorithm using these tools and a hierarchy of meshes, going from the entire box containing all the particles to a desired refinement level. This hierarchy allows one to determine recursively the contribution to the potential at a given particle p_0 of all particles outside the finest box containing p_0 and its neighbors. At the end, one only needs to add the contribution of nearby particles, which is done by direct computation.

The good performance of the method depends particularly on the fact that the number of particles in each box at the finest level is roughly constant. It is therefore important to derive adaptive methods in which the refinement process preserves that property even in the case of a highly nonuniform distribution of particles. Such an algorithm is given in great detail in the book, and numerical experiments are reported which prove its efficiency. Also the problem of boundary conditions is discussed, although the proposed method is limited to the case of a simple geometry, where image particles more or less reduce the problem to a free space problem.

The next part of the book is devoted to the 3D case. The basic idea is the same but the expansions involve spherical harmonics. This makes the translation and addition techniques much more involved.

Finally, the author mentions several important applications of these new algorithms, ranging from Astrophysics to numerical solutions of Integral Equations. In Fluid Mechanics those ideas have already led to a more systematic use of the so-called vortex methods: the reconstruction of the velocity field from the vorticity carried by the particles can now be achieved in $O(N)$ operations without using an intermediate grid. It can also be hoped that in Plasma Physics such grid-free methods will be used as an alternative to the usual particle-grid methods, overcoming, for instance, aliasing difficulties introduced by the grid.

It also seems that some other, related, ideas of Rokhlin concerning the fast solution of potential equations could be very helpful for solving integral equations arising in various fields.

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11[34A55, 15A18, 15A99].—G. M. L. GLADWELL, *Inverse Problems in Vibration*, Mechanics: Dynamical Systems, Martinus Nijhoff Publishers, Dordrecht, 1986, x + 263 pp., $24\frac{1}{4}$ cm. Price \$79.50/Dfl. 175.00.

Inverse problems are in fashion, so the appearance of Gladwell's book is well timed. The author is not concerned with inverse scattering nor does he ask whether you can hear the shape of a drum. He is interested in whether you could reconstruct the vocal chords of your favorite opera star if you listened well. Idealized sopranos have one-dimensional vocal chords.

Free from the difficulties associated with geometry, there has been considerable progress in discovering how to reconstruct one-dimensional systems that possess designated spectral properties. Half the problem is to elucidate what eigenvalue information is needed to specify uniquely the vibrating system. Eigenvalues are just natural frequencies, or overtones, in disguise.

Gladwell provides a user-friendly account of this body of work. He approaches the subject gently, since the book could be used as a text. There are exercises at the end of all the earlier sections. In fact, the student will learn several topics that most graduates in engineering and mathematics seem to miss these days: Perron's theorem on positive matrices, Gantmacher and Krein's theory of oscillation matrices, and the small transverse vibrations of a beam. The final chapters give an account of the Gelfand-Levitan method for reconstructing the potential (or density) in Sturm-Liouville systems. The material here would be heavy going for a student whose background was weak enough to warrant study of the first two chapters: elementary matrix analysis and vibration of masses connected by springs. So much for the end conditions; the middle of the book, nearly half of it, in fact, presents a nice exposition of oscillation matrices and their use. Gladwell shares the enthusiasm of Gantmacher and Krein for determinants.

A revealing clue that this book was conceived as a text rather than a research monograph is the absence of an index. It is not easy to dip into one of the later chapters. There is no pointer to where certain symbols (such as S_ν^+) are defined. The surprise here is that the author is in an engineering department and engineers are usually punctilious in collecting all their symbols in an obvious place. The complete absence for any numerical data to illustrate the efficacy of the reconstruction techniques for discrete problems makes me suspect that the author is an applied mathematician dressed in engineer's clothing. That possibility would be consistent with the author's interest in Pascal—the philosopher, not the language.

This well-focussed study presents material that is not easy to locate elsewhere. It provides a gateway to the world of inverse problems.

B.P.

12[65–06, 68–06].—J. S. KOWALIK & C. T. KITZMILLER (Editors), *Coupling Symbolic and Numerical Computing in Expert Systems*, II, North-Holland, Amsterdam, 1988, viii + 274 pp., 23 cm. Price \$73.75/Dfl. 140.00.

This volume contains 19 papers presented at a second workshop on the subject held July 20–22, 1987 in Bellevue, Washington. For the first workshop, see [1]. The contributions reflect the interdisciplinary nature of the workshop, drawing from such fields as artificial intelligence, symbolic and numerical computation, and software development. About half of the papers address specific applications in science and engineering. The three papers with the strongest numerical analysis component discuss expert systems related issues in the stable evaluation of symbolically generated mathematical expressions and in finite difference methods and grid generation for partial differential equations.

W.G.

1. J. S. KOWALIK (ed.), *Coupling Symbolic and Numerical Computing in Expert Systems*, North-Holland, Amsterdam, 1986.